Structural Engineering and Mechanics, Vol. 41, No. 5 (2012) 575-589
DOI: http://dx.doi.org/10.12989/sem.2012.41.5.575

Reliability analysis of tested steel I-beams with web openings

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(Received September 7, 2011, Revised January 19, 2012, Accepted January 28, 2012)

Abstract. This paper presents a reliability analysis of steel I-beams with rectangular web openings, based on a combination of the common probabilistic reliability methods, such as RSM, FORM and SORM and using data obtained from experimental tests performed at the Istanbul Technical University. A procedure is proposed to obtain the optimum design load that can be applied to this type of structural members, by taking into account specified target values of reliability indices for ultimate and serviceability limit states. The goal of the paper is to present an algorithm to obtain more realistic and economical design of beams and to demonstrate that it can be applied efficiently to steel I-beams with web openings. Finally, a sensitivity analysis is performed allowing to ranking the random variables according to their importance in the reliability analysis.

Keywords: random variable; reliability; target; performance function; failure mechanism; steel I-beams; ultimate; serviceability

1. Introduction

Most of the available information on structural engineering applications is based on the assumption of complete determinacy of structural parameters. In reality, however, there are many uncertainties such as geometric properties (cross-sectional properties and dimensions), material mechanical properties (modulus and strength, etc.), analytical models, load magnitude and distribution, etc. in design variables. During the last decades, several techniques have been developed to perform reliability analysis of the structures. These techniques may be classified mainly in three categories as : (1) First/Second Order Reliability Methods (FORM/SORM) (Zhao and Ono 1999, 2001, Rackwitz 2001), (2) Response Surface Method (RSM) (Bucher and Bourgund 1990, Rajashekhar and Ellingwood 1993, Gavin and Yau 2008, Allaix and Carbone 2011), and (3) Monte Carlo Simulation (MCS) (Harbitz 1983, Shinozuka 1983, Hohenbichler and Rackwitz 1988, Karamchandani *et al.*1989, Liu and Moses 1994).

The literatures on reliability analysis and design as well as their various structural engineering applications have greatly increased in recent years. Sakurai *et al*. (2001) have investigated the effect of variability in connection stiffness on the stochastic response of steel frames with partially restrained (PR) beam-to-column connections by using structural analysis. The mean and the

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standard deviation are determined by the perturbation method and MCS in this paper. Huh and Haldar (2002) have proposed an algorithm to estimate the reliability of nonlinear steel structures with and without PR connections subjected to seismic loading in time domain by using a hybrid method consisting of RSM, FORM and an iterative linear interpolation scheme. The algorithm is verified by using MCS. Foschi *et al.* (2002) have presented an approach to reliability calculations in performance-based design in earthquake engineering applications by using the combinations of the advantages of RSM, FORM and Monte Carlo Importance Sampling. Skejic *et al.* (2008) have studied the reliability of welded beam-to-column joints designed according to the component method adopted in Eurocode 3 (2005) by using FORM, SORM and the adaptive sampling by taking into account the variability of the basic variables of resistance and action. Cheng *et al.* (2004) have proposed an improved MCS method for the probabilistic determination of initial cable forces of cable-stayed bridges with parametric uncertainties under dead loads incorporating RSM. The method is also used to investigate the effects of various parameters on the initial cable forces of cable-stayed bridges. Cheng and Xiao (2005) have suggested a reliability analysis method to estimate the serviceability reliability of cable-stayed bridges by using a combination of the advantages of RSM, FORM and importance sampling updating method. Cheng *et al.* (2005) have carried out the flutter reliability analysis of long-span suspension bridges by using the same methods. The effects of various parameters on the flutter reliability of suspension bridges are also investigated in the paper. Buonopane and Schafer (2006) have estimated the structural reliabilities of the steel frames designed by nonlinear analysis using both first-order and importance sampling techniques. Two failure criteria which are plastic collapse and first plastic hinge are considered in the nonlinear analysis. Chen and Li (2007) have elaborated a reliability evaluation approach based on the development process of the structural nonlinearity. The approach is accordingly proposed without using the concept of failure modes. On the other hand, structural design specifications and codes have also been recently revised to incorporate probabilistic analysis in some extent.

In this paper, a reliability analysis of steel I-beams with rectangular web openings is investigated, based on a combination of the common probabilistic reliability methods, such as RSM, FORM and SORM, and using data obtained from experimental tests (Bayramoglu 1991) performed at the Istanbul Technical University. The procedure is proposed to obtain the optimum design load that can be applied to this type of structural members, taking into account specified target values of reliability indices for the ultimate and serviceability limit states. The goal of the paper is to show the potential of the proposed algorithm to obtain more realistic and economical design of beams and to demonstrate that it can be applied efficiently to steel I-beams with web openings. A sensitivity analysis is also performed to identify the relative importance of individual random variables employed in the beam model. In this study, the sensitivity measures with respect to the mean value and to the standard deviation of each random variable are of particular interest.

2. Proposed algorithm

Strengths and deflections (i.e., vertical displacements in beams, lateral displacements in frames) conduct the design of the structural systems at the ultimate and serviceability limit states, respectively. A structural system or a structural member should also be designed to satisfy the target reliability indices for both the ultimate and serviceability limit states simultaneously.

The ultimate and serviceability performance functions of statistically independent random

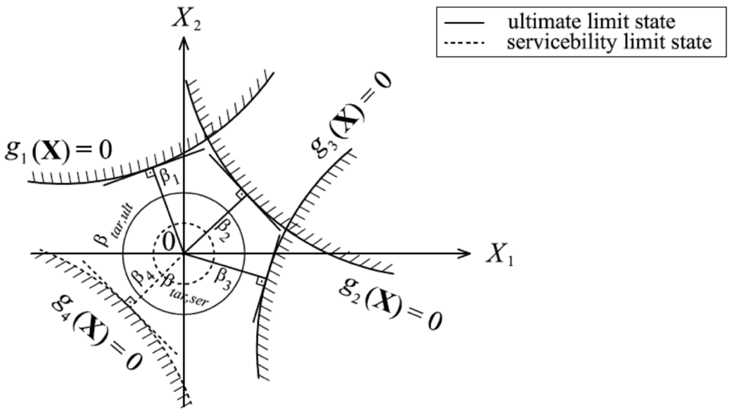

Fig. 1 Reliability indices for multiple performance functions in the standard normal space

variables of a structural system or a structural member, $g_i(\mathbf{X}) = 0$ where $i = 1, \ldots, n$, define hyper surfaces as shown in Fig. 1. The reliability index values $\beta_{i,ult}$ and $\beta_{i,ser}$ define the minimum distances from the origin of the standard normal space to the hyper surfaces for each limit state. The optimum design load P_{opt} of the structural members corresponds to the minimum value of these reliability indices, which are greater than or equal to the radius of the target hyper spheres $\beta_{tar,ult}$ and $\beta_{tar,ser}$.

The steps of the proposed procedure to obtain the optimum design load P_{opt} can given as follows:

1. Choose the target reliability index values $\beta_{tar,ult}$ and $\beta_{tar,ser}$ for the ultimate and the serviceability limit states, respectively.
2. Determine the statistical parameters of the random variables X_i.
3. Construct the performance functions $g_i(\mathbf{X})$ for each limit state.
4. Perform a deterministic structural analysis using the mean values of random variables, if necessary.
5. Apply probabilistic reliability methods to the performance functions.
6. Calculate the reliability index values $\beta_{i,ult}$ and $\beta_{i,ser}$ for each limit state.
7. Check whether $\beta_{i,ult} \geq \beta_{tar,ult}$ and $\beta_{i,ser} \geq \beta_{tar,ser}$.
8. Determine the smallest values of the reliability indices $\beta_{i,ult}$ and $\beta_{i,ser}$; $\beta_{ult} = \min(\beta_{i,ult})$ and $\beta_{ser} = \min(\beta_{i,ser})$.
9. Determine the target ultimate load $P_{tar,ult}$ and the target service load $P_{tar,ser}$, which correspond to the target value of the reliability index $\beta_{tar,ult}$ and $\beta_{tar,ser}$, respectively.
10. Obtain the optimum design load $P_{opt} = \min(P_{tar,ult}, P_{tar,ser})$.

A flow chart for the proposed algorithm is given in Fig. 2.

3. Experimental study

Load capacity and structural behaviour, including failure mechanism of the six full-size built-up steel I-beams having rectangular web openings are determined experimentally at the Structural Material Laboratory of Civil Engineering Faculty of Istanbul Technical University. The results of this study are used as an application of the algorithm presented above. In the following a brief summary is presented to give the main points of the experimental study.

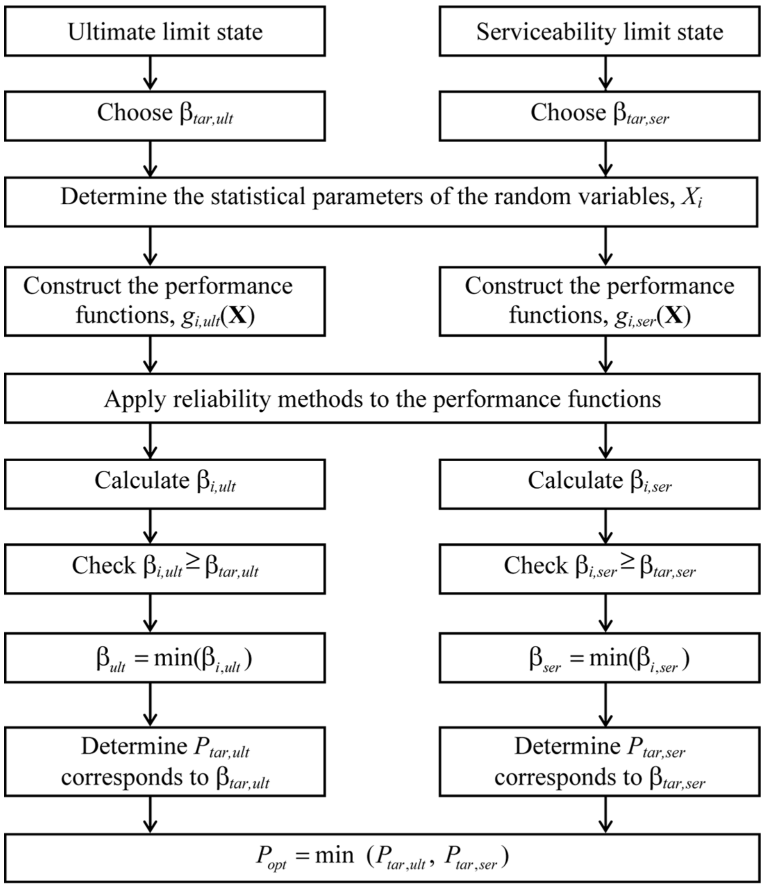

Fig. 2 Flow chart for the proposed algorithm

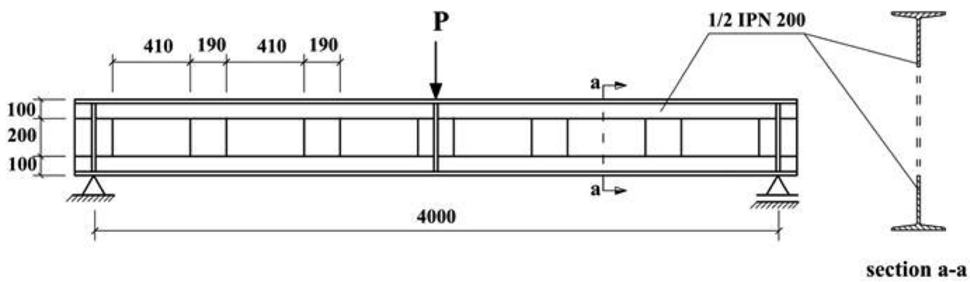

Fig. 3 Geometry and loading of the tested beams (in mm)

Fig. 3 shows the geometry of the tested beams which were geometrically and mechanically identical. The beams were first fabricated by cutting IPN 200 profiles in two equal T-parts along their longitudinal direction. These parts were then fixed to the vertical narrow web plates by butt welds. The thickness of each web plate was 8 mm, and the height was 200 mm. Thus the total height of the beams became 400 mm. The span length of the beams was 4 m. All parts of the beam specimens were made of S235 steel; however, the steel material samples taken from the flanges and

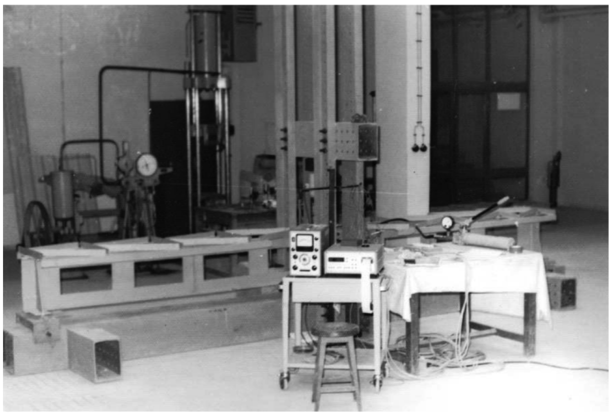

Fig. 4 A photograph of the test setup

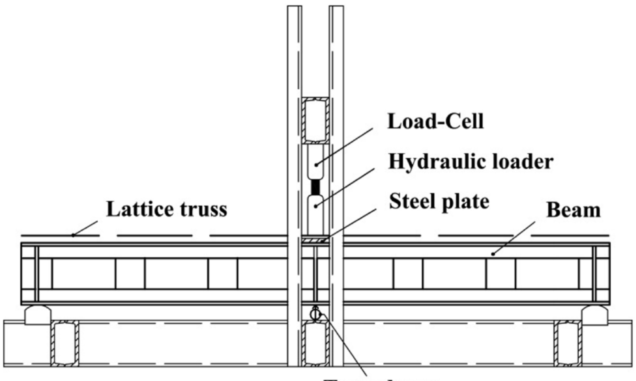

Fig. 5 Test setup

the webs of IPN 200 profiles were also tested in tension to determine their mechanical properties.

A photograph and a schematic view of the test setup are shown in Figs. 4 and 5, respectively. The testing frame was anchored on the floor of the laboratory to resist the applied loads at the supports. Lateral buckling of the top chord of the beam specimens was prevented by a lattice truss mounted on the chords in the horizontal plane by bolts. To prevent buckling of the vertical web plates, the vertical web stiffeners were placed at the supports and at the mid-span.

In the test programme, the simply supported beam specimens were loaded with one static concentrated force at their mid-span, as shown in Fig. 5. The load was increased incrementally until the failure of the beams. The vertical deflections at the mid-span were recorded at each loading level.

4. Test results

4.1 Material properties

The histogram and the fitted density curve of the measured yield stresses of the material samples in the tension tests are shown in Fig. 6.

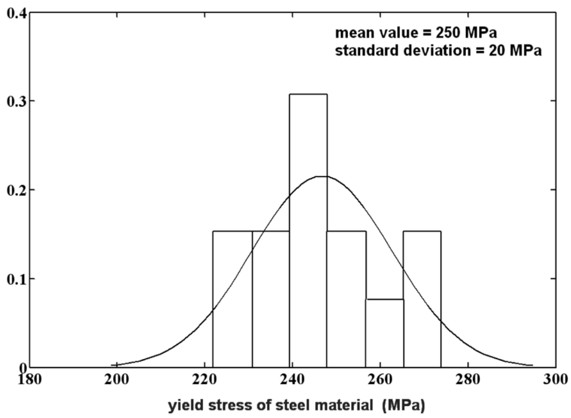

Fig. 6 Histogram and density curve for the yield stresses of the steel material

4.2 Experimental ultimate strength

The histogram and the fitted density curve of the recorded ultimate strengths of the full-size beam specimens are shown in Fig. 7. The applied load-deflection values of the tested beams (Bayramoglu 1996), marked in asterisks, and their fitted curve, are shown in Fig. 8. The load-displacement curves obtained by employing the elasto plastic structural analysis and the plastic hinge method are also plotted in Fig. 8. It is clearly seen that the experimental ultimate strength is slightly above the others due to the strain-hardening effect of the steel material.

The failure mechanism and the calculated stress distributions at the middle part of the beams are shown in Figs. 9 and 10, respectively. As seen in Fig. 10, the beams failed by forming plastic zones at the chords on each corner of the rectangular web openings, which decrease from the mid-span to the supports.

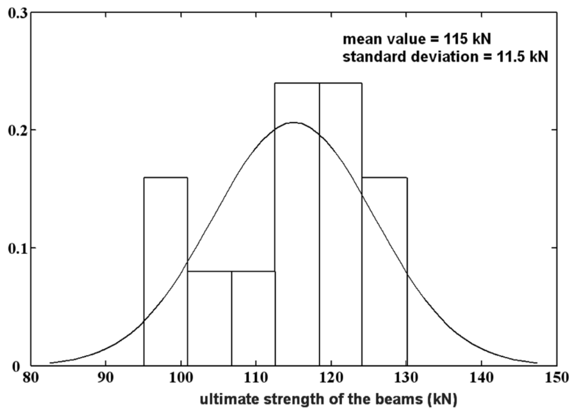

Fig. 7 Histogram and density curve for ultimate strengths of the beams

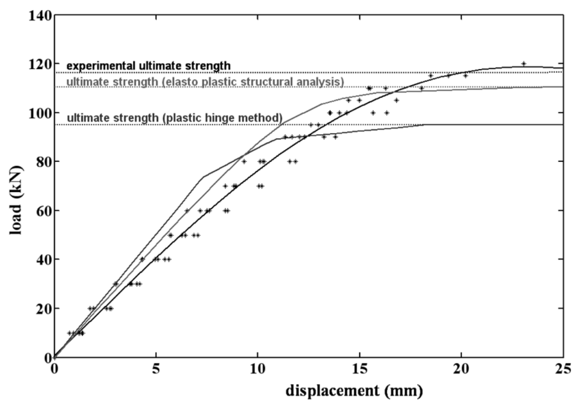

Fig. 8 Load-displacement curves

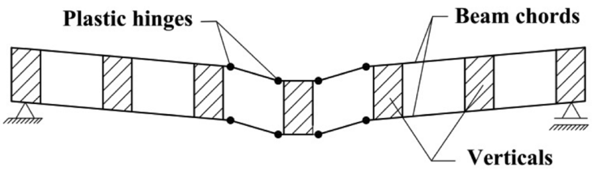

Fig. 9 Failure mechanism of the tested beams and configuration of the plastic hinges

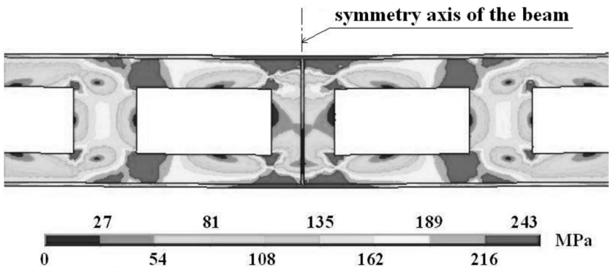

Fig. 10 Elasto plastic stress distributions at the middle part of the beam

5. Performance functions

In order to estimate the reliability of the tested beams, the performance functions are established in the following forms for the ultimate and the serviceability limit states:

5.1 Ultimate limit state

The beam with web openings may fail in one of the three failure modes which are given below. Accordingly, three performance functions are defined for the ultimate limit states.

5.1.1 Plastic hinge mechanism in the chords

The beam chords yield due to the axial and shear forces and the bending moments. In this case the performance function with four random variables is written in implicit form as follows

$$g_{1,ult}(P, s_o, f_y, L) = \frac{4[M_{u1}(P, s_o, f_y, L) + M_{u2}(P, s_o, f_y, L)]}{s_o} - P \tag{1}$$

Here, P is the beam load, s_o is the width of the web opening, f_y is the yield stress and L is the beam length. M_{u1} and M_{u2} are the ultimate moments of the beam chord at the left and right corners of the web opening, as shown in Fig. 11.

5.1.2 Axial force mechanism in the chords

The beam chords yield owing to the pure axial forces. In this case the performance function with four random variables is written in explicit form as follows

$$g_{2,ult}(A_c, f_y, P, L) = A_c f_y - \frac{PL}{4h_g} \tag{2}$$

Here, A_c is the cross-sectional area of the beam chord, and h_g is the distance between the gravity

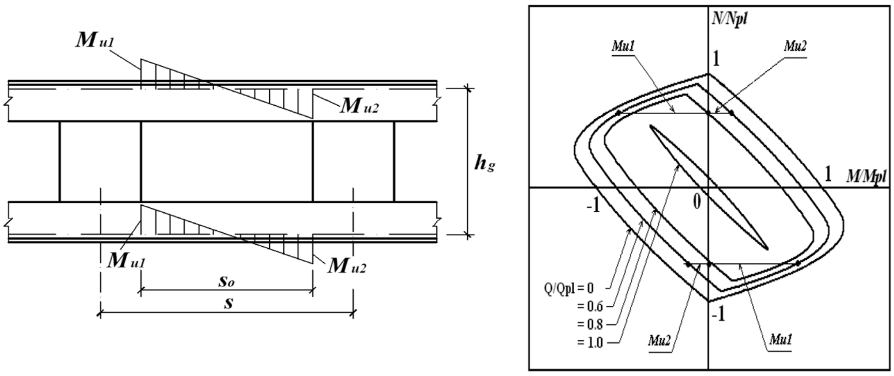

Fig. 11 Plastic hinge mechanism and interaction curves for the chords

centres of the upper and lower beam chords; h_g is treated as deterministic (in the tested beams h_g = 349.2 mm). The other three random variables, f_y, P and L, are defined in Section 5.1.1.

5.1.3 Shear force mechanism in the verticals

Assuming the simplified static system with the hinges in the middle of the beam chords and verticals, the horizontal shear force in the vertical is obtained from the moment equilibrium condition at the upper T-part in the middle of the beam. The beam verticals yield due to the horizontal shear forces. In this case the performance function with four random variables is written in explicit form as follows

$$g_{3,ult}(A_v, f_y, P, s) = \frac{A_v f_y}{\sqrt{3}} - \frac{Ps}{2h_g} \quad (3)$$

Here, A_v is the cross-sectional area of the vertical, and s is the distance between the gravity centres of the verticals, i.e., the step length of the web opening; f_y, P and h_g are defined in Section 5.1.1.

5.2 Serviceability limit state

The performance function of the tested beams for the serviceability limit state can be written in terms of various deflection limits $\delta_{lim} = L/K$ and four random variables in implicit form as follows

$$g_{ser}(E, A_c, P, L) = \delta_{lim} - \delta(E, A_c, P, L) \quad (4)$$

Here, K is a constant varying from 150 to 400, E is the elasticity modulus and A_c is the cross-sectional area of the beam chord. The other two random variables, P and L, are defined in Section 5.1.1.

6. Reliability assessment of the beams and discussion

The design working life of the structural system is assumed to be 50 years; thus the reliability

Table 1 Statistical parameters of the random variables

Variable	Mean value	Standard deviation	Dimension	Distribution
E	210000	10500	MPa	Lognormal
f_y	250	20	MPa	Lognormal
A_c	1675	84	mm^2	Normal
A_v	1520	76	mm^2	Normal
s	600	30	mm	Normal
s_o	410	21	mm	Normal
L	4	0.2	m	Normal
P	115	11.5	kN	Normal

index β and the failure probability p_f which are related by the cumulative standard distribution function Φ by the equation $p_f = 1 - \Phi(\beta)$ are based on this reference period. In this study the target values of the reliability indices and the corresponding failure probability are assumed as $\beta_{tar,ult} = 3.8$ $(p_f = 7.0 \times 10^{-5})$ and $\beta_{tar,ser} = 1.5$ $(p_f = 6.68 \times 10^{-2})$ for ultimate and serviceability limit states, respectively (Eurocode 2002).

The statistical descriptions of the random variables in the performance functions, Eqs. (1)-(4) are given in Table 1.

6.1 Ultimate limit state

The FORM algorithm is applied to the three performance functions, Eqs. (1)-(3), in order to estimate the reliability indices $\beta_{1,ult}$, $\beta_{2,ult}$ and $\beta_{3,ult}$ for the ultimate limit states (Melchers 1999, Haldar and Mahadevan 2000).

The ultimate load-reliability index curves obtained for possible mechanisms of failure are plotted in Fig. 12. As seen in Fig. 12, the reliability indices for the plastic hinge mechanism in the chords of the beams are smaller than the others, and the plastic hinge mechanism in the chords is the most influential failure mechanism for the reliability index and the failure probability of the beams. It is noted that the beam specimens tested also failed in this mechanism mode during the tests. The

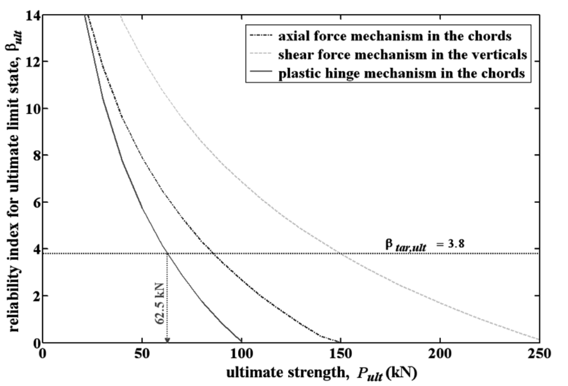

Fig. 12 Ultimate load-reliability index curves for possible failure mechanisms

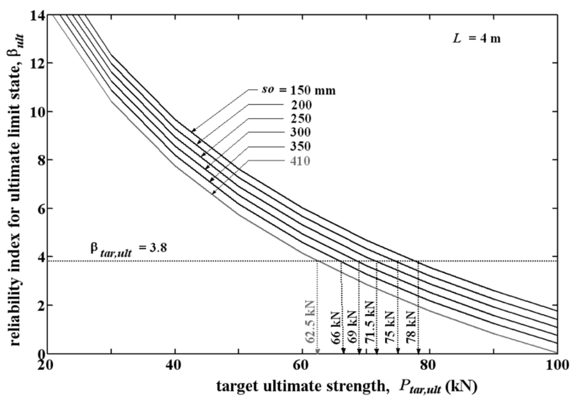

Fig. 13 Target ultimate load-reliability index curves at various widths of the web opening for the governed failure mode

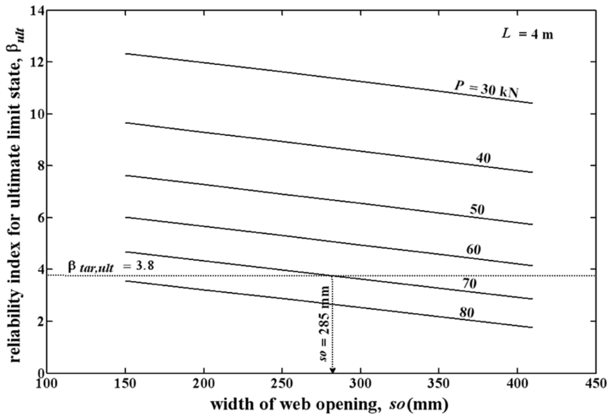

Fig. 14 Width of the web opening-reliability index curves for various beam loads

target value of the reliability index $\beta_{tar,ult}$ = 3.8 is also indicated in the same figure. It is seen that the target ultimate load of the beams $P_{tar,ser}$ is 62.5 kN. The beams are safe at loads of $P < 62.5$ kN.

On the other hand, for the various values of the web opening s_o, the target values of the ultimate loads $P_{tar,ult}$ of the beams are plotted in Fig. 13 for the plastic hinge mechanism in the chords. It is also seen that the target ultimate load of the beams $P_{tar,ult}$ is 62.5 kN.

For the various values of the beam load *P*, the influence of different widths of the web opening s_o on the reliability index of the beams is shown in Fig. 14. It is clearly seen that the beams are not safe for the width of the web opening $s_o > 285$ mm at loads of $P > 70$ kN.

6.2 Serviceability limit state

RSM and FORM (Haldar and Mahadevan 2000) are applied to the above serviceability performance function, Eq. (4), for each deflection limit δ_{lim} in order to estimate the reliability index β_{ser}.

The service load-reliability index curves obtained for various deflection limits of the beams *L/K*

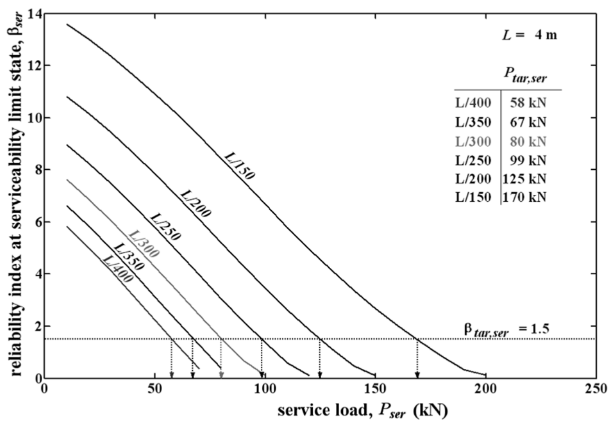

Fig. 15 Service load-reliability index curves for various deflection limits

where $K = 150\sim400$ are plotted in Fig. 15. The target value of the reliability index $\beta_{tar,ser} = 1.5$ is also indicated in this figure. As seen in Fig. 15, the target service load of the beams $P_{tar,ser}$ for a deflection limit of $L/300$ is obtained as 80 kN, and for $L/400$ as 58 kN.

6.3 Optimum design load

The optimum design load of the beams for both the ultimate and serviceability limit states are determined by using Figs. 12 and 15 together. The minimum value of $P_{tar,ult}$ and $P_{tar,ser}$ corresponds to the target value of the optimum design load, which is also the economical load of the beams. In this study, the target optimum design load of the beams with a deflection limit of $L/300$ is obtained as 62.5 kN, thus the ultimate load controls the target reliability of the beams. On the other hand, that with a deflection limit of $L/400$ is obtained as 58 kN, thus the service load controls the target reliability of the beams.

6.4 Sensitivity analysis

An important step in the structural reliability analysis is the sensitivity analysis of reliability indices. In the proposed algorithm the values of the sensitivity factors are measures of the sensitivity of the reliability index of the X_i random variables. These factors allow the ranking of the random variables according to their importance in the reliability analysis.

The sensitivity analysis is used to identify the influences of the random variables on the reliability of steel I-beams for the governed failure mode which is the plastic hinge mechanism in the beam chords. Fig. 16 gives the sensitivity factors a_i of the random variables. The figure shows that the sensitivity factors of both the random variables f_y and P have the greatest influence on the reliability index with $\alpha_{fy} = -0.6339$ and $\alpha_P = 0.6131$, which have approximately the same effects. Although the bending moments of the beam chord at the corners of the web opening are directly dependent on the width of the web opening s_o and the beam load P for this failure mode, the random variable s_o has the third greatest influence on the reliability index. It can be seen also from Fig. 16 that the effect of the width of the web opening s_o on the beam reliability remains unchanged, even if the loads increase. Among random variables, as expected, the random variable L has the least influence on the reliability index, with $\alpha_L = -0.1179$ for the governed failure mode.

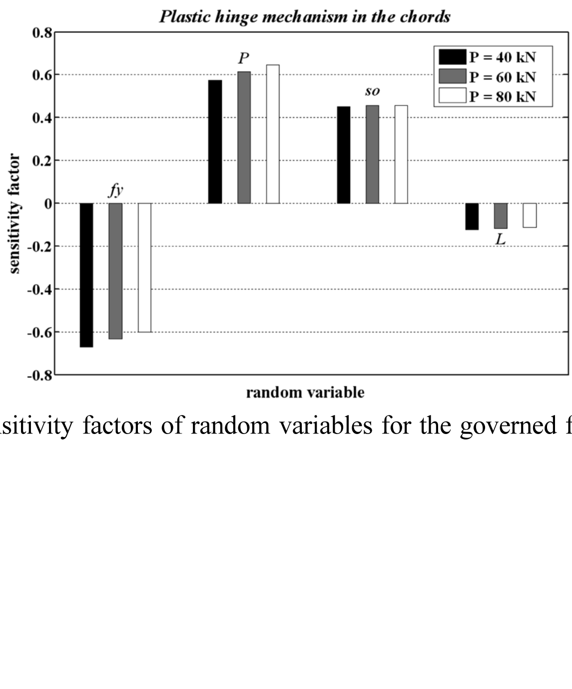

Fig. 16 Sensitivity factors of random variables for the governed failure mode

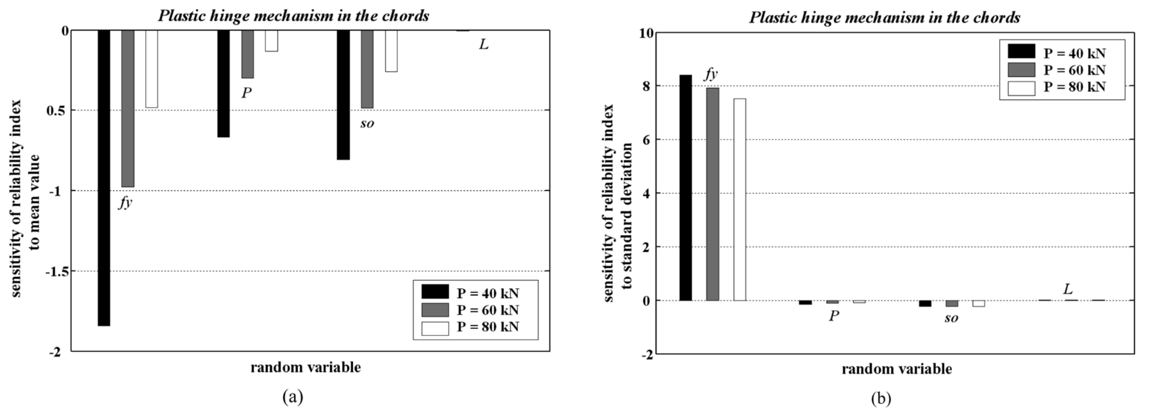

Fig. 17 Sensitivity of the reliability index to (a) the mean value and (b) the standard deviation of random variables for the governed failure mode

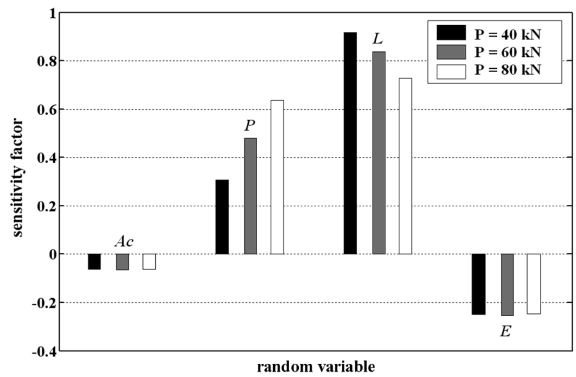

Fig. 18 Sensitivity factors of random variables for a deflection limit of L/300

Fig. 17 shows the sensitivities of the reliability index with regard to the mean value and to the standard deviation of the random variables for the governed failure mode. The highest sensitivity of the reliability index is obtained for the standard deviation of the yield stress f_y, which is extremely high in comparison with the reliability index sensitivities of the other random variables. Furthermore, it is noted that the reliability index sensitivities to the standard deviations of the other random variables have almost no influences on the beam reliability.

A high sensitivity of the reliability index is obtained for the mean value of the yield stress f_y. Furthermore, the sensitivities of the reliability index of the beam load P and the width of the web opening s_o are noticeable, and the reliability index sensitivity of the beam length L has almost no influence on the beam reliability. The obtained results point to the importance of the material quality of the steel beams.

Finally, sensitivity analysis is carried out for the serviceability performance function in Fig. 18. As expected, the sensitivity factor of the random variable L has the greatest influence on the reliability index with $\alpha_L = 0.8365$. It is noticed that the random variable P has the second greatest influence on the reliability index with $\alpha_P = 0.4807$. It can also be seen from Fig. 18 that as the loads increase, the sensitivity factor of the beam length L on the beam reliability reduces, and the sensitivity factor of the beam load P increases; the sensitivity factors of A_c and E do not display any significant variation. Among the random variables, the random variable A_c has the least influence on the reliability index with $\alpha_{AC} = -0.0643$.

7. Conclusions

In this paper, the reliability analysis of the tested full-size steel I-beams with rectangular web openings is investigated by using an algorithm, based on a combination of the common probabilistic reliability methods, such as RSM, FORM and SORM. The performance functions written in implicit and explicit forms are employed. Based on the statistical data obtained from the test results, the reliability index values of the beams β are calculated for the ultimate and serviceability limit states. In the numerical study, the target values of the reliability indices are assumed as $\beta_{tar,ult} = 3.8$ and $\beta_{tar,ser} = 1.5$ for the ultimate and serviceability limit states, respectively.

The ultimate load-reliability index curves are plotted for three possible failure mechanisms of the beam which are plastic hinge mechanism in the chords, axial force mechanism in the chords and shear force mechanism in the verticals. The reliability indices for the plastic hinge mechanisms in the chords are smaller than the others, thus this mechanism is the most influential failure mechanism for the reliability index and the failure probability of the beams. The target ultimate load $P_{tar,ult}$ is determined by intersecting $\beta_{tar,ult}$ line and the curve of the plastic hinge mechanism. Choosing the beam load P as a parameter, the influence of different widths of web opening s_o on the beam reliability is also studied. On the other hand, the target service load of the beams $P_{tar,ser}$ is obtained for various deflection limits by plotting the service load-reliability index curves. The optimum design load P_{opt} is the target ultimate load for the beams with a deflection limit of $L/300$, and is the target service load for the beams with a deflection limit of $L/400$. In the first case the target reliability of the beams is controlled by the ultimate load, and in the second case is by the service load.

In the sensitivity analysis performed for the governed mode which is the plastic hinge mechanism in the beam chords, the beam load P and the yield stress f_y have the greatest influence on the reliability index, whereas the width of the web opening s_o has the second greatest influence and the beam length L has the least influence on the beam reliability. In the sensitivity analysis performed for the serviceability limit state, the beam length L has the greatest influence on the reliability index, whereas the beam load P has the second greatest influence and the elasticity modulus E has the third greatest influence on the reliability of the beams.

Although the proposed algorithm is illustrated for the specific structural application, it can be applied other type of the structural design problems in order to estimate the structural target reliability, provided that the performance functions are defined and the statistical values of the random variables are known.

Acknowledgements

The author wishes to thank Prof. Dr. Y. Alpay Ozgen for his kind interest and comments on this paper.

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