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Multiple damages detection in beam based approximate waveform capacity dimension

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Abstract. A number of mode shape-based structure damage identification methods have been verified by numerical simulations or experiments for on-line structure health monitoring (SHM). However, many of them need a baseline mode shape generated by the healthy structure serving as a reference to identify damages. Otherwise these methods can hardly perform well when multiple cracks conditions occur. So it is important to solve the problems above. By aid of the fractal dimension method (FD), Qiao and Wang proposed a generalized fractal dimension (GFD) to detect the delamination damage. As a modification of GFD, Qiao and Cao proposed the approximate waveform capacity dimension (AWCD) technique to simplify the calculation of fractal and overcome the false peak appearing in the high mode shapes. Based on their valued work, this paper combined and applied the AWCD method and curvature mode shape data to detect multiple damages in beam. In the end, the identification properties of the AWCD for multiple damages have been verified by groups of Monte Carlo simulations and experiments.

Keywords: curvature mode shape; fractal dimension; waveform dimension; multiple damages; damage detection

1. Introduction

In recent years, structural damage detection has become an intensely investigated subject due to its practical importance. To avoid a catastrophic failure for structures such as aircrafts and planes, damage detection in the early stage becomes more and more significant. Compared with the damage detection techniques based on natural frequency (Chaudhari 2000, Nandwana 1997, Chinchalkar 2001), the damage detection techniques based on structure's modal performs better in damage localization. But classical techniques for damage identification often require the mode shape of healthy structures to serve as a reference for damaged structures, and these healthy mode shapes cannot be easily obtained in most practical conditions. To solve this problem, wavelet transform is combined with damage detection techniques based on modal data, Rucka (2006) used continuous wavelet transform in vibration based damage detection for beams and plates, Fan (2006) used 2D continuous wavelet transform of mode shape data to detect damage for plate structures. However, damage detections using wavelet transform for mode shape require a number of measured points, which limits the application of this method. Compared with wavelet transform, the fractal dimension

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Table 1 Capabilities of five comparative damage detection algorithms (Fan 2011)

Algorithm	Single damage detection	Multiple damages detection	Large-area damage detection	Noise immunity	Sensor spacing tolerance
SDI	Yes	No	No	N/A	N/A
GFD	Yes	No	No	Excellent	Fair
MSC	Yes	Yes	Yes	Good	Good
GSM	Yes	Yes	No	Fair	Good
DIM	Yes	Yes	Yes	Fair	Fair

(FD) based methods, only requires a small quantity of measured points, which makes it ideal for online data processing and application (Qiao 2007, Wang 2007, Qiao and Cao 2008, Fan 2011).

The original work of FD application for damage detection was proposed by Wang and Qiao (2007). In order to overcome the false peaks appearing in high mode shape when FD is used in damage detection directly, they proposed a scale parameter S, forming the generalized fractal dimension (GFD) method. Combine the GFD with uniform load surface (ULS), they applied this technology to detect a single damage in beam successfully (Wang 2007). By aid of mode shape curvature (MSC), Qiao *et al.* (2007) extended the GFD method to delamination detection for composite laminated plates. As an improvement of GFD, Qiao and Cao (2008) proposed a novel waveform fractal dimension-based damage identification algorithm named approximate waveform capacity dimension (AWCD). They had validated several basic characteristics of AWCD on irregularity detection of mode shapes, such as crack localization, crack quantification, noise immunity, and the detection case of a single crack in beam is studied. Recently, Fan and Qiao (2011) made a systematic comparative study for these methods. In the study, they mentioned GFD can not provide a good detection for multiple damages, but the conventional MSC method could provide a detection for the case shown in Table 1 (Fan 2011). Similar to GFD, the AWCD method based on mode shape data also can hardly give a satisfying multiple damages detection. Referencing Qiao *et al.* (2007), this paper considers MSC as an input of AWCD to detect multiple damages in beam-type structures.

Based on AWCD method and curvature mode shape, this paper proposed multiple damages detection based on AWCD. In section 2, the model of cracked beam is constructed by finite element method and rotational spring model. Section 3 gives a brief introduction of AWCD and curvature mode shape. The simulation results are showed in section 4. At last, experiments are presented to validate the present method for multiple damages detection.

2. Finite element model of multiple damages beam

In Fig. 1(a), we give a cracked model of cantilever beam with dimensions of length L and uniform cross-section $b \times h$ (where b is the width and h is the depth of beam), several open cracks of depth a_i locates at L_{ci} away from the clamped end, where the subscript i expresses the serial number of cracks. Since the linear rotational spring model can describe open crack effectively, present work is based on this model (as show in Fig. 1(b)). The stiffness of these rotational springs K_t can be written as (Nandwana 1997)

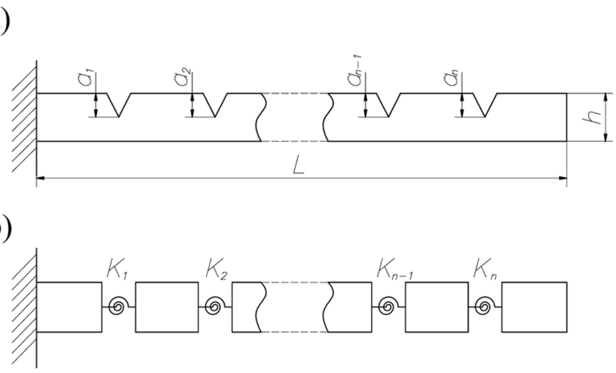

Fig. 1 Cantilever beam with multiple cracks: (a) the cracked cantilever beam, (b) the rational spring model

$$K_t = \frac{bh^2E}{72\pi(a/h)^2 f(a/h)} \tag{1}$$

where E is the modulus of elasticity of beams, h is the depth of beams and $f(a/h)$ is a dimensionless local compliance function expressed as follows

$$f\left(\frac{a}{h}\right) = 0.6384 - 1.035\frac{a}{h} + 3.7201\left(\frac{a}{h}\right)^2 - 5.1773\left(\frac{a}{h}\right)^3 + 7.553\left(\frac{a}{h}\right)^4 - 7.332\left(\frac{a}{h}\right)^5 + 2.4909\left(\frac{a}{h}\right)^6 \tag{2}$$

To express briefly, we denote two dimensionless parameters to describe cracks: relative crack size $\alpha_i = a_i/h$ and normalized location $\beta_i = L_{ci}/L$.

3. Damage identification algorithm

3.1 A brief introduction of AWCD

The term "fractal dimension" (FD), which is deemed as one of the three important factors of fractal by Mandelbrot (1983), has been frequently used in signal processing. Katz (1988) proposed a method to estimate waveform's FD, this procedure implies a waveform's FD can be approximately measured by sampling N points. Wang and Qiao (2007) introduced this method into damage detection, forming the GFD. In order to simplify the computation of GFD, the AWCD is proposed by Qiao and Cao (2008). In this reference, they give the formulation of AWCD as

$$\mathrm{AWCD} = \lim_{l\to 0}\frac{\log(L/l)}{\log(1/l)} \approx \frac{\log(L) + \log(N-1)}{\log(N-1)} = 1 + \frac{\log(N)}{\log(N-1)} \tag{3}$$

where L is the length of waveform, l is the basic mesh of L, and N is the number of segments.

Compared with classical methods, the apparent merit of AWCD is its efficiency. Qiao and Cao (2008) used AWCD to identify single crack location, getting a satisfying result. Since the higher mode shapes are more sensitive to cracks than the lower ones, it is more important to get the AWCD of higher mode shapes. However, inflexions appearing in higher mode shapes would cause a number of false peaks covering up the peaks induced by damages. Qiao and Cao (2008) verified that using a specific bijective linear mapping from vector space **Z** to $\mathbf{Z}^*$ as shown in Eq. (4), these false peaks can be inhibited well.

$$\begin{bmatrix} x_i^* \\ y_i^* \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ A & B \end{bmatrix} \begin{bmatrix} x_i \\ y_i \end{bmatrix} \tag{4}$$

where $A \propto 1/B$, and A is a constant bigger than 1. For example, A and B can be chosen as $\sin\alpha$ and $\cos\alpha$ or $1/\varepsilon$ and α ($\alpha < 1$) and so on. This paper uses $\sin\alpha$ and $\cos\alpha$ as the A and B.

3.2 Multiple damages detection using waveform fractal dimension based curvature mode

Damages usually appear at different places simultaneously, so the detection of multiple damages is important. Due to the nonlinearity of FD, the smaller crack will be shaded by the bigger ones easily. Using AWCD based mode shape will also cause such a problem (as shown in Table 1). We will illustrate this phenomenon by numerical examples in section 4.

Curvature mode has been widely used for damage identification (Pandey 1991, Ratcliffe 2000, Catbas 2008, Tomaszewska 2011). Regarding a beam-type structure, the curvature of neutral surface can be expressed as

$$v = \frac{M}{EI} = \frac{w''}{[1+(w')^2]^{3/2}} \tag{5}$$

where v denotes the curvature of neutral surface, M stands for flexural moment. Using central difference method, v can be approximated as

$$v_i = \frac{w_{i+1} - 2w_i + w_{i-1}}{h^2} \tag{6}$$

where the subscript i indicates the node number of deflection w in finite element modeling or sampling points.

By aid of the sensibility for small change in mode shape, the faint signal of damage can be reinforced. Consider this factor, modal curvature is a good preliminary treatment for AWCD. However, modal curvature will einforce the damage signal while it will reinforce the measured noise too. The treatment of noise before the computation of fractal dimension is also an important part for multiple damages detection. Combine these terms, a flowchart for multiple damages detection is shown in Fig. 2.

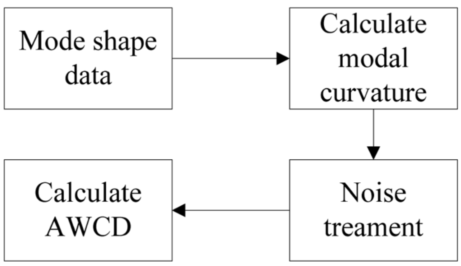

Fig. 2 The flowchart for Multiple damages detection using AWCD

4. Numerical examples

4.1 Analysis of AWCD with mode shape and modal curvature

Under the noise-free circumstances, nine crack cases locating at β = 0.1, 0.2,…, 0.9 away from the clamped end with α = 0.3 are deemed to be cracked conditions. Then AWCD are employed to detect damages as shown in Fig. 3. It can be seen that AWCD based curvature mode shape gives a more clear localization than AWCD based mode shape. Moreover, the peaks obtained by mode shape decline sharply when the crack moves from the clamped end to the free end of beam. Near the free end the feature of the AWCD peak nearly disappears in Fig. 3(a). However, AWCD of modal curvature as presented in Fig. 3(b) holds a steady peak along the beam. This property allows AWCD to avoid the influence caused by damage location.

In order to show the influence of crack depth on AWCD peak values, a group of cases with α ranging from 0.05 to 0.5 locating at β = 0.6 are given in Fig. 4. With the comparison between Fig. 4(a) and Fig. 4(b), such a conclusion is obtained: AWCD of mode shape is more sensitive to crack depth than AWCD of curvature mode shape. The peak of AWCD based mode shape increases with the increscent α, and the AWCD of curvature mode shape keep increases slowly with the

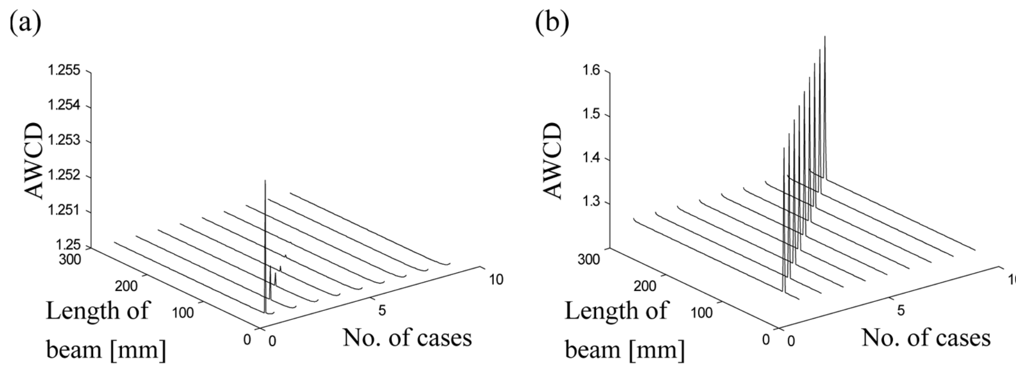

Fig. 3 AWCD of different crack locations of α = 0.3 cracked beam: (a) AWCD of mode shape, (b) AWCD of curvature mode shape

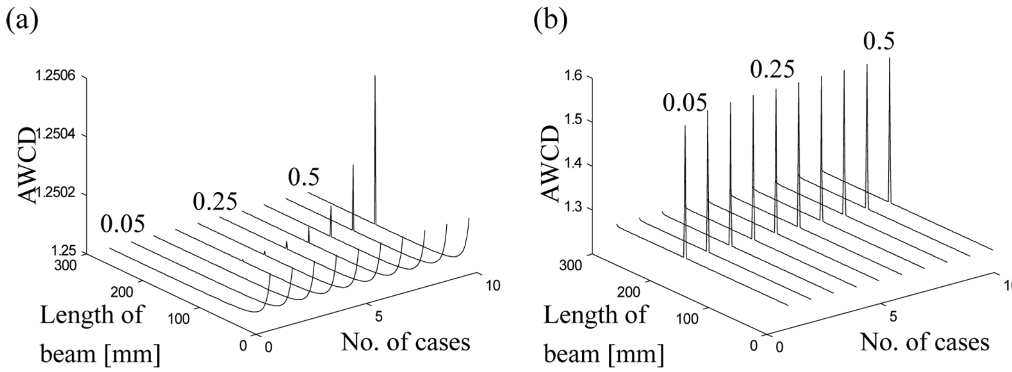

Fig. 4 AWCD of different relative depth of crack for β = 0.6: (a) AWCD of mode shape, (b) AWCD of curvature mode shape

increscent α. Though this property of curvature mode shape weakens the quantification of AWCD, it allows AWCD of curvature mode shape to show the localization of small damage steadily. So it is an appropriate method for the detection of multiple damages.

Fig. 5 is given here to verify the conclusion we get above. There are two damages locating at the

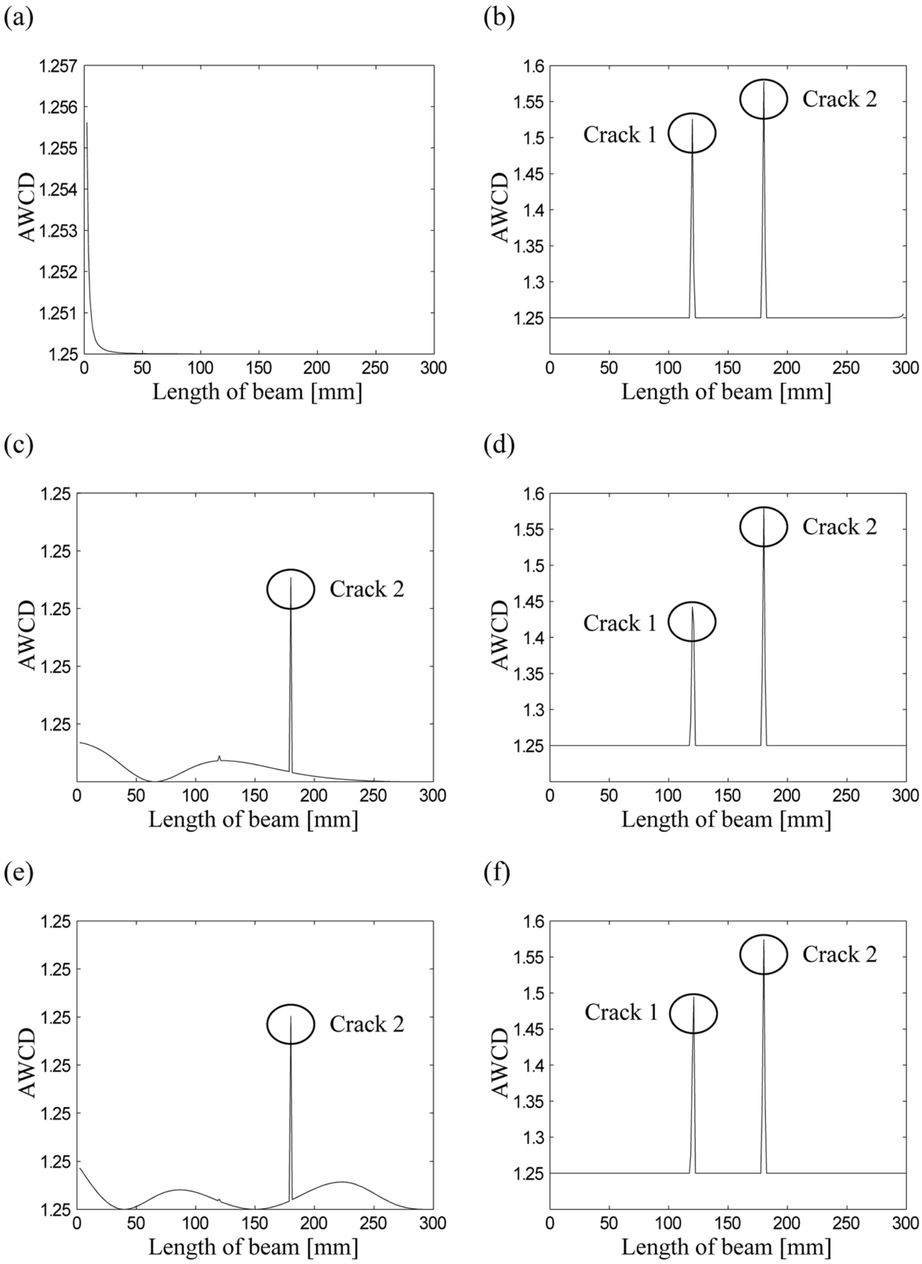

Fig. 5 AWCD of the 1st-3rd mode shapes and curvatures mode shape (α_1 = 0.1, β_1 = 0.4, α_2 = 0.3, β_2 = 0.6): (a) the 1st mode shape, (b) the 1st curvature mode shape, (c) the 2nd mode shape, (d) the 2nd curvature mode shape, (e) the 3rd mode shape, (f) the 3rd curvature mode shape

case of Fig. 5 (α_1 = 0.1, β_1 = 0.4, α_2 = 0.3, β_2 = 0.6). It is obvious that the curvature mode shape's AWCD performs well.

4.2 Noise treatment

In practical crack identification, noise immunity is a vital property. Double damages condition

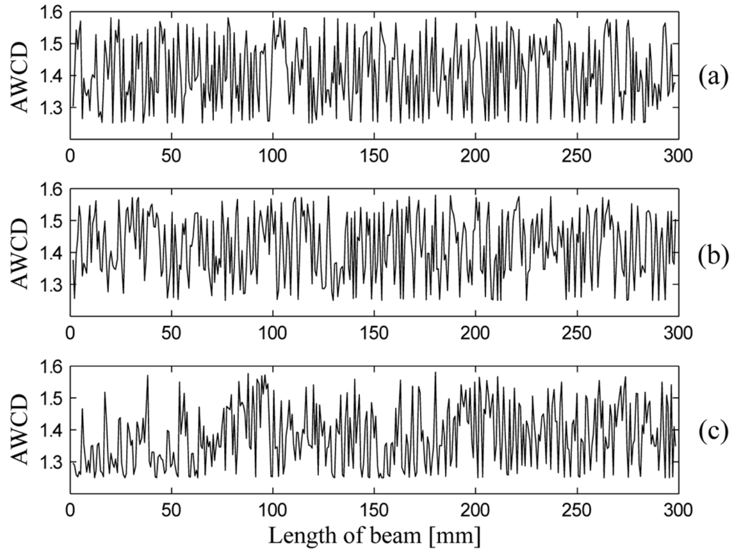

Fig. 6 Calculate AWCD by curvature mode shape directly: (a) 1st mode shape, (b) 2nd mode shape, (c) 3rd mode shape

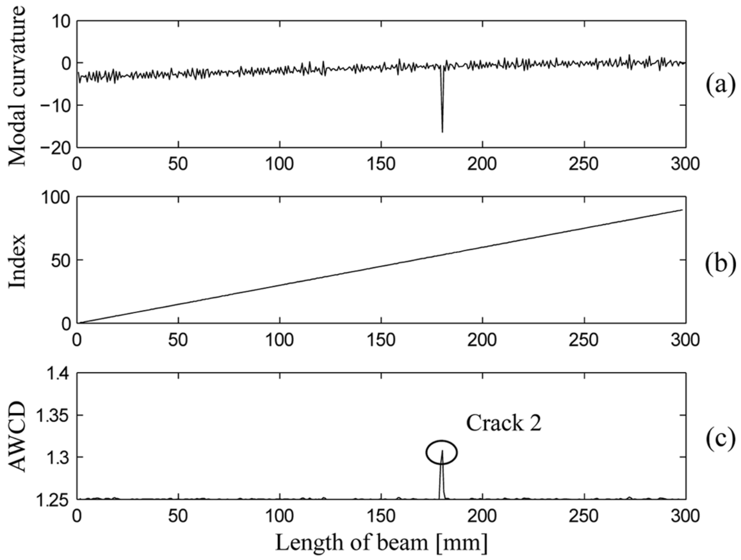

Fig. 7 A typical sample of Monte Carlo simulation (SNR = 100 dB 1st mode shape): (a) modal curvature, (b) modal curvature after linear mapping, (c) 1st curvature mode shape AWCD

($\alpha_1 = 0.1$, $\beta_1 = 0.4$, $\alpha_2 = 0.3$, $\beta_2 = 0.6$), which is the simplest multiple damages condition, is studied in the following examples, and more damages condition can be extended easily. The signal-to-noise ratio (SNR) of modal data in Monte Carlo simulations is 100 dB.

Fig. 6 presents the AWCD calculated by curvature mode shape data directly. There is nearly no

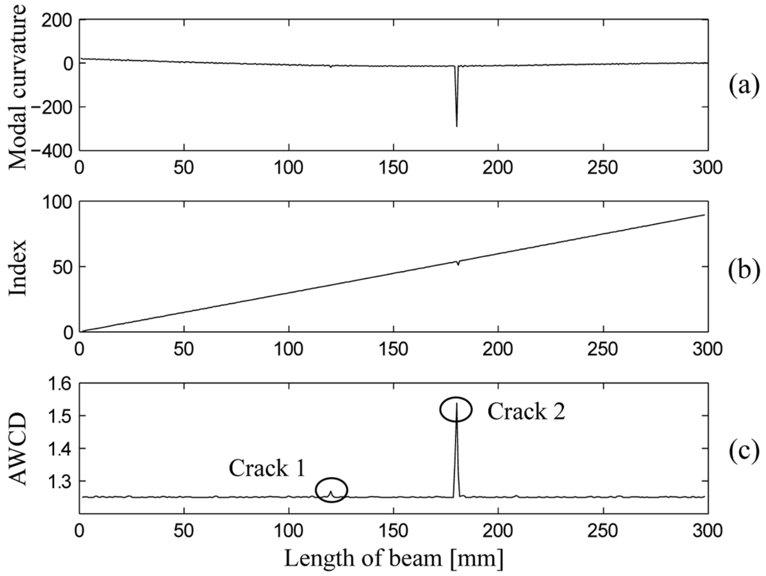

Fig. 8 A typical sample of Monte Carlo simulation (SNR = 100 dB 2nd mode shape): (a) modal curvature, (b) modal curvature after linear mapping, (c) 2nd curvature mode shape AWCD

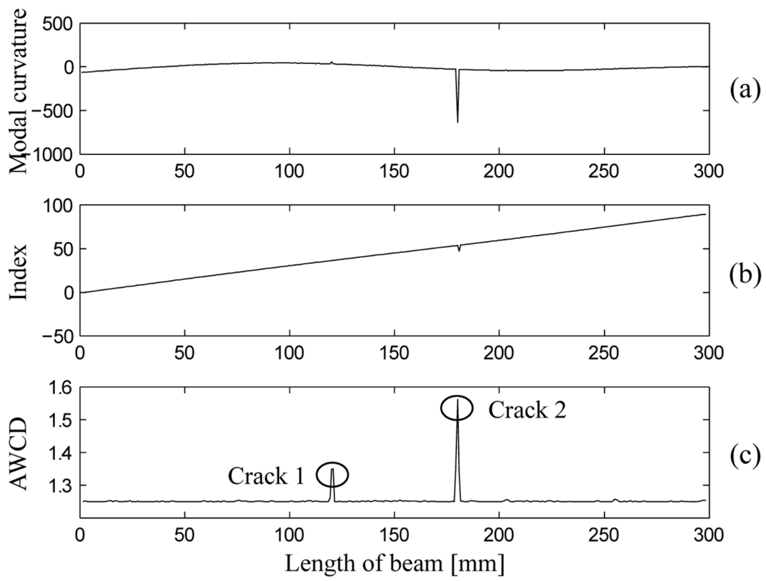

Fig. 9 A typical sample of Monte Carlo simulation (SNR = 100 dB 3rd mode shape): (a) modal curvature, (b) modal curvature after linear mapping, (c) 3rd curvature mode shape AWCD

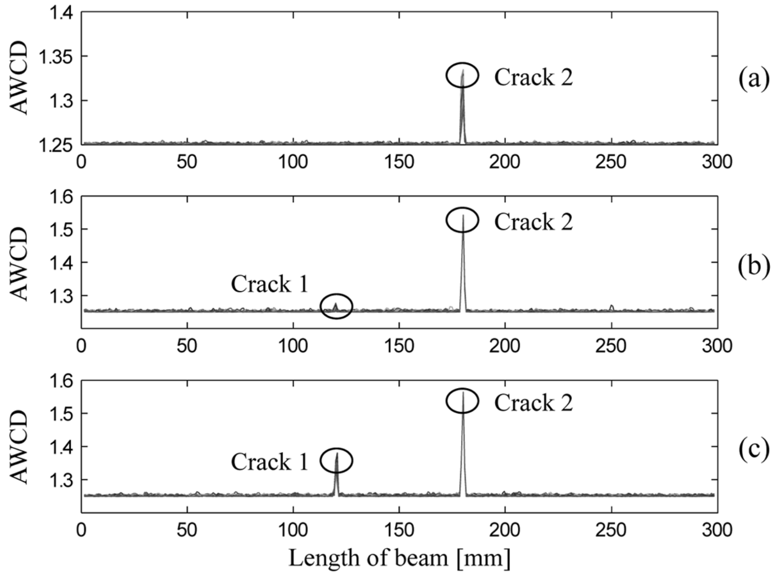

Fig. 10 All samples of Monte Carlo simulation (SNR = 100 dB): (a) 1st curvature mode shape AWCD, (b) 2nd curvature mode shape AWCD, (c) 3rd curvature mode shape AWCD

information about damage location. As mentioned above, the modal curvature not only amplifies the damage feature but also the noise, so a noise treatment is needed. Qiao and Cao (2008) proposed to use a linear mapping as shown in Eq. (4) to solve the inflexion problems. Here we notice that this method is also an effectual technology to restrain the noise for measured data if the α is selected as a constant near 89^{o}. A group of simulation results of the 1st to 3rd curvature mode shape AWCD with a noise treatment are shown in Fig. 7-Fig. 9. The comparisons of Fig. 7-Fig. 9 also present that the higher curvature mode shape is more sensitive to damage, using the higher curvature mode shape data to compute AWCD can improve the noise immunity too. Fig 10 gives all the Monte Carlo samples, which verifies the conclusion noticed above further.

5. Experimental validations

To validate the results obtained by simulations, a group of experiments are conducted using steel beams in this part as shown in Fig. 11(a). Polytec laser vibrometer is used to measure the displacement of the specimen. By selecting an appropriate frequency for function generator, the actuator near the free end of beam will lead a resonance for specimen, so a mode shape can be measured.

Fig. 11(b) gives arrangement of measured points. Two uniform cracks locate at the different places of beam. The AWCD of the first two mode shapes and curvature mode shapes are given in Fig. 12. Figs. 12(a) and (b) both give the right localization of cracks, and the higher mode shape is used as shown in Figs. 12(c) and (d). Curvature mode shape AWCD presents a more steady detection of two damage condition.

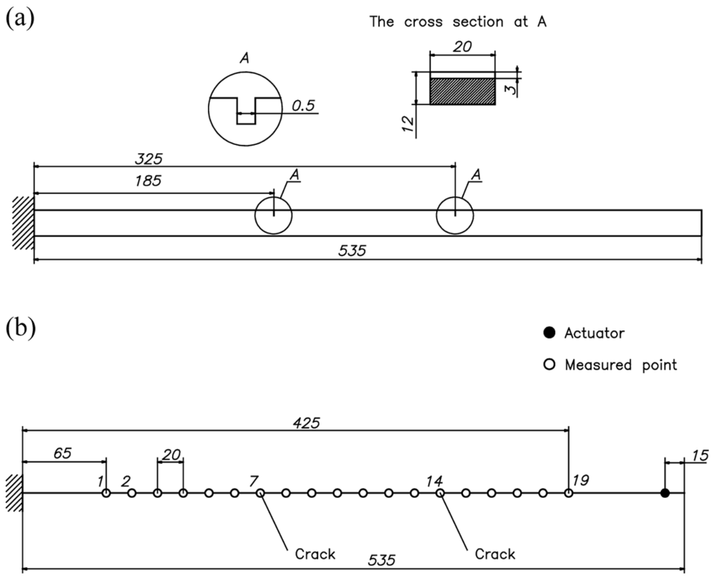

Fig. 11 Experiment for double cracks: (a) specimen for two cracks, (b) arrangement of measured points

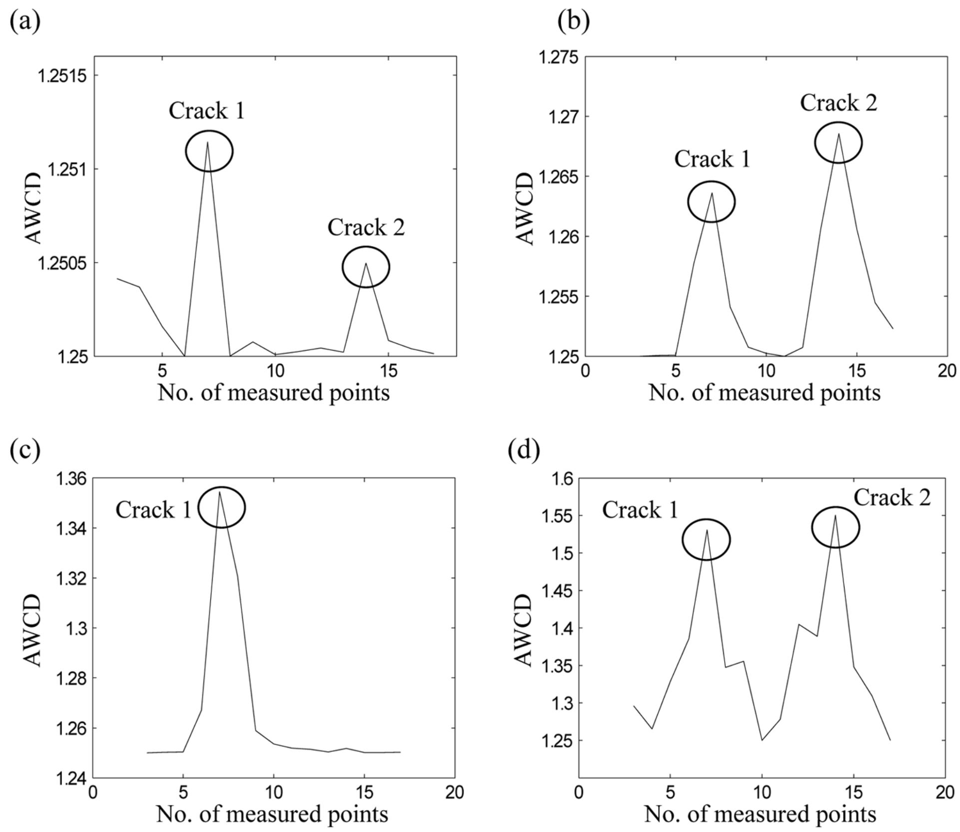

Fig. 12 Damage index of the experiment results: (a) the 1st mode shape, (b) the 1st curvature mode shape, (c) the 2nd mode shape, (d) the 2nd curvature mode shape

6. Conclusions

In this paper, a method of multiple damages detection in beam based approximate waveform capacity dimension is presented. By means of finite element method, a group of simulations verify the good applicability of the AWCD for this problem. Monte Carlo simulation is used to prove the noise immunity of the present method. The multiple damages condition can be identified clearly based the waveform capacity dimension. Furthermore, the conclusions obtained by simulations are validated by experiments. The results demonstrate that the present method provide a feasible and effective way for multiple damages detection.

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