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Evaluating the effective spectral seismic amplification factor on a probabilistic basis

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Abstract. All contemporary seismic Codes have adopted smooth design acceleration response spectra, which have derived by statistical analysis of many elastic response spectra of natural accelerograms. The above smooth design spectra are characterized by two main branches, an horizontal branch that is 2.5 times higher than the peak ground acceleration, and a declining parabolic branch. According to Eurocode EN/1998, the period range of the horizontal, flat branch is extended from 0.1 s, for rock soils, up to 0.8 s for softer ones. However, from many natural recorded accelerograms of important earthquakes, the real spectral amplification factor appears to be much higher than 2.5 and this means that the spectrum leads to an unsafe seismic design of the structures. This point is an issue open to question and it is the object of the present study. In the present paper, the spectral amplification factor of the smooth design acceleration spectra is re-calculated on the grounds of a known “reliability index” for a desired probability of exceedance. As a pilot scheme, the seismic area of Greece is chosen, as it is the most seismically hazardous area in Europe. The accelerograms of the 82 most important earthquakes, which have occurred in Greece during the last 38 years, are used. The soil categories are taken into account according to EN/1998. The results that have been concluded from these data are compared with the results obtained from other strong earthquakes reported in the World literature.

Keywords: maximum spectral amplification factor; effective spectral amplification factor; standard normal probability density function; probability of exceeding; reliability index; predominant period; Eurocode EN/1998

1. Introduction

It is known that the magnitude of ground accelerations and the frequency content are affected by the soil conditions. For this reason, Eurocode EN/1998, which is one of the contemporary seismic Codes with high scientific documentation, proposes the soil factor S that is increasing Peak Ground Acceleration (PGA) with respect to soil categories. Moreover, EN/1998 proposes different boundaries of the period range of the flat branch of the smooth design acceleration spectra, for each soil category. At present, in order to draw a smooth design acceleration spectrum, a collection of response acceleration spectra of recorded natural accelerograms for each soil category was taken place. After that, the final smooth design acceleration spectra are drawn using a statistical

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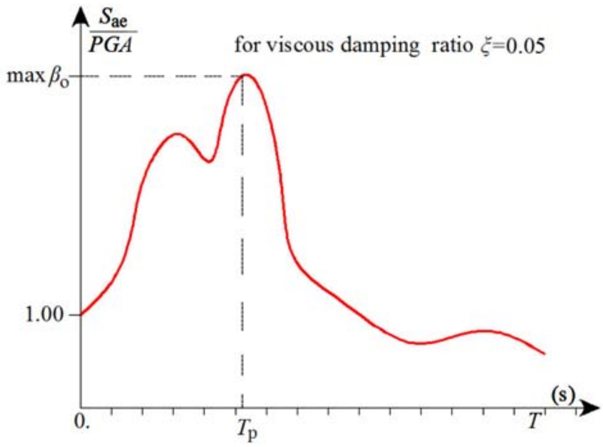

Fig. 1 Maximum spectral amplification factor max β_o of a response acceleration spectrum (T_p: the predominant period)

elaboration. Note here that, according to EN/1998, the spectral amplification factor maxβ_o = S_{ae}/ PGA, where S_{ae} is the extreme elastic spectral acceleration for the predominant period T_p (Fig. 1), can be considered that is "rather independent" from the various soil categories and for this reason, the maxβ_o-parameter is equal to 2.50 for all soil categories. However, taking into account a great number of natural accelerograms as shown in the next, we can ascertain that the maxβ_o-parameter has often a much higher value than 2.50, much is recommended by the contemporary seismic Codes.

In the past, many articles have been published about the erroneous evaluation of the spectral amplification factor. Mylonakis and Gazetas (2000) evidenced that smooth design spectra of the seismic Codes along with the increased fundamental period and effective damping due to soil-structure interaction lead invariably to reduced seismic loadings on the structure. Similarly, Xu and Xie (2004) developed a unique average bi-normalized spectrum of 206 natural strong motion accelerograms from the Chi-Chi (1999) earthquake and they arrived in similar conclusions. Similar doubts about the role of the soil-structure interaction were set by Gazetas (2006). In other paper, using an extended large parametric analysis via 2500 non-linear analyses of ideal soil columns, it was concluded that the peak ground acceleration of the natural spectra can be maintained at the smooth response spectra, when the periods have been divided with the predominant period (Ziotopoulou and Gazetas 2010). With this way a bi-normalized mean spectrum are given and according to the last papers, the spectral amplification factor has resulted 3.75 which is quite different from the conventional smooth design spectra of the seismic Codes that propose the value 2.50. Moreover, this bi-normalized mean spectrum is not affected by the various soil categories or the used method of analysis (equivalent linear or inelastic). On the other hand, in order to use this bi-normalized acceleration spectrum, the predominant period must be calculated, firstly. In another paper, suitable artificial accelerograms, where their response spectra are compatible with seismic design spectrum with 5% viscous damping ratio, have been developed using suitable spectral amplification factor and phase angles (Jun 2010). The spectral amplification factor with the peak ground acceleration, peak ground velocity, spectral displacement, and the spectrum intensity are the most important parameters in order to predict the spatial distribution of seismic demands in specific structures (Bradley *et al.* 2010, Tothong and Luco 2007, Beyer and Bommer 2007). Therefore, the issue of the estimation of the spectral amplification factor is an issue open to question.

A probabilistic estimation of the maxβ_o-parameter can be achieved, applying another alternative way, using the concept of the "reliability index". From the point of view of Eurocode EN/1990, and this constitutes the object of the present paper. The reliability index defines the characteristic value of each variable parameter. According to Statistic Theory, in the case of the Standard Normal Probability Density Function $F(x, \mu, \sigma)$, the "confidence interval" is defined from zero to $(\mu + z \cdot \sigma)$ for a known safe probability $P_s = 1 - P_f$. Note that, P_f is the probability of exceedance (according to section C5 of Annex C of EN/1990), μ is the arithmetic mean of the parameter in question, σ is the "standard deviation", n is the number of the observations and c is a suitable constant. Finally, z is the "reliability index", where used in order to re-calculate the maxβ_o-parameter, in the present paper.

It is worth noting that, according to Eurocode EN/1990, for the design of the structures, three "reliability classes" are defined, namely RC1, RC2 and RC3. The majority of the structures that are designed according to Eurocodes belongs to the intermediate category RC2; except for when the owner of the structure chooses a higher "reliability category". According to Annex C of EN/1990 (i.e., C5 paragraph), the "reliability" of the parameters can be measured by the "reliability index z", that corresponds to a given specific probability of exceeding of the used parameter in the lifetime of the structure. Following my previous point, according to Annex C of EN/1990, all individual design parameters (strength & actions) of a structure have to the same exactly "reliability index z". Thus, in order to design a structure against to earthquakes, the used maxβ_o-parameter must be correlated with a desired "reliability index z". Besides, a similar analogous happens in all others characteristic design parameters such as strengths, gravity loadings, etc of a structure.

In the present paper, the natural accelerograms of the 82 most important independent earthquakes that have occurred in the three seismically hazardous areas of Greece during last 38 years have been used. From the statistic processing of the above earthquakes, it shown that it is possible to define an ideal special Standard Normal Probability Density Function $F(x, \mu, \sigma)$ about the maxβ_o-parameter, from which the statistic mean value of β can be derived for a desired reliability index. The results that have been concluded from this elaboration are in agreement with the results obtained from a set of 1009 cases of accelerograms, studied by Ziotopoulou and Gazetas (2010).

2. Evaluation of the spectral amplification factor of smooth design spectra for known reliability index

This paper is based on the instrumental records of the seismic ground accelerations of 82 earthquakes with a $PGA_h \geq 0.08$ g, that have occurred in all Hellenic hazard seismic zones during the last 38 years. The relevant data was collected from the Hellenic database (Theodoulidis *et al.* 2004) that reports all accelerograms of earthquakes that have occurred in Greece in the period 1973-1999, and also from all additional acceleration records pertaining to the most important earthquakes from 1999 to 2011, which were recorded by the Hellenic Institute of Engineering Seismology & Earthquake Engineering (ITSAK). The main objective of this procedure is to calculate two compatible Standard Normal Probability Density Functions of the maxβ_{oh} and maxβ_{ov}-parameters, for horizontal and vertical seismic component, respectively.

After dividing the maxβ_{oh}-parameter of the horizontal seismic components of the above 82

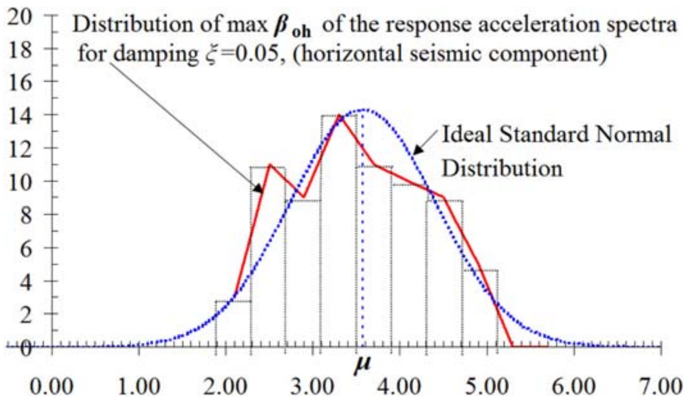

Fig. 2 Distribution of spectral amplification factors max β_{oh}, for horizontal seismic component of Hellenic Earthquakes

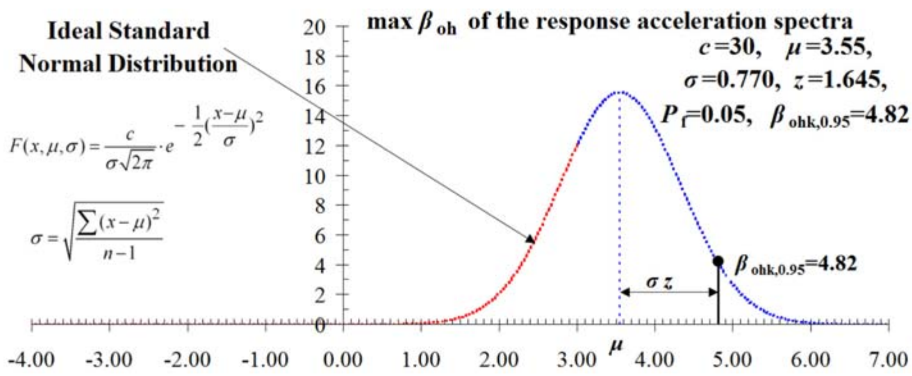

Fig. 3 Ideal Standard Normal Probability Density Function of maxβ_{oh}, for horizontal seismic component of Hellenic Earthquakes and viscous damping ratio 0.05

earthquakes into groups per 0.4 as shown in Fig. 2, the frequency histograms of this maxβ_{oh} is calculated. Approximation, this distribution is a part of an ideal Standard Normal Probability Density Function. The data of μ, σ and c are founded as indicated in Fig. 3.

As it can be seen in Fig. 3, for probability of exceeding $P_f = 5\%$, the reliability index is $z = 1.645$, thus, the "absolute uncertainty" $z \cdot \sigma$ is equal to $1.645 \times 0.770 = 1.27$. Therefore, the characteristic value of the maximum spectral amplification factor maxβ_{oh}, is $3.55 + 1.27 = 4.82$, where 3.55 is the statistic arithmetic mean value μ (= mean value of maxβ_{oh}) of the ideal standard normal distribution. With similar way, is possible to calculate the maxβ_{oh} for every desired probability of exceedance.

For the needs of the common seismic design of the structures and into the frame of a "common smoothing" of the design spectra, we can use of an "*effective value*" of the spectral amplification factor instead of the maxβ_{oh}. This replacement is rational because, the seismic response of structures is non-linear and post-elastic. It means that the fundamental period of a multi-degree of freedom structure is changed continuously, due to reduction of its global lateral stiffness, and for this reason, the phenomenon of co-ordination between the structure and the "seismic excitation of the base" is relatively removed. Besides, the horizontal, lateral vibration of each structure must be characterized by ductile seismic behavior. This "*effective value*" of the spectral amplification factor cannot be taken less than $1/\sqrt{2} = 0.707$ of the maximum value maxβ_{oh}, since, a drop of 50% (and more) of the "*power spectral density*" of the accelerograms appears, which is unacceptable according to the Theory of Signal Proceecing (Bendat and Piersol 2000). Therefore, a more rational *effective value* (effβ_{oh}) is about 80% of the maxβ_{oh}. We conclude that, the *effective value* of the spectral amplification factor according to the above example is effβ_{oh} = 0.80(maxβ_{oh}) = 0.80 × 4.82 = 3.85, which is 1.54 times higher than 2.50.

With reference to the range of the predominant periods of a smooth design acceleration spectrum, with analogous way of elaboration of the above 82 earthquakes, the statistic arithmetic mean value of the predominant period T_{ph} (that is the period where the maxβ_{oh} has been appeared as shown in Fig. 1) is found equal to 0.25 s. Moreover, the absolute uncertainty is found equal to $z \cdot \sigma$ = 0.145 s for soil categories from B to E, thus the respective characteristic period T_C becomes equal to sum of both abovementioned values (0.25 + 0.145), namely 0.40 s. This characteristic value is in good agreement with Eurocode EN/1998, where the characteristic periods T_B & T_C (Table 3.2 by EN/1998) provide a very good estimation (Fig. 4).

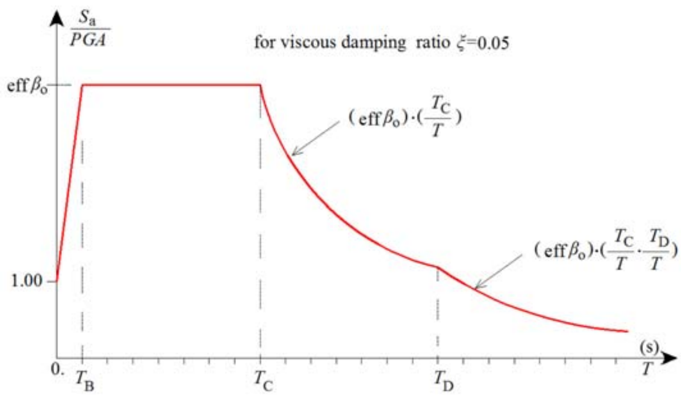

Fig. 4 Elastic response acceleration spectral form, by EN/1998

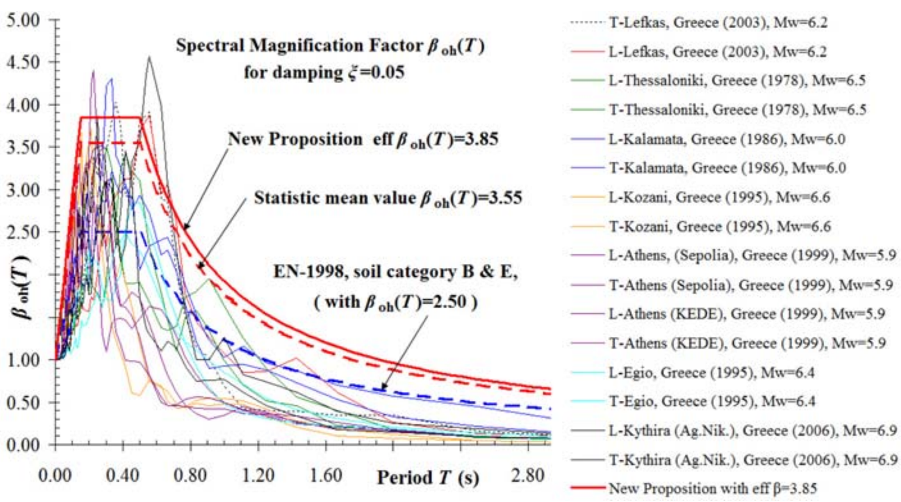

Fig. 5 New proposition of effective spectral amplification factor effβ_{oh} = 3.85 for the horizontal seismic components in comparison with the most important Hellenic Earthquakes

The spectral amplification curves of the more important Hellenic earthquakes (with $M_w \geq 5.9$, from 1978 to the present) are shown in Fig. 5. As we can see in Fig. 5, the design acceleration response spectrum that is proposed by EN/1998 and its Hellenic National Annex, i.e., β_{oh} = 2.50, underestimates the spectral amplification of many natural spectra. According to our proposal, the effective spectral amplification factor effβ_{oh} = 3.85, for probability of exceeding 5%, represents a safe approach.

Working in a similar way the Standard Normal Probability density function of the maximum spectral amplification factor, maxβ_{ov}, for the vertical seismic component can be drawn (Figs. 6, 7). Note that, in the case of the vertical seismic component, the concept of effective value of the spectral amplification factor has no rational basis, because the vertical vibration of the structure is not characterized by ductile seismic behavior. The statistic arithmetic mean value of the predominant period T_{pv}, where the maxβ_{ov} is appeared, is 0.16 s with absolute uncertainty 0.101s. According to the present data, the range of the predominant periods T_{pv} of the vertical seismic components is not affected by the soil categories.

In an attempt to summarize the above, the parameters of the Standard Normal Probability Density Function have been inserted into Table 1. Using the parameters in Table 1, a variety of Design

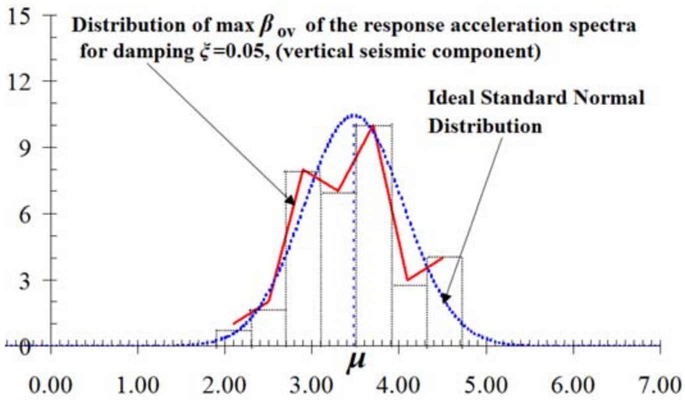

Fig. 6 Distribution of spectral amplification factors maxβ_{ov}, for vertical seismic component of Hellenic Earthquakes

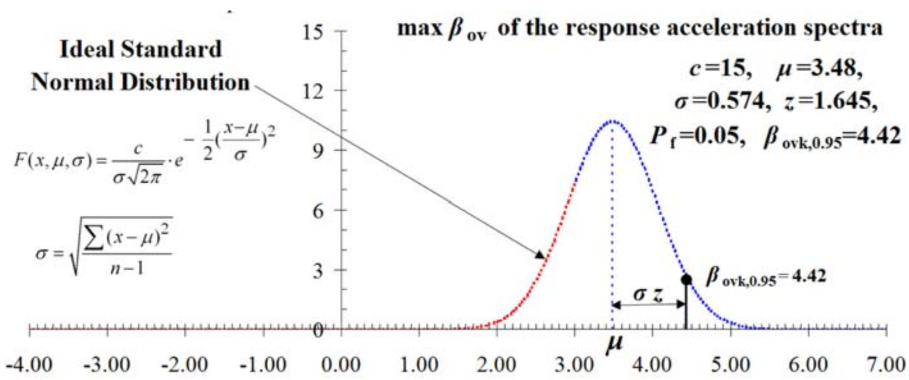

Fig. 7 Ideal Standard Normal Probability Density Function of maxβ_{ov}, for vertical seismic component of Hellenic Earthquakes and viscous damping ratio 0.05

Basis Earthquakes can be calculated for each Hellenic seismic hazard zone, using different values of the “reliability index *z*” or the corresponding probability of exceedance, thus, Table 2 includes specific information for seismic actions.

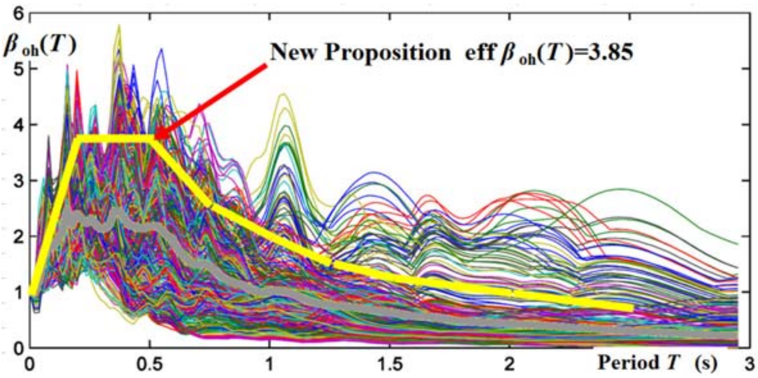

Fig. 8 The present preposition about the effβ_{oh} = 3.85 for the horizontal seismic components is compatible and in accordance with the elastic response spectra by 1009 accelerograms of the World (Ziotopoulou and Gazetas 2010)

Table 1 Special parameters of the Standard Normal Probability density function

	c	μ	σ
Maximum Spectral Amplification factor, maxβ_{oh}, for horizontal seismic component and damping ratio $\zeta = 0.05$	30.00	3.55	0.77
Predominant period T_{ph} (s) of horizontal seismic components for soil categories *B-E*	-	0.25	0.145
Maximum Spectral Amplification factor, maxβ_{ov} for vertical seismic component and damping ratio $\zeta = 0.05$	15.00	3.48	0.574
Predominant period T_{pv} (s) of vertical seismic components for all soil categories	-	0.16	0.101

Table 2 Values of various design earthquakes for various level of Probability of exceeding P_f or the reliability index z from the Standard Normal Probability Density Function

P_f =	0.15	0.10	0.05	0.02	0.01	0.001
P_s =	0.85	0.90	0.95	0.98	0.99	0.999
z =	1.037	1.283	1.645	2.054	2.326	3.097
Effective spectral amplification factor, effβ_{oh} for horizontal seismic component & damping ratio $\zeta = 0.05$	3.48	3.63	3.85	4.10	4.27	4.75
Maximum spectral amplification factor, maxβ_{ov} for vertical seismic component & damping ratio $\zeta = 0.05$	4.08	4.22	4.42	4.66	4.82	5.26
Design Basis Earthquakes (D.B.E.) & Maximum Capable Earthquakes (M.C.E.)			D.B.E.			M.C.E.

According to our proposal for a probability of exceeding 5% (that means reliability index z = 1.645), the effective value of spectral amplification factor for horizontal seismic component is effβ_{oh} = 3.85. This compares well results with the demonstrated by Ziotopoulou and Gazetas (2010), where a group of 1009 accelerograms from all over the World is examined for other purposes (Fig. 8). As we can see in the last figure, the new proposition matches well with the majority of the elastic response acceleration spectra of 1009 records, from various soil categories, while, on the contrary, the value 2.50 of the design spectral amplification that is proposed by contemporary seismic Codes is fully inadequate. For various probabilities of exceedance which mean different reliability indices, Figs. 3, 7 give the effβ_{oh} and maxβ_{ov} spectral amplification value for horizontal and vertical seismic components, respectively. These values are suitable for all soil categories *A, B, C, D* and *E* that are defined by Eurocode EN/1998. Some results are shown in Table 2.

Another important point for discuss is the following: As it can be seen in Fig. 5, the spectral accelerations for period T greater than 1.5 s have very low values, so the new factor (equal to 3.85) proposed by the present work is rather conservative. However, on the other hand, as it can be seen in Fig. 8, by taking into account 1009 accelerograms taken by other authors, the new proposition is very good. This issue is a known fact and due to capabilities of accelerographs, which were installed in Greece up to 2000. These accelerographs were proportional (analogist) and, therefore, the small frequencies (below of the 0.67 Hz) had been removed by these machines since the later had been played the role of filters. In Greece, the abovementioned old accelerographs have been replaced by digital ones, after 2000 year, but the seismic records of the last 10 years are very restricted.

3. Conclusions

In the present paper, a new estimation of the maximum spectral amplification factor based on statistical processing of important Hellenic earthquakes during last 38 years has been taken place. The soil categories are taken into account according to EN-1998. In order to re-calculate the maximum spectral amplification factor of the smooth design acceleration spectra, a known "reliability index" for a desired probability of exceedance is taken into account. With reference to horizontal seismic component, a Standard Normal Probability Density Function has been calculated in order to describe the distribution of the maximum spectral amplification factor. Next, using a desired probability of exceedance, its reliability index has been calculated and afterwards the characteristic design value of the above spectral factor has been estimated. In addition, the effective spectral amplification factor of horizontal seismic component has been found as 3.85 for a probability of exceeding 5%. Thus, this value is proposed instead of the design value 2.50 that has been proposed by the contemporary seismic Codes. The effective spectral amplification factor has been verified by a large number (of 1009 records from the World) of elastic response spectra, using relative literature. Moreover, with respect to vertical seismic component, another Standard Normal Probability Density Function has been calculated in order to describe the distribution of the maximum spectral amplification factor, too. According to present proposition, the characteristic design value of the maximum spectral amplification factor of the vertical seismic component has been calculated 4.42 for a probability of exceeding 5%. This value is proposed instead of the design value 3.00 that has been proposed by the Eurocode EN/1998. In other words, the characteristic values of spectral amplification factor must be taken into account per 50% about higher than the proposition of the Eurocode EN/1998, for both the horizontal and the vertical seismic component. With analogous way, the period range of the constant-acceleration plateau of the smooth, design, acceleration, response spectra has been calculated. The recommendations of EN/1998 about this period range constitute a very good estimation in relation to the results of the present paper.

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