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Strain distribution between CFRP strip and concrete at strengthened RC beam against shear

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Abstract. In recent years, CFRP material usage in strengthening applications gradually became widespread. Especially, the studies on the strengthening of shear deficient reinforced concrete beams with CFRP strips are chosen as a subject to numerous experimental studies and research on this subject are increased rapidly. The most important variable, that is affected on the failure mode of CFRP strips and that is needed for determining the shear capacity of the strengthened reinforced concrete beams, is the strain distribution between CFRP strips and concrete. Numerous experimental studies are encountered in the literature about the determination of strain distribution between CFRP strips and concrete. However, these studies mainly focused on the CFRP strips under axial tension. There are very limited numbers of experimental and analytic studies examining the strain distribution between concrete and CFRP strips, which are under combined stresses due to the effects of shear force and bending moment. For this reason, existing experimental study in the literature is used as model for ANSYS finite element software. Nonlinear finite element analysis of RC beams strengthened against shear with CFRP strips under reverse cyclic loading is performed. The strain distributions between CFRP strips and concrete that is obtained from finite element analysis are compared with the results of experimental measurements. It is seen that the experimental results are consisted with the results derived from the finite element analysis and important findings on the strain distribution profile are reached by obtaining strain values of many points using finite element method.

Keywords: CFRP; Nonlinear Finite Element Analysis; ANSYS

1. Introduction

CFRP is being commonly used for strengthening of RC structures in the last decade because of advantages as being light in weight, ease of application, resistance to corrosion and environmental conditions, firm geometrical sizes of the members, and being available in the market with many different kinds and sizes which are convenient for different types of strengthening details (Teng *et al.* 2002).

With the widespread experimental studies on strengthening of reinforced concrete with CFRP, the analytical models have been spread out as well. Behavior, performance and developed analytical models are closely related to the failure modes of the structures that have been strengthened with CFRP. When the experimental studies in the literature are surveyed, three modes of failure can be

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seen in the RC structures strengthened with CFRP (Chen and Teng 2001, Smith and Teng 2002a, b, 2003, Oehlers and Moran 1990, Teng *et al.* 2003, Momamed Ali *et al.* 2001, 2002, Oehlers 2003). The observed failures are; debonding of concrete from the CFRP, epoxy failure due to exceeding shear strength of adhesive concrete interface and rupture of the CFRP. Since CFRP has very high axial tension strength, CFRP rupture is a very rare case. In most cases, CFRP debonds from concrete surface or adhesive concrete interface reaches its shear stress capacity and fails before CFRP rupture. The most common mode of failure is the debonding of CFRP from concrete, just a couple of millimeters below the surface. The strength and the stiffness of the strengthened specimen depend on the failure mode. If the structure fails due to debonding of CFRP from concrete, the strain distribution between CFRP and concrete must be known to calculate the capacity of the strengthened structure. Hence, the studies of determining the strain between CFRP and concrete became popular in the recent years (Baran and Anil 2010, Teng *et al.* 2002, Chen and Teng 2001, Smith and Teng 2002a, b, 2003, Teng *et al.* 2003, Chen and Teng 2003, Yao *et al.* 2005). In all these studies, some knowledge about the strain distribution is necessary for calculating the capacity of the structure.

In all of the above mentioned studies, the strain distribution between the CFRP strips under axial tension and the concrete surface is being investigated experimentally or analytically. Among these studies, some studies include finite element modeling which examine various crack models. H.B. Pham, R. Al-Mahaidi, and V. Saouma examined the adherence between CFRP and concrete by applying single pull test to the specimens in their study. By using synthesis of smeared and discrete crack models, they modeled 12 specimens. It is observed that the finite element model gave compatible results considering the bearing capacity, the failure of the crack model and CFRP strain distribution (Pham *et al.* 2006). X.Z. Lu, L.P. Ye, J.G. Teng, and J.J. Jiang used finite element modeling to verify the bond-slip model that is obtained from the results of single pull test. The strain distributions are used for the examination of the crack distribution between CFRP and the boundary layer in the concrete interface. By comparing with the experimental study, they verified the correctness of finite element modeling and thus they examined the rupture mechanisms as well (Lu *et al.* 2005a, b).

In the literature review, no studies are found about the examination of the strain distribution between the CFRP strips, which are used for strengthening the RC beams against shear, and the concrete surface. For this reason, a finite element model is created by using the results of an experimental study conducted by Anil (2008). T-sectioned RC beams, strengthened against shear by using CFRP strips, are modeled nonlinearly by ANSYS finite element software. The model is verified by comparing the results of the finite element analysis with the experiment results. Then, strain measurements taken from CFRP strips during the experiments and strain distributions obtained from the finite element analysis are compared and comments are made about this subject. The compatibility of the strain distribution obtained from the finite element analysis and the real experimental behavior is examined.

2. The experimental study which forms a foundation for the finite element model

A total of 5 T-cross sectioned beams were tested under reversed cyclic loading in experimental program. The geometrical details and reinforcement schemes of the test specimens are given in Fig. 1. Cross sections and longitudinal reinforcement of all specimens are identical.

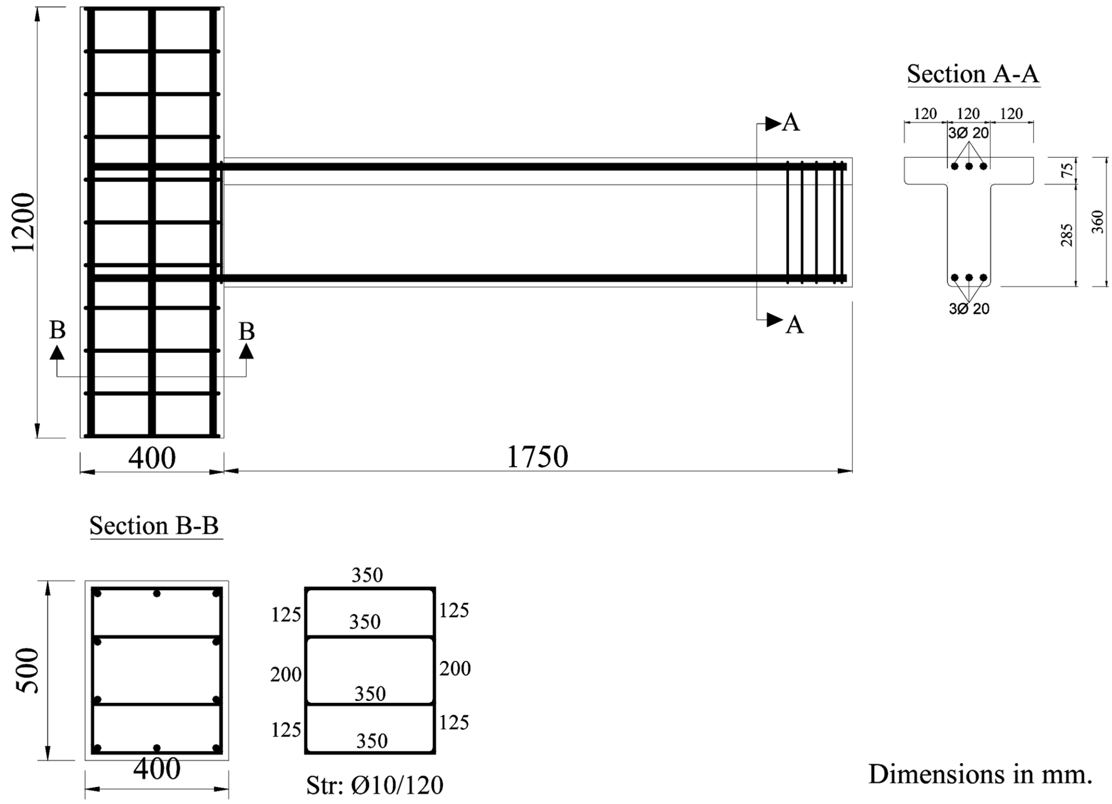


Fig. 1 Test specimen reinforcement and geometric dimension details (Anil 2008)

Table 1 Properties of test specimens

Test Specimen #	a/d	f_c (MPa)	Properties of CFRP strips		
			w_f Width (mm)	s_f* Spacing (mm)	Bonding form
Beam-1	5.0	15.0	-	-	-
Beam-2	5.0	14.0	50	80	*U* type
Beam-3	5.0	14.5	100	130	*U* type
Beam-4	5.0	14.8	1750	-	Only two side faces
Beam-5	5.0	14.2	1750	-	*U* type

*The distance between two consecutive CFRP axes. (Ref. Fig. 2)

The properties of the test specimens are summarized in Table 1. Mean standard cylinder compressive strength values of concrete for the test beams are determined. All compressive strength values of the test specimens are below 15 MPa as seen in Table 1. The test specimens have low concrete compression strength intentionally.

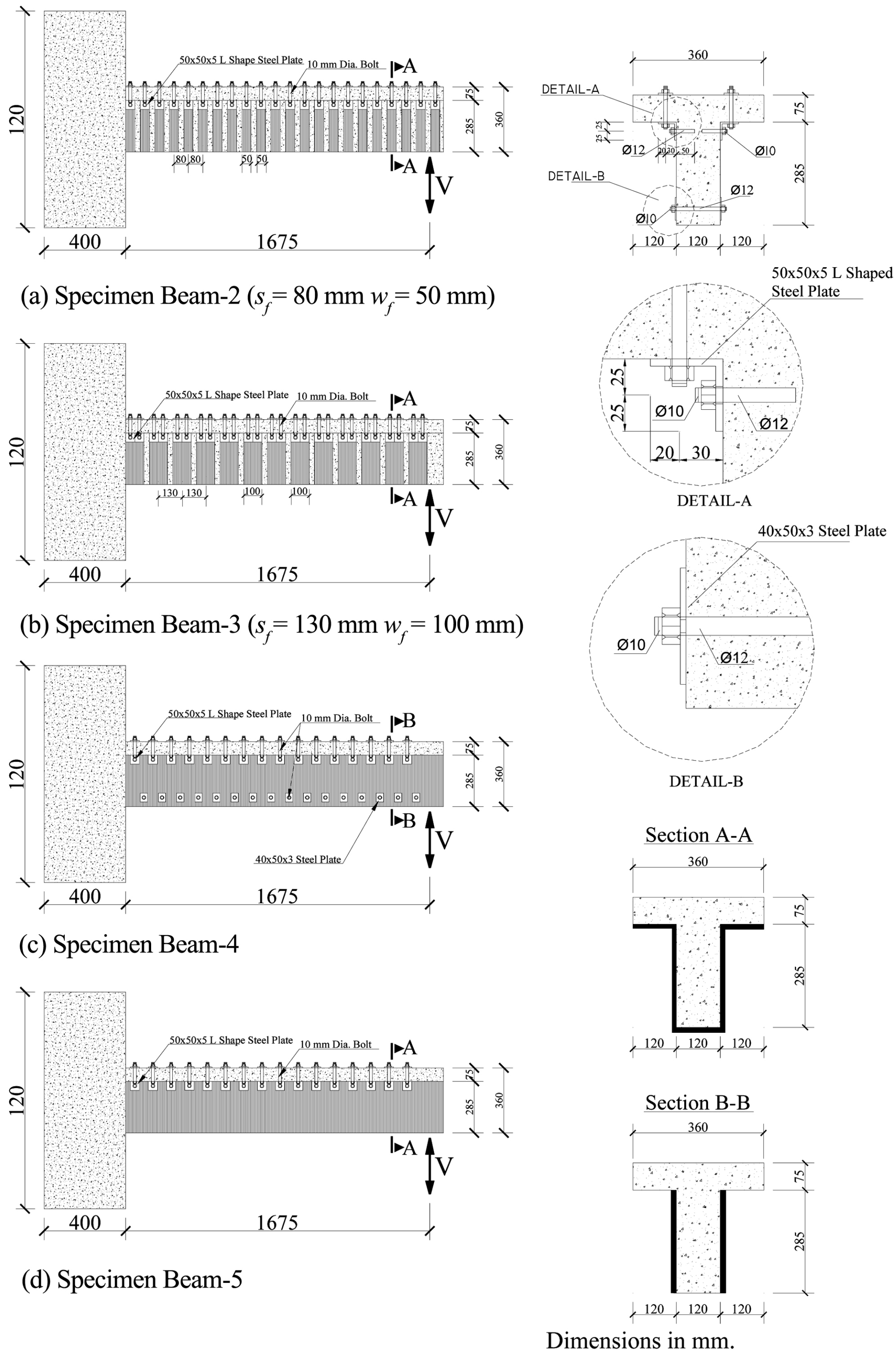


Fig. 2 The strengthening details applied to the specimens (Anil 2008)

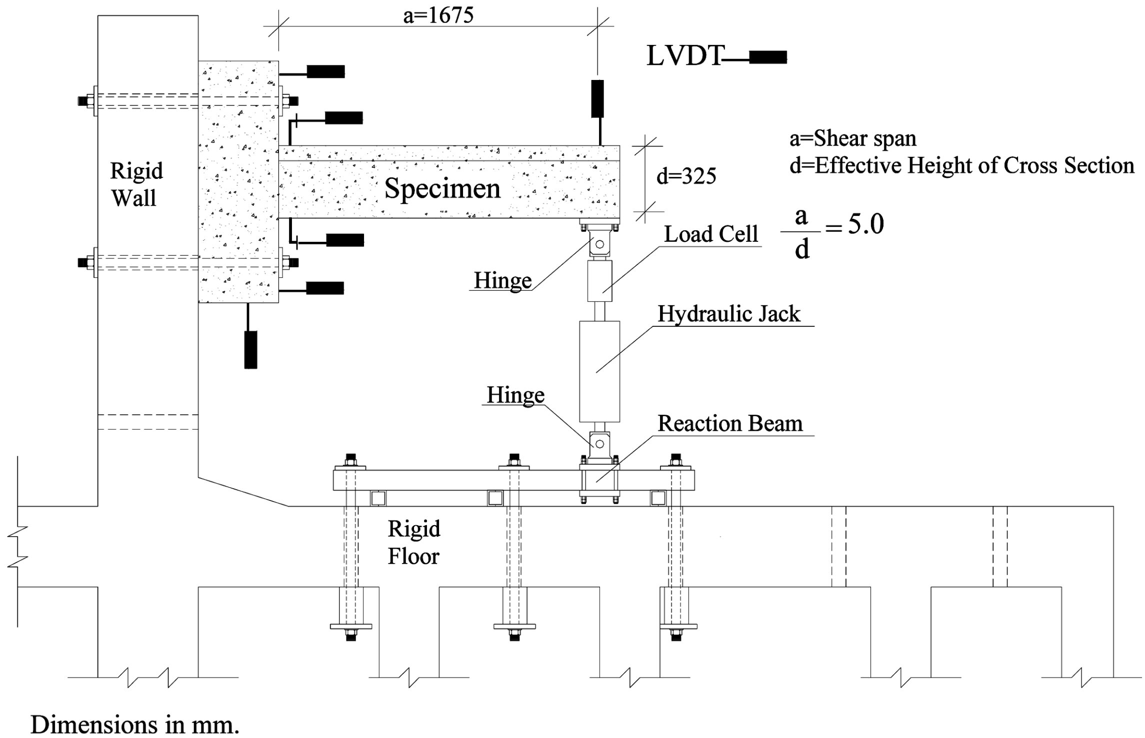


Fig. 3 Test setup and instrumentation (Anil 2008)

Beam-1 is a control specimen and it has no strengthening. The beams without shear reinforcement are strengthened with CFRP strips that are applied along to shear span of the beam. Beam-2 and Beam-3 specimens are strengthened with CFRP strips and Beam-4 and Beam-5 specimens are strengthened with CFRP plates in various arrangements. The strengthening details of the specimens are given in Fig. 2. The experimental setup and the schematic view of the measurement equipments are given in Fig. 3. Test specimens are fixed to the rigid wall in the laboratory by using high strength steel bolts with a radius of 45 mm, which can be considered as fixed. Reverse cyclic loading is applied to the specimens with a loading column, which has hinged supports at both ends mounted between the rigid slab and the specimen. For all specimens, the ratio between the shear span length ($a = 1675$ mm) and the effective height ($d = 335$ mm) is $a/d = 5.0$. The displacement values at the end-point of the beam and the curvature value at the maximum moment value region of the beam are all recorded by LVDT's. The strain values on the CFRP strips are also recorded electronically by using strain gauges. The location of the strain values measurements from the CFRP is along the shear span of the beam and is 300-1000 mm away from the rigid support, approximately. Strain is measured in the main fiber direction of the CFRP. Maximum strain-load curves obtained from the measurements during tests can be seen in Fig. 4. The location, where strain measurements are taken, and the strip, where maximum strain is measured, are indicated on Fig. 4. Detailed information about the experimental results can be found in the reference (Anil 2008).

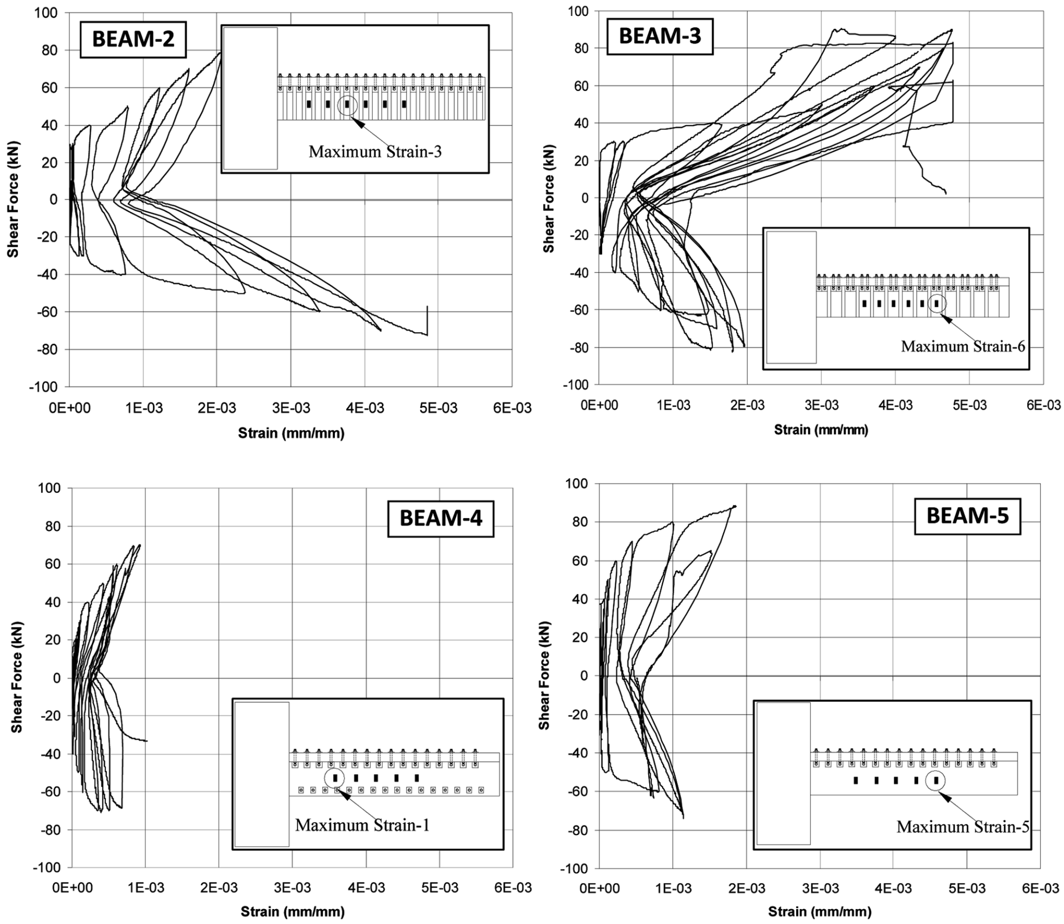


Fig. 4 Load-maximum CFRP strain curves of test specimens (Anil 2008)

3. Modeling studies

Finite element software ANSYS is chosen to form the model. ANSYS finite element software had been developed by Swanson Analysis System Company in 1971. Until then the software has been updated many times up to nowadays. The software is widely used and approved for engineering applications and academic studies. The software ANSYS has many element types, which can compute nonlinear behaviors of materials. Besides a component named "Solid65 element type" has been annexed to the body of the program that considers the nonlinear behavior of a concrete element. Those features are the reasons why ANSYS finite element software is used to model the beams strengthened with CFRP in this study. The steps of analysis conducted in ANSYS Finite Element Software are given in Fig. 5.

It is possible to model systems with complex geometry as group of elements with simple geometry by finite element method. Similarly, in this study, beams are modeled by gathering many volumes together to form the model instead of modeling it as a single member. By doing this, the suitable layout of the CFRP strips for experimental study is achieved; and by modeling the

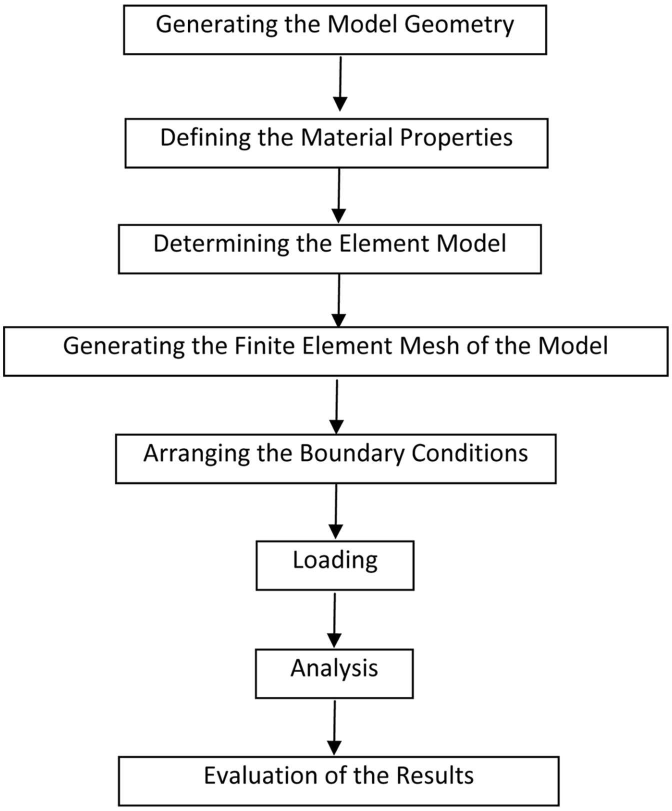


Fig. 5 Steps of analysis in ANSYS

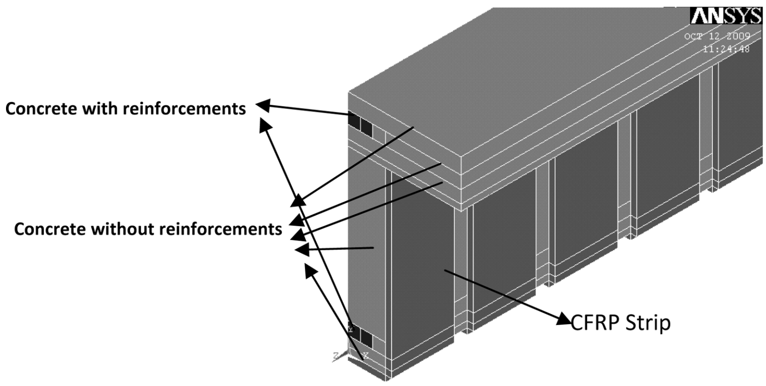


Fig. 6 Beam-3; Model volumes used for the specimen

reinforcement as a separate volume, concrete cover is being considered as well. Option of defining the reinforcements as a volumetric ratio in concrete mass is selected (smeared modeling). In order to decrease the amount of operations, only half of beam is modeled by taking advantage of the symmetry property of T-section beams. As an example, the finite element mesh of test Specimen 3 can be seen in Fig. 6.

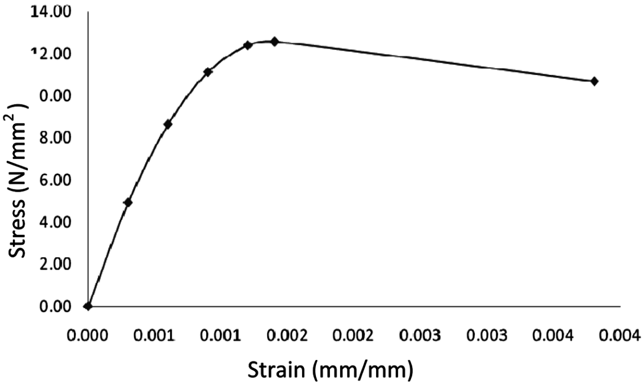


Fig. 7 The material model used for concrete in the finite element model (Hognestad model)

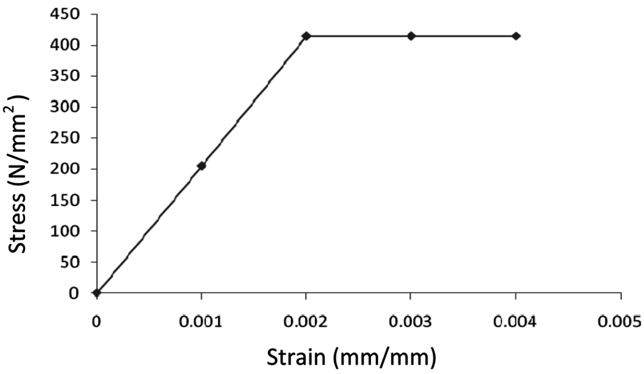


Fig. 8 The material model used for reinforcement in the finite element model

Concrete is a heterogeneous construction material whose properties may be affected by lots of parameters. Being a nonlinear material with variations in behavior depending on time and impact load, it is not possible to model the concrete depending on only one parameter. Thus, the behavior of concrete beyond the linear region and in failure zone is considered in the study. For the modeling of the concrete, which is prepared without confinement reinforcement, Hognestad concrete model is used (Fig. 7) (Ersoy 2000). William-Warnke failure zone criterion is considered for the behavior of concrete under multi-axial stresses (William and Warnke 1974). In addition, since the behavior of concrete will change under impact load due to increase in cracks, the modeling of closed cracks depending on shear transfer is done. Hognestad concrete model and William-Warnke failure criterion with closed crack behavior are transferred to the model by using the parameters available in ANSYS.

For the steel of the reinforcement and the loading plate, linear-elastic material property is defined; also yield strength values and Poisson ratios are taken into consideration. Although CFRP is an anisotropic material, in formation of the model, it is accepted as an isotropic material and it is modeled as linear-elastic material. The stress-strain relationship for the reinforcement can be seen in Fig. 8, while the stress-strain relationship for the CFRP strips can be seen in Fig. 9.

In most of the programs that perform analysis based on finite element method, when a specific

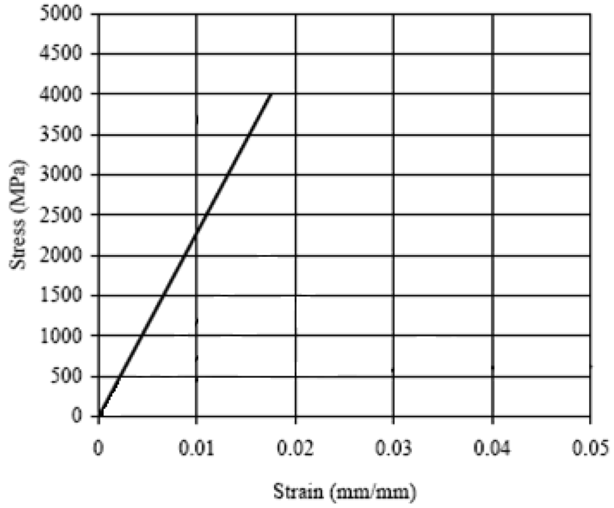
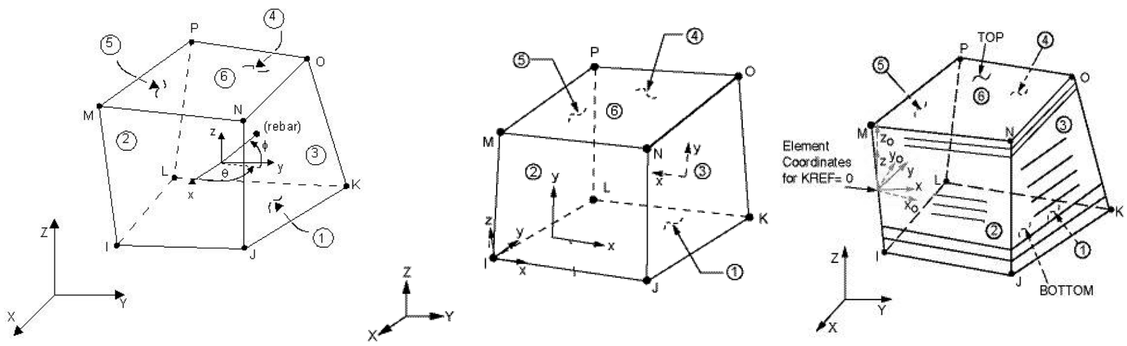


Fig. 9 The material model used for CFRP in the finite element model



(a) Concrete (Solid65) (b) Steel Plate (Solid45) (c) CFRP (Solid46)

Fig. 10 Element types used for the finite element model

ratio of dimensions between elements linked to each other are exceeded, convergence problems can be seen in analysis. Thus, the dimensions of CFRP strips are modeled larger than real dimensions but, the modulus of elasticity values are decreased at a rate corresponding to the increase in the dimensions.

Selecting the most suitable model, formed by using finite element method, is important for structural member or system behavior. In the model, concrete beams, CFRP strips and loading plates are defined with separate elements suitable to the material properties and behaviors. For concrete Solid65, for loading plate Solid45 and for CFRP strips Solid46 elements are used. The fig.s and degrees of freedom are given in Fig. 10.

Solid65 element is the only element developed for modeling of concrete in the ANSYS elements library. Solid65 element is plastically deformable and it has cracking, crushing and creep properties. In this study, Solid65, which has the ability to model the concrete reinforced or unreinforced, is used in three different types taking the amount and location of the reinforcement of the beam into consideration. The location of reinforcement is indicated angular and defined with volumetric ratio by Solid65 element. Although Solid45 element does not involve cracking and crushing properties like Solid65 element, it is an element with creep and shrinkage behaviors. Solid46 is an element used for modeling the materials with thick covers or layered materials. In this study, it is assumed that the CFRP strips are single layered and modeled as so.

After the geometric modeling of the beams and element selections are completed, one of the most important points is the formation of the mesh model. Depending on the properties of the employed element, mesh formation is separated into two as free mesh formation, in which the dimensions of the elements are randomly selected, and map mesh formation, in which the dimensions of the elements are either same as or similar to each other. For usage of Solid65 element, only map mesh formation can be formed and therefore, map mesh is formed in the modeling studies. One of the most delicate points in mesh formation is the determination of mesh fineness. Since mesh fineness being at the optimum level is influential on the duration and results of analysis, various model meshes are created by using elements with different thicknesses in the modeling study. Since using excessive amount of elements has a negative effect on the duration of analysis, the thickness of elements are determined so that the number of elements are at an optimum level.

After formation of model mesh is completed, one end of the beams is fixed by the nodal points for convenience of experimental study. Besides, due to half modeling, the nodal points at the symmetry axis of the beams are rearranged in a manner that the boundary conditions of symmetry are satisfied. For convenience of experimental study, loading is applied to the beams gradually from many points instead of applying from one point at the free end of the beams.

In order for modeling studies to reach their goals, they should reflect the behavior of the structure in the most appropriate way. In reinforced concrete members under increasing loading, when the elastic limit is exceeded, irreversible plastic deformations occurs, and therefore, the behavior of members differs from linearity. Besides, due to the elastic limit being exceeded, the stiffness of the member differs from the initial stiffness and starts to decrease. In cases, where sensitive analyses are required, using the initial mechanical properties of the members during the whole analysis process cannot be possible because of the previously mentioned reasons. Hence, modeling a reinforced concrete member with one of the methods that uses nonlinear analysis is essential, because better and more accurate results about the behavior of the member can be achieved.

Because of the abovementioned reasons, the analyses of the beams are done by using nonlinear methods. ANSYS uses Newton-Raphson method for the nonlinear analysis. Newton-Raphson, which is one of the sequential approaches, depends on the principal of application of loading step-by-step. The method is divided into three as, the full Newton-Raphson, in which the stiffness matrices are updated after all iterations or at every sub-step, the modified Newton-Raphson, and the initial stiffness. In analyses, besides the number of steps and sub-steps that will be used in the analyses, the Newton-Raphson method and the convergence criterion can also be predefined by the user. The convergence criterion is determined by the assumptions on both the displacement, and the stress and moment values separately. The analyses continue until the convergence criterion is achieved.

At the first analyses in the scope of this study, the number of steps and sub-steps is taken small intentionally, in order to prevent the duration of the analyses from being long. However, in the analyses performed this way, it is observed that the RC beams had linear behavior until the failure point. Therefore, in the subsequent analyses, the applied loading steps are decreased gradually by increasing the number of steps; by this way, nonlinear behavior can be observed in the beams as well.

4. Comparison of the results of the experiment and finite element analysis

The locations of the strain measurements taken from the test specimens Beam-2, 3, 4, and 5 and the graphics of the measured maximum strain values are given in Fig. 4. Totally, six strain measurements are taken from Beam 2 and 3, while 5 strain measurements are taken from Beam 4 and 5. The measurements are taken from the exact mid-point between the CFRP strip, which is used for strengthening, and the beam web height along the direction of the fibers of the load carrying plates. The result of the performed finite element analysis showed that the strain between the adhered CFRP strips and the concrete surface is not obtained from only one point instead; strain values are obtained from many points depending on the fineness of the mesh of finite element model. Thus, by using the finite element analysis, information is gathered about the strain profiles of the CFRP strengthening members, which are adhered to the concrete surface. Firstly, the comparison of the strain values, which are measured from the CFRP strips and plates, and the

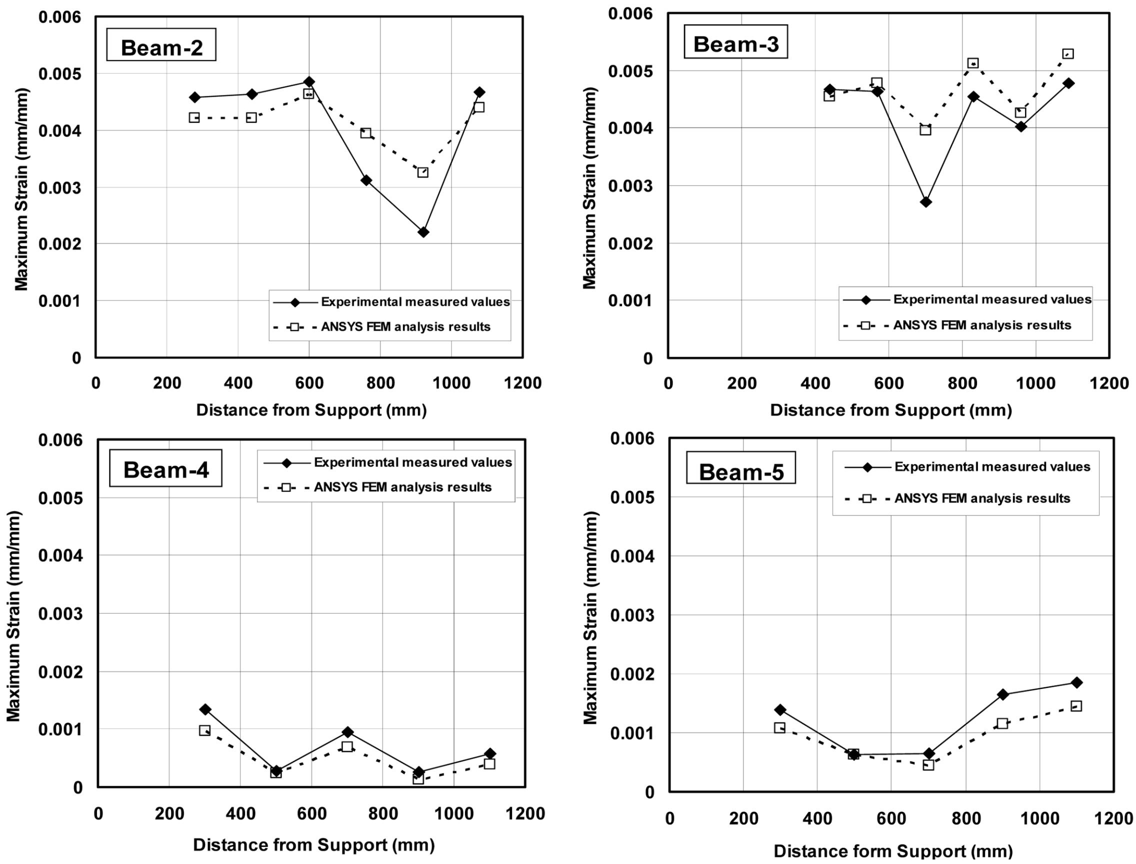


Fig. 11 Comparison of the measured and the calculated maximum strain along to Beam

results obtained from finite element analysis is done. Throughout the beam, the graphics at Fig. 11 showing the distribution of the maximum measured strain values of test specimens and the maximum strain values obtained from the analysis for the mid-point of the beam web height.

When the graphics given in Fig. 11 are examined, it is found that the measured maximum strain values and the maximum strain values obtained from the finite element analysis are matching to each other. The difference between the calculated and the measured values for members Beam-2 and Beam-3, which are strengthened with CFRP strips, are very little as the measured values are obtained averagely 6.21% smaller than the calculated values. For the test specimens Beam-4 and Beam-5, which are strengthened with CFRP plates, the measured values are obtained averagely 35.62% bigger than the calculated values. However, the general behavior of the measured strain values throughout the beam axis for all test specimens are compatible with the profile obtained from the finite element analysis. It is thought that the reasons for the larger difference between the measured values and the analysis results for the CFRP plates are the complexity of the strain distribution under the plates and the effects of shear and flexural cracks on this distribution. Although the model in ANSYS finite element program reflects the real behavior of the cracks in concrete, when CFRP is glued to the concrete surface by epoxy, not all the cracks that occurs in real case observed during the experimental study, could be obtained in finite element model. This finding becomes more prominent especially for the case in which the CFRP plate covers the entire surface of the RC beam. More cracks occurred in the experiments, where the CFRP plates are used for strengthening, compared to the finite element model and this caused the experimentally measured strain values to be bigger than the results obtained from the analysis. The CFRP material glued as a plate to the surface of the beam caused the RC beam to be stiffer in the finite element model compared to the experimental study and caused less cracks to occur. For the test specimens strengthened with CFRP strips, the crack distribution obtained from test specimens and the distribution obtained from the finite element analysis are more compatible with each other. The strain values of the test specimens strengthened by the CFRP strips obtained from the analysis and the experimental values are very close to each other because of the compatibility of the cracks and the strain distributions in the CFRP plates being independent of each other.

Experimental information could not be attained about the strain distribution through the beam web height, because experimentally, the only measurement taken from the CFRP strips and plates is obtained from the mid-point of the beam web height. These single measurements taken from the mid-point of the beam web height and the results of the FEM analysis are compared and it is seen that they are compatible. This finding showed that the strain distribution obtained from the results of the FEM analysis and the experimental results can be compatible, and used in verifying the correctness of the results of the analysis. The strain distributions, obtained by FEM analysis, of two of the selected test specimens, strengthened with CFRP strips and plates, are given in Fig. 12 for Beam-2 and in Fig. 13 for Beam-5. The damage and crack distribution of the test specimens are indicated on the figures and the strain distribution obtained by the FEM analysis are given as graphics at the point where the maximum CFRP strain value measured experimentally. At the location where the crack passes, the strain distribution shows concentration like the corner points and strain value increases to create concentration at that point. Also again, similar to the cracking edge, at the bottom corner of the beam, the strain value reaches to peak and a concentration point is created. Furthermore, CFRP strip is ruptured at the corner point where the strain reaches to maximum value. It is thought that the strain value at the point of rupture is smaller than the value anticipated by the producer firms for the CFRP material, because the applied stresses are not only

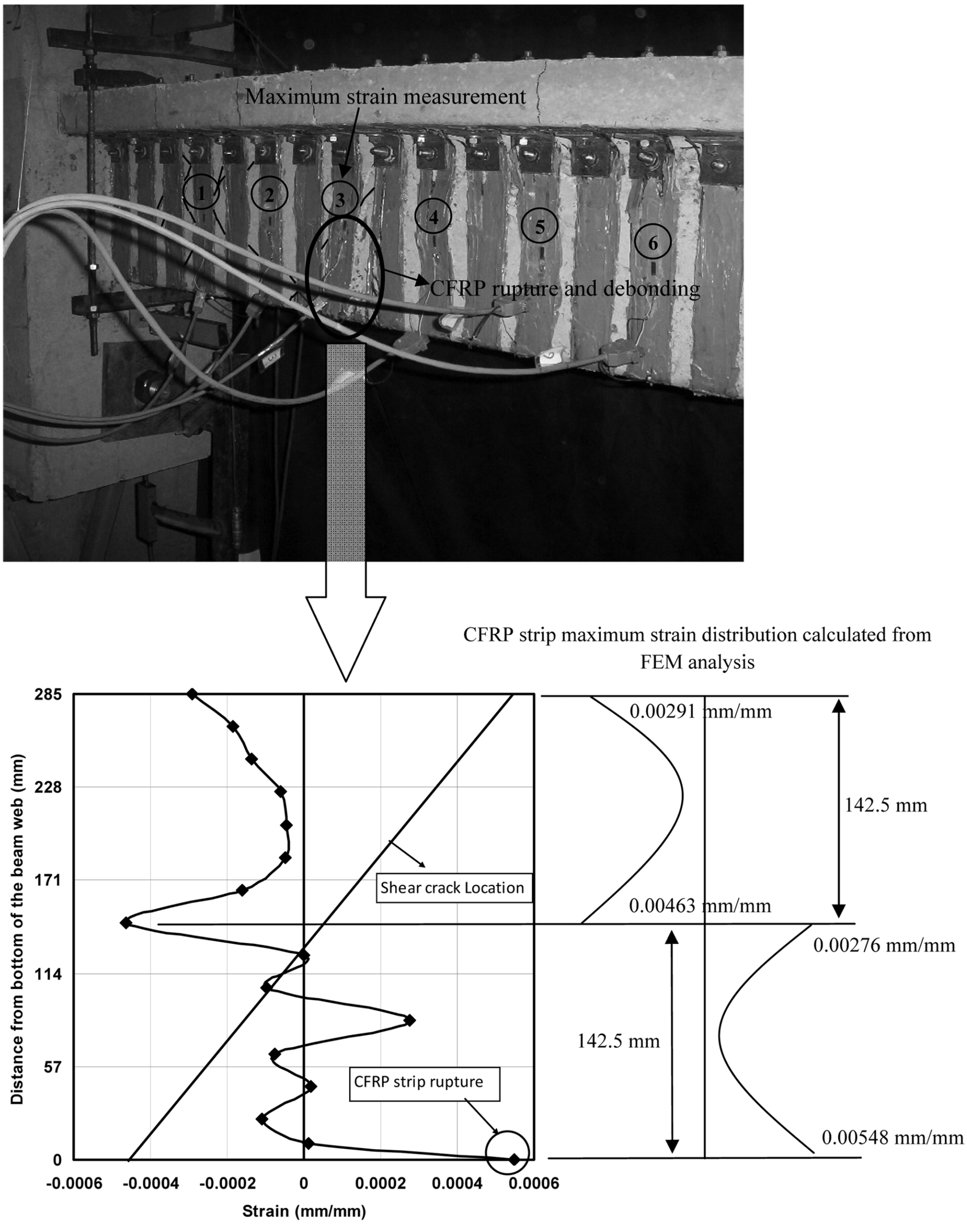


Fig. 12 Beam 2 maximum CFRP strain distribution calculated from FEM analysis

axial tension and the strip experiences also flexure.

The strain distribution obtained from the FEM analysis of the test specimen Beam-5 that is strengthened with CFRP plates, since no shear cracks formed under the CFRP plate, occurred only at the bottom edge of the web of beam. At this point the plate is under flexure and the maximum

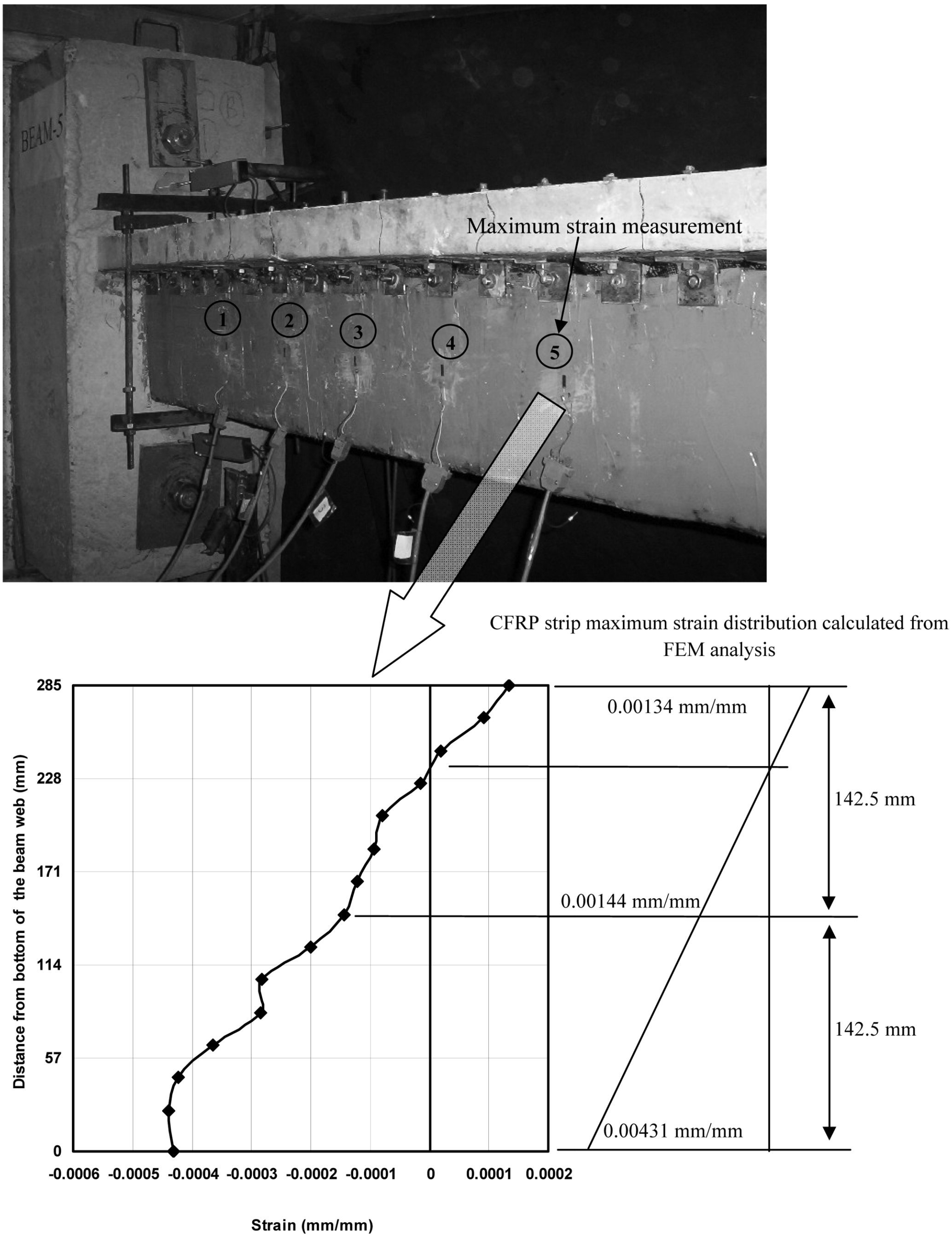


Fig. 13 Beam 5 maximum CFRP strain distribution calculated from FEM analysis

strain value is reached. In the FEM analysis, the CFRP plate avoided the formation of shear cracks and made the beam stiffer. For this reason, the obtained strain values are smaller than the strain values of the test specimens strengthened with CFRP strips. However, as mentioned before at the part where the maximum strain values are compared, for the test specimens strengthened with the CFRP plates, the difference between the measured strain values and the values obtained from analysis is greater. Although the crack model used by the finite element software is successful in modeling the effects of cracks under crushing and tension, it is thought that not all the cracks occurring in real cases are acquired by the model when CFRP material with high tensile strength is bonded to the concrete surface by epoxy. Hence, for the test specimens strengthened with CFRP plates, the strain values obtained from the experiments are smaller than that of the analysis. The authors think that constituting a new crack model, which includes the effects of epoxy and CFRP, within the body of ANSYS finite element program will be useful. A behavior, closer to the real case, can be achieved with a material model that will be defined for epoxy and contact parameters defined between CFRP and the concrete surface. However, it is thought that defining contact surface will increase the duration of analysis by making the model more complex and may also cause convergence problems.

5. Conclusions

Within the scope of the studies conducted, nonlinear analysis of the T-sectioned RC beams strengthened with CFRP strips and plates are executed by using ANSYS finite element analysis program. With the results of the analysis, the strain distribution between the RC beam surface and CFRP members is examined. For verification of the correctness of the finite element model, the results of an experimental study conducted by Anil (2008) are used. The results of the study are summarized below;

- Determination of the strain distribution between the concrete surface and CFRP members, used for strengthening against shear, is extremely important for calculation of the contribution of the details of CFRP and consequently, for calculation of the capacity of the strengthened structural members or systems. The failure mode of the CFRP member used for strengthening and determination of its capacity can be made with the strain distribution, which is determined in a manner that the behavior is the closest behavior to the real case; and by this way, the calculation of the capacity can be compatible with the real behavior.
- It is observed that the strain distribution between the CFRP members and the concrete surface for the RC beams, strengthened against shear by using the CFRP strips and plates, is different and more complex than the distribution occurring under the effects of axial forces only. Changes in strain distribution occur at lots of points such as the points where the shear and flexural cracks occurs beneath CFRP members, the inflections caused by CFRP members at the corners or the point that the member ends; and strain accumulation occurs at these points.
- It is found that the maximum strain values of CFRP strips and plates used for strengthening against shear, obtained from the nonlinear analysis performed by using ANSYS finite element program, are compatible with the experimental results and considerably close to the real case. Especially, in all of the performed analysis, the distributions of the calculated and measured maximum strain values are very well matching with each other. For CFRP strips, the measured values are obtained averagely 6.21% smaller than the calculated values. For the test specimens strengthened with CFRP plates, the measured values are obtained averagely 35.62% bigger than the calculated values.
- It is thought that the reasons for the larger difference between the measured values and the

analysis results for the CFRP plates are the complexity of the strain distribution under the plates and the effects of shear and flexural cracks on this distribution.

• Although the crack model used by the finite element software is successful in modeling the effects of cracks under crushing and tension, it is examined that not all the cracks occurring in real cases are acquired by the model when CFRP material with high tensile strength is bonded to the concrete surface by epoxy. This finding becomes more prominent especially for the case in which the CFRP plate covers the entire surface of the RC beam. In the experiments in which CFRP plates are used for strengthening, a lot more cracks occurred compared to finite element model and caused the experimentally measured strain values be greater than the strain values obtained from the analysis. The CFRP plate bonded to the surface made the RC beam stiffer than the experimental study and caused less cracks to occur.

• When the strain distributions of the CFRP strips are examined, it can be seen that the shear crack passing under the CFRP strip and the CFRP strip inflection point at the bottom corner of the beam web, the strain distribution affected significantly. At the location where the crack passes, the strain distribution shows concentration like the corner point and strain value increases to create concentration at that point. Similar to the cracking edge, at the bottom corner of the beam where the CFRP strip makes inflection, the strain value reaches to peak and a concentration point is created.

• Although the crack model used in finite element software reflects the real behavior of the cracks in concrete occurring due to crushing and tension, it is thought that the model causes fewer cracks to occur than the real case when CFRP material with high tensile strength is bonded to the concrete surface by epoxy. For this reason, the strain values obtained from the analysis are smaller than the experimental results for the test specimens strengthened with CFRP plates. The authors think that constituting a new crack model, which includes the effects of epoxy and CFRP, within the body of ANSYS finite element program will be useful.

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