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Thermomechanical postbuckling of imperfect moderately thick plates on two-parameter elastic foundations

Hui-Shen Shen†

Department of Civil Engineering, Shanghai Jiao Tong University, Shanghai 200030, China

Abstract. A postbuckling analysis is presented for a simply supported, moderately thick rectangular plate subjected to combined axial compression and uniform temperature loading and resting on a two-parameter elastic foundation. The two cases of thermal postbuckling of initially compressed plates and of compressive postbuckling of initially heated plates are considered. The initial geometrical imperfection of the plate is taken into account. The formulations are based on the Reissner-Mindlin plate theory considering the first order shear deformation effect, and including the plate-foundation interaction and thermal effect. The analysis uses a deflection-type perturbation technique to determine the buckling loads and postbuckling equilibrium paths. Numerical examples cover the performances of perfect and imperfect, moderately thick plates resting on Winkler or Pasternak-type elastic foundations. Typical results are presented in dimensionless graphical form.

Key words: structural stability; thermomechanical postbuckling; moderately thick plate; Pasternak-type elastic foundation; perturbation method

1. Introduction

The postbuckling response of a moderately thick plate subjected to combined axial loads and thermal loading is current interest to engineers engaged in civil and other structural engineering practice. These plates are often supported by an elastic medium and may have significant and unavoidable initial geometrical imperfections. Therefore, there is a need to understand the thermomechanical buckling and postbuckling behavior of imperfect moderately thick plates resting on elastic foundations.

Although considerable literature has been devoted to the postbuckling and thermal postbuckling analyses of isotropic and anisotropic thin plates subjected to either mechanical loads or thermal loading, published literature on postbuckling or thermal postbuckling of thick plates is very limited and addresses only perfect plates. Thermal buckling loads for initially stressed transversely isotropic and antisymmetric cross-ply laminated thick plates were evaluated using the Galerkin method by Chen *et al.* (1982) and by Yang and Shieh (1988). Thermal buckling analyses of composite laminated thick plates subjected to uniform or nonuniform temperature loading have been made by Tauchert (1987). Sun and Hsu (1990) and Chen *et al.* (1991).

Recently, Librescu and Souza (1993) analyzed postbuckling of an imperfect, shear-deformable,

† Professor

transversely isotropic plate under combined thermal and compressive edge loading.

For elastic foundations, the simplest of which is that of Winkler model, which assumes that the surface displacement of the elastic foundation at every point is directly proportional to the load applied at that point and completely independent of the load applied at any other point, even in its neighbourhood. The Winkler model does not accurately represent the characteristics of many practical foundations, this leads to the development of so-called two-parameter model. Ariman (1969) studied the compressive buckling of a moderately thick plate on a Winkler elastic foundation. Raju and Rao (1988) calculated the thermal postbuckling response of a thin isotropic square plate resting on a Winkler elastic foundation by the finite element method. Dumir (1988) analyzed the thermal postbuckling of a thin isotropic rectangular plate resting on a Pasternak-type elastic foundation using the Galerkin method, but his numerical results were only for Winkler elastic foundation case.

More recently, Shen (1995a, 1995b, 1995c) analyzed the postbuckling of perfect and imperfect, isotropic and anisotropic, thin and moderately thick plates resting on two-parameter elastic foundations, from which results for Winkler elastic foundations follow as a limiting case. To the author's knowledge, there are no research works dealing with the postbuckling of moderately thick plates under combined axial and thermal loading and resting on two-parameter elastic foundations.

A postbuckling analysis of perfect and imperfect, moderately thick plates under compressive loads or thermal loading has been presented by Shen(1990) and Shen and Zhu (1995) using a deflection-type perturbation technique. This work is extended here to the case of perfect and imperfect, moderately thick plates subjected to combined axial loads and uniform temperature loading and resting on two-parameter elastic foundations. The present analysis is based on the four assumptions of:

(1) Reissner-Mindlin plate theory, i.e. the first order shear deformation effects are put into consideration;
(2) uniform temperature distribution throughout the plate thickness;
(3) the longitudinal edges are immovable and;
(4) the material properties being independent of temperature.

The initial geometrical imperfection of the plate is taken into account but, for simplicity, its form is taken as the buckling mode of the plate.

2. Analytical formulation

Consider a moderately thick rectangular plate of length a, width b and thickness t which is subjected to uniaxial compression P_x and a uniform temperature rise T_0 and rests on a two-parameter elastic foundation. Let $\overline{U}$, $\overline{V}$ and $\overline{W}$ be the plate displacements parallel to a right-hand set of axes (X, Y, Z), where X is longitudinal and Z is perpendicular to the plate. Then the load-displacement relationship of the foundation is assumed to be $p=\overline{K}_1\overline{W}-\overline{K}_2\nabla^2\overline{W}$, where p is the force per unit area, $\overline{K}_1$ is the Winkler foundation stiffness and $\overline{K}_2$ is a constant showing the effect of the shear interactions of the vertical elements, and ∇^2 is the Laplace orerator in X and Y. Denoting the initial deflection by $\overline{W}^*$ (X, Y), let $\overline{W}$ (X, Y) be the additional deflection, and $\overline{F}$ (X, Y) be the stress function for the stress resultants, so that $N_x=\overline{F}_{,yy}$, $N_y=\overline{F}_{,xx}$ and $N_{xy}=-\overline{F}_{,xy}$.

From the Reissner-Mindlin plate theory considering the first order shear deformation effect, including the plate-foundation interaction and thermal effect, the governing differential equations are

$$D\nabla^4\overline{W}+\nabla^2 M^T=\left(1-\frac{D}{\kappa^2 Gt}\nabla^2\right)[L(\overline{W}+\overline{W}^*,\ \overline{F})-(\overline{K}_1\overline{W}-\overline{K}_2\nabla^2\overline{W})] \tag{1}$$

$$\nabla^4\overline{F}+(1-\nu)\nabla^2 N^T=-\frac{1}{2}Et\,L(\overline{W}+2\overline{W}^*,\ \overline{W}) \tag{2}$$

where

$$\nabla^4=\frac{\partial^4}{\partial X^4}+2\frac{\partial^4}{\partial X^2\partial Y^2}+\frac{\partial^4}{\partial Y^4}$$

$$\nabla^2=\frac{\partial^2}{\partial X^2}+\frac{\partial^2}{\partial Y^2}$$

$$L(\,)=\frac{\partial^2}{\partial X^2}\frac{\partial^2}{\partial Y^2}-2\frac{\partial^2}{\partial X\partial Y}\frac{\partial^2}{\partial X\partial Y}+\frac{\partial^2}{\partial Y^2}\frac{\partial^2}{\partial X^2}$$

in which D is flexural rigidity and $D=Et^3/12(1-\nu^2)$. E is Young's modulus, G is the shear modulus and ν is Poisson's ratio. Also κ^2 is the shear factor, which accounts the non-uniformly of the shear strain distribution through the plate thickness, and for Reissner plate theory $\kappa^2=5/6$ while for Mindlin plate theory $\kappa^2=\pi^2/12$.

The thermal force and moment are defined by

$$(N^T,\ M^T)=\frac{E\alpha}{1-\nu}\int_{-t/2}^{t/2}(1,\ Z)T_0\,dZ \tag{3}$$

in which α is thermal expansion coefficient for a plate. Because of Eq. (3) it is noted that the thermal moment $M^T=0$ and $\nabla^2 N^T=0$.

The unit end-shortening relationships are

$$\begin{aligned}\frac{\Delta_x}{a}&=-\frac{1}{abt}\int_{-t/2}^{t/2}\int_o^b\int_o^a\frac{\partial\overline{U}}{\partial X}dXdYdZ\\&=-\frac{1}{ab}\int_o^b\int_o^a\left[\frac{1}{Et}\left(\frac{\partial^2\overline{F}}{\partial Y^2}-\nu\frac{\partial^2\overline{F}}{\partial X^2}\right)-\frac{1}{2}\left(\frac{\partial\overline{W}}{\partial X}\right)^2-\frac{\partial\overline{W}}{\partial X}\frac{\partial\overline{W}^*}{\partial X}+\frac{1}{Et}(1-\nu)N^T\right]dXdY\end{aligned} \tag{4a}$$

$$\begin{aligned}\frac{\Delta_y}{b}&=-\frac{1}{abt}\int_{-t/2}^{t/2}\int_o^a\int_o^b\frac{\partial\overline{V}}{\partial Y}dYdXdZ\\&=-\frac{1}{ab}\int_o^a\int_o^b\left[\frac{1}{Et}\left(\frac{\partial^2\overline{F}}{\partial X^2}-\nu\frac{\partial^2\overline{F}}{\partial Y^2}\right)-\frac{1}{2}\left(\frac{\partial\overline{W}}{\partial Y}\right)^2-\frac{\partial\overline{W}}{\partial Y}\frac{\partial\overline{W}^*}{\partial Y}+\frac{1}{Et}(1-\nu)N^T\right]dYdX\end{aligned} \tag{4b}$$

All the edges are assumed to be simply supported and the longitudinal edges are restrained against expansion in the Y-direction (immovable), so the boundary conditions are

$$X=0,\ a;\ \overline{W}=0 \tag{5a}$$

$$N_{xy}=0,\ \overline{M}_x=0 \tag{5b}$$

$$\int_0^b N_x dY+\sigma_x b\,t=0 \tag{5c}$$

$$Y=0,\ b;\ \overline{W}=0 \tag{5d}$$

$$N_{xy}=0,\ \overline{M}_y=0 \tag{5e}$$

$$\overline{V}=0 \tag{5f}$$

where σ_x is the average axial stress and $\overline{M}_x$ and $\overline{M}_y$ are, respectively, the bending moments per unit width and per unit length of the plate.

Eqs. (1)~(5) are the governing equations describing the required large deflection postbuckling response of the plate.

3. Analytical method and asymptotic solutions

Introducing the dimensionless quantities (in which the alternative forms k_1 and k_2 are not needed until the numerical examples are considered)

$$\begin{aligned}
&x=\pi X/a,\ y=\pi Y/b,\ \beta=a/b,\ \gamma=\pi^2 D/\kappa^2 a^2 Gt,\\
&(W,\ W^*)=(\overline{W},\ \overline{W}^*)\sqrt{12(1-\nu^2)/t},\ F=\overline{F}/D,\\
&(M_x,\ M_y)=(\overline{M}_x,\ \overline{M}_y)a^2\sqrt{12(1-\nu^2)}/\pi^2 Dt,\\
&(K_1,\ k_1)=(a^4,\ b^4)\overline{K}_1/\pi^4 D,\ (K_2,\ k_2)=(a^2,\ b^2)\overline{K}_2/\pi^2 D,\\
&\lambda_T=12(1-\nu^2)b^2\alpha T_0/\pi^2 t^2,\ \lambda_x=\sigma_x b^2 t/4\pi^2 D\\
&(\delta_x,\ \delta_y)=(\Delta_x/a,\ \Delta_y/b)12(1-\nu^2)b^2/4\pi^2/4\pi^2 t^2
\end{aligned} \tag{6}$$

enables the nonlinear Eqs. (1) and (2) to be written in dimensionless form as

$$\overline{\nabla}^4 W+(1-\gamma\overline{\nabla}^2)(K_1 W-K_2\overline{\nabla}^2 W)=\beta^2(1-\gamma\overline{\nabla}^2)L(W+W^*,\ F) \tag{7}$$

$$\overline{\nabla}^4 F=-\frac{1}{2}\beta^2 L(W+2W^*,\ W) \tag{8}$$

where

$$\overline{\nabla}^4=\frac{\partial^4}{\partial x^4}+2\beta^2\frac{\partial^4}{\partial x^2\partial y^2}+\beta^4\frac{\partial^4}{\partial y^4}$$

$$\overline{\nabla}^2=\frac{\partial^2}{\partial x^2}+\beta^2\frac{\partial^2}{\partial y^2}$$

$$L(\,)=\frac{\partial^2}{\partial x^2}\frac{\partial^2}{\partial y^2}-2\frac{\partial^2}{\partial x\partial y}\frac{\partial^2}{\partial x\partial y}+\frac{\partial^2}{\partial y^2}\frac{\partial^2}{\partial x^2}$$

and the unit end-shortening relationships become

$$\delta_x=-\frac{1}{4\pi^2\beta^2}\int_0^\pi\int_0^\pi\left[\left(\beta^2\frac{\partial^2 F}{\partial y^2}-\nu\frac{\partial^2 F}{\partial x^2}\right)-\frac{1}{2}\left(\frac{\partial W}{\partial x}\right)^2-\frac{\partial W}{\partial x}\frac{\partial W^*}{\partial x}+\lambda_T\beta^2\right]dxdy \tag{9a}$$

$$\delta_y=-\frac{1}{4\pi^2\beta^2}\int_0^\pi\int_0^\pi\left[\left(\frac{\partial^2 F}{\partial x^2}-\nu\beta^2\frac{\partial^2 F}{\partial y^2}\right)-\frac{1}{2}\beta^2\left(\frac{\partial W}{\partial y}\right)^2-\beta^2\frac{\partial W}{\partial y}\frac{\partial W^*}{\partial y}+\lambda_T\beta^2\right]dydx \tag{9b}$$

(Note that Eqs. (7) and (8) are identical to those of moderately thick plates under pure axial compression and resting on two-parameter elastic foundations (Shen 1995c), but that Eq. (9) contains terms in λ_T.) The boundary conditions of Eq. (5) become

$$x=0,\ \pi;\ W=0, \tag{10a}$$

$$F_{,xy}=0,\ M_x=0 \tag{10b}$$

$$\frac{1}{\pi}\int_0^\pi\beta^2\frac{\partial^2 F}{\partial y^2}\,\mathrm{d}y+4\lambda_x\beta^2=0 \tag{10c}$$

$$y=0,\ \pi;\ W=0, \tag{10d}$$

$$F_{,xy}=0,\ M_y=0 \tag{10e}$$

$$\delta_y=0 \tag{10f}$$

To construct an asymptotic solution for the moderately thick plate, the additional deflection and stress functions in Eqs. (7) and (8) are taken as the preturbation expansions

$$W(x,\ y,\ \varepsilon)=\sum_{j=1}\varepsilon^j w_j(x,\ y);\ F(x,\ y,\ \varepsilon)=\sum_{j=0}\varepsilon^j f_j(x,\ y) \tag{11}$$

where ε is a small perturbation parameter, and the first term of $w_j\ (x,\ y)$ is assumed to have the form

$$w_1(x,\ y)=A_{11}^{(1)}\ \sin mx\sin ny \tag{12}$$

The initial geometrical imperfection is assumed to have a similar form to $w_1(x,\ y)$, i.e.

$$W^*(x,\ y,\ \varepsilon)=\varepsilon A_{11}^*\ \sin mx\ \sin ny=\varepsilon\mu A_{11}^{(1)}\ \sin mx\ \sin ny \tag{13}$$

where the imperfection parameter $\mu=A_{11}^*/A_{11}^{(1)}$

Substituting Eq. (11) into Eqs. (7) and (8), we get a system of perturbation equations. By using Eqs. (12) and (13) to solve these perturbation equations of each order, the amplitudes in terms of $w_j(x,\ y)$ and $f_j(x,\ y)$ can be determined step by step, and hence the asymptotic solutions are obtained as

$$W=\varepsilon[A_{11}^{(1)}\ \sin mx\ \sin ny]+\varepsilon^3[A_{13}^{(3)}\ \sin mx\ \sin 3ny+A_{31}^{(3)}\ \sin 3mx\ \sin ny]+O(\varepsilon^4) \tag{14}$$

$$\begin{aligned}F=&-B_{00}^{(0)}\frac{y^2}{2}-b_{00}^{(0)}\frac{x^2}{2}+\varepsilon^2[-B_{00}^{(2)}\frac{y^2}{2}-b_{00}^{(2)}\frac{x^2}{2}+B_{20}^{(2)}\cos 2mx\\&+B_{02}^{(2)}\cos 2ny]+\varepsilon^4[-B_{00}^{(4)}\frac{y^2}{2}-b_{00}^{(4)}\frac{x^2}{2}+B_{20}^{(4)}\cos 2mx\\&+B_{02}^{(4)}\ \cos 2ny+B_{22}^{(4)}\ \cos 2mx\ \cos 2ny+B_{40}^{(4)}\ \cos 4mx+B_{04}^{(4)}\ \cos 4ny\\&+B_{24}^{(4)}\ \cos 2mx\ \cos 4ny+B_{42}^{(4)}\ \cos 4mx\ \cos 2ny]+O(\varepsilon^5)\end{aligned} \tag{15}$$

It has been shown by Shen (1990, 1995c) and Shen and Zhu (1995) that all coefficients in Eqs.(14) and (15) are related and can be written as function of $A_{11}^{(1)}$.

Next, substituting Eqs. (14) and (15) into boundary conditions Eqs. (10c) and (10f) enable the interactive postbuckling equilibrium path to be written as

$$\frac{\lambda_x}{\lambda_{cr}^a}+\frac{\lambda_T}{\lambda_{cr}^T}=S_0+S_2 W_m^2+S_4 W_m^4+\cdots \tag{16}$$

in which W_m is the dimensionless form of the maximum deflection of the plate, which is assumed to be at the point $(x, y)=(\pi/2m, \pi/2n)$, and λ_{cr}^a and λ_{cr}^T are the critical values of the non-dimensional uniaxial compressive stress and thermal stress, respectively, such that

$$\lambda_{cr}^a=\frac{\Theta_1}{4\beta^2(m^2+\nu n^2\beta^2)};\ \lambda_{cr}^T=\frac{\Theta_1}{n^2\beta^4} \tag{17}$$

where the Θ_1, S_0, S_2, S_4, δ_0, δ_2, δ_4, and δ_i, of Eqs. (18)~(21) are given in detail in the Appendix.

From Eq. (17), equations for the critical value of compressive load P_{cr} or temperature rise T_{cr} can easily be found. Then substituting $\lambda_T/\lambda_{cr}^T=T_0/T_{cr}$ in Eq. (16) for the initilly heated plate case gives the postbuckling equilibrium path as

$$\lambda_x=\frac{\Theta_1}{4\beta^2(m^2+\nu n^2\beta^2)}\left[S_0+S_2 W_m^2+S_4 W_m^4-\frac{T_0}{T_{cr}}\right] \tag{18}$$

and

$$\delta_x=\delta_0\lambda_x+\delta_2 W_m^2+\delta_4 W_m^4-\delta_i\frac{T_0}{T_{cr}} \tag{19}$$

Similarly, substituting $\lambda_x/\lambda_{cr}^a=P_x/P_{cr}$ in Eq. (16) for the initially compressed plate case gives thermal postbuckling equilibrium path as

$$\lambda_T=\frac{\Theta_1}{n^2\beta^4}\left[S_0+S_2 W_m^2+S_4 W_m^4-\frac{P_x}{P_{cr}}\right] \tag{20}$$

and

$$\delta_x=\delta_0\lambda_T+\delta_2 W_m^2+\delta_4 W_m^4+\delta_i\frac{P_x}{P_{cr}} \tag{21}$$

Eqs. (16)-(21) can be employed to obtain numerical results for the postbuckling load-deflection or load-shortening curves of moderately thick plates under uniform thermal loading combined with uniaxial compression, specially for the two cases of:

(1) thermal postbuckling of initially compressed thick plates; and

(2) postbuckling of initially heated thick plates.

In the second case, if the initial thermal stress is zero, Eqs. (18)-(19) are reduced to the equations for postbuckling equilibrium path of moderately thick plates loaded in uniaxial compression with longitudinal edges restrained and resting on two-parameter elastic foundations. The buckling load of perfect plates can also readily be obtained numerically, by setting $\mu=0$ (or $\overline{W}^*t=0$), while taking $W_m=0$ (or $\overline{W}/t=0$). In all cases, the minimum buckling load is determined by considering Eq. (18) or (20) for various values of the buckling mode (m, n), which determine the number of half-waves in the X-and Y-directions. As expected, there are three special cases:

(1) if $K_2=0.0$, Eqs. (16)-(21) are valid for the thermomechanical postbuckling of moderately thick plates resting on Winkler elastic foundations;
(2) if $K_1=K_2=0.0$, Eqs. (16)-(21) reduce to thermomechanical postbuckling equilibrium paths of moderately thick plates without any elastic foundations and;
(3) if the plate is thinner enough, then γ approaches to zero, Eqs. (16)-(21) are brought into a form suitable for the solutions of von Karman plate.

4. Numerical results and discussion

A postbuckling analysis has been presented for moderately thick plates under combined axial and thermal loads and resting on two-parameter elastic foundations. A number of examples were solved to illustrate their application to the performance of perfect and imperfect, moderately

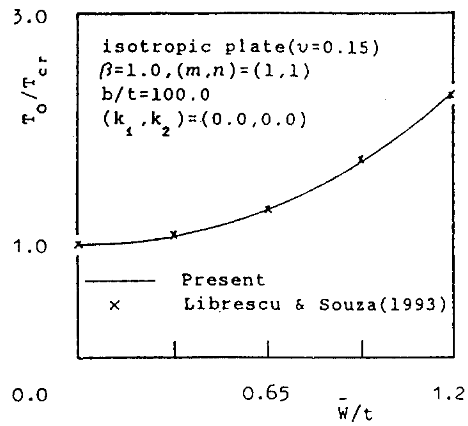


Fig. 1 Comparisons of thermal postbuckling load-deflection curves of a single-layer isotropic square plate without an elastic foundation.

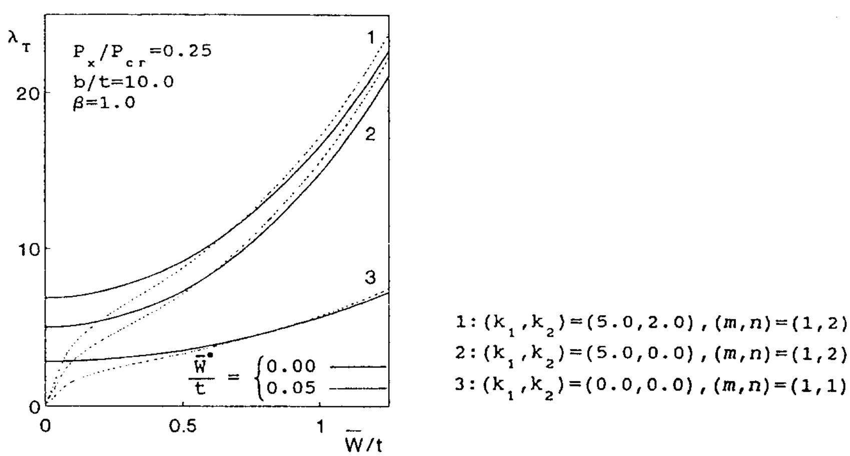


Fig. 2 Thermal postbuckling load-deflection curves of intially compressed moderately thick plates with and without elastic foundations.

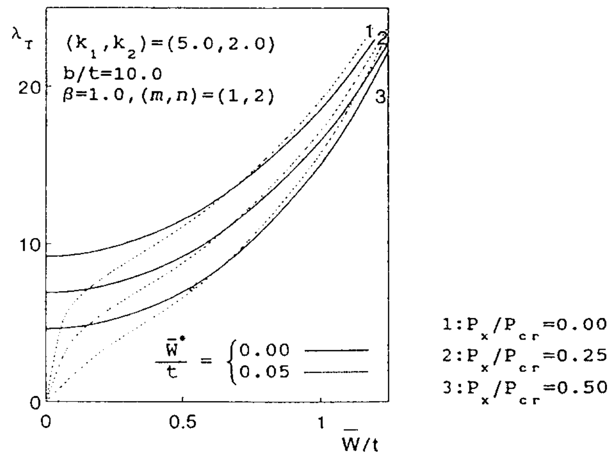


Fig. 3 Effect of initial compressive load proportion P_x/P_{cr} on the thermal postbuckling of moderately thick plates on two-parameter elastic foundations.

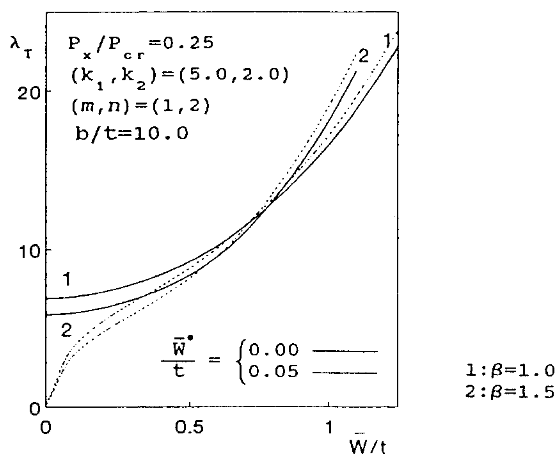


Fig. 4 Effect of plate aspect ratio β on the thermal postbuckling of initially compressed moderately thick plates on two-parameter elastic foundations.

thick rectangular plates resting on Winkler or two-parameter elastic foundations. Throughout these numerical illustrations $\nu=0.3$, $\alpha=1.0\times10^{-6}/℃$ and the transverse shear correction factor was considered $\kappa^2=\pi^2/12$.

To validate the present method, the thermal postbuckling load-deflection curve of an isotropic square plate with its longitudinal edges restrained is compared in Fig. 1 with results given by Librescu and Souza(1993), from which the good agreement is apparent.

Fig. 2 gives the thermal postbuckling load-deflection curves of initially compressed moderately thick plates with $b/t=10.0$ either without foundations or resting on Winkler or two-parameter elastic foundations. The stiffnesses are $(k_1, k_2)=(5.0, 2.0)$ for the two-parameter elastic foundation, $(k_1, k_2)=(5.0, 0.0)$ for the Winkler elastic foundation, and $(k_1, k_2)=(0.0, 0.0)$ for the plate without an elastic foundation. It can be seen that the elastic foundation increases the thermal buckling load and it has a singnificant effect on thermal postbuckling behavior. The buckling modes

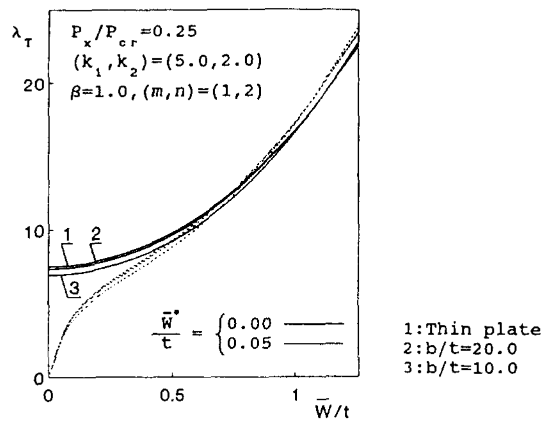


Fig. 5 Effect of transverse shear deformations on the thermal postbuckling of initially compressed moderately thick plates on two-parameter elastic foundations.

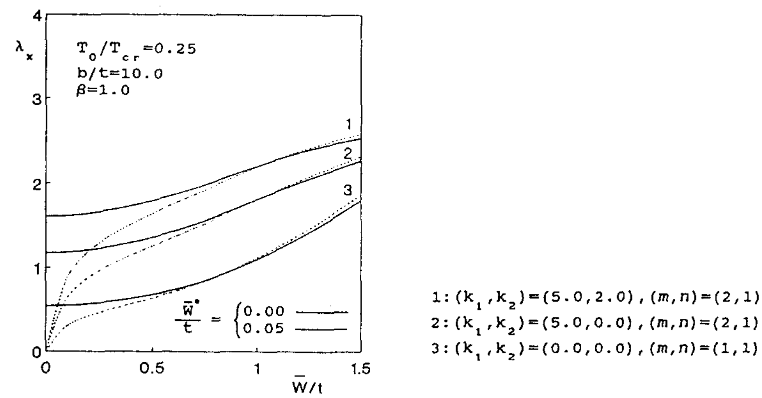


Fig. 6 Postbuckling load deflection curves of initially heated moderately thick plates with and without elastic foundations.

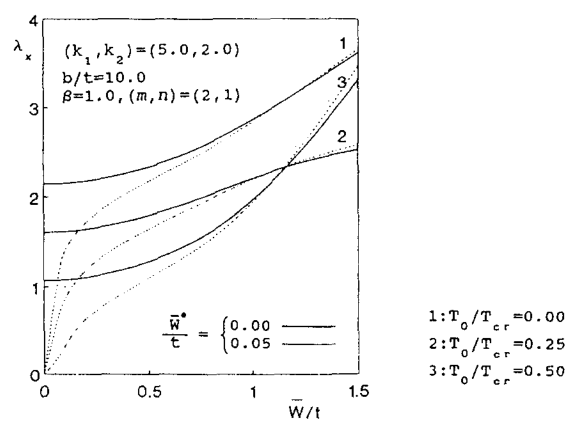


Fig. 7 Effect of initial thermal load proportion T_0/T_{cr} on the postbuckling of moderately thick plates on two-parameter elastic foundations.

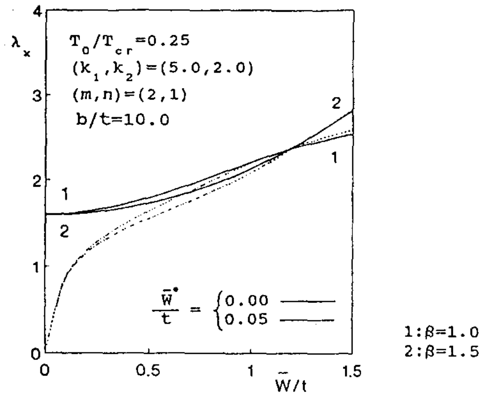


Fig. 8 Effect of plate aspect ratio β on the postbuckling of initially heated moderately thick plates on two-parameter elastic foundations.

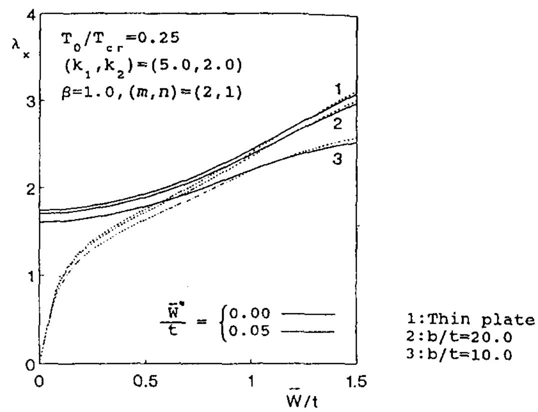
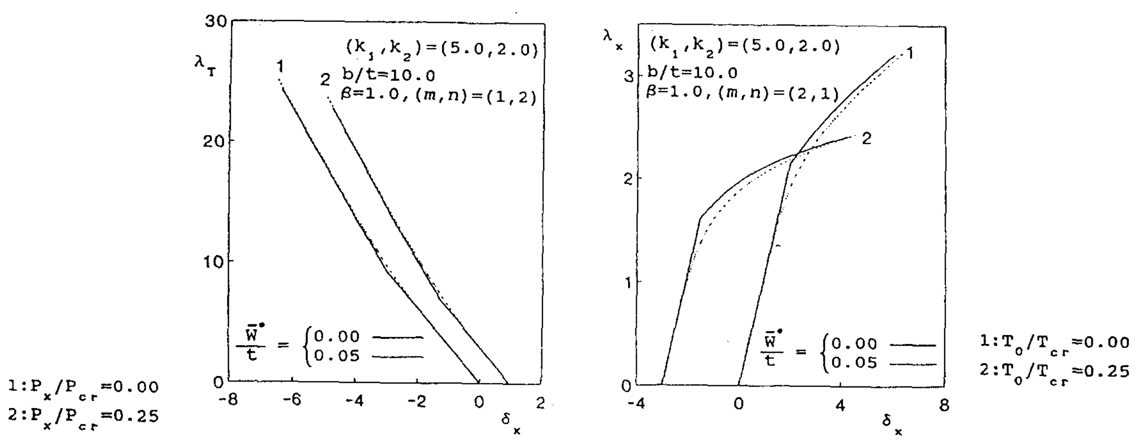


Fig. 9 Effect of transverse shear deformations on the postbuckling of initially heated moderately thick plates on two-parameter elastic foundations.



(a) initially compressed (b) initially heated

Fig. 10 Postbuckling load-shortening curves of initially compressed and initially heated moderately thick plates on two-parameter elastic foundations.

can also be seen to change as the foundation stiffness is increased, e.g. $(m, n)=(1, 2)$ for the two-parameter and Winkler elastic foundation cases, whereas $(m, n)=(1, 1)$ for the foundationless case.

Fig. 3 shows the thermal load-deflection curves of moderately thick square plates with $b/t=10.0$ for the different values of initial compressive load P_x shown, when the plates are supported by two-parameter elastic foundations. Clearly the initial compressive stress decreases the thermal buckling load and affects the postbuckling response singificantly.

Fig. 4 shows the effect of plate aspect ratio β $(=1.0, 1.5)$ on the thermal postbuckling response of initially compressed moderately thick plates with $b/t=10.0$ resting on two-parameter elastic foundations. As expected, these results show that the thermal buckling load and postbuckling strength are increased by decreasing the plate aspect ratio β.

The effect of transverse shear deformation on thermal postbuckling response of initially compressed moderately thick plates resting on two-parameter elastic foundations is shown in Fig. 5. It is found that the thermal buckling load of thick plates with $b/t=10.0$ is 7.8% lower than that of thin plates. It can be seen that, like moderately thick plates without any foundations (see Shen and Zhu 1995), in the postbuckling range $(\overline{W}/t<1.0)$ the deflection of thick plates is larger than that predicted by thin plate theory, and in the deep postbuckling range the effect of transverse shear deformation is insignificant. In Figs. 4 and 5 (as in Fig. 2), the compressive load is 25% of P_{cr}, and in Figs. 3-5 the foundation stiffness is characterized by $(k_1, k_2)=(5.0, 2.0)$.

Figs. 6-9 are postbuckling results for initially heated plates analogous to the thermal postbuckling results of Figs. 2-5. Note that now the compressive postbuckling equilibrium path becomes flatter and flatter with the increase in foundation stiffness. In contrast, the effect of transverse shear deformation is more significant in the deep postbuckling range (see Fig. 9).

Fig. 10 shows the load-shortening curves of initially compressed and initially heated moderately thick plates resting on two-parameter elastic foundation with $(k_1, k_2)=(5.0, 2.0)$. The results show that the initially compressed plate exhibits negative end-shortening, i.e. extension, in the thermal postbuckling region.

Postbuckling load-deflection (or load-shortening) curves for initially compressed and initially heated imperfect moderately thick plates have been plotted, along with the perfect plate results, in Figs. 2-10. The imperfect curves show that the effect of initial geometrical imperfection on moderately thick plates of initial geometrical imperfection on moderately thick plates under combined axial and thermal loading is substantial, as was already known to be the case (see Shen, 1990, 1995c) for moderately thick plates under pure axial compression with or without elastic foundations.

5. Conclusions

The postbuckling of moderately thick plates subjected to uniaxial compression combined with a uniform temperature rise and resting on two-parameter elastic foundations has been studied by a perturbation method. The two cases of thermal postbuckling of initially compressed plates and of compressive postbuckling of initially heated plates have been considered. The numerical examples presented principally relate to the performance of perfect and imperfect, moderately thick plates resting on Winkler or two-parameter elastic foundations. They show that the characteristics of postbuckling are significantly influenced by foundation stiffness, initial geometricl imperfection and the amount of initial compressive or thermal load present for, respectively, thermal and compressive buckling. In contrast plate aspect ratio β and the plate transverse shear deformation have rather less effect.

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Appendix

In Eqs. (16)-(21)

$$S_0 = 1/(1+\mu),\ S_2 = \frac{1}{16}\Theta_2(1+2\mu)/\Theta_1,$$

$$S_4 = \frac{1}{256}(m^2 + v n^2\beta^2)[1+\gamma(m^2+n^2\beta^2)](C_2 - C_4)/\Theta_1 \tag{22a}$$

where

$$\Theta_1 = \Theta_{11}/[1+\gamma(m^2+n^2\beta^2)]$$
$$\Theta_2 = (m^4 + 3n^4\beta^4)$$
$$\Theta_{11} = (m^2+n^2\beta^2)^2 + [K_1 + K_2(m^2+n^2\beta^2)][1+\gamma(m^2+n^2\beta^2)]$$
$$C_2 = 2(1+\mu)^2(1+2\mu)^2\Theta_2[m^4[1+\gamma(m^2+9n^2\beta^2)]/g_{13} + n^4\beta^4[1+\gamma(9m^2+n^2\beta^2)]/g_{31}]$$
$$C_4 = (1+\mu)(1+2\mu)[2(1+\mu)^2+(1+2\mu)][m^8[1+\gamma(m^2+9n^2\beta^2)]/g_{13}$$

$$+n^8\beta^8[1+\gamma(9m^2+n^2\beta^2)]/g_{31}]$$
$$g_{13}=\Theta_{13}(m^2+\nu n^2\beta^2)[1+\gamma(m^2+n^2\beta^2)](1+\mu)-\Theta_{11}C_{13}[1+\gamma(m^2+9n^2\beta^2)]$$
$$g_{31}=\Theta_{31}(m^2+\nu n^2\beta^2)[1+\gamma(m^2+n^2\beta^2)](1+\mu)-\Theta_{11}C_{31}[1+\gamma(9m^2+n^2\beta^2)]$$
$$\Theta_{13}=(m^2+9n^2\beta^2)^2+[K_1+K_2(m^2+9n^2\beta^2)]\,[1+\gamma(m^2+9n^2\beta^2)]$$
$$\Theta_{31}=(9m^2+n^2\beta^2)^2+[K_1+K_2(9m^2+n^2\beta^2)]\,[1+\gamma(9m^2+n^2\beta^2)]$$
$$\delta_2=\frac{1}{32}\frac{(m^2+\nu n^2\beta^2)}{\beta^2}(1+2\mu)$$
$$\delta_4=\frac{1}{256}(1+\mu)^2(1+2\mu)^2\frac{(m^2+\nu n^2\beta^2)^2}{\beta^2}[1+\gamma(m^2+n^2\beta^2)]$$
$$\times[m^4[1+\gamma(m^2+9n^2\beta^2)/g_{13}+n^4\beta^4[1+\gamma(9m^2+n^2\beta^2)]/g_{31}] \quad (22b)$$

in above equations, for the case of initially compressed plates

$$C_{13}=(m^2+9\nu n^2\beta^2)+8\left[1-\frac{P_x}{P_{cr}}(1+\mu)\right]m^2$$
$$C_{31}=(9m^2+\nu n^2\beta^2)-8\left[1-\frac{P_x}{P_{cr}}(1+\mu)\right]m^2$$
$$\delta_0=-\frac{1}{4}(1+\nu)\,\lambda_T$$
$$\delta_i=\frac{(1-\nu^2)\Theta_1}{4\beta^2(m^2+\nu n^2\beta^2)} \quad (22c)$$

and for the case of initially heated plates

$$C_{13}=(m^2+9\nu n^2\beta^2)-\frac{T_o}{T_{cr}}(1+\mu)\;[(m^2+9\nu n^2\beta^2)-9(m^2+\nu n^2\beta^2)]$$
$$C_{31}=(9m^2+\nu n^2\beta^2)-\frac{T_o}{T_{cr}}(1+\mu)\;[(9m^2+\nu n^2\beta^2)-(m^2+\nu n^2\beta^2)]$$
$$\delta_0=(1-\nu^2)\lambda_x$$
$$\delta_i=\frac{(1+\nu)\Theta_1}{4n^2\beta^4} \quad (22d)$$

Notations

a, b	plate length and breadth
D	flexural rigidity for a plate
E	elastic modulus for a plate
$\bar{F}$, F	stress function and its dimensionless form
G	shear modulus for a plate
$\bar{K}_1$, K_1, k_1	Winkler elastic foundation stiffness and its two alternative dimensionless forms
$\bar{K}_2$, K_2, k_2	Pasternak elastic foundation stiffness and its two alternative dimensionless forms
t	thickness of a plate
$\bar{W}$, W	deflection of plate and its dimensionless form
$\bar{W}^*$, W^*	geometrical imperfection of plate and its dimensionless form
α	thermal expansion coefficient for a plate
β	aspect ratio of plate, $=a/b$
Δ_x, δ_x	end-shortening and its dimensionless form
ε	a small perturbation parameter

κ^2	shear factor for a moderately thick plate
λ_T	dimensionless form of thermal stress
λ_x	dimensionless form of compressive stress
μ	imperfection parameter
ν	Poisson's ratio
σ_x	average axial stress in the X-direction