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# Free vibration of an axially functionally graded pile with pinned ends embedded in Winkler-Pasternak elastic medium

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**Abstract.** In the present study, free vibration of an axially functionally graded (AFG) pile embedded in Winkler-Pasternak elastic foundation is analyzed within the framework of the Euler-Bernoulli beam theory. The material properties of the pile vary continuously in the axial direction according to the power-law form. The frequency equation is obtained by using Lagrange's equations. The unknown functions denoting the transverse deflections of the AFG pile is expressed in modal form. In this study, the effects of material variations, the parameters of the elastic foundation on the fundamental frequencies are examined. It is believed that the tabulated results will be a reference with which other researchers can compare their results.

**Keywords:** vibration; pile; functionally graded material; Winkler-Pasternak foundation

## 1. Introduction

Functionally graded materials (FGMs) are special composites whose material properties vary continuously through their thickness. FGMs are usually made of mixture of ceramic and metal, and can thus resist high-temperature environments while maintaining toughness. The technology of FGMs was an original material fabrication technology proposed in Japan in 1984 by Sendai Group. FGMs are used in very different applications, such as reactor vessels, fusion energy devices, biomedical sectors, aircrafts, space vehicles, defense industries and other engineering structures. Because of the wide material variations and applications of FGMs, it is important to study the static and dynamic analysis of FG structures, such as beams and plates. Sankar (2001) gave an elasticity solution based on the Euler-Bernoulli beam theory for functionally graded beam subjected to static transverse loads by assuming that Young's modulus of the beam vary exponentially through the thickness. Chakraborty *et al*. (2003) proposed a new beam finite element based on the first-order shear deformation theory to study the thermoelastic behavior of functionally graded beam structures. Aydogdu and Taskin (2007) investigated the free vibration behavior of a simply supported FG beam

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by using Euler-Bernoulli beam theory, parabolic shear deformation theory and exponential shear deformation theory. In a recent study by Yang *et al*. (2008), free and forced vibrations of cracked FG beams subjected to an axial force and a moving load were investigated by using the modal expansion technique. Pradhan and Sarkar (2009) carried out bending, buckling and vibration analyses of functionally graded tapered beam using Eringen's nonlocal elasticity theory and Rayleigh-Ritz method. Şimşek and Kocatürk (2009) have investigated the free and forced vibration characteristics of an FG Euler-Bernoulli beam under a moving harmonic load. Şimşek (2009) studied the static analysis of an FG beam under uniformly distributed load within the framework of the higher-order shear deformation beam theory by Ritz method. Thermal post-buckling behavior of uniform slender FGM beams is investigated independently using the classical Rayleigh-Ritz (RR) formulation and the versatile Finite Element Analysis (FEA) formulation developed in this paper by Anandrao *et al*. (2010). In a recent study, Şimşek (2010a) has studied the dynamic deflections and the stresses of an FG simply-supported beam subjected to a moving mass by using Euler-Bernoulli, Timoshenko and the parabolic shear deformation beam theory. Şimşek (2010b) studied the free vibration of FG beams having different boundary conditions by using different higher order shear deformation beam theories. Şimşek (2010c) performed the non-linear dynamic analysis of a functionally graded beam with immovable supports under a moving harmonic load. Kocatürk *et al*. (2011) investigated the large displacement static analysis of a cantilever Timoshenko beam composed of functionally graded material.

It is anticipated that axially functionally graded (AFG) materials will be developed in the near future, they will allow a tailored fit to a special purpose, i.e., with the static deflection not exceeding a specific level, or the buckling load not being less than a pre-specified level, or the natural frequency either exceeding or to being less than a pre-specified frequency (Elishakoff 2005). It is anticipated that the best functional grading will combine that both in axial and thickness directions (Elishakoff 2005). In spite of the fact that there exist many studies on the analysis of FG beams in thickness direction, however, the research effort devoted to free vibration of AFG beams has been very limited. For instance, the functional grading in the axial direction (i.e., the variation of the elastic modulus along the axis) in the free vibration analysis of beams was studied by Candan and Elishakoff (2001), Elishakoff and Candan (2001). Wu *et al*. (2005) used the semi-inverse method to find the solutions to the dynamic equation of axially functionally graded simply supported beams. Aydogdu (2008) analyzed the vibration and buckling of axially functionally graded simply-supported beam by using semi-inverse method. Huang and Li (2010) presented a new approach for free vibration of axially functionally non-uniform graded beams. Alshorbagy *et al*. (2011) have investigated the dynamic characteristics of non-uniform graded beams with material graduation in axially or transversally thorough the thickness. Shahba *et al*. (2011) studied free vibration and stability analysis of AFG tapered Timoshenko beams by using finite element method. Şimşek *et al*. (2011) investigated the dynamic behavior of an AFG beam under a moving harmonic load.

The analysis of structures on elastic foundations is of considerable interest and widely used in several engineering fields, such as foundation, pavement and railroad, pipeline, and some aero-space structures applications (Civalek and Öztürk 2010). Many problems in the engineering related to soil-structure interaction can be modeled by means of a beam or a beam-column on an elastic foundation. Winkler foundation model is extensively used by engineers and researchers because of its simplicity. Generally, the foundation is considered to be an array of springs uniformly distributed along the length of the beam. There are many studies on beam-column type structures on elastic foundation in the literature (i.e., Doyle and Pavlovic 1982, Zhaohua and Cook 1983, Yankelevsky and Eisenberger 1986, Yokoyama 1991, Matsunaga 1999, Celep and Demir 2007, Vu and Leon 2008, Kim 2009, Yesilce and Catal 2009, Balkaya *et al*. 2010, Ozturk and Coskun 2011).

This paper is motivated by the lack of the contribution of the existing literature to the axially functionally graded beam-type structures on elastic foundation. In the present study, the free vibration of an AFG pile embedded in Winkler-Pasternak elastic foundation is investigated within the framework of the Euler-Bernoulli beam theory. The material properties of the pile vary continuously in the axial direction according to the power-law form. The frequency equation is obtained by using Lagrange's equations. In this study, the effects of material variations, the parameters of the elastic foundation on the fundamental frequencies are examined.

## 2. Formulation

### 2.1 Functionally graded materials

An axially functionally graded (AFG) pile having the length $L$, the diameter $d$ is shown in Fig. 1 and the pile is embedded in two-parameter elastic medium modeled as a Winkler-Pasternak foundation with spring constant $k_w$ and $k_p$.

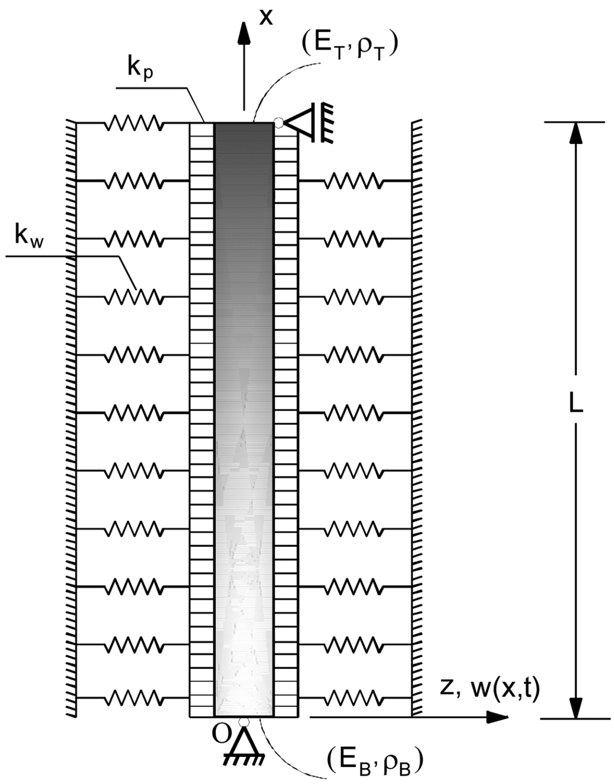


Fig. 1 A simply supported AFG pile embedded in Winkler-Pasternak elastic medium

In this study, it is assumed that the material properties of the axially functionally graded (AFG) pile vary continuously in the axial direction according to a function of the volume fractions of the constituents. According to the rule of mixture, the effective material properties $P$ (i.e., Young's modulus $E$ and mass density $\rho$) can be expressed as

$$P = P_B V_B + P_T V_T \tag{1}$$

where $P_B$, $P_T$ are the effective material properties of the AFG pile at the bottom and the top end of the pile, and $V_B$ and $V_T$ are the volume fractions of the constituents and related by

$$V_B + V_T = 1 \tag{2}$$

The effective material properties of the AFG pile are defined by the power-law form. The volume fraction of the constituent at the bottom end of the pile is assumed by

$$V_B = \left(1 - \frac{x}{L}\right)^k \tag{3}$$

where $k$ is the non-negative parameter (power-law exponent) which dictates the material variation profile along the length of the pile. Fig. 2 shows variation of the volume fraction $V_B$ along the length of the pile.

Therefore, from Eqs. (1)-(3), the effective material properties of the AFG pile can be expressed as

$$E(x) = (E_B - E_T)\left(1 - \frac{x}{L}\right)^k + E_T \tag{4a}$$

$$\rho(x) = (\rho_B - \rho_T)\left(1 - \frac{x}{L}\right)^k + \rho_T \tag{4b}$$

It is evident from Eq. (4) that when $x = 0$, $E = E_B$, $\rho = \rho_B$ and when $x = L, E = E_T, \rho = \rho_T$.

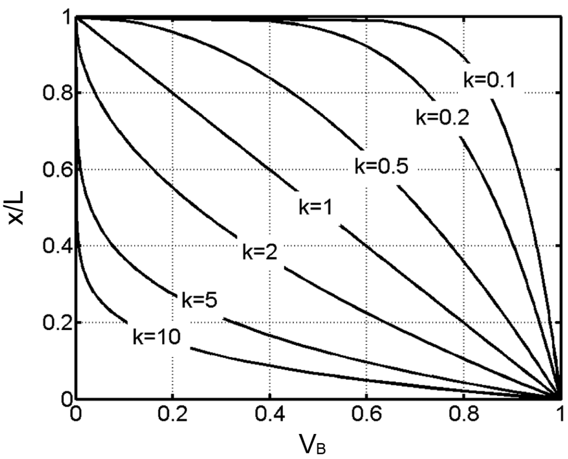


Fig. 2 The variation of the volume fraction of the constituent at the bottom end of the AFG pile

## 2.2 Frequency equation

Based on Euler-Bernoulli beam theory, the potential energy of the AFG pile can be written in the following form

$$U_{\text{int}} = \frac{1}{2}\int_0^L E(x)I\left(\frac{\partial^2 w(x,t)}{\partial x^2}\right)^2 dx \tag{5}$$

where $w$ is the transverse deflections of any point on the neutral axis, $E$ is the modulus of elasticity, $I$ is the second moment of the beam cross-section and $t$ denotes time. The potential energy induced by the elastic medium is given by

$$U_{em} = \frac{1}{2}\int_0^L k_w(w(x,t))^2 dx + \frac{1}{2}\int_0^L k_p\left(\frac{\partial w(x,t)}{\partial x}\right)^2 dx \tag{6}$$

where $k_w$ and $k_p$ are the spring constants of the Winkler and Pasternak elastic medium, respectively. Considering the rotary inertia, the kinetic energy of the pile can be expressed as

$$K_e = \frac{1}{2}\int_0^L \rho(x)A\left(\frac{\partial w(x,t)}{\partial t}\right)^2 dx + \frac{1}{2}\int_0^L \rho(x)I\left(\frac{\partial^2 w(x,t)}{\partial x \partial t}\right)^2 dx \tag{7}$$

where $\rho$ is the mass density of the beam and $A$ is the area of the cross-section. Frequency equation of the problem will be derived by using Lagrange's equations. It is well-known that Hamilton's principle can be expressed as Lagrange's equations when the functions of infinite dimensions can be expressed in terms of generalized coordinates $q_i(t)$. Therefore, the transverse displacement of the pile can be approximated as

$$w(x,t) = \sum_{i=1}^{N} q_i(t)\phi_i(x) \tag{8}$$

where $q_i(t)$ are the unknown generalized coordinates to be determined and $\phi_i(x)$ are the test functions which are expressed for pinned-pined pile as

$$\phi_i(x) = \sin\left(\frac{i\pi x}{L}\right) \qquad i = 1,2,3,\ldots \tag{9}$$

The Lagrange's equations are given as follows

$$\frac{d}{dt}\left(\frac{\partial K_e}{\partial \dot{q}_i}\right) + \frac{\partial U_{\text{int}}}{\partial q_i} + \frac{\partial U_{em}}{\partial q_i} = 0 \qquad i = 1,2,3,\ldots \tag{10}$$

where the overdot stands for the partial derivative with respect to time. After substituting Eq. (9) into Eq. (8) and then using the Lagrange's equations given by Eq. (10) yields the following equations of motion

$$[K]\{q\} + [M]\{\ddot{q}\} = \{0\} \tag{11}$$

where $[K]$ is the stiffness matrix and $[M]$ is the mass matrix. For free vibration analysis, the time-dependent generalized displacement coordinates can be expressed as follows

$$q_i(t) = \bar{q}_i \sin\omega t \tag{12}$$

where $\omega$ is the natural frequency of the beam. Introducing the following non-dimensional variables

$$\bar{w} = \frac{w}{L},\ \bar{x} = \frac{x}{L},\ \xi = \frac{L}{d},\ E_{\text{ratio}} = \frac{E_B}{E_t},\ \rho_{\text{ratio}} = \frac{\rho_B}{\rho_t},\ K_W = \frac{k_w L^4}{E_T I},\ K_P = \frac{k_p L^2}{E_T I},\ \lambda^2 = \frac{\omega^2 \rho_T A L^4}{E_T I} \tag{13}$$

and after some mathematical manipulations with the aid of Eq. (12), the following algebraic equation (frequency equation) is obtained which can be expressed in the following matrix form

$$[\bar{K}]\{\bar{q}\} - \lambda^2[\bar{M}]\{\bar{q}\} = \{0\} \tag{14}$$

In Eq. (14), the following abbreviations have been introduced

$$\bar{K}_{ij} = \int_0^1 [(E_{\text{ratio}}-1)(1-\bar{x})^k + 1]\phi_i''\phi_j'' d\bar{x} + \int_0^1 K_W \phi_i \phi_j d\bar{x} + \int_0^1 K_P \phi_i' \phi_j' d\bar{x} \tag{15a}$$

$$\bar{M}_{ij} = \int_0^1 [(\rho_{\text{ratio}}-1)(1-\bar{x})^k + 1]\phi_i \phi_j d\bar{x} + \int_0^1 [(\rho_{\text{ratio}}-1)(1-\bar{x})^k + 1]\phi_i' \phi_j' d\bar{x} \tag{15b}$$

The non-dimensional frequencies (eigenvalues) $\lambda$ are found from the condition that the determinant of the coefficients' matrix of Eq. (14) must vanish.

## 3. Numerical results and discussion

In numerical analysis, free vibration frequencies of AFG pile with pinned ends are given for various values of modulus ratio $E_{\text{ratio}}$, the power-law exponent $k$, Winkler and Pasternak parameters for the slenderness ratio $\xi = 20$. Although convergence study performed is not shown here, when more than 12 terms are used in the displacement function, the numerical accuracy of the responses is satisfactory. Therefore, the number of terms in the displacement function is set to 14 in the subsequent calculations.

The numerical results are compared with the previous works to demonstrate the performance of the present study. To this end, the first three natural frequencies of the simply-supported AFG beam are compared with results of Alshorbagy *et al.* (2011) obtained by Finite Element method (FEM). Computations have been carried out for various values of modulus ratio and the power-law exponent. The foundation parameters are taken as zero for the comparison purpose. As seen from Tables 1-3, the present results are in good agreement with that the results of Alshorbagy *et al.* (2011). It is seen from these tables that as $E_{\text{ratio}}$ increases, the dimensionless frequencies increase, and $E_{\text{ratio}}$ is more effective on the dimensionless frequencies for small values of $k$ than large values of $k$. As the values of $k$ increase, the effect of increase of $E_{\text{ratio}}$ on the natural frequencies decreases. For instance, when $k = 0$, $\lambda_1$ is 2.2203 and 4.4406 for $E_{\text{ratio}} = 0.25$ and $E_{\text{ratio}} = 4$, respectively. On the other hand, when $k = 10$, $\lambda_1$ is 3.1266 and 3.1725 for the same values of $E_{\text{ratio}}$. As an expected result, the beam is homogeneous for $E_{\text{ratio}} = 1$, and the frequencies are independent of the power-law exponent.

Fig. 3 shows the variation of the first non-dimensional frequency with the power-law exponent for

Table 1 The first dimensionless frequency parameters $\lambda_1$ for different values of the modulus ratio and the power-law exponent and for $\rho_{\text{ratio}} = 1$, $\xi = 20$, $K_W = K_P = 0$

| $E_{\text{ratio}}$ | Source | $k = 0$ | $k = 0.1$ | $k = 0.2$ | $k = 0.5$ | $k = 1$ | $k = 2$ | $k = 5$ | $k = 10$ |
|---|---|---|---|---|---|---|---|---|---|
| 0.25 | Present | 2.2203 | 2.3285 | 2.4106 | 2.5821 | 2.7532 | 2.9278 | 3.0834 | 3.1266 |
| | Alshorbagy *et al*. (2011) | 2.2203 | 2.3285 | 2.4106 | 2.5821 | 2.7533 | 2.9278 | 3.0834 | 3.1265 |
| 0.50 | Present | 2.6403 | 2.6867 | 2.7257 | 2.8147 | 2.9104 | 3.0122 | 3.1052 | 3.1316 |
| | Alshorbagy *et al*. (2011) | 2.6404 | 2.6868 | 2.7258 | 2.8148 | 2.9104 | 3.0122 | 3.1052 | 3.1316 |
| 1.0 | Present | 3.1399 | 3.1399 | 3.1399 | 3.1399 | 3.1399 | 3.1399 | 3.1399 | 3.1399 |
| | Alshorbagy *et al*. (2011) | 3.14 | 3.14 | 3.14 | 3.14 | 3.14 | 3.14 | 3.14 | 3.14 |
| 2.0 | Present | 3.7340 | 3.6987 | 3.6653 | 3.5757 | 3.4611 | 3.3243 | 3.1922 | 3.1530 |
| | Alshorbagy *et al*. (2011) | 3.7341 | 3.6988 | 3.6653 | 3.5758 | 3.4611 | 3.3244 | 3.1923 | 3.1531 |
| 4.0 | Present | 4.4406 | 4.3768 | 4.3144 | 4.1387 | 3.8937 | 3.5794 | 3.2667 | 3.1725 |
| | Alshorbagy *et al*. (2011) | 4.4406 | 4.3768 | 4.3144 | 4.1387 | 3.8937 | 3.5795 | 3.2668 | 3.1726 |

Table 2 The second dimensionless frequency parameters $\lambda_2$ for different values of the modulus ratio and the power-law exponent and for $\rho_{\text{ratio}} = 1$, $\xi = 20$, $K_W = K_P = 0$

| $E_{\text{ratio}}$ | Source | $k = 0$ | $k = 0.1$ | $k = 0.2$ | $k = 0.5$ | $k = 1$ | $k = 2$ | $k = 5$ | $k = 10$ |
|---|---|---|---|---|---|---|---|---|---|
| 0.25 | Present | 4.4338 | 4.6692 | 4.8373 | 5.1674 | 5.4729 | 5.7675 | 6.0639 | 6.1991 |
| | Alshorbagy *et al*. (2011) | 4.4338 | 4.6693 | 4.8374 | 5.1675 | 5.4730 | 5.7674 | 6.0636 | 6.1987 |
| 0.50 | Present | 5.2727 | 5.3751 | 5.4572 | 5.6326 | 5.8047 | 5.9739 | 6.1459 | 6.2264 |
| | Alshorbagy *et al*. (2011) | 5.2727 | 5.3752 | 5.4573 | 5.6327 | 5.8048 | 5.9739 | 6.1459 | 6.2263 |
| 1.0 | Present | 6.2703 | 6.2703 | 6.2703 | 6.2703 | 6.2703 | 6.2703 | 6.2703 | 6.2703 |
| | Alshorbagy *et al*. (2011) | 6.2703 | 6.2703 | 6.2703 | 6.2703 | 6.2703 | 6.2703 | 6.2703 | 6.2703 |
| 2.0 | Present | 7.4567 | 7.3774 | 7.3039 | 7.1176 | 6.9030 | 6.6782 | 6.4482 | 6.3363 |
| | Alshorbagy *et al*. (2011) | 7.4567 | 7.3774 | 7.3039 | 7.1176 | 6.9031 | 6.6783 | 6.4483 | 6.3365 |
| 4.0 | Present | 8.8675 | 8.7236 | 8.5854 | 8.2114 | 7.7399 | 7.2208 | 6.6900 | 6.4299 |
| | Alshorbagy *et al*. (2011) | 8.8676 | 8.7236 | 8.5853 | 8.2113 | 7.7399 | 7.2209 | 6.6902 | 6.4302 |

Table 3 The third dimensionless frequency parameters $\lambda_3$ for different values of the modulus ratio and the power-law exponent and for $\rho_{\text{ratio}} = 1$, $\xi = 20$, $K_W = K_P = 0$

| $E_{\text{ratio}}$ | Source | $k = 0$ | $k = 0.1$ | $k = 0.2$ | $k = 0.5$ | $k = 1$ | $k = 2$ | $k = 5$ | $k = 10$ |
|---|---|---|---|---|---|---|---|---|---|
| 0.25 | Present | 6.6338 | 6.9948 | 7.2468 | 7.7329 | 8.1753 | 8.5992 | 9.0270 | 9.2318 |
| | Alshorbagy *et al.* (2011) | 6.6338 | 6.9949 | 7.2470 | 7.7330 | 8.1753 | 8.5989 | 9.0262 | 9.2305 |
| 0.50 | Present | 7.8890 | 8.0471 | 8.1714 | 8.4316 | 8.6813 | 8.9246 | 9.1710 | 9.2905 |
| | Alshorbagy *et al.* (2011) | 7.8890 | 8.0472 | 8.1715 | 8.4317 | 8.6814 | 8.9246 | 9.1708 | 9.2901 |
| 1.0 | Present | 9.3816 | 9.3816 | 9.3816 | 9.3816 | 9.3816 | 9.3816 | 9.3816 | 9.3816 |
| | Alshorbagy *et al.* (2011) | 9.3817 | 9.3817 | 9.3817 | 9.3817 | 9.3817 | 9.3817 | 9.3817 | 9.3817 |
| 2.0 | Present | 11.156 | 11.033 | 10.920 | 10.638 | 10.323 | 10.000 | 9.6733 | 9.5136 |
| | Alshorbagy *et al*. (2011) | 11.157 | 11.033 | 10.920 | 10.638 | 10.324 | 10.001 | 9.6735 | 9.5139 |
| 4.0 | Present | 13.267 | 13.043 | 12.829 | 12.258 | 11.561 | 10.814 | 10.059 | 9.6937 |
| | Alshorbagy *et al*. (2011) | 13.268 | 13.043 | 12.829 | 12.258 | 11.562 | 10.815 | 10.059 | 9.6942 |

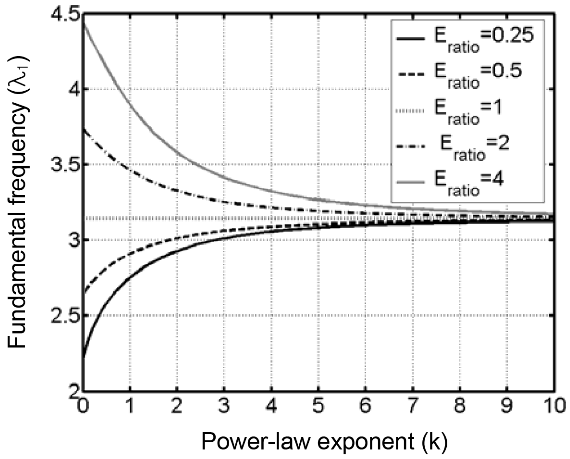


Fig. 3 Variation of non-dimensional fundamental frequency with power-law exponent $k$ for the different Young's modulus ratio, $\rho_{\text{ratio}} = 1$, $\xi = 20$, $K_W = K_P = 0$

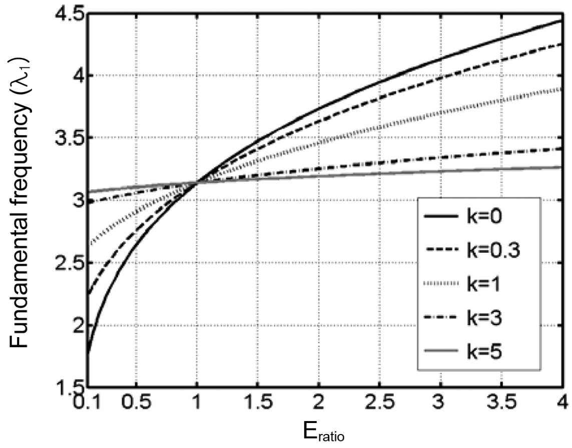


Fig. 4 Variation of the non-dimensional fundamental frequency with Young's modulus ratio for the different power-law exponent $k$, $\rho_{\text{ratio}} = 1$, $\xi = 20$, $K_W = K_P = 0$

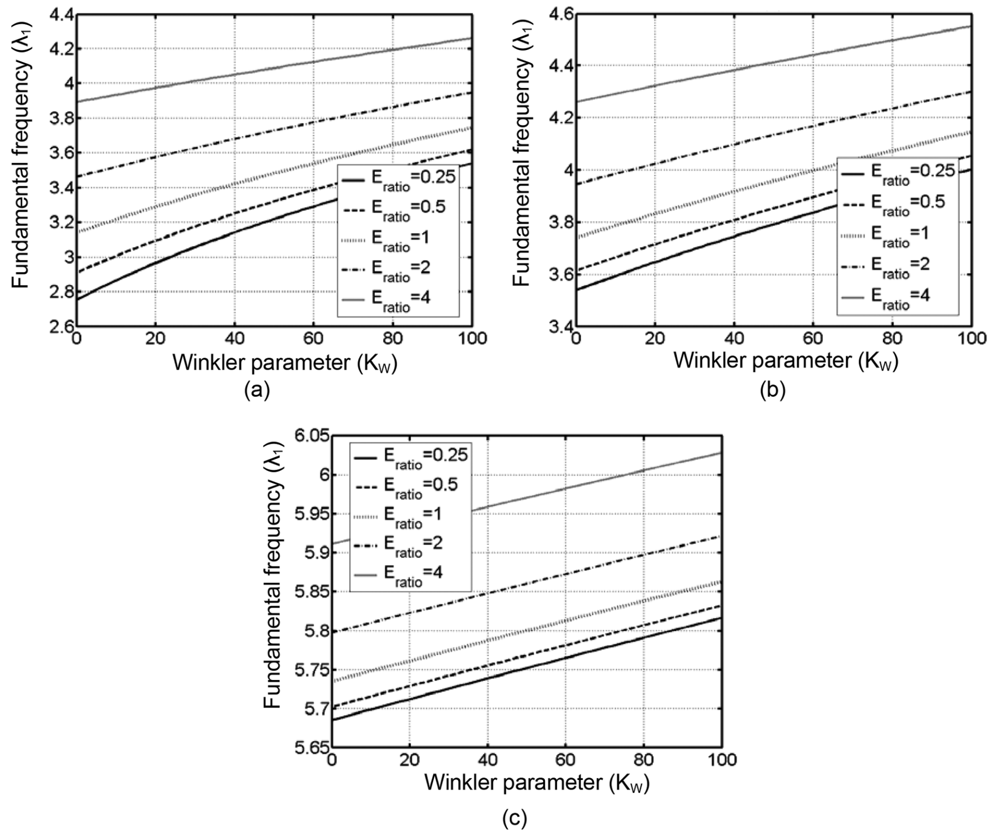


Fig. 5 Variation of the non-dimensional fundamental frequency with the Winkler parameter for the different Young's modulus ratio and for $k = 1$, $\rho_{\text{ratio}} = 1$, $\xi = 20$, (a) $K_P = 0$, (b) $K_P = 10$, (c) $K_P = 100$

different values of $E_{\text{ratio}}$. It is seen from Fig. 3 that the first dimensionless frequency increases with increase in the power-law exponent $k$ when $E_{\text{ratio}}$ is smaller than unity, whereas the first dimensionless frequency decreases as the power-law exponent increases when $E_{\text{ratio}}$ is larger than unity. For both situations, it is clear that the first dimensionless frequency of the AFG beam approaches the first dimensionless frequency of the homogeneous beam as the power-law exponent $k$ increases. In Fig. 4, the variation of the first frequency with $E_{\text{ratio}}$ for the different values of the power-law exponent $k$ is displayed. Similar figures can be given for the two other consecutive frequencies. It is clearly seen from this figure that dimensionless frequencies increases with increase in $E_{\text{ratio}}$, as discussed before and the curves for various variable $k$ intersect each other when $E_{\text{ratio}} = 1$.

Fig. 5 shows the variation of the fundamental frequency of the AFG pile with the Winkler parameter for the various values of Young's modulus ratio. It is obviously observed from Fig. 5 that the increasing the value of $K_W$ has an important effect on the frequencies, and increase in the Winkler parameter $K_W$ of the pile causes increase in the fundamental frequency. This is because increasing the Winkler parameter makes the pile becomes stiffer. Also, note the variation of the fundamental frequency with the Winkler parameter is nearly linear.

Fig. 6 displays the variation of the fundamental frequency with the Pasternak parameter for the various values of Young's modulus ratio. It is shown that the increase in the Pasternak parameter $K_P$

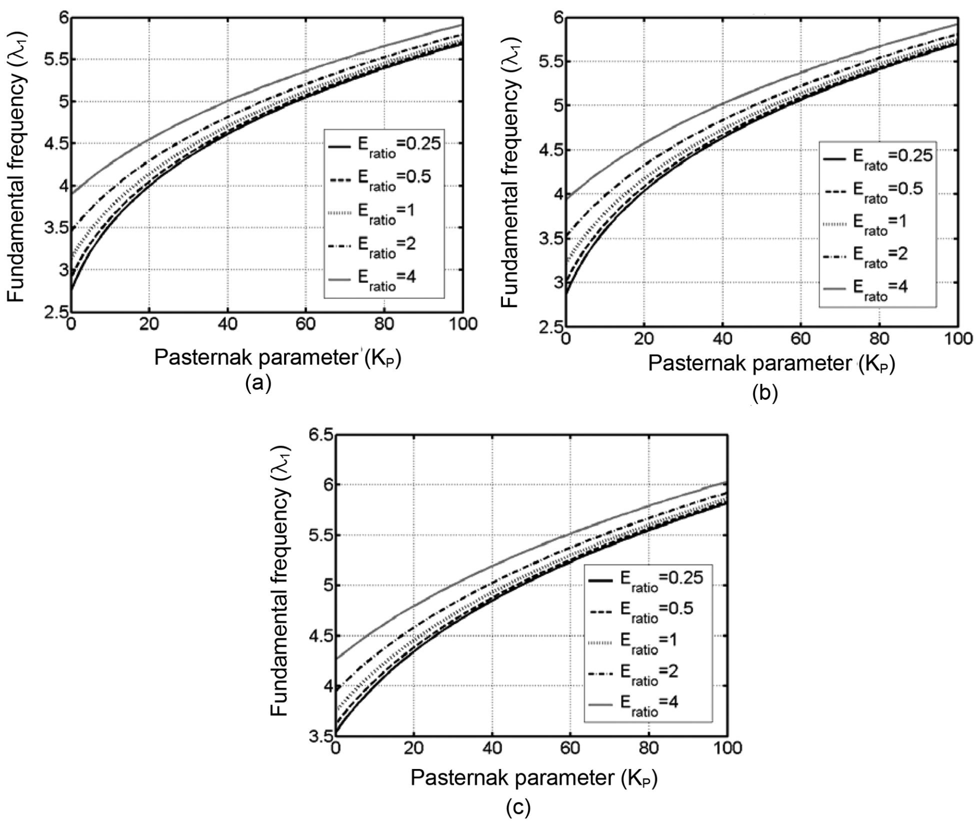


Fig. 6 Variation of the non-dimensional fundamental frequency with the Pasternak parameter for the different Young's modulus ratio and for $k = 1$, $\rho_{\text{ratio}} = 1$, $\xi = 20$, (a) $K_W = 0$, (b) $K_W = 10$, (c) $K_W = 100$

Table 4 The first three dimensionless frequency parameters for different values of the foundation parameters, $E_{ratio} = 2$, $\rho_{ratio} = 1$, $k = 1$, $\xi = 20$

| Foundation parameters | | Present results | | |
|---|---|---|---|---|
| $K_W$ | $K_P$ | $\lambda_1$ | $\lambda_2$ | $\lambda_3$ |
| 0 | 0 | 3.4611 | 6.9030 | 10.323 |
| 1 | 1 | 3.5249 | 6.9335 | 10.344 |
| 10 | 10 | 3.9853 | 7.1913 | 10.520 |
| 100 | 100 | 5.9215 | 8.9097 | 11.926 |
| 1000 | 1000 | 10.239 | 14.354 | 17.771 |

of the pile causes also increase in the fundamental frequency. The most important observation from this figure that as the Pasternak parameter increases, the effect of $E_{ratio}$ on the frequencies decreases. The variation of the fundamental frequency with the Pasternak parameter is nonlinear. Also, comparing Fig. 5 and Fig. 6 reveals that the effect of the Winkler parameter on frequencies is less than the Pasternak parameter.

In Table 4, the first three frequency parameters of the AFG pile with $E_{ratio} = 2$, $\rho_{ratio} = 1$, $k = 1$ are given for various values of the foundation parameters. It is seen that the foundation parameters play an important role on the dynamic behavior of the pile.

## 4. Conclusions

In the present study, free vibration of an AFG pile embedded in Winkler-Pasternak elastic foundation is investigated within the framework of the Euler-Bernoulli beam theory. The material properties of the pile vary continuously in the axial direction according to the power-law form. The frequency equation is obtained by using Lagrange's equations. In this study, the effects of material variations, the parameters of the elastic foundation on the fundamental frequencies are examined. From the numerical results, the above-mentioned effects have a great influence on the dynamic characteristics of the AFG pile.

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