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Dynamic response of pile groups in series and parallel configuration

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Abstract. Basic problem of pile foundation is three dimensional in nature. Three dimensional finite element formulation is employed for the analysis of pile groups. Pile, pile-cap and soil are modeled using 20 node element, whereas interface between pile or pile cap and soil is modeled using 16 node surface element. A parametric study is carried out to consider the effect of pile spacing, number of piles, arrangement of pile and soil modulus on the response of pile group. Results indicate that the response of pile group is dependent on these parameters.

Keywords: pile; pile-cap; spacing; series arrangement; parallel arrangement

1. Introduction

Pile foundations involve three dimensional interaction between pile-cap, soil and piles. In the past, the problem has often been solved by making assumptions regarding geometry and material properties. However, to account for the realistic nature of the problem, it is necessary to allow for three dimensional geometry, interface effects and soil properties. Lateral loading is an aspect of the problem that introduces additional complexity and is encountered in offshore foundations as well as other applications. Structures like offshore platforms are subjected to lateral loading due to wind and wave action indicating dynamic nature of load.

Recent devastating earthquakes have shown that the collapse of many buildings was due to the failure of supporting pile foundation. Substantial research has been carried out on the dynamic analysis of single piles in the frequency domain. Novak *et al*. (1978) presented the analysis based on plane strain conditions in the frequency domain. Gazetas (1984) reported seismic response of end-bearing single piles in the frequency domain. Analysis is extended to consider pile-soil-pile interaction in the frequency domain (Makris and Gazetas 1992). A few researchers have performed time domain analyses. Matlock *et al*. (1978) developed a unit load transfer curve approach, also known as p-y curves, for the time domain nonlinear analysis. Using one-dimensional finite element formulation Badoni and Makris (1996) considered macroscopic model that consists of distributed hysteretic springs and frequency dependent dashpots to model the lateral soil reaction and presented the nonlinear response of single piles under dynamic lateral loads.

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Nogami and Konagai (1986, 1988) have developed a time domain analysis method for axial and flexural response of single piles respectively based on the assumptions of the Winkler soil model. Based on the Winkler hypothesis, El Naggar and Novak (1995, 1996) presented a nonlinear analysis for pile groups in the time domain. The soil is modeled as near field consists of non linear spring and dashpot and far field consists of linear spring and dashpot. However, it is difficult to properly represent damping and inertia effects of continuous semi-infinite soil media when using such systems. Further, full coupling in the axial and lateral direction may not be considered.

Rao *et al*. (1998) considered the pile as a shear beam, soil is treated as a linear elastic continuum (modeled using eight- noded brick elements), and the pile cap is treated as a thin plate connecting the pile heads. Soil is represented by the soil modulus, which has been related to the undrained compressive strength of clay (C_u). An interface element having intermediate properties between pile and soil in contact is used to model the separation and slip between the soil and pile. Rajashree and Sitharam (2001) developed a finite element model to analyse piles under static and cyclic lateral loads. The pile was idealised as beam elements and soil as elasto-plastic spring elements. Non-linear soil behaviour was represented by a hyperbolic relation for static load condition and a modified hyperbolic relation which included both degradation and gap for a cyclic load condition.

Material nonlinearity of the soil caused by plasticity and work hardening was considered in the dynamic analysis of single pile by Maheshwari *et al*. (2005). An advanced plasticity based hierarchical single surface (HiSS) soil model was incorporated in a finite element technique. It was found that the nonlinearity of soil has significant effects on the pile response for lower and moderate frequencies of excitations, while at higher frequencies its effects are not as significant. Dewaikar *et al*. (2007) presented non linear static three dimensional finite element analysis incorporating the no tension behavior, gapping in the soil pile contact and yielding of the soil. The soil was modeled using the Von Mises yield criteria and pile was treated linear elastic. 20 noded isoparametric continuum elements were used to model the soil and pile; the pile soil contact was modeled using the 8 noded interface element. Similar technique is used by Karthigeyan *et al*. (2007) to study the influence of vertical loads on the lateral response of piles installed in sandy soils. The soil was idealized using Von Mises constitutive model with associated flow rule for clayey soils and the Drucker-Prager constitutive model with non associated flow rule for sandy soils. Fan and long (2005) carried out non-linear finite element analysis to study the soil response of piles subjected to lateral loads in sand. He modeled the soil using HiSS elastic–plastic model Proposed by Desai *et al*. (1991) with an isotropic hardening rule and non-associative flow rule. The pile soil interface was modelled using the interface element of finite thickness possessing the similar properties as the surrounding elements.

Cai *et al.* (2000) investigated the seismic response of soil–pile–structure systems using three-dimensional finite element subsystem methodology with an advanced plasticity-based constitutive model (HiSS) for soils. The structure subsystem is represented by space frame elements while the pile–soil subsystem is idealized as an assemblage of eight-node hexahedral elements. A successive-coupling, incremental solution scheme in the time domain is created to take account of both inertial and kinematic soil–pile–structure interactions simultaneously. The boundaries were assumed to be fixed. To account for the radiation effect Maheshwari *et al.* (2004, 2005) used the Kelvin elements at the boundaries to simulate radiation effects. Analyses are carried out for free-field response and pile head response of end-bearing single piles. Both harmonic and transient excitations are considered in the analyses. Maheshwari and Emani (2008) and Emani and Maheshwari (2009) presented the dynamic impedances for the pile groups with caps embedded in isotropic homogeneous elastic soils. A general three-dimensional finite element procedure is developed. The system is sub-structured into bounded near-field and an unbounded far-field. The pile-soil system of the near-field is modeled using solid 8 node finite elements, and the unbounded elastic soil system of the far-field is modeled using the consistent infinitesimal finite element cell method (CIFECM) in the frequency domain. For the near-field, material-nonlinearity due to plasticity and strain hardening has been considered.

Chore *et al*. (2010) highlighted the effect of pile diameter, pile spacing and arrangement on the response of pile group employing three dimensional finite element analysis. Reddy and Rao (2011) carried out static vertical load tests on a model building frame supported by pile groups embedded in cohesionless soil. The experimental results have been compared with those obtained from the nonlinear finite element analysis. Soil is modelled as Winkler springs.

Based on the literature review, it is observed that most of the researchers have neglected the dynamic nature of the forces. Few studies have been reported on the dynamic analysis of pile groups. Therefore, in the present study it is aimed to use three dimensional finite element formulation for the dynamic analysis of pile group. The soil-pile system is idealized as an assemblage of 20 node solid elements with the analysis performed in the time domain. A parametric study is carried out to consider the effect of pile spacing, number of piles, arrangement of pile and soil modulus on the response of pile group.

2. Finite element formulation

Full three-dimensional geometric models were used to represent the soil-pile system. Taking advantage of symmetry, only one-half of the actual model was built, thus dramatically improving efficiency of computation. The pile is completely embedded in the soil. The pile, soil and pile-cap system is idealized as an assemblage of 20-node isoparametric continuum elements. Each node of the elements has three translational degrees of freedom, in the X, Y, and Z coordinate directions. These elements are suitable for modeling the response of a system dominated by bending deformations. The interface between pile or pile cap and soil is modeled using 16 node isoparametric surface elements with zero thickness. These interface elements are useful in simulating the mechanics of stress transfer along the interface. The displacement at each time step is evaluated by using Newmark-Beta integration method.

2.1 Continuum element

Relation between strains and nodal displacements is expressed as

$$\{\varepsilon\}_e = [B]\{\delta\}_e \tag{1}$$

where, $\{\varepsilon\}_e$ is strain vector, $\{\delta\}_e$ is vector of nodal displacements, and $[B]$ is strain displacement transformation matrix.

The stress-strain relation is given by

$$\{\sigma\}_e = [D]\{\varepsilon\}_e \tag{2}$$

where, $\{\sigma\}_e$ is stress vector, and $[D]$ is constitutive relation matrix.

The stiffness matrix of an element, $[K]_e$ is expressed as

$$[K]_e = \int_V [B]^T[D][B]dv \tag{3}$$

2.2 Interface element

Relative displacements (strains) between the surface of soil and structure induce stresses in the interface element. These relative displacements are given as

$$\{\varepsilon\}_e = [B]_f \{\delta\}_e \tag{4}$$

where, $[B]_f$ represents the strain displacement transformation matrix.

The element stiffness matrix is obtained by the usual expression

$$[K]_e = \int_S [B]_f^T[D]_f [B]_f \, ds \tag{5}$$

where, $[D]_f$ is the constitutive relation matrix for the interface.

2.3 Equivalent nodal force vector

The lateral or vertical force (F_H or F_V), acting on pile cap, is considered as uniformly distributed force over the pile cap. The intensity of this uniformly distributed force is, $q = F/A$, where A is the area of pile-cap. Equivalent nodal force vector, $\{Q)_e$, is then expressed as

$$\{Q\}_e = \int_A q[N]^T dA \tag{6}$$

where, $[N]$ represents matrix of shape functions.

2.4 Time history analysis

Dynamic force equilibrium equation in incremental form

$$[M]\{\Delta\ddot{q}_i\} + [C]\{\Delta\dot{q}_i\} + [K]\{\Delta q_i\} = \{\Delta F_i\} \tag{7}$$

In which $\{\Delta q_i\}$, $\{\Delta\dot{q}_i\}$, $\{\Delta\ddot{q}_i\}$ are vectors of incremental displacement, velocity and acceleration, $[K]$ assembled stiffness matrix, $[M]$ consistent mass matrix and $[C]$ damping matrix given by following equation, and $\{\Delta F_i\}$ is dynamic load increment at current time step.

$$[K] = \sum \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} [B]^T[D][B]|J| d\xi \, d\eta \, d\zeta$$

$$[M] = \sum \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} [N]^T m[N]|J| d\xi \, d\eta \, d\zeta$$

$$[C] = \sum \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} [N]^T c[N]|J| d\xi \, d\eta \, d\zeta \tag{8}$$

The displacement at each time step can be evaluated by applying Newmark-Beta integration method.

$$\dot{q}_{i+1} = \dot{q}_i + \{(1-\alpha)\ddot{q}_i + \alpha\,\ddot{q}_{i+1}\}\Delta t \tag{9}$$

$$q_{i+1} = q_i + \dot{q}_i \Delta t + \left\{\left(\frac{1}{2} - \beta\right)\ddot{q}_i + \beta\,\ddot{q}_{i+1}\right\}\Delta t^2 \tag{10}$$

In the Eqs. (9) and (10) the value of *a* and *b* are taken considering average acceleration method as 0.5 and 0.25 respectively. The average acceleration method is stable for any size of time step.

After application of Newmark-Beta integration method, Dynamic force equilibrium equation in incremental form is given as follows.

$$\left(\frac{1}{\beta\Delta t^2}[M] + \frac{\alpha}{\beta\Delta t}[C] + [K]\right)\{\Delta q_i\} = \{\Delta F_i\} + \left(\frac{1}{\beta\Delta t}[M] + \frac{\alpha}{\beta}[C]\right)\{\dot{q}_{i-1}\} + \left\{\frac{1}{2\beta}[M] + \Delta t\left(\frac{\alpha}{2\beta} - 1\right)[C]\right\}\{\ddot{q}_{i-1}\} \tag{11}$$

Solution of the above equation yields incremental displacement $\{\Delta q_i\}$ from which displacement at current time step are updated by $\{q_{i+1}\} = \{q_i\} + \{\Delta q_i\}$, Velocity and accelerations ($\{\dot{q}_i\}$ and $\{\ddot{q}_i\}$) are evaluated using Eqs. (9) and (10).

For next time step the values of $\{\Delta q_{i+1}\}$, $\{\Delta \dot{q}_{i+1}\}$, $\{\Delta \ddot{q}_{i+1}\}$ are set as $\{\Delta q_i\}$, $\{\Delta \dot{q}_i\}$, $\{\Delta \ddot{q}_i\}$ and analysis is performed for next dynamic load increment.

3. Validation

Prakash (1962) carried out an experimental study for studying the behaviour of long pile embedded in sand and subjected to lateral load. The bottom boundary was considered rigid and rough and lateral boundary, rigid and smooth. The pile soil contact surface was assumed smooth for which contact shear stiffness was taken zero and contact normal stiffness had a very high value. The section for the pile was hollow circular with diameter of 1.6 inch. However, for the sake of convenience, the hollow circular section of pile is converted into an equivalent solid circular section of 1.6 inch in the numerical modeling here. An equivalent modulus of elasticity of sand was defined. As detailed information pertaining to stress-strain behaviour was not available, this modulus was approximately computed from the relation $E = J\gamma z$ as given by Terzaghi and Peck (1967), where, z is the depth from surface, γ is density of soil and J is the dimensionless parameter whose value is taken as 350. For the pile, Young's modulus and Poisson's ratio were taken as 43520 psi and 0.2, respectively. An equivalent modulus of elasticity of sand was defined. The unit weight of sand was taken as 18.9 kN/m^3. Poisson's ratio for sand was considered as 0.25. Lateral load of magnitude 2.75 lb was applied at the top of the pile. Employing judiciously the geometrical and material properties used by Prakash (1962) in the experimental study of a long pile embedded in the sand in the proposed numerical procedure, an analysis is performed. The comparison of the displacements and variation of the bending moment obtained by F.E.M. employed here and those obtained by Prakash (1962) experimentally are presented in Fig. 2(a) and 2(b), respectively. The variation obtained in displacement between either result is in the range of 4-10% and that in moment is in the range of 3-8%. The variation indicates close agreement in the results.

4. Parametric study

A parametric study is carried out considering different configurations of pile group. The dynamic load of the type $P_0 \sin(\omega t)$ is applied, where the external frequency (ω) is varied between 0.5 to 50 rad/sec. All piles considered in the analysis are square concrete piles. Square piles are more convenient for mesh generation, however circular piles can also be considered as square piles with equivalent cross sectional area and flexural rigidity. Trend shown by the results would not be different. Material properties of pile, soil and interface media are described in Table 1. When the direction of loading is parallel to the line joining piles, it is referred to as a series arrangement. On the other hand, if the lateral loading is acting in a direction perpendicular to the line joining piles, it is referred to as a parallel arrangement (Fig. 1(a)). Fig. 1(b) shows the typical discretization scheme used for the two piles in series arrangement. The hatched portion shows the piles and the thick line around the piles shows the location of sixteen node interface elements. Following Four types of pile configurations are considered in the present parametric study:

- Group of two piles in series arrangement (G2PS)
- Group of three piles in series arrangement (G3PS)
- Group of two piles in parallel arrangement (G2PP)
- Group of three piles in parallel arrangement (G3PP)

In each case spacing between piles is varied as 2D, 3D, 4D, 5D and 7D. Top displacements in the pile in case of lateral loading are considered for comparison. Effect of pile spacing, number of piles and arrangement of pile on top displacements is presented in graphical form.

Table 1 Material properties of pile, soil and interface media

Pile	Soil	Interface
$E_p = 1.237\times10^8$ kN/m^2	$E_s = 4267$ kN/m^2	$k_s = 1000$ kN/m^3
$\nu = 0.30$ $c = 5\%$	$\nu = 0.40$ $c=5\%$	$k_n = 1.0\times10^6$ kN/m^3
Width 1 m, Length 25 m		
Pile-cap thickness 0.50 m		

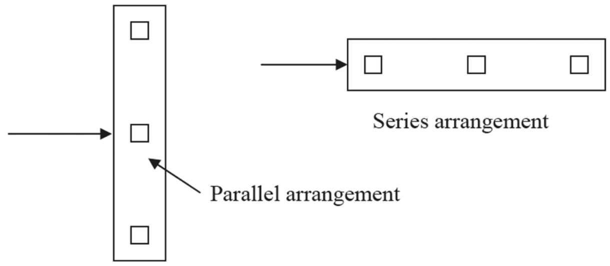

Fig. 1(a) Arrangements of pile in groups

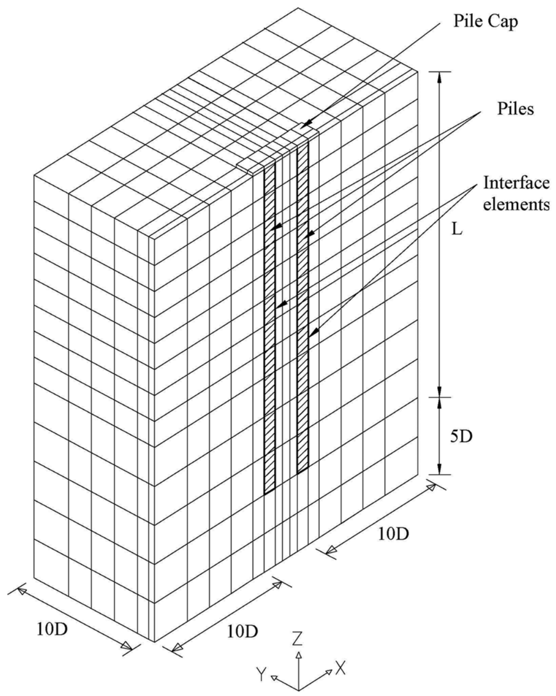

Fig. 1(b) Typical descritization scheme for two piles in series arrangement

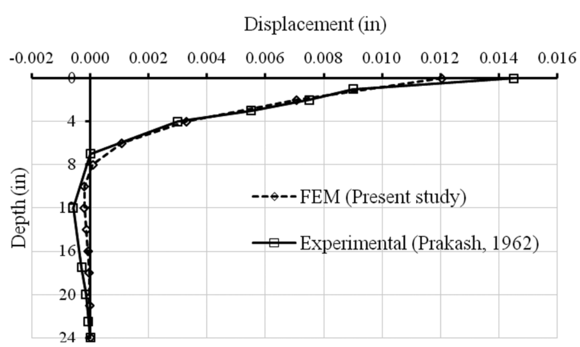

Fig. 2(a) Comparison of displacement of pile

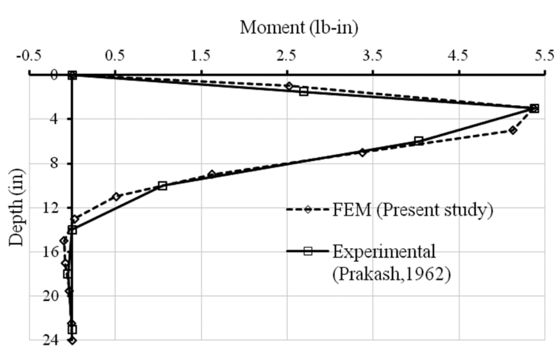

Fig. 2(b) Comparison of B.M. distribution in pile

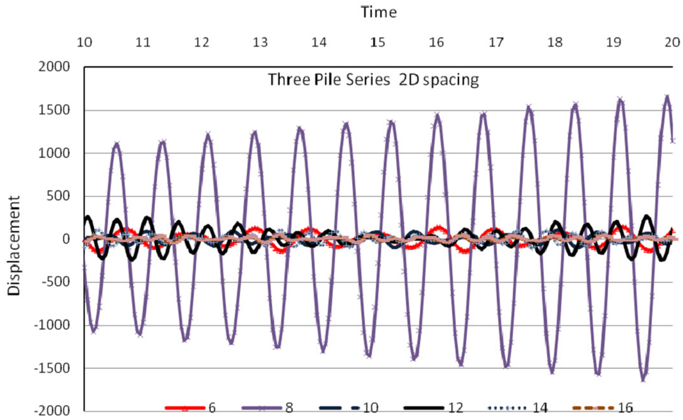

Fig. 3 Typical time history response

5. Results and discussion

Fig. 3 shows top displacement of the pile versus time. A nearly steady state response is observed for all the frequencies and resonance is found to occur at the frequency of 7-8 rad/sec.

Figs. 4 to 7 compares the variation in maximum amplitude with frequency at different spacing for specified pile configurations. Fig. 4 shows the response for 2- pile series configuration. When the external frequency is matching with natural frequency of the system, a clear peak in the response is observed. Two or three clear peaks are observed corresponding to initial frequencies. The fundamental frequency corresponding to maximum amplitude at the pile spacing of 2D is observed to be 9.5 rad/sec, which is decreasing with increase in pile spacing and the same reduced to 7 rad/sec at 7D spacing. Though overall stiffness of pile-soil system is increasing with increase in pile spacing due to reduction in the overlapping zone of individual piles, fundamental frequency is decreasing. This may be attributed to the increase in the mass of pile-soil system taking part in vibration at higher spacing. Natural frequency being inversely proportional to the mass, fundamental frequency is decreasing with increase in pile spacing. Second peak is observed at 13 rad/sec for 2D spacing which is reduced to 12.5 rad/sec at higher spacing. For three pile group series arrangement, similar trend is observed. The fundamental frequency at the pile spacing of 2D is 9.5 rad/sec, which is reduced to 5 rad/sec at 7D spacing. Even results for two pile parallel configuration (Fig. 6) show similar pattern. The first frequency at the pile spacing of 2D is 13, which is reduced to 10 rad/sec at 7D spacing.

Generally in the case of static analysis, top displacement is reducing with increase in the pile spacing, but such a definite pattern is not observed in the dynamic analysis for maximum amplitudes in top displacement with different pile spacing. Maximum amplitude is a complex phenomenon. It is not only a function of stiffness of pile soil system, but is also dependent on the external frequency as well as natural frequency. Comparison of response for different pile configurations is reported in Table 2.

For 3-pile parallel configuration (Fig. 7) slightly different trend is observed. The fundamental frequency at 2D is 12.5 rad/sec, which is reduced to 11.75 rad/sec at 3D spacing. But the same is increased to 12.5 rad/sec and 14 rad/sec at spacing 4D and 5D respectively. Maximum amplitude is observed to be decreasing with increase in pile spacing. These two observations on fundamental

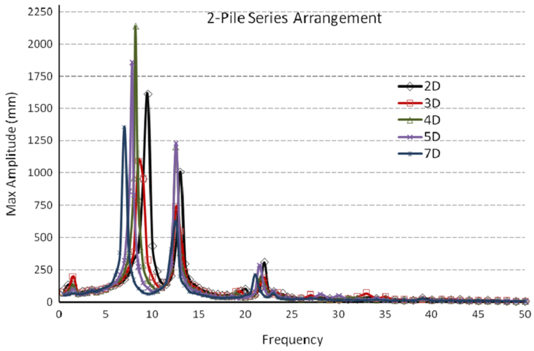

Fig. 4 Frequency-amplitude curve for 2-pile series

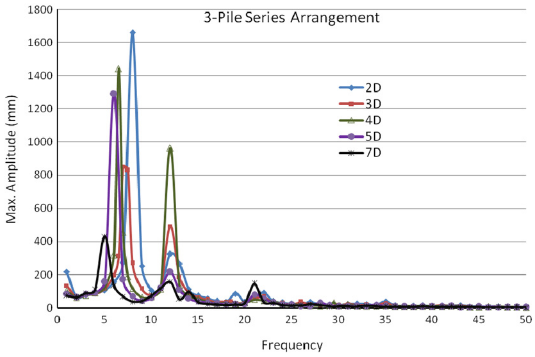

Fig. 5 Frequency-amplitude curve for 3-pile in series configuration

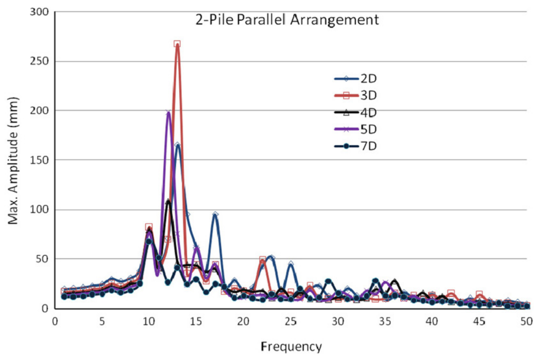

Fig. 6 Frequency-amplitude curve for 2-pile in parallel configuration

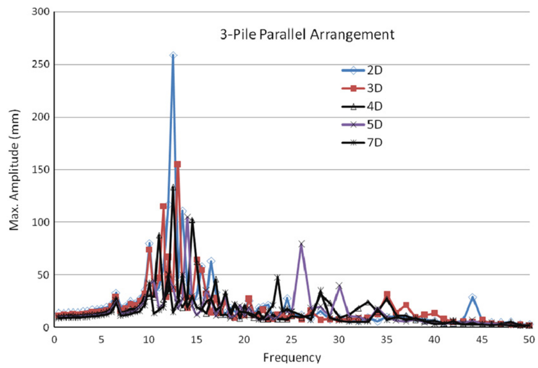

Fig. 7 Frequency-amplitude curve for 3-pile in parallel configuration

Table 2 Comparison of response for different pile configurations

Pile configuration	Spacing 2D	Spacing 3D	Spacing 4D	Spacing 5D	Spacing 7D
	Maximum amplitude (mm)				
Two Pile Series	1609.695	1092.575	2136.919	1857.168	1356.303
Three Pile Series	1658.295	851.090	1441.935	1292.177	430.594
Two Pile Parallel	165.136	267.253	108.597	197.615	67.796
Three Pile Parallel	258.748	190.301	133.466	104.642	50.471
	Fundamental frequency (rad/sec)				
Two Pile Series	9.5	8.5	8.2	7.8	7
Three Pile Series	8	7	6.5	6	5
Two Pile Parallel	13	13	12	12	10
Three Pile Parallel	12.5	11.75	12.5	14	11.99
	Second frequency (rad/sec)				
Two Pile Series	13	12.5	12.5	12.5	12.5
Three Pile Series	12	12	12	12	12
Two Pile Parallel	17	17	17	15	13
Three Pile Parallel	13.5	13	14.5	16	13.5

Table 3 Comparison of response for different Soil Modulus Configuration (G2PS)

Soil modulus	Spacing 2D	Spacing 3D	Spacing 4D	Spacing 5D	Spacing 7D
kN/m^3	Maximum amplitude (mm)				
4267	1609.695	1092.575	2136.919	1857.168	1356.303
21335	546.605	564.169	414.582	574.125	227.265
42670	504.515	428.071	312.103	211.541	339.938
	Fundamental frequency (rad/sec)				
4267	9.5	8.5	8.2	7.8	7
21335	19	18	17	16.5	14.5
42670	25	24	22.5	21.5	20
	Second frequency (rad/sec)				
4267	13	12.5	12.5	12.5	12.5
21335	25	25	24.5	24.5	24
42670	34.5	33	32	31	30

frequency and maximum amplitude suggest that in case of three-pile parallel configuration increase in pile-soil system stiffness is significant with increase in pile spacing. Stiffness is a dominant factor here in governing the response as compare to mass involved in vibration.

Maximum amplitudes and natural frequencies in case of three pile series configuration are smaller as compare to those in two pile series configuration. Due to higher stiffness, there is reduction in maximum amplitudes. Increase in the mass involved in vibration with number of piles yields lower value of natural frequencies. Comparison between two pile parallel configuration and three pile parallel configuration also demonstrates same effect of number of piles.

Comparison of all four configurations shows that three pile parallel configuration offer smaller amplitude and higher natural frequency owing to their higher stiffness. For other configurations, mass involved in vibration is major governing factor than the stiffness of pile-soil system. It can be concluded that parallel configurations are stiffer as compare to the series one as indicated by much smaller maximum amplitude and higher natural frequency (Fig. 10). Effect of stiffer behavior is more significant in the dynamic response.

To study the effect of soil modulus on dynamic response two more value of soil modulus are considered (21335 kN/m^2 and 42670 kN/m^2). Results obtained for two-pile series configuration are presented in graphical form in Figs. 8 and 9. Trend is very much similar to those observed for soil modulus 4267 kN/m^2. Table 3 compares the effect of soil modulus on dynamic response in view of maximum amplitude, and natural frequencies. For same spacing smaller values of maximum amplitudes are observed for higher soil modulus indicating higher stiffness. Also at the same pile spacing, higher values of fundamental frequency (Fig. 11) and second frequency are obtained for higher modulus suggesting higher stiffness.

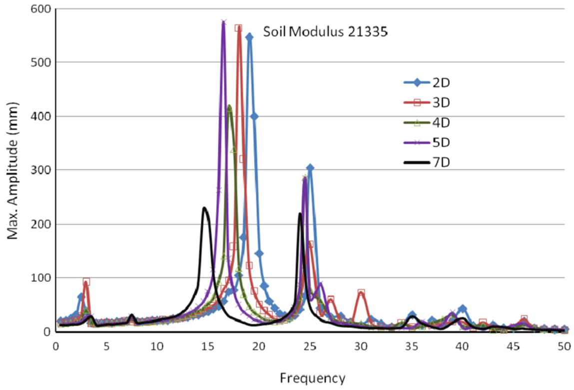

Fig. 8 Frequency-amplitude curve for soil modulus 21335 kN/m^2

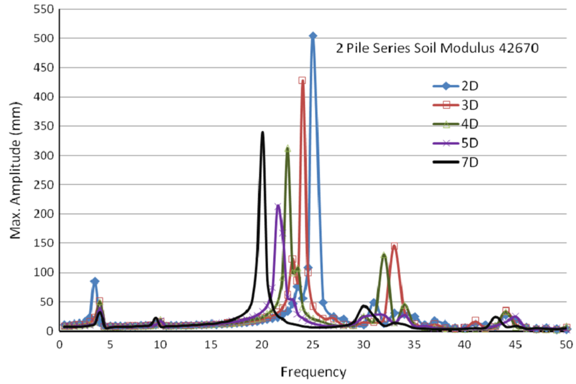

Fig. 9 Frequency-amplitude curve for soil modulus 42670 kN/m^2

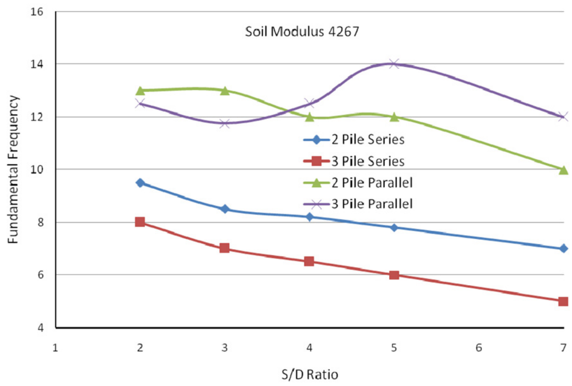

Fig. 10 Comparison of response for different pile configurations

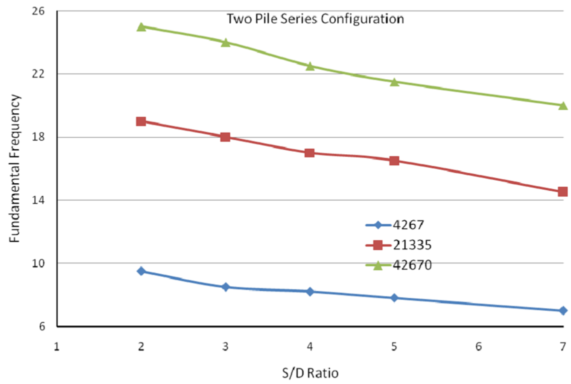

Fig. 11 Comparison of response for different soil modulus

6. Conclusions

1. Except for 3-pile parallel configuration, fundamental frequency is decreasing with increase in pile spacing. Maximum amplitude is a complex phenomenon and depends on stiffness of pile soil system, the external frequency and natural frequency. Mass involved in vibration is major governing factor.
2. For 3-pile parallel configuration, maximum amplitude is observed to be decreasing with increase in pile spacing. A complex pattern is observed for natural frequency. Stiffness is a dominant factor here in governing the response as compare to mass involved in vibration.
3. Maximum amplitude and natural frequencies decreases with increase in number of piles.
4. It can be concluded that parallel configurations are stiffer as compare to the series one as indicated by much smaller maximum amplitude and higher natural frequency.
5. For same spacing smaller values of maximum amplitudes are observed for higher soil modulus indicating higher stiffness. Also at the same pile spacing, higher values of fundamental frequency and second frequency are obtained for higher modulus suggesting higher stiffness.

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