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Response of forced Euler-Bernoulli beams using differential transform method

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Abstract. In this paper, forced vibration differential equations of motion of Euler-Bernoulli beams with different boundary conditions and dynamic loads are solved using differential transform method (DTM), analytical solutions. Then, the modal deflections of these beams are obtained. The calculated modal deflections using DTM are represented in tables and depicted in graphs and compared with the results of the analytical solutions where a very good agreement is observed.

Keywords: differential transform method; forced vibration; partial differential equations; natural frequencies

1. Introduction

Engineering problems are usually described by ordinary and partial differential equations. Mostly these problems cannot be solved or are difficult to solve analytically. Alternatively, the numerical methods can provide approximate solution rather than the analytical solutions of problems. Forced vibration equations of Euler-Bernoulli beams are fourth-order partial differential equation.

The dynamic analysis of forced beams is one of the important topics in engineering. The behavior of the beam has been investigated by many researchers in the past. Clough and Penzien have solved the forced vibration equation of the Euler-Bernoulli beam with simply supported and subjected to concentrated constant force at mid-span using modal analysis (Clough and Penzien 1993). Chopra has derived mathematical expressions for the dynamic response of simply supported beam to a step-function force (Chopra 1995). Paz has determined the modal steady – state response of the fixed beam subjected to harmonic load (Paz 1997). Paz and Dung have derived general stiffness matrix for Euler-Bernoulli beam elements using power series (Paz and Dung 1975). Kai-Yuan *et al*. have investigated the dynamic response of Euler-Bernoulli beams with arbitrary nonhomogenity and arbitrary variable cross-section (Kai-yuan *et al*. 1992). Fan *et al*. have proposed a method of analysis for the forced vibration of a beam with viscoelastic boundary supports (Fan *et al*. 1998). Hilal has obtained the dynamic response of prismatic damped Euler-Bernoulli beams subjected to distributed and concentrated dynamic load using Green Functions (Hilal 2003). Oniszczuk has

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determined the steady-state response of the simply supported double-beam system using modal analysis (Oniszcczuk 2003). Celebi *et al*. have obtained displacements of non-uniform rods subjected to dynamic axial load using Laplace transformation (Celebi *et al*. 2011).

Catal has calculated natural frequencies of beam on elastic soil using DTM (Catal 2006). Catal and Catal have calculated the critical buckling loads of partially embedded pile in elastic soil using DTM (Catal and Catal 2006). Catal has investigated free vibration of beam on elastic soil using DTM as well (Catal 2008). Yesilce and Catal have calculated the natural frequencies of axially loaded Reddy-Bickford beam on elastic soil using DTM (Yesilce 2009). DTM has been used to find the non-dimensional natural frequencies of the tapered cantilever Euler-Bernoulli and Timoshenko beams by Ozgumus and Kaya (2006, 2010). Yesilce has calculated natural frequency of semi-rigid connected Reddy-Bickford beam and a moving beam using DTM (Yesilce 2011, 2010). DTM has been used to find natural frequencies of elastically supported Timoshenko columns with a tip mass by Demirdag and Yesilce (2011). Yesilce and Catal have calculated natural frequencies of semi-rigid connected Reddy-Bickford beams resting on elastic soil using DTM (Yesilce and Catal 2011). Chen and Ho have proposed a method to solve eigen - value problems for the free and transverse vibration problems of rotating twisted Timoshenko beam under axial loading by using DTM (Chen and Ho 1996, 1999). DTM has been applied to solve a second-order non-linear differential equation that describes the under dumped and over damped motion of a system subjected to external excitations by Jang and Chen (1997). DTM has applied to eigen-value problems and Sturm-Liouville eigen-value problem by Hassan (2002a, b). The DTM was first proposed by Zhou in 1986 and was applied to solve electric circuit analysis (Zhou 1986). In this study, a new transformation called differential transform has introduced to solve the forced vibration equations of Euler-Bernoulli beams.

2. Formulation of problem

The forced vibration equation of a uniform elastic undamped Euler-Bernoulli beam acted by lateral forces $P(x, t)$ is described by the partial differential equation as (Paz 1997)

$$EIy^{iv}(x,t) + m\ddot{y}(x,t) = P(x,t) \tag{1}$$

where EI is the flexural rigidity; t is time variable; m is the mass per unit length of the beam; x is position, $y(x, t)$ is lateral displacement function of the beam; $y^{iv}(x,t) = \partial^4 y(x,t)/\partial x^4$; $\ddot{y}(x,t) = \partial^2 y(x,t)/\partial t^2$ the lateral displacement function of the beam at position x and time t is written using separable of variables method as

$$y(x,t) = \sum_{n=1}^{\infty} Y_n(x)T_n(t) \tag{2}$$

where $Y_n(x)$ and $T_n(t)$ are the n-th normal mode of the beam, n-th generalized deflection of the beam, respectively. Substituting Eq. (2) in Eq. (1) gives

$$EIY_n^{iv}(x)T_n(t) + mY_n(x)\ddot{T}_n(t) = P(x,t) \tag{3}$$

Multiplying each term of Eq. (3) by the j-th normal mode and integrating between 0 and length of beam L yields

$$\int_0^L EIY_n^{iv}(x)Y_j(x)T_n(t)dx + \int_0^L mY_n(x)Y_j(x)\ddot{T}_n(t)dx = \int_0^L Y_j(x)P(x,t)dx \tag{4}$$

According to the ortogonality properties of normal modes, Eq. (4) is written as (Chopra 1995)

$$M_n\ddot{T}_n(t) + K_nT_n(t) = P_n \tag{5}$$

where, $\ddot{T}_n(t) = \dfrac{\partial^2 T_n(t)}{\partial t^2}$, $M_n = \int_0^L mY_n^2(x)dx$, $K_n = \int_0^L EIY_n^{iv}(x)Y_n(x)dx$, and $P_n = \int_0^L Y_n(x)P(x,t)dx$ are the generalized mass stiffness and force, respectively. The differential equation of free vibration depends on the n-th normal mode of the beam is given as (Clough and Penzien 1993, Chopra 1995, Paz 1997)

$$Y_n^{iv}(x) - \lambda_n^4 Y_n(x) = 0 \tag{6}$$

where $\lambda_n^4 = \dfrac{m\omega_n^2}{EI}$; ω_n is the natural frequency of the n-th normal mode of the beam.

3. Differential transformation

The differential transformation technique, which was first proposed by Zhou in 1986, is one of the numerical methods for ordinary and partial differential equations that use the form of polynomials as the approximation to the exact solutions that are sufficiently differentiable. $f(x, t)$ function that will be solved and the calculation of following derivatives necessary in the solution become more difficult when the order increases. This is in contrast with the traditional high-order Taylor series method. Instead, the differential transform technique provides an iterative procedure to obtain higher-order series. Therefore, it can be applied to the case of high order (Çatal 2008).

Basic definitions and operations of differential transformation are introduced and also differential transformation of the function $y(x)$ is defined as follows

$$\bar{y}(k) = \frac{1}{k!}\left[\frac{d^k y(x)}{dx^k}\right]_{x=x_0} \tag{7}$$

In Eq. (7), $y(x)$ is the original function and $\bar{y}(k)$ is transformed function which is called the T-function (it is also called the spectrum of the $y(x)$ at $x = x_0$, in the K domain). The differential inverse transformation of $\bar{y}(k)$ is defined as

$$y(x) = \sum_{k=0}^{\infty}(x-x_0)^k\bar{y}(k) \tag{8}$$

from Eq. (7) and Eq. (8), we get

$$y(k) = \sum_{k=0}^{\infty}\frac{(x-x_0)^k}{k!}\left[\frac{d^k y(x)}{dx^k}\right]_{x=x_0} \tag{9}$$

Eq. (9) implies that the concept of the differential transformation is derived from Taylor's series expansion, but the method does not evaluate the derivatives symbolically. However, relative derivative are calculated by iterative procedure that are described by the transformed equations of the original functions.

From the definitions of Eq. (7) and Eq. (8), it is easily proven that the transformed functions comply with the basic mathematical operations shown in below. In real applications, the function $y(x)$ in Eq. (8) is expressed by a finite series and can be written as

$$y(x) = \sum_{k=0}^{N} (x-x_0)^k \bar{y}(k) \tag{10}$$

Eq. (10) implies that $\sum_{k=N+1}^{\infty} (x-x_0)^k \bar{y}(k)$ is negligibly small and N is decided by the converge of the displacements, where N is series size.

3.1 Some basic mathematical operations of the differential transformation

The fundamental mathematical operations performed by differential transformation are listed, where the transformed functions $\bar{y}(k)$ are related to the known original function $y(x)$, as follows

Original function $y(x)$	Transformed function $\bar{y}(k)$	
$ay(x)$	$a\bar{y}(k)$	(11.1)
$y_1(x) \pm y_2(x)$	$\bar{y}_1(k) \pm \bar{y}_2(k)$	(11.2)
$dy(x)/dx$	$(k+1)\bar{y}(k+1)$	(11.3)
$d^2y(x)/dx^2$	$(k+1)(k+2)\bar{y}(k+2)$	(11.4)
$d^3y(x)/dx^3$	$(k+1)(k+2)(k+3)\bar{y}(k+3)$	(11.5)
$d^4y(x)/dx^4$	$(k+1)(k+2)(k+3)(k+4)\bar{y}(k+4)$	(11.6)
$a\sin(\omega t)$	$a\dfrac{\omega^k}{k!}\sin\left(\dfrac{\pi}{2}k\right)$	(11.7)

3.2 Using differential transformation to solve motion equations

The n-th normal mode and generalized deflection of the both ends simply supported and one end fixed and other end simply supported beams are calculated from Eq. (6) and Eq. (5) by using DTM.

3.2.1 Both ends simply supported Euler-Bernoulli beam

The boundary conditions of the both ends simply supported beam subject to a step-function force P_0 at distance ξ form the left end and time t

$$Y_n(x=0) = 0 \tag{12.1}$$

$$Y_n(x=L) = 0 \tag{12.2}$$

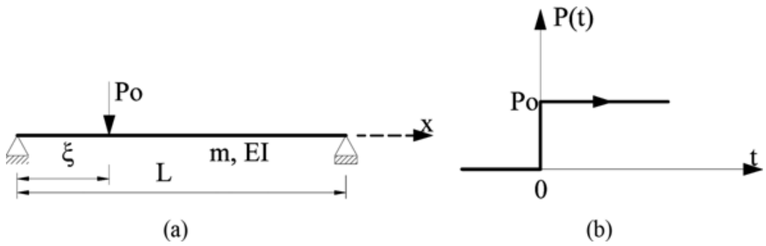

Fig. 1 (a) Both ends simply supported beam, b) the step-function force

$$\left.\frac{d^2Y_n(x)}{dx^2}\right|_{x=0} = 0 \tag{12.3}$$

$$\left.\frac{d^2Y_n(x)}{dx^2}\right|_{x=0} = 0 \tag{12.4}$$

$$Tn(t=0) = 0 \tag{12.5}$$

$$\left.\frac{dT_n(t)}{dt}\right|_{t=0} = 0 \tag{12.6}$$

Substituting $P(x, t) = P_0\,\delta(x-\xi)$ in the generalized force gives (Chopra 1995)

$$P_n = P_0\ Y_n(\xi) \tag{13}$$

where $\delta(x-\xi)$ is the Dirac delta function centered at ξ. Substituting Eq. (13) in Eq. (5) gives

$$M_n\ddot{T}_n(t) + K_nT_n(t) = P_0Y_n(\xi) \tag{14}$$

Applying the differential transform method to Eqs. (6)-(14) and using the theorems introduced Eqs. (11.1)-(11.6), the following expression are obtained.

$$\bar{Y}_n(k+4) = \frac{\lambda^4\bar{Y}_n(k)}{(k+1)(k+2)(k+3)(k+4)}, \quad (k = 0, 1, 2, 3, \ldots) \tag{15}$$

$$\bar{T}_n(k+2) = -\left[\frac{P_0Y_n(\xi)\beta(k) - \lambda^4\bar{T}_n(k)}{(k+1)(k+2)}\right], \quad (k = 0, 1, 2, 3, \ldots) \tag{16}$$

where $\beta(k) = \begin{cases} 1, & \text{if } k=0 \\ 0, & \text{if } k\neq 0 \end{cases}$.

Boundary conditions (12.1) and (12.2) are written using Eq. (10) for near the $x_0 = 0$ point as in following respectively

$$\text{for } x = 0;\quad \bar{Y}_n(0) = 0 \tag{17}$$

$$\text{for } x = L;\quad \bar{Y}_n(1)\sum_{k=0}^{N}\frac{L^{(4k+1)}}{(4k+1)!}(\lambda_n^4)^k + 3!\bar{Y}_n(3)\sum_{k=0}^{N}\frac{L^{(4k+3)}}{(4k+3)!}(\lambda_n^4)^k = 0 \tag{18}$$

Boundary conditions (12.3) and (12.4) are written using the function obtained by derivating Eq. (10) twice as in following

$$\text{for } x = 0;\quad \overline{Y}_n(2) = 0 \tag{19}$$

$$\text{for } x = L;\quad (\lambda_n^4)\overline{Y}_n(1)\sum_{k=0}^{N}\frac{L^{(4k+3)}}{(4k+3)!}(\lambda_n^4)^k + 3!\,\overline{Y}_n(3)\sum_{k=0}^{N}\frac{L^{(4k+1)}}{(4k+1)!}(\lambda_n^4)^k = 0 \tag{20}$$

Boundary condition (12.5) is written using Eq. (10) for near the $t_0 = 0$ time as in following

$$\text{for } t = 0;\quad \overline{T}_n(0) = 0 \tag{21}$$

Boundary condition (12.6) is written using the function obtained by derivating Eq. (10) as in following

$$\text{for } t = 0;\quad \overline{T}_n(1) = 0 \tag{22}$$

Thus,

$$\overline{T}_n(t) = P_0 Y_n(\xi)\sum_{k=0}^{N}\frac{t^{(2k+2)}}{(2k+2)!}(\lambda_n^4)^k(-1)^{(k+1)} \tag{23}$$

The matrix expression is obtained using Eqs. (18) and (20)

$$\begin{bmatrix} A_n & B_n \\ \lambda^4 B_n & A_n \end{bmatrix}\begin{Bmatrix} \overline{Y}_n(1) \\ 3!\,\overline{Y}_n(3) \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{24}$$

The frequency equation of the beam is obtained using determinant of the coefficients matrix of Eq. (24) as in the following

$$f_n = A_n^2 - \lambda^4 B_n^2 = 0 \tag{25}$$

Solving Eq. (25) we get $\omega = \omega_i^{(N)},\ i = 1, 2, 3, \ldots$ where $A_n = \sum_{k=0}^{N}\frac{L^{(4k+1)}}{(4k+1)!}(\lambda^4)^k$, $B_n = \sum_{k=0}^{N}\frac{L^{(4k+3)}}{(4k+3)!}(\lambda^4)^k$, $\omega_i^{(N)}$, is the N-th estimated ω natural frequency corresponding to N, and N is indicated by

$$\left|\frac{\omega_i^{(N)} - \omega_i^{(N-1)}}{\omega_i^{(N)}}\right| \le \varepsilon \tag{26}$$

where $\omega_i^{(N-1)}$ is the i-th estimated circular frequency corresponding to N–1 and ε is a positive and small value (i.e., $\varepsilon = 0{,}0001$).

Substituting Eqs. (15), (17) and (19) in Eq. (10) gives

$$Y_n(x) = \sum_{k=0}^{N}(\lambda^4)^k\left[\frac{x^{(4k+1)}}{(4k+1)!}\overline{Y}_n(1) + 3!\sum_{k=0}^{N}\frac{x^{(4k+3)}}{(4k+3)!}\overline{Y}_n(3)\right] \tag{27}$$

$\overline{Y}_p(3)$ is obtained using Eq. (24) as in the following

$$\bar{Y}_p(3) = -\frac{A_n + \lambda^4 B_n}{3!(A_n + B_n)}\bar{Y}_p(1) \tag{28}$$

Substituting Eq. (28) in Eq. (27) gives

$$Y_n(x) = \bar{Y}_n(1)\left\{\sum_{k=0}^{N}(\lambda^4)^k\left[\frac{x^{(4k+1)}}{(4k+1)!} - \frac{A_n + \lambda^4 B_n}{A_n + B_n}\frac{x^{(4k+3)}}{(4k+3)!}\right]\right\} \tag{29}$$

where, value of $\bar{Y}_n(1)$ is arbitrary.

The natural frequencies, values of $Y_n(\xi)$ and $y(x, t)$ are calculated using Eq. (25), Eq. (29) and Eq. (2), respectively.

3.2.2 One end fixed and other end simply supported Euler-Bernoulli beam

The boundary conditions of the one end fixed and other end simply supported beam subjected to a harmonic load $P(x,t) = P_0\sin(\varpi t)$ uniformly distributed along span in (Fig. 2a-2b) are given below.

$$Y_n(x=0) = 0 \tag{30.1}$$

$$\left.\frac{dY_n}{dx}\right|_{x=0} = 0 \tag{30.2}$$

$$Y_n(x=L) = 0 \tag{30.3}$$

$$\left.\frac{d^2Y_n}{dx^2}\right|_{x=L} = 0 \tag{30.4}$$

Dividing each term of Eq. (5) by M_n yields.

$$\ddot{T}_n(t) + \omega_n^2 T_n(t) = \frac{P_0}{m}\sin(\varpi t)C_n \tag{31}$$

where ϖ is circular frequency of the harmonic load

$$C_n = \frac{\int_0^L Y_n(x)dx}{\int_0^L Y_n^2(x)dx}$$

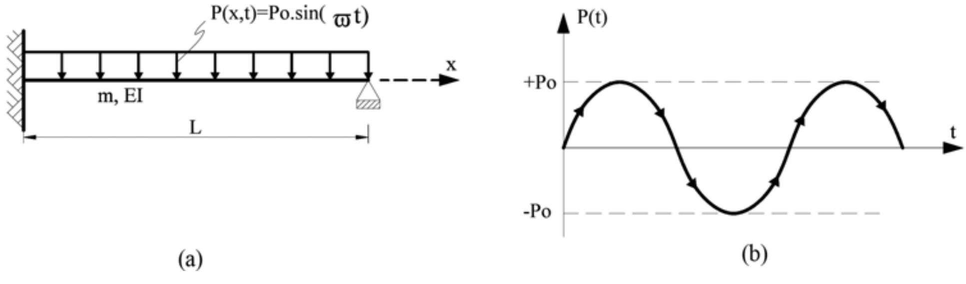

Fig. 2 (a) One end fixed and other end simply supported beam, (b) the harmonic load

The values of C_n (C_1, C_2, …, C_5) for the first five modes of the beam were calculated by Paz as below (Paz 1997)

$$C_1 = 0{,}8600;\ C_2 = 0.826;\ C_3 = 0.3345;\ C_4 = 0.0435;\ C_5 = 0.2076$$

The differential transform method is applied to Eq. (31) by using theorems introduced Eq. (11.1)-(11.7), the following expression is obtained

$$\overline{T}_n(k+2) = \frac{1}{(k+1)(k+2)}\left[\frac{C_n P_0(\omega_n)^k}{m(k!)}\sin\left(\frac{\pi}{2}k\right) - (\omega)^2\overline{T}_n(k)\right], \quad (k = 0, 1, 2, 3, \ldots) \tag{32}$$

Boundary conditions (30.1) and (30.3) are written using Eq. (10) for near the $x_0 = 0$ point as following respectively

$$\text{For } x = 0;\quad \overline{Y}_n(0) = 0 \tag{33}$$

$$\text{For } x = L;\quad 2!\overline{Y}_n(2)\sum_{k=0}^{N}(\lambda^4)^k\frac{L^{(4k+2)}}{(4k+2)!} + 3!\overline{Y}_n(3)\sum_{k=0}^{N}\frac{L^{(4k+3)}}{(4k+3)!}(\lambda^4)^k = 0 \tag{34}$$

Boundary conditions (30.2) and (30.4) are written using the function obtained by derivating Eq. (10) twice as in the following

$$\text{For } x = 0;\quad \overline{Y}_n(1) = 0 \tag{35}$$

$$\text{For } x = L;\quad 2!\overline{Y}_n(2)\sum_{k=0}^{N}(\lambda^4)^k\frac{L^{4k}}{(4k)!} + 3!\overline{Y}_n(3)\sum_{k=0}^{N}\frac{L^{(4k+1)}}{(4k+1)!}(\lambda^4)^k = 0 \tag{36}$$

According to boundary conditions (12.5) and (12.6), $\overline{T}_n(0) = 0$ and $\overline{T}_n(1) = 0$ are calculated. Thus,

$$T_n(t) = \frac{C_n P_0}{m}\left\{\sum_{k=1}^{N}(-1)^k\frac{t^{(2k+1)}}{(2k+1)!}\left[\sum_{h=1}^{N}(\varpi)^{(2k-2h+1)}\omega_n^{(2n-2)}\right]\right\} \tag{37}$$

The matrix expression is obtained using Eqs. (34) and (36)

$$\begin{bmatrix} D_n & F_n \\ G_n & H_n \end{bmatrix}\begin{Bmatrix} 2!\overline{Y}_n(2) \\ 3!\overline{Y}_n(3) \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{38}$$

The frequency equation of the beam is obtained using determinant of the coefficients matrix of Eq. (38) as in the following

$$F_n = D_n \cdot H_n - G_n \cdot F_n \tag{39}$$

Solving Eq. (39) we get $\omega = \omega_i^{(N)}$, $i = 1, 2, 3, \ldots$ where $D_n = \sum_{k=0}^{N}\frac{L^{(4k+2)}}{(4k+2)!}(-\lambda^4)^k(-1)^k$, $F_n = \sum_{k=0}^{N}\frac{L^{(4k+3)}}{(4k+3)!}(-\lambda^4)^k(-1)^k$, $G_n = \sum_{k=0}^{N}\frac{L^{(4k)}}{(4k)!}(-\lambda^4)^k(-1)^k$, $H_n = \sum_{k=0}^{N}\frac{L^{(4k+1)}}{(4k+1)!}(-\lambda^4)^k(-1)^k$, $\omega_i^{(N)}$ is the N-th estimated ω natural frequency corresponding to N substituting Eqs. (15), (33) and (35) in Eq. (10) gives

$$Y_n(x) = \sum_{k=0}^{N}(\lambda^4)^k\left[2!\frac{x^{(4k+2)}}{(4k+2)!}\bar{Y}_n(2)+3!\frac{x^{(4k+3)}}{(4k+3)!}\bar{Y}_n(3)\right] \tag{40}$$

$\bar{Y}_p(3)$ is obtained using Eq. (38) as in the following

$$\bar{Y}_p(3) = -\frac{D_n+G_n}{3(F_n+H_n)}\bar{Y}_p(2) \tag{41}$$

Substituting Eq. (41) in Eq. (40) gives

$$Y_n(x) = \bar{Y}_n(2)\left\{\sum_{k=0}^{N}(\lambda^4)^k\left[2!\frac{x^{(4k+2)}}{(4k+2)!}-2!\left(\frac{D_n+G_n}{F_n+H_n}\right)\frac{x^{(4k+3)}}{(4k+3)!}\right]\right\} \tag{42}$$

where, value of $\bar{Y}_p(2)$ is arbitrary.

The natural frequencies and $y(x, t)$ of the beam are calculated Eq. (39) and Eq. (2), respectively.

4. Analytical solution of equations of motion

The n-th normal modes and generalized deflections of both ends simply supported beam subject to a step function force P_0 were calculated by Chopra as below (Chopra 1995)

$$Y_n(x) = \sin\left(\frac{n\pi x}{L}\right) \tag{43}$$

$$T_n(x) = \frac{2P_0L^3}{\pi^4 EI}\frac{Y_n(\xi)}{n^4}[1-\cos(\omega_n t)] \tag{44}$$

where $\omega_n = n^2\pi^2\sqrt{\frac{EI}{mL^4}}$

Solving Eq. (6) we get

$$Y_n(x) = A_1\sin(\lambda x)+A_2\cos(\lambda x)+A_3\sinh(\lambda x)+A_4\cosh(\lambda x) \tag{45}$$

where A_1, …, A_4 are constants of integration.

The constants of integration of one end fixed and other end simply supported beam are calculated using Eqs. (30.1)-(30.4) as below

$$A_3 = -A_1; \quad A_2 = -A_4; \quad A_2 = -\frac{\cos(\lambda L)-\cosh(\lambda L)}{\sin(\lambda L)-\sinh(\lambda L)} \tag{46}$$

Substituting Eq. (46) in Eq. (45) gives the n-th normal mode of one end fixed and other end simply supported beam.

$$Y_n(x) = A_1\left\{\cos(\lambda x)-\cosh(\lambda x)-\frac{\cos(\lambda L)-\cosh(\lambda L)}{\sin(\lambda L)-\sinh(\lambda L)}[\sin(\lambda L)-\sinh(\lambda L)]\right\} \tag{47}$$

If Eq. (31) is calculated using boundary conditions (12.5) and (12.6), the n-th generalized deflection of one end fixed and other end simply supported beam is obtained as

$$T_n(t) = \frac{P_0 C_0}{m(\omega_n^2 - \varpi^2)}\left[\sin(\varpi t) - \frac{\varpi}{\omega_n}\sin(\omega_n t)\right] \tag{48}$$

where ω_n is natural frequencies of the beam (n = 1, 2, 3, …).

The values of ω_n were calculated for the first five modes of the beam by Chopra as below (Chopra 1995)

$$\omega_1 = 15.4118\alpha \tag{49.1}$$

$$\omega_2 = 49.9648\alpha \tag{49.2}$$

$$\omega_3 = 104.2477\alpha \tag{49.3}$$

$$\omega_4 = 178.2697\alpha \tag{49.4}$$

$$\omega_5 = 272.0309\alpha \tag{49.5}$$

where $\alpha = \sqrt{\frac{EI}{mL^4}}$.

The lateral displacement function of both ends simply supported beam and one end fixed and other end simply supported are calculated using Eq. (43), Eq. (44), Eq. (47), Eq. (48), Eq. (2), respectively.

5. Numerical examples

For numerical examples, both ends simply supported and one end fixed and other end simply supported beams shown in Fig. 1 and Fig. 2 are considered in this study. The normal modes, the generalized deflections and the lateral displacements of the both ends simply supported beam subjected to a step-function force, P_0 = 9.806 kN, at distance ξ = 2.5 m and ξ = 5.0 m from the left end are calculated for the first seven modes using DTM and analytical method. The normal modes, the generalized deflections and the lateral displacements of one end fixed and other end simply supported beam subjected to a harmonic load $P(x, t)$ = 29.418 Sin(5t) kN/m uniformly distributed along the span are calculated for the first five modes using DTM and analytical method. For calculations, a computer program is established by author.

The numerical results are obtained based on a uniform beam with the following data as:

$$I = 5.73 * 10^{-6}\ \mathrm{m}^4\ ;\ EI = 1179.955\ \mathrm{kNm}^2\ ;\ m = 19.612\ \mathrm{kNs}^2/\ \mathrm{m}^2\ ;\ L = 10\ \mathrm{m}$$

The lateral displacements at x = 1 m, 2 m, ..., 10 m points and t = 0.1, 0.2, ..., 1.0 sn time of the both ends simply supported and beam subjected to a step-function force (P_0 = 9.906 kN) at distance ξ = 2.5 m and ξ = 5.0 m from the left end are presented in (Tables 1, 2) by using DTM and analytical method, respectively.

Both analytical solution and DTM solution obtained the both ends simply supported beam at N = 50 are plotted in Figs. 3-6 for ξ = 2.5 m and ξ = 5.0 m, respectively.

The lateral displacements at x = 1 m, 2 m, ..., 10 m points of one end fixed and other end simply supported beam subjected to $P(x, t)$ = 29.418 Sin(5t) kN/m are presented in Table 3 by using DTM and analytical method.

Both the analytical solution and DTM solution obtained as N = 50 of one end fixed and other end simply supported beam are plotted in Figs. 7, 8, respectively.

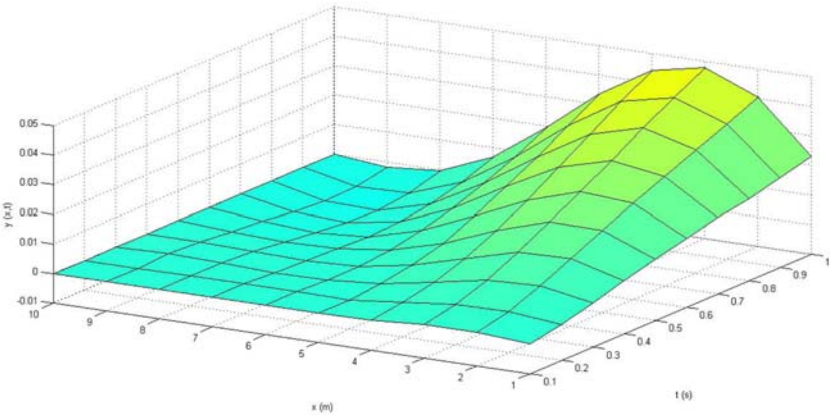

Fig. 3 Variation of the lateral displacements of both ends simply supported beam due to time and position for $\xi = 2.5$ m (DTM)

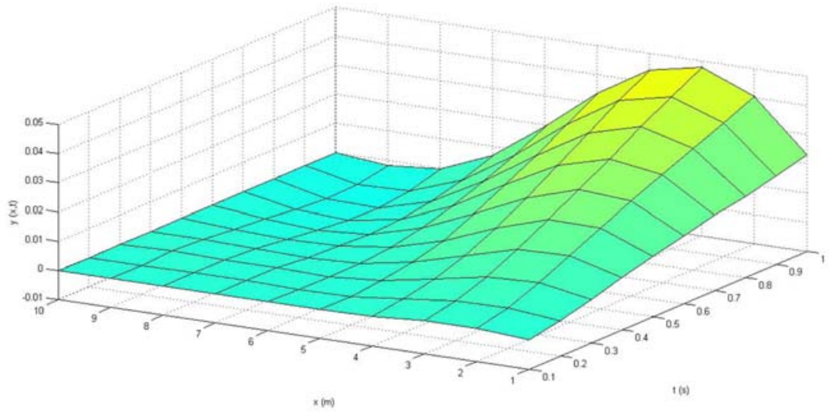

Fig. 4 Variation of the lateral displacements of both ends simply supported beam due to time and position for $\xi = 2.5$ m (Analytical Method)

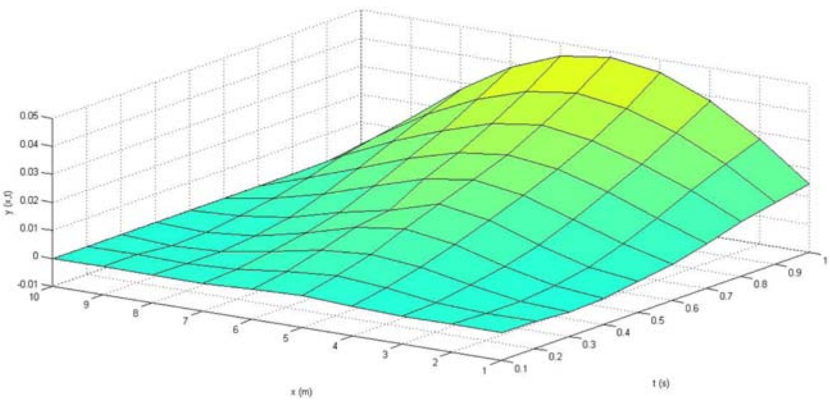

Fig. 5 Variation of the lateral displacements of both ends simply supported beam due to time and position for $\xi = 5.0$ m (DTM)

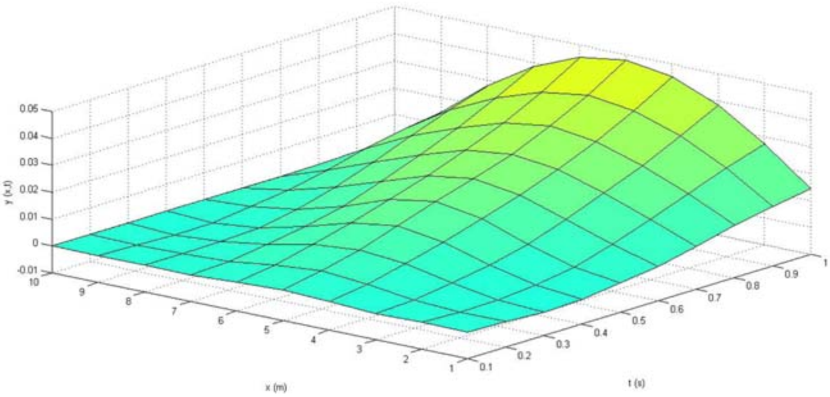

Fig. 6 Variation of the lateral displacements of both ends simply supported beam due to time and position for ξ = 5.0 m (Analytical Method)

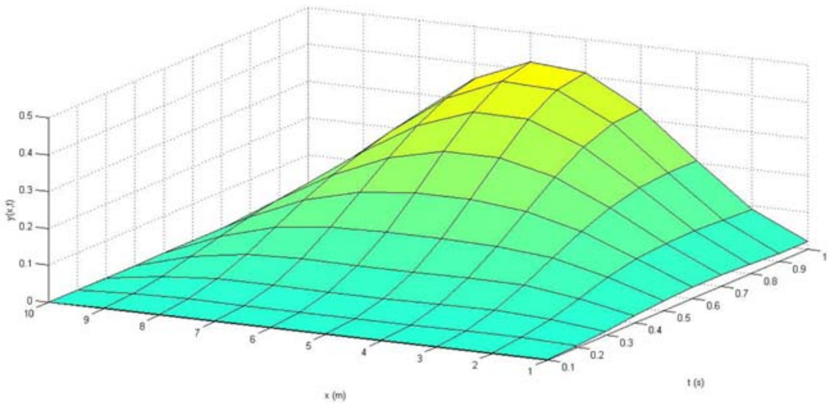

Fig. 7 Variation of the lateral displacements of one ends fixed and other end simply supported beam due to time and position (DTM)

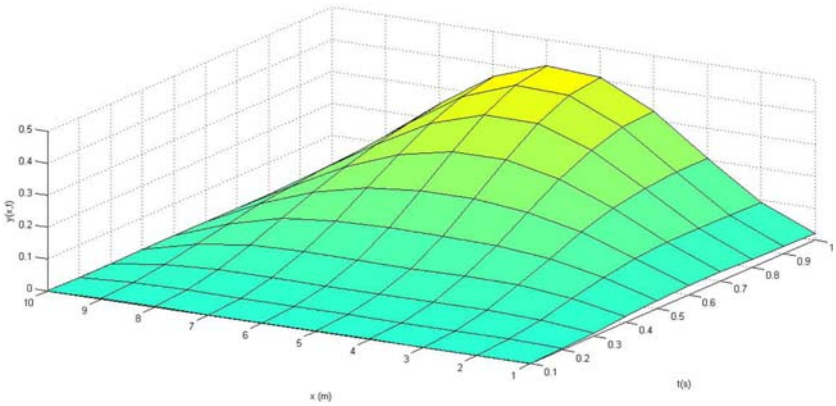

Fig. 8 Variation of the lateral displacements of one ends fixed and other end simply supported beam due to time and position (Analytical Method)

Table 1(a) The lateral displacement of the both ends simply supported beam subjected to P_0 = 9.806 kN at ξ = 2.5 m

x (m)	Method	N	t = 0.1 s.	t = 0.2 s.	t = 0.3 s.	t = 0.4 s.	t = 0.5 s.	t = 0.6 s.	t = 0.7 s.	t = 0.8 s.	t = 0.9 s.	t = 1.0 s.
1.0	DTM	10	0.000123	0.002152	0.004347	-0.56492	-104.987	-7148.13	-246473	-5184.156	-7499046	-80942353
		12	0.000102	0.002145	0.004958	-0.08071	-40.8075	-5928.36	-388614	-142617.7	-33683488	-56.31e+8
		30	0.000102	0.002145	0.004989	0.007854	0.010614	0.013263	0.014110	-8.713450	-15664.57	-126504.1
		40	0.000102	0.002145	0.004989	0.007854	0.010614	0.013263	0.015880	0.017566	0.017926	-16.09983
		42	0.000102	0.002145	0.004989	0.007854	0.010614	0.013263	0.015880	0.017566	0.020255	-0.553427
		45	0.000102	0.002145	0.004989	0.007854	0.010614	0.013263	0.015880	0.017566	0.020311	0.025693
		50	0.000102	0.002145	0.004989	0.007854	0.010614	0.013263	0.015880	0.017566	0.020311	0.022914
		51	0.000102	0.002145	0.004989	0.007854	0.010614	0.013263	0.015880	0.017566	0.020311	0.022914
	Analytic Method		0.000103	0.002146	0.004991	0.007855	0.010614	0.013262	0.015880	0.017566	0.020310	0.022914
2.0	DTM	10	0.001214	0.003863	0.008841	0.684444	123.1117	8379.441	288904.5	6076162	878876.04	9485742
		12	0.001239	0.003872	0.008124	0.116826	47.91429	6956.355	455991.2	167342.06	3952234.1	66.06e+8
		30	0.001239	0.003872	0.008088	0.012898	0.017700	0.022588	0.028967	10.27962	18386.604	148486.2
		40	0.001239	0.003872	0.008088	0.012898	0.017700	0.022588	0.028967	0.031455	0.038624	18.964788
		42	0.001239	0.003872	0.008088	0.012898	0.017700	0.022588	0.026888	0.031455	0.035890	0.716940
		45	0.001239	0.003872	0.008088	0.012898	0.017700	0.022588	0.026888	0.031455	0.035822	0.037188
		50	0.001239	0.003872	0.008088	0.012898	0.017700	0.022588	0.026888	0.031455	0.035822	0.040442
		51	0.001239	0.003872	0.008088	0.012898	0.017700	0.022588	0.026888	0.031455	0.035822	0.040442
	Analytic Method		0.001239	0.003873	0.008089	0.012898	0.017699	0.022588	0.026888	0.031454	0.035821	0.040442
3.0	DTM	10	0.001251	0.003718	0.007121	-0.20068	-39.0147	-2656.03	-91540.3	-1924.573	-278288.2	-30027402
		12	0.001243	0.003715	0.007348	-0.02105	-15.3739	-2235.31	-146517	-5376.627	-126975.6	-21.22e+8
		30	0.001243	0.003715	0.007360	0.012342	0.018147	0.023861	0.029140	-3.261580	-5916.892	-4778.39
		40	0.001243	0.003715	0.007360	0.012342	0.018147	0.023861	0.029808	0.036357	0.041119	-6.04225
		42	0.001243	0.003715	0.007360	0.012342	0.018147	0.023861	0.029804	0.036352	0.041998	-0.169958
		45	0.001243	0.003715	0.007360	0.012342	0.018147	0.023861	0.029804	0.036352	0.042014	0.048791
		50	0.001243	0.003715	0.007360	0.012342	0.018147	0.023861	0.029804	0.036352	0.042014	0.047736
		51	0.001243	0.003715	0.007360	0.012342	0.018147	0.023861	0.029804	0.036352	0.042014	0.047736
	Analytic Method		0.001242	0.003714	0.007358	0.012340	0.018146	0.023861	0.029804	0.036352	0.042014	0.047736

Table 1(b) The lateral displacement of the both ends simply supported beam subjected to $P_0 = 9.806$ kN at $\xi = 2.5$ m

x (m)	Method	N	$t = 0.1$ s.	$t = 0.2$ s.	$t = 0.3$ s.	$t = 0.4$ s.	$t = 0.5$ s.	$t = 0.6$ s.	$t = 0.7$ s.	$t = 0.8$ s.	$t = 0.9$ s.	$t = 1.0$ s.
4.0	DTM	10	0.000151	0.001728	0.003539	-0.39664	-74.1017	-5054.98	-174002	-3660.090	-529475.2	-57152880
		12	0.000134	0.001722	0.003969	-0.05707	-29.8064	-4330.91	-283902	-104190.4	-24607984	-41.13e+8
		30	0.000134	0.001722	0.003990	0.007625	0.012542	0.018200	0.023660	-6.345713	-11441.43	-9239.902
		40	0.000134	0.001722	0.003990	0.007625	0.012542	0.018200	0.024955	0.031450	0.036480	-11.73191
		42	0.000134	0.001722	0.003990	0.007625	0.012542	0.018200	0.024960	0.031450	0.038182	-0.376767
		45	0.000134	0.001722	0.003990	0.007625	0.012542	0.018200	0.024954	0.031444	0.038225	0.046224
		50	0.000134	0.001722	0.003990	0.007625	0.012542	0.018200	0.024954	0.031444	0.038216	0.044190
		51	0.000134	0.001722	0.003990	0.007625	0.012542	0.018200	0.024954	0.031444	0.038216	0.044190
	Analytic Method		0.000133	0.001720	0.003989	0.007624	0.012540	0.018199	0.024955	0.031443	0.038216	0.044190
5.0	DTM	10	-0.00017	-0.00010	0.001926	0.839119	153.3074	10434.80	359736.4	7565237	10941771	11.81e+8
		12	-0.00015	-0.00010	0.001018	0.112031	50.40083	7319.140	479764.7	176063.75	4158156.9	69.50e+8
		30	-0.00015	-0.00010	0.000980	0.002680	0.005440	0.010199	0.017752	10.80443	19346.570	15623.878
		40	-0.00015	-0.00010	0.000980	0.002680	0.005440	0.010198	0.015567	0.021200	0.0303690	19.945815
		42	-0.00015	-0.00010	0.000980	0.002680	0.005440	0.010195	0.015565	0.021199	0.0274920	0.745237
		45	-0.00015	-0.00010	0.000980	0.002680	0.005440	0.010195	0.015565	0.021197	0.0127423	0.029995
		50	-0.00015	-0.00010	0.000980	0.002680	0.005440	0.010195	0.015565	0.021197	0.0274190	0.033418
		51	-0.00015	-0.00010	0.000980	0.002680	0.005440	0.010195	0.015565	0.021197	0.0274190	0.033418
	Analytic Method		-0.00015	-0.00010	0.000980	0.002679	0.005438	0.010195	0.015564	0.021196	0.0274190	0.033418
6.0	DTM	10	0.000158	-0.00045	0.000339	0.771767	141.520	9634.115	332176.4	6986494.58	10105816	10.91e+8
		12	0.000032	-0.00049	-0.00061	-0.06373	-29.1780	-4237.55	-277756	-1019267	-2407141	-40.23e+8
		30	0.000032	-0.00049	-0.00059	-0.00042	0.000652	0.002781	0.004372	-6.27009	-112266.8	-90988.94
		40	0.000032	-0.00049	-0.00059	-0.00042	0.000652	0.002781	0.005645	0.009750	0.012354	-11.57682
		42	0.000032	-0.00049	-0.00059	-0.00042	0.000652	0.002781	0.005645	0.009750	0.014029	-0.394966
		45	0.000032	-0.00049	-0.00059	-0.00042	0.000652	0.002781	0.005645	0.009750	0.014070	0.021570
		50	0.000032	-0.00049	-0.00059	-0.00042	0.000652	0.002781	0.005645	0.009750	0.014070	0.019575
		51	0.000032	-0.00049	-0.00059	-0.00042	0.000652	0.002781	0.005645	0.009750	0.014070	0.019575
	Analytic Method		0.000032	-0.00048	-0.00059	-0.00042	0.000651	0.002780	0.005645	0.009749	0.014068	0.019571

Table 1(c) The lateral displacement of the both ends simply supported beam subjected to $P_0 = 9.806$ kN at $\xi = 2.5$ m

x (m)	Method	N	$t = 0.1$ s.	$t = 0.2$ s.	$t = 0.3$ s.	$t = 0.4$ s.	$t = 0.5$ s.	$t = 0.6$ s.	$t = 0.7$ s.	$t = 0.8$ s.	$t = 0.9$ s.	$t = 1.0$ s.
7.0	DTM	10	0.001004	0.000336	0.011430	10.41507	1908.874	129930.5	4479296.8	941989.4	13.62e+8	14.70e+9
		12	-1e-6	-0.00002	-0.00084	-0.03320	-14.7392	-2140.54	-140319	-5149.687	-12162785	-20.33e+8
		30	-1e-6	-0.00002	-0.00083	-0.00122	-0.00144	-0.00203	-0.00218	-3.412167	-6122.095	-4944.083
		40	-1e-6	-0.00002	-0.00083	-0.00122	-0.00144	-0.00203	-0.00149	0.000123	0.001967	-6.294236
		42	0.000000	-0.00002	-0.00083	-0.00122	-0.00144	-0.00203	-0.00149	0.000123	0.002877	-0.218328
		45	0.000000	-0.00002	-0.00083	-0.00122	-0.00144	-0.00203	-0.00149	0.000123	0.002899	0.008006
		50	0.000000	-0.00002	-0.00083	-0.00122	-0.00144	-0.00203	-0.00149	0.000123	0.002899	0.006922
		51	0.000000	-0.00002	-0.00083	-0.00122	-0.00144	-0.00203	-0.00149	0.000123	0.002899	0.006922
	Analytic Method		0.000000	-0.00002	-0.00083	-0.00122	-0.00144	-0.00203	-0.00149	0.000124	0.002900	0.006923
8.0	DTM	10	0.008977	0.003333	0.114029	97.04974	17785.47	1210602	417351.6	87768848	12.69e+9	13.69E+10
		12	0.000018	0.000149	-0.00005	0.127456	59.03450	8573.979	562012.04	20624.412	48708771	81.42e+8
		30	-3e-6	0.000142	-0.00010	-0.00066	-0.00188	-0.00331	-0.00233	10.279172	18450.779	149004.8
		40	-3e-6	0.000142	-0.00010	-0.00066	-0.00188	-0.00331	-0.00442	-0.004778	-0.000497	18.990044
		42	-3e-6	0.000142	-0.00010	-0.00066	-0.00188	-0.00331	-0.00442	-0.004788	-0.003241	0.678468
		45	-3e-6	0.000142	-0.00010	-0.00066	-0.00188	-0.00331	-0.00442	-0.004788	-0.003302	-0.003658
		50	-3e-6	0.000142	-0.00010	-0.00066	-0.00188	-0.00331	-0.00442	-0.004788	-0.003302	-0.000381
		51	-3e-6	0.000142	-0.00010	-0.00066	-0.00188	-0.00331	-0.00442	-0.004788	-0.003302	-0.000381
	Analytic Method		-3e-6	0.000141	-0.00011	-0.00067	-0.00188	-0.00331	-0.00442	-0.004787	-0.003302	-0.000381
9.0	DTM	10	0.090505	0.032136	1.140994	969.6117	177691.3	120949	41696851	87.68e+8	12.68E+10	13.68E+11
		12	2e-6	-0.00006	0.000540	0.138415	63.79072	9264.95	607324.7	222880.03	52639304	87.99e+8
		30	2e-6	-0.00006	0.000412	-0.00018	-0.00127	-0.00216	-0.05210	-8.67285	-15552.78	-125601.1
		40	2e-6	-0.00006	0.000412	-0.00018	-0.00127	-0.00216	-0.00345	-0.004159	-0.006240	-16.00940
		42	2e-6	-0.00006	0.000412	-0.00018	-0.00127	-0.00216	-0.00345	-0.004159	0.003862	-0.573967
		45	2e-6	-0.00006	0.000412	-0.00018	-0.00127	-0.00216	-0.00345	-0.004159	-0.003862	0.001020
		50	2e-6	-0.00006	0.000412	-0.00018	-0.00127	-0.00216	-0.00345	-0.004159	-0.003862	-0.001724
		51	2e-6	-0.00006	0.000412	-0.00018	-0.00127	-0.00216	-0.00345	-0.004159	-0.003862	-0.001724
	Analytic Method		2e-6	-0.00006	0.000410	-0.00018	-0.00127	-0.00216	-0.00345	-0.004157	-0.003865	-0.001724

Table 1(d) The lateral displacement of the both ends simply supported beam subjected to $P_0 = 9.806$ kN at $\xi = 2.5$ m

x (m)	Method	N	$t = 0.1$ s.	$t = 0.2$ s.	$t = 0.3$ s.	$t = 0.4$ s.	$t = 0.5$ s.	$t = 0.6$ s.	$t = 0.7$ s.	$t = 0.8$ s.	$t = 0.9$ s.	$t = 1.0$ s.
10.0	DTM	10	1.827004	0.650265	23.061252	19605.87	3592975	24456282	84.31E+8	17.73e+10	25.64e+11	27.67e+12
		12	0.004108	0.001460	0.003418	4.882911	2248.515	326561.2	2140590	78555246	18.55e+9	31.013e+10
		30	0.000000	-2e-6	5e-6	0.00000	-6e-6	-0.00001	-0.00006	-0.199219	-357.3772	-288610.5
		40	0.000000	-2e-6	5e-6	0.00000	-6e-6	-0.00001	-0.00002	-0.000027	-0.000087	-0.367851
		42	0.000000	-2e-6	5e-6	0.00000	-6e-6	-0.00001	-0.00002	-0.000007	-0.000008	-0.013173
		45	0.000000	-2e-6	5e-6	0.00000	-6e-6	-0.00001	-0.00002	-0.000007	-0.000008	0.000039
		50	0.000000	-2e-6	5e-6	0.00000	-6e-6	-0.00001	-0.00002	-0.000007	-0.000008	-0.000004
		51	0.000000	-2e-6	5e-6	0.00000	-6e-6	-0.00001	-0.00002	-0.000007	-0.000008	-0.000004
	Analytic Method		0.000000	0.000000	0.000000	0.00000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

Table 2(a) The lateral displacement of the both ends simply supported beam subjected to $P_0 = 9.806$ kN at $\xi = 5$ m.

x (m)	Method	N	$t = 0.1$ s.	$t = 0.2$ s.	$t = 0.3$ s.	$t = 0.4$ s.	$t = 0.5$ s.	$t = 0.6$ s.	$t = 0.7$ s.	$t = 0.8$ s.	$t = 0.9$ s.	$t = 1.0$ s.
1.0	DTM	10	0.000042	-0.00031	-0.00221	-0.95477	-174.857	-11902.1	-410320	-862.9023	-12480354	-13.47e+8
		12	0.00026	-0.00031	-0.00118	-0.12558	-57.5441	-8357.51	-547829	-201042.3	-47480843	-79.37e+8
		30	0.000026	-0.00031	-0.00113	-0.00071	0.001030	0.002835	0.003422	-12.30349	-22091.24	-17840.45
		40	0.000026	-0.00031	-0.00113	-0.00071	0.001030	0.002841	0.005915	0.009565	0.008851	-22.72296
		42	0.000026	-0.00031	-0.00113	-0.00071	0.001030	0.002857	0.005917	0.009565	0.012136	-0.798381
		45	0.000026	-0.00031	-0.00113	-0.00071	0.001030	0.002867	0.005927	0.009575	0.012228	0.018333
		50	0.000026	-0.00031	-0.00113	-0.00071	0.001030	0.002867	0.005927	0.009575	0.012228	0.014435
		51	0.000026	-0.00031	-0.00113	-0.00071	0.001030	0.002867	0.005927	0.009575	0.012228	0.014435
	Analytic Method		0.000026	-0.00031	-0.00113	-0.00071	0.001030	0.002869	0.005928	0.009579	0.012230	0.014436
2.0	DTM	10	-0.00006	-0.00041	0.000990	1.121090	205.3083	13974.48	481766.3	101315.2	14653458	15.81e+8
		12	-0.00004	-0.00040	-0.00023	0.147479	67.54782	9809.772	643024.5	235976.7	5573143	93.16e+8
		30	-0.00004	-0.00040	-0.00027	0.000940	0.003330	0.007407	0.015226	14.470358	25929.985	209405.15
		40	-0.00004	-0.00040	-0.00027	0.000936	0.003332	0.007410	0.012310	0.017718	0.026927	26.715991
		42	-0.00004	-0.00040	-0.00027	0.000917	0.003340	0.007421	0.012310	0.017729	0.023071	0.981663
		45	-0.00004	-0.00040	-0.00027	0.000925	0.003343	0.007425	0.012310	0.017730	0.022999	0.023031
		50	-0.00004	-0.00040	-0.00027	0.000925	0.003343	0.007425	0.012310	0.017730	0.022999	0.027637
		51	-0.00004	-0.00040	-0.00027	0.000925	0.003343	0.007425	0.012310	0.017730	0.022999	0.027654
	Analytic Method		-0.00004	-0.00040	-0.00027	0.000925	0.003345	0.007425	0.012317	0.017731	0.023006	0.027655
3.0	DTM	10	-0.00014	0.000588	0.002197	-0.35278	-65.5480	-4462.19	-153833	-3235.112	-467901.8	-50497477
		12	-0.00015	0.000586	0.002585	-0.04226	-21.7278	-3156.80	-206927	-7593.813	-17934569	-29.98e+8
		30	-0.00015	0.000590	0.002601	0.004903	0.008162	0.013100	0.017720	-4.626617	-8344.424	-6738814
		40	-0.00015	0.000590	0.002601	0.004903	0.008162	0.013100	0.018660	0.024350	0.029882	-8.550131
		42	-0.00015	0.000590	0.002601	0.004903	0.008162	0.013100	0.018663	0.024354	0.031123	-0.268632
		45	-0.00015	0.000590	0.002601	0.004903	0.008162	0.013100	0.018668	0.024380	0.031179	0.039862
		50	-0.00015	0.000590	0.002601	0.004903	0.008162	0.013100	0.018668	0.024380	0.031179	0.038410
		51	-0.00015	0.000590	0.002601	0.004903	0.008162	0.013100	0.018668	0.024380	0.031179	0.038410
	Analytic Method		-0.00015	0.000591	0.002609	0.004914	0.008176	0.01311	0.018668	0.024380	0.031181	0.038411

Table 2(b) The lateral displacement of the both ends simply supported beam subjected to $P_0 = 9.806$ kN at $\xi = 5$ m

x (m)	Method	N	$t = 0.1$ s.	$t = 0.2$ s.	$t = 0.3$ s.	$t = 0.4$ s.	$t = 0.5$ s.	$t = 0.6$ s.	$t = 0.7$ s.	$t = 0.8$ s.	$t = 0.9$ s.	$t = 1.0$ s.
4.0	DTM	10	0.000693	0.002874	0.005294	-0.66268	-123.249	-8390.17	-289249	-6082.904	-8797844	-94949147
		12	0.000679	0.002873	0.006021	-0.08134	-42.0146	-6103.99	-400114	-146833.7	-346782.2	-57.96e+8
		30	0.000680	0.002872	0.006053	0.009860	0.014960	0.018675	0.022007	-8.963690	-16135.48	-130307.2
		40	0.000680	0.002872	0.006053	0.009860	0.014960	0.018675	0.023820	0.029809	0.034493	-16.56196
		42	0.000680	0.002872	0.006053	0.009860	0.014960	0.018675	0.023830	0.029809	0.036892	-0.548182
		45	0.000680	0.002872	0.006053	0.009860	0.014960	0.018675	0.023840	0.029829	0.036969	0.048348
		50	0.000680	0.002872	0.006053	0.009860	0.014960	0.018675	0.023840	0.029829	0.036969	0.045500
		51	0.000680	0.002872	0.006053	0.009860	0.014960	0.018675	0.023840	0.029829	0.036969	0.045500
	Analytic Method		0.000682	0.002873	0.006061	0.009865	0.014969	0.018671	0.023847	0.029830	0.036968	0.045506
5.0	DTM	10	0.001475	0.004232	0.009276	1.409783	256.1348	17433.17	601003.2	126390.7	1828018.1	19.72e+8
		12	0.001475	0.004232	0.007740	0.166605	71.10925	10325.04	676799	24837.12	58658694	98.05e+8
		30	0.001475	0.004232	0.007686	0.012345	0.017021	0.021241	0.029016	15.239438	27283.821	220338.33
		40	0.001475	0.004232	0.007686	0.012345	0.017021	0.021241	0.025934	0.032206	0.043501	28.12977
		42	0.001475	0.004232	0.007686	0.012345	0.017021	0.021241	0.025934	0.032206	0.039444	1.051836
		45	0.001475	0.004232	0.007686	0.012345	0.017021	0.021241	0.025934	0.032206	0.039346	0.043154
		50	0.001475	0.004232	0.007686	0.012345	0.017021	0.021241	0.025934	0.032206	0.039346	0.043154
		51	0.001475	0.004232	0.007686	0.012345	0.017021	0.021241	0.025934	0.032206	0.039346	0.047988
	Analytic Method		0.001475	0.004232	0.007686	0.012345	0.017021	0.021241	0.025934	0.032206	0.039347	0.047990
6.0	DTM	10	0.000880	0.002949	0.007612	1.297032	235.8851	16055.02	553491.9	116399.1	16835071	18.16e+8
		12	0.000686	0.002880	0.006036	-0.07952	-41.1843	-5983.41	-392210	-143933.2	-33993185	-56.82e+8
		30	0.000683	0.002875	0.006060	0.009875	0.014174	0.018670	0.022051	-8.826427	-15889.24	-128318.6
		40	0.000683	0.002875	0.006060	0.009875	0.014174	0.018676	0.023831	0.029826	0.034546	-16.30851
		42	0.000683	0.002875	0.006060	0.009875	0.014174	0.018675	0.023835	0.029826	0.036908	-0.539106
		45	0.000683	0.002875	0.006060	0.009875	0.014174	0.018663	0.023836	0.029806	0.036945	0.048321
		50	0.000683	0.002875	0.006060	0.009875	0.014174	0.018663	0.023836	0.029806	0.036945	0.045490
		51	0.000683	0.002875	0.006060	0.009875	0.014174	0.018663	0.023836	0.029806	0.036945	0.045490
	Analytic Method		0.000682	0.002874	0.006059	0.009870	0.014176	0.018665	0.023830	0.029807	0.066948	0.045489

Table 2(c) The lateral displacement of the both ends simply supported beam subjected to $P_0 = 9.806$ kN at $\xi = 5$ m.

x (m)	Method	N	$t = 0.1$ s.	$t = 0.2$ s.	$t = 0.3$ s.	$t = 0.4$ s.	$t = 0.5$ s.	$t = 0.6$ s.	$t = 0.7$ s.	$t = 0.8$ s.	$t = 0.9$ s.	$t = 1.0$ s.
7.0	DTM	10	0.001508	0.001185	0.023101	17.40862	3189.407	217092.56	7484206.4	15739254	22.76e+8	24.56e+9
		12	-0.00014	0.000595	0.002603	-0.04013	-20.7598	-3016.215	-197712.1	-7255.626	-17135860	-28.64+8
		30	-0.00014	0.000594	0.002615	0.004919	0.008178	0.013104	0.017709	-4.787865	-8633.759	-6972.475
		40	-0.00014	0.000594	0.002615	0.004919	0.008178	0.013104	0.018685	0.024373	0.029858	-8.847901
		42	-0.00014	0.000594	0.002615	0.004919	0.008178	0.013104	0.018675	0.024373	0.031142	-0.279250
		45	-0.00014	0.000594	0.002615	0.004919	0.008178	0.013104	0.018665	0.024353	0.031173	0.039942
		50	-0.00014	0.000594	0.002615	0.004919	0.008178	0.013104	0.018665	0.024353	0.031173	0.038400
		51	-0.00014	0.000594	0.002615	0.004919	0.008178	0.013104	0.018665	0.024353	0.031173	0.038400
	Analytic Method		-0.00014	0.000590	0.002611	0.004916	0.008176	0.013108	0.018665	0.024352	0.031152	0.038394
8.0	DTM	10	0.014888	0.004922	0.190354	162.1400	29713.66	2022517.2	69725.732	14.66e+8	21.20e+9	22.88e+10
		12	-0.00001	-0.00039	-0.00019	0.181713	83.30792	12098.674	793060.69	291036.9	68735184	11.49e+9
		30	-0.00004	-0.00040	-0.00026	0.000927	0.003331	0.007413	0.015254	14.520851	26020.544	210136.49
		40	-0.00004	-0.00040	-0.00026	0.000927	0.003331	0.007413	0.012305	0.017726	0.026965	26.809231
		42	-0.00004	-0.00040	-0.00026	0.000927	0.003331	0.007413	0.012305	0.017725	0.023096	0.985027
		45	-0.00004	-0.00040	-0.00026	0.000927	0.003331	0.007413	0.012305	0.017720	0.023000	0.023047
		50	-0.00004	-0.00040	-0.00026	0.000927	0.003331	0.007413	0.012305	0.017720	0.023000	0.027655
		51	-0.00004	-0.00040	-0.00026	0.000927	0.003331	0.007413	0.012305	0.017720	0.023000	0.027655
	Analytic Method		-0.00004	-0.00040	-0.00027	0.000922	0.003332	0.007413	0.012305	0.017716	0.022994	0.027654
9.0	DTM	10	0.150520	0.053364	1.903868	1619.880	296859.7	202063.2	69660761	14.64e+9	21.18e+10	22.86e+11
		12	0.000294	-0.00022	-0.00096	0.194722	89.95493	13064.38	856362.46	3142673.4	74221599	12.40e+9
		30	0.000026	-0.00031	-0.00114	-0.00072	0.001017	0.002859	0.003449	-12.21558	-21933.55	-177131.1
		40	0.000026	-0.00031	-0.00114	-0.00072	0.001017	0.002860	0.005927	0.009583	0.008899	-22.56063
		42	0.000026	-0.00031	-0.00114	-0.00072	0.001017	0.002860	0.005927	0.009583	0.012161	-0.792552
		45	0.000026	-0.00031	-0.00114	-0.00072	0.001017	0.002860	0.005930	0.009593	0.012239	0.018333
		50	0.000026	-0.00031	-0.00114	-0.00072	0.001017	0.002860	0.005930	0.009593	0.012239	0.014469
		51	0.000026	-0.00031	-0.00114	-0.00072	0.001017	0.002860	0.005930	0.009593	0.012239	0.014469
	Analytic Method		0.000026	-.000312	-0.00113	-0.00072	0.001017	0.002862	0.005935	0.009594	0.012255	0.014466

Table 2(d) The lateral displacement of the both ends simply supported beam subjected to $P_0 = 9.806$ kN at $\xi = 5$ m

x (m)	Method	N	$t = 0.1$ s.	$t = 0.2$ s.	$t = 0.3$ s.	$t = 0.4$ s.	$t = 0.5$ s.	$t = 0.6$ s.	$t = 0.7$ s.	$t = 0.8$ s.	$t = 0.9$ s.	$t = 1.0$ s.
10.0	DTM	10	3.040940	1.84671	38.519120	32754.61	6002615	40857955	14.08e+9	29.62e+10	42.8e+11	46.23e+12
		12	0.005791	0.002059	0.004799	6.888230	3171.94	460674.9	301969.7	11.08e+8	26.17e+9	43.75e+10
		30	0.000000	-2e-6	-0.00001	-0.00001	-9e-6	-0.00001	-0.00006	-0.280903	-503.9970	-407017.9
		40	0.000000	-2e-6	-0.00001	-0.00001	-9e-6	-0.00001	0.000000	0.000011	0.000012	-0.518726
		42	0.000000	-2e-6	-0.00001	-0.00001	-9e-6	-0.00001	0.000000	0.000011	0.000012	-0.018531
		45	0.000000	-2e-6	-0.00001	-0.00001	-9e-6	-0.00001	0.000000	0.000011	0.000012	0.000101
		50	0.000000	-2e-6	-0.00001	-0.00001	-9e-6	-0.00001	0.000000	0.000011	0.000012	0.000012
		51	0.000000	-2e-6	-0.00001	-0.00001	-9e-6	-0.00001	0.000000	0.000011	0.000012	0.000012
	Analytic Method		0.000000	0.00000	0.00000	0.00000	0.000000	0.000000	0.000000	0.00000	0.000000	0.000000

Table 3(a) The lateral displacement of one end fixed and other end simply supported beam subjected to $P(x, t)$ = 29.418.sin(5t) kN/m

x (m)	Method	N	t = 0.1 s.	t = 0.2 s.	t = 0.3 s.	t = 0.4 s.	t = 0.5 s.	t = 0.6 s.	t = 0.7 s.	t = 0.8 s.	t = 0.9 s.	t = 1.0 s.
1.0	DTM	5	0.000769	0.004595	0.011689	0.048041	0.484692	4.196953	26.13474	125.1333	490.62539	1647.4996
		8	0.000766	0.004581	0.010886	0.018612	-0.01357	-1.12259	-19.4026	-220.1487	-1843.229	-12171.00
		10	0.000766	0.004568	0.010850	0.018728	0.023598	-0.12344	-4.87652	-97.24722	-1336.588	-13760.01
		20	0.000766	0.004568	0.010850	0.018664	0.026005	0.029410	0.029198	0.026275	-0.006797	-2.614767
		30	0.000766	0.004568	0.010850	0.018664	0.026005	0.029410	0.029198	0.026310	0.022557	0.021032
		36	0.000766	0.004568	0.010850	0.018664	0.026005	0.029410	0.029198	0.026310	0.022557	0.021032
	Analytic Method		0.000765	0.004568	0.010850	0.018664	0.025949	0.029412	0.029021	0.026311	0.022560	0.021032
2.0	DTM	5	0.001503	0.010016	0.027542	0.077313	0.506671	4.031390	24.79243	118.4622	464.3264	1559.236
		8	0.001433	0.009740	0.026531	0.049430	0.145014	-0.69970	-13.2193	-150.7991	-1263.203	-8341.622
		10	0.001430	0.009700	0.026427	0.049416	0.071087	-0.01489	-3.26204	-66.53032	-915.4396	-9425.095
		20	0.001430	0.009700	0.026427	0.049207	0.072430	0.089460	0.097700	0.098562	0.074859	-1.713191
		30	0.001430	0.009700	0.026427	0.049207	0.072430	0.089460	0.097700	0.098354	0.094763	0.092142
		36	0.001430	0.009700	0.026427	0.049207	0.072430	0.089460	0.097700	0.098354	0.094763	0.092142
	Analytic Method		0.001428	0.009704	0.026423	0.049203	0.072433	0.089465	0.097700	0.098357	0.094762	0.092141
3.0	DTM	5	0.001651	0.011782	0.034671	0.084324	0.358456	2.415643	14.38502	68.270403	267.10406	896.33013
		8	0.001257	0.010271	0.032791	0.067892	0.145014	1.234922	18.52561	208.08304	1740.547	11491.563
		10	0.001253	0.010219	0.032637	0.067320	0.109920	0.291637	4.817340	92.184809	1264.1733	13012.35
		20	0.001253	0.010219	0.032637	0.067005	0.107098	0.146316	0.177549	0.197952	0.234568	2.698904
		30	0.001253	0.010219	0.032637	0.067005	0.107098	0.146316	0.177549	0.197170	0.206344	0.206231
		36	0.001253	0.010219	0.032637	0.067005	0.107098	0.146316	0.177549	0.197170	0.206344	0.206231
	Analytic Method		0.001250	0.010219	0.032636	0.067002	0.107099	0.146314	0.177542	0.197170	0.206342	0.206230
4.0	DTM	5	0.003126	0.017071	0.044365	0.280025	3.331516	29.27856	182.4298	873.1222	3422.074	11487.726
		8	0.001140	0.009439	0.031577	0.070417	0.142671	0.759163	9.945070	110.21784	920.6161	6077.205
		10	0.001133	0.009390	0.031440	0.069850	0.124125	0.261297	2.723425	49.46197	675.9541	6955.7962
		20	0.001133	0.009390	0.031440	0.069845	0.122470	0.183421	0.243111	0.292174	0.336392	1.659259
		30	0.001133	0.009390	0.031440	0.069845	0.122470	0.183421	0.243111	0.291563	0.321190	0.327010
		36	0.001133	0.009390	0.031440	0.069845	0.122470	0.183421	0.243111	0.291563	0.321190	0.327010
	Analytic Method		0.001133	0.009391	0.031443	0.069848	0.122469	0.183423	0.243113	0.291563	0.321190	0.327015

Table 3(b) The lateral displacement of one end fixed and other end simply supported beam subjected to $P(x, t)$ = 29.418.sin(5t) kN/m

x (m)	Method	N	t = 0.1 s.	t = 0.2 s.	t = 0.3 s.	t = 0.4 s.	t = 0.5 s.	t = 0.6 s.	t = 0.7 s.	t = 0.8 s.	t = 0.9 s.	t = 1.0 s.
5.0	DTM	5	0.011607	0.048979	0.098638	1.264264	18.45872	166.4664	1041.466	4988.7505	19557.18	65658.107
		8	0.001322	0.009514	0.029713	0.067756	0.084250	-1.02579	-20.3944	-233.8909	-1960.564	-12947.57
		10	0.001313	0.009461	0.29645	0.067798	0.123854	0.042280	-4.77837	-99.93325	-1377.559	-14185.19
		20	0.001313	0.009461	0.29645	0.067794	0.126267	0.199931	0.278910	0.349377	0.365221	-2.308988
		30	0.001313	0.009461	0.29645	0.067794	0.126267	0.199931	0.278910	0.349390	0.395487	0.408519
		36	0.001313	0.009461	0.29645	0.067794	0.126267	0.199931	0.278910	0.349390	0.395487	0.408519
	Analytic Method		0.001312	0.009468	0.29642	0.067794	0.126262	0.199838	0.278910	0.349392	0.395489	0.408519
6.0	DTM	5	0.054692	0.214263	0.383837	6.148683	93.26521	844.9059	5289.981	25343.771	99358.36	333574.41
		8	0.001286	0.009592	0.030400	0.068371	0.105888	-0.39791	-9.80029	-113.8592	-955.7575	-6313.221
		10	0.001242	0.009440	0.030297	0.068571	0.125466	0.146926	-1.41304	-33.20760	-460.7263	-4746.778
		20	0.001242	0.009439	0.030297	0.068680	0.126490	0.199946	0.279664	0.351744	0.391071	-0.488667
		30	0.001242	0.009439	0.030297	0.068680	0.126490	0.199946	0.279664	0.351660	0.401417	0.420285
		36	0.001242	0.009439	0.030297	0.068680	0.126490	0.199946	0.279664	0.351660	0.401417	0.420285
	Analytic Method		0.001243	0.009439	0.030297	0.068689	0.126498	0.199949	0.279666	0.351660	0.401417	0.420282
7.0	DTM	5	0.276569	1.064507	1.850677	31.364468	479.5340	4348.105	27227.62	130448.6	511416.98	1716972
		8	0.001372	0.010610	0.033453	0.071749	0.130383	0.391297	3.751191	40.028551	332.91903	2196.184
		10	0.001148	0.009770	0.032756	0.071299	0.125223	0.349121	5.562690	105.7436	1449.227	14916.196
		20	0.001147	0.009759	0.032740	0.071990	0.122958	0.183797	0.244545	0.299006	0.370059	3.218744
		30	0.001147	0.009759	0.032740	0.071990	0.122958	0.183797	0.244545	0.298958	0.338510	0.359374
		36	0.001147	0.009759	0.032740	0.071990	0.122958	0.183797	0.244545	0.298958	0.338510	0.359374
	Analytic Method		0.001143	0.009759	0.032741	0.071291	0.122951	0.183798	0.245458	0.298959	0.338519	0.359373
8.0	DTM	5	1.372021	5.256139	9.07617	156.07416	2390.411	21678.58	135753.8	650404.3	2549875	8560649
		8	0.002595	0.015116	0.036392	0.065692	-0.07046	-5.06331	-87.6715	-995.1664	-8332.646	-55020.98
		10	0.001389	0.010490	0.032059	0.065654	0.105684	0.171700	1.016960	16.80407	228.2705	2347.864
		20	0.001382	0.010590	0.032056	0.065644	0.105659	0.145703	0.180696	0.208948	0.236127	0.709027
		30	0.001382	0.010530	0.032056	0.065644	0.105659	0.145703	0.180695	0.208925	0.231435	0.249105
		36	0.001382	0.010530	0.032056	0.065644	0.105659	0.145703	0.180695	0.208925	0.231435	0.249105
	Analytic Method		0.001384	0.010531	0.032193	0.065643	0.105656	0.145704	0.180695	0.208928	0.231435	0.249105

Table 3(c) The lateral displacement of one end fixed and other end simply supported beam subjected to $P(x, t)$ = 29.418.sin(5t) kN/m

x (m)	Method	N	t = 0.1 s.	t = 0.2 s.	t = 0.3 s.	t = 0.4 s.	t = 0.5 s.	t = 0.6 s.	t = 0.7 s.	t = 0.8 s.	t = 0.9 s.	t = 1.0 s.
9.0	DTM	5	6.060872	23.19128	39.98981	690.232391	10575.5	95912.43	600616.9	2877590.5	1128143.1	378749.1
		8	0.007376	0.031393	0.043347	0.041515	-0.90803	-28.6691	-484.847	-5494.401	-45997.74	-303719.9
		10	0.001203	0.007907	0.021455	0.041264	0.060450	-0.09168	-5.46129	-110.0615	-1513.639	-15583.45
		20	0.001199	0.007910	0.021420	0.041248	0.063520	0.081987	0.095070	0.104124	0.079983	-2.808733
		30	0.001199	0.007910	0.021420	0.041248	0.063520	0.081987	0.095070	0.104738	0.113004	0.123017
		36	0.001199	0.007910	0.021420	0.041248	0.063520	0.081987	0.095070	0.104738	0.113004	0.123017
	Analytic Method		0.001202	0.007915	0.021423	0.041245	0.063525	0.081987	0.095069	0.104730	0.113005	0.123018
10.0	DTM	5	18.47153	70.65592	121.8028	2104.5013	32247.24	292460.8	1831430	8774485	34399863	11548990
		8	0.024589	0.093547	0.087610	0.004534	-3.68995	-109.323	-1843.97	-20892.59	-174904.8	-115.4882
		10	0.000023	0.000089	0.000089	0.000074	-0.00012	-0.01344	-0.43264	-8.578843	-117.8796	-1213.534
		20	0.000000	0.000002	0.000008	0.000017	0.000019	0.000023	0.000027	0.000039	0.000047	0.000050
		30	0.000000	0.000002	0.000008	0.000017	0.000019	0.000023	0.000027	0.000039	0.000047	0.000050
		36	0.000000	0.000002	0.000008	0.000017	0.000019	0.000023	0.000027	0.000039	0.000047	0.000050
	Analytic Method		0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

6. Discussions

In application of DTM, the lateral displacements of one end fixed and the other end simply and both ends simply supported beam are calculated by increasing series size N. In (Tables 1-3), converges of the lateral displacements are introduced. Tables 1 and 2 indicate that change in results of the lateral displacements of both ends simply supported Euler-Bernoulli beam are negligible within the range of t = 0.1s-0.2s and series size, N, larger than 12, and within the range of t = 0.3s-0.6s and N larger then 30. Besides, Table 3 indicates that change in results of the lateral displacements of one end fixed and other end simply supported beam are negligible within the range of t = 0.1s-0.25s and series size N larger then 10, and within the range of t = 0.3s-0.7s and N larger then 20, within the range of t = 0.8s-1.0s and N larger then 30.

The values of the lateral displacements of the beams using DTM become fixed for the case that the series is taken higher than a definite value. Tables 1, 2, 3 and Figs. 1, 2, 3 are achieved that the results of DTM agree well with the results of analytical solutions.

7. Conclusions

In this study, lateral displacements of one end fixed and the other end simply and both ends simply supported beams are calculated using DTM. Then, the calculated displacements are compared with the result of the analytical solutions. It is seen from the results of DTM and analytical solution that rate of convergence and accuracy of DTM is very good, that the lateral displacements of both ends simply supported and one end fixed other end simply supported beams using analytical method and obtained DTM for N = 50 and N = 30 overlap, respectively.

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