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Performance of multi-storey structures with high damping rubber bearing base isolation systems

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Abstract. Base isolation, having quite simple contents, aims to protect the buildings from earthquake-induced damages by installing structural components having low horizontal stiffness between substructure and superstructure. In this study, an appropriate base isolation system for 2-D reinforced concrete frame is investigated. For different structural heights, the structural systems of 2, 3 and 4 bays are modeled by applying base isolation systems and results are compared with conventional structural systems. 1999 Marmara earthquake data is used for applying the model by time history method in SAP2000 package. Results of various parameters such as base shear force, structure drift ratio, structure period and superstructure acceleration are discussed for all models.

Keywords: base isolation; high damping rubber bearing; time history method; 1999 Marmara earthquake

1. Introduction

Seismic isolation technology has been applied to protect civil structures from earthquake damage for over two decades. Observed performance of this technology in several cases revealed the success of seismic isolation technique in reducing earthquake-induced forces (Kelly 1998, 1999a, Buckle and Mayes 1990, Jangid and Datta 1995, Stewart *et al.* 1999, Hino *et al.* 2008, Morgan and Mahin 2008). Many seismic isolation devices were developed to meet the increasing demand for practical applications (Naeim and Kelly 1999, Kunde and Jangid 2003). Among those well developed seismic isolation devices, the lead rubber bearing (LRB) (Turkington *et al.* 1988, Asher *et al.* 1997, Nagarajaiah and Sun 2000, 2001, Jangid 2007, Providakis 2008), friction pendulum system (FPS) (Lin and Hone 1993, Lee *et al.* 2003a, b, Wu *et al.* 2004, Wang 2005, Jangid 1997) and high damping rubber bearing (HDRB) (Amin *et al.* 2002, Fujita *et al.* 1990, Yoshida *et al.* 2004, Dall'Asta *et al.* 2006, Pan and Yang 1996) are major devices for practical use.

In a conventional structural design, the superstructure should be as light as possible and the base should be heavy. In addition to this, the structure should have the sufficient stiffness and ductility, and the natural period of the structure should be set properly as to be different from the dominant period of the expected earthquake. The fundamental requirement of a qualified design is through its high capacity of energy absorption to reduce effects of the earthquake. However, all these conditions may not always be met in practice. In order to increase the stiffness of the structure, the dimensions

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of the members or the amount of the reinforcement can be increased. Yet, in this case the ductility of the structure will decrease while the cost of the structure and weight of the superstructure will increase. In case of increasing the ductility of the structure, the strength and stiffness of the structure will also decrease. Therefore, it is a very complex and difficult task to ensure a good trade off between these requirements in a structure (Karabork 2001).

Besides, in a conventional structural design, the structures under the influence of severe earthquakes are constructed with the only principle of “not to collapse”. Situations such as collapsing of the walls, cracking of the bearing members of the structure and damage to the goods within the structure are not taken into consideration.

Vibration control systems that are endurable against earthquakes are based on the principle of reducing the effects of earthquake induced forces. In order to reduce the effects of earthquakes on structures, special structural members are used to absorb the energy and control the vibrations of the structure through seismic isolation. It was tried to protect the structure from the lateral effects of severe ground motions by placing structural members whose horizontal stiffness are rather low, between the base and the superstructure. The properties of these structural members set the natural frequency of the structure. This frequency is much lower than the dominant frequency of the ground movement and initial basic frequency of the structure (Karabork 2007, Ristic 1993).

In this study, non-linear analysis of base isolated structures with HDBR is investigated. 2-D structures with different bays and heights are tested by using HDBR with various mechanical properties.

2. High Damped Rubber Bearings (HDRB)

In 1982, a new material was developed by “Malaysian Rubber Producers Research Union” by increasing the damping feature of the natural rubber bearings. Damping was increased by adding carbon blocks and materials such as resin to the natural rubber bearings. When the damping rate reached to 100%, transformation rate is between 10%-20%. For shear transformation rates higher than 20%, the material exhibits non-linear behavior. In this case, structures show high stiffness and damping properties against the effects of less intensive earthquakes and/or wind. For greater shear transformation rates, energy absorption rate also increases since the rubber has the feature of crystallization. The damping of the HDBR is somewhere in between viscose damping and hysteric damping (Elmas *et al.* 2002, Skinner *et al.* 1993).

HDBR’s, as in layered rubber bearings, extend the period of the structure systems by means of lateral motions. They also have damping property, which helps them to dissipate the energy of the earthquake by acting like a damper.

3. Mechanical features of the HDBR

Rubber is capable of carrying a huge amount of pressure load and it accompanies one or more movements in case of a shear. Rubber has low shear modulus. For this reason, torsion flexibility of the rubber is decreased by placing steel plates into it, and consequently shearing stiffness increases. Rubber, produced at a few mm or cm thicknesses as a roll is cut as rings and steel plates of a few mm thicknesses are placed between those rings. The metal surfaces are burnished and glued for the

Fig. 1 High damping rubber bearing

steel plates to adhere well to the rubber. After placing the rubber and plates side by side in a steel container at desired amounts, the top and bottom layers being rubber, it is left at 135ºC temperature for 14 hours and rubber – steel composite bearing is produced. The rubber – steel composite bearing produced in this way is illustrated in Fig. 1.

Steel plates increase the vertical load bearing capacity of the HDRB, and resulting, the vertical stiffness becomes greater than horizontal stiffness. Horizontal stiffness depends on the number and thickness of the rubber plates. Increasing the number of plates decreases the stiffness. With the increase of the height, buckling may occur. Therefore, height should be kept limited (Soberon *et al.* 1996, Kelly 1999b, Akkar 1994).

Some difficulties in use of this kind of energy dissipating device arise from its complex dynamic behaviour, which makes it difficult to predict the behaviour of equipped structures accurately and to give design values (Dall'Asta 2008). More specifically, the material behaviour is strongly non-linear and both stiffness and damping properties vary with the amplitude of strain and also they depend on the strain rate (Lion 1997, Haupt and Sedlan 2001). Furthermore, the presence of filler added to natural rubber, makes the response of HDRB "strain history-dependent" and causes a transient behaviour in which stiffness and damping change remarkably. The phenomenon, usually known as the "Mullin effect" or "scragging", is a consequence of damage of the microstructure, which occurs during the process (Govindjee and Simo 1991).

Recent studies (Govindjee and Simo 1992) show that the transient response is related to the maximum strain experienced by material and is influenced by the strain rate. The initial properties of the material may however be recovered and the healing times depend on the considered material and on the temperature.

There are two basic methods for the solution of the system, one is linear and the other is non-linear. In this study, the non-linear solution is used. First, the system is defined as single-degree-of-freedom system and then the solution method is defined by writing it a matrix form for multi degree-of-freedom systems.

4. Movement relations of multi-storey structures in those base is isolated by using HDBRs

HDBR's exhibit non-linear behaviour and their superstructure behaviour are elastic. B-Spline comparison method is used for the solution of their movement relations (Karabork 2001). This

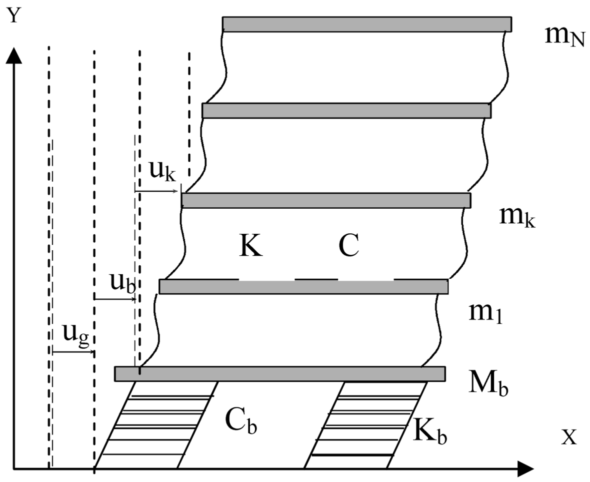

Fig. 2 Mathematical model of multi-storey structures in which base is isolated

method provides a solution algorithm by using different integral relations. Basic movement relations of an N-storey structure, whose base is isolated, can be described as the following. The mathematical model used is shown in Fig. 2 (Naeim 1999, Pan and Yang 1996, Karabork 2001).

$$M\ddot{u} + C\dot{u} + Ku = -Mr(\ddot{u}_b + \ddot{u}_g) \quad (1)$$

$$r^T M\ddot{u} + (r^T Mr + M_b)\ddot{u}_b + C_b\dot{u}_b + K_b u_b = -(r^T Mr + M_b)\ddot{u}_g \quad (2)$$

Where, M: Superstructure mass, C: Superstructure damping, K: Superstructure stiffness matrix, M_b: Mass of the base isolation system, C_b: damping of the base isolation system, K_b: Stiffness matrix of the base isolation system, r: influence matrix, and $\ddot{u}_g$: horizontal storey acceleration vector at the x direction.

For simplification of two-dimensional structure, it can be assumed that the storeys are rigid diaphragm and structure mass are concentrated mass at storey levels. With these assumptions and simplifications, mass and stiffness matrix of N-storey superstructure can be described as

$$K = \text{diag}[M_k] \qquad k = 1, 2, 3, \ldots, N \quad (3)$$

$$K = \begin{bmatrix} K_1 + K_2 & -K_2 & & & \\ -K_2 & K_2 + K_3 & -K_3 & & \\ & \ldots & \ldots & \ldots & \\ & & -K_{N-1} & K_{N-1} + K_N & -K_N \\ & & & -K_N & K_N \end{bmatrix} \quad (4)$$

Where, $[M_k]$ is the mass matrix and $[K_k]$ is the stiffness matrix for the storey k and

$$M_k = \begin{bmatrix} m_k & 0 & -y_{kc}m_k \\ 0 & m_k & x_{kc}m_k \\ -y_{kc}m_k & x_{kc}m_k & j_{kc}+(x_{kc}^2+y_{kc}^2)m_k \end{bmatrix} \quad (5)$$

$$K_k = \sum_i T_{ki}^T \begin{bmatrix} k_{kix} & 0 \\ 0 & k_{kiy} \end{bmatrix} T_{ki} \quad (6)$$

Where, M_k: storey mass, x_{kc} and y_{kc}: x and y coordinates of the mass center, J_{kc}: polar moment of inertia about the vertical direction of the center of mass k_{kix} and k_{kiy}: the horizontal stiffness in the x and y direction between the k–1 and k storeys; and T_{ki}: is the transformation matrix.

Similarly, mass, stiffness and damping matrices can be formed for the base isolation system, as shown below

$$M_b = \begin{bmatrix} m_b & 0 & -y_{bc}m_b \\ 0 & m_b & x_{bc}m_b \\ -y_{bc}m_b & x_{bc}m_b & j_{bc}+(x_{bc}^2+y_{bc}^2)m_b \end{bmatrix} \quad (7)$$

$$K_b = \sum_i T_{bi}^T \begin{bmatrix} K(u_{bi}, \dot{u}_{bi}) & 0 \\ 0 & K(u_{bi}, \dot{u}_{bi}) \end{bmatrix} T_{bi} \quad (8)$$

$$C_b = \sum_i T_{bi}^T \begin{bmatrix} C(u_{bi}, \dot{u}_{bi}) & 0 \\ 0 & C(u_{bi}, \dot{u}_{bi}) \end{bmatrix} T_{bi} \quad (9)$$

5. Analysis method for the HDRB at structural analysis program

In the SAP 2000 program, "Rapid non-linear Analysis Method" developed by Wilson is used for the non-linear analysis of the HDBR in the time-history method. This method is used for the defined limited number of linear members and for the analysis of the structural systems. SAP 2000 program defines the rubber bearings as non-linear members called Nllink (Computers & Structures 1984).

Dynamic balance relations can be constituted as

$$K_L u(t) + C\dot{u}(t) + M\ddot{u}(t) + r_N(t) = r(t) \quad (10)$$

Where, K_L: is the stiffness matrix of the members apart from Nllink elements; C: damping matrix, M: diagonal mass matrix; r_N: force vectors formed depending on the degree of freedom of the non-linear Nllink elements, $u, \dot{u}, \ddot{u}$: relative displacement, velocity and accelerations respectively; and r: the influence matrix.

In this analysis, one linear acting stiffness is defined for the degree of freedom of each non-linear element. The effective stiffness in the non-linear degree of freedom can change arbitrarily; yet, it is generally between the maximum non-linear stiffness and zero. Therefore, the general balance relations become

$$Ku(t) + C\dot{u}(t) + M\ddot{u}(t) = r(t) - [r_N(t) - K_N u(t)] \tag{11}$$

$$K = K_L + K_N \tag{12}$$

Where, K_L: is the stiffness of all the linear elements and Nllink elements at their linear degree of freedom and K_N: is the linear-effective stiffness matrix of all the non-linear degree of freedom.

During this analysis, Ritz-vector method is used. If balance relations in the modal formcan be written by using standard techniques, the relation

$$\Omega^2 a(t) + \Lambda\dot{a}(t) + I\ddot{a}(t) = q(t) - q_N(t) \tag{13}$$

is obtained. Here, Ω^2: is the diagonal matrix giving the structural frequency.

$$\Omega^2 = \Phi^2 K\Phi \tag{14}$$

Λ: Diagonal modal damping matrix.

$$\Lambda = \Phi^T C\Phi \tag{15}$$

I: Unit matrix.

$$I = \Phi^T M\Phi \tag{16}$$

$q(t)$: Modal load vector.

$$q(t) = \Phi^T r(t) \tag{17}$$

$q_N(t)$: Modal force vectors for non-linear elements.

$$q_N(t) = \Phi^T[r_N(t) - K_N u(t)] \tag{18}$$

$a(t)$: Modal displacement vector.

$$u(t) = \Phi a(t) \tag{19}$$

Φ: Mod patterns matrix.

The non-linear modal relations are solved at each stage of the analysis. Program assumes that the statements at the right side of the relation change linearly for each stage space. These relations are solved by closed form integration at each iteration. The iteration continues until reaching a convergence for the analysis. If any convergence cannot be obtained, the program tries to obtain it by dividing the time stages to sub-stages.

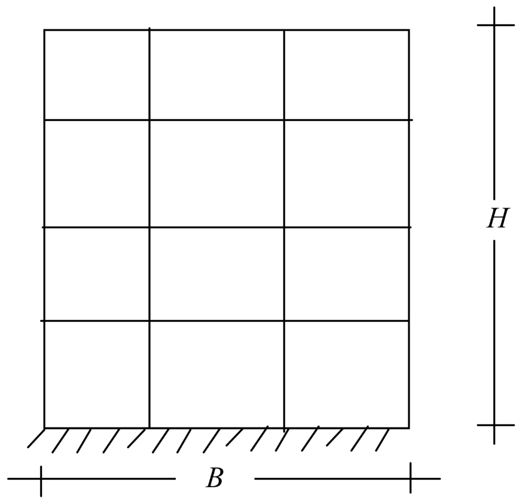

Fig. 3 2D frame model whose dynamic analysis is performed

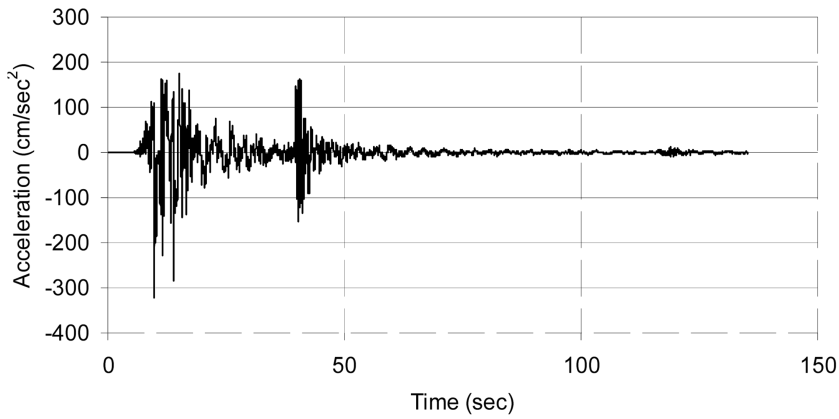

Fig. 4 The records of 17 August 1999 Marmara Earthquake Yarimca-Petkim N-S acceleration component

6. Numerical analysis

In this study, the appropriate base isolation system for 2 dimensional reinforced concrete framed structures is searched. For this purpose, all the models are analyzed with SAP 2000 program by using time – history analysis.

The base width of the considered frame structure is B = 12 m and it is modeled as having 2, 3 and 4 bays (Fig. 3). For the definition of the models, the column cross- sections are selected to be 40 × 40 cm, beam cross- sections 25 × 60 cm and storey heights 3 m. The models are prepared by changing the ratio of the structure height to the base width (Structure Selender) H/B. In the analysis, the records of 17 August 1999 Marmara earthquake Yarımca-Petkim N-S acceleration components are used (Fig. 4). By using 4 different base isolation systems, dynamic analyses are performed for those records with time-history analysis. HDRB behavior is accepted to be non-linear and the behaviour of superstructure is considered to be linear. The linear and non-linear physical properties of the used high- damped bearings are given in Table 1; and formulated models are given in Table 2.

In order to evaluate the results, the parameters given in Table 3 are considered.

As a result of the analysis, the maximum values for base shear force are obtained for (*H/B*) = 1, 2 and 3. The maximum relative displacement of the structure, structure periods and the maximum last storey acceleration of the structure are given in Figs. 5-16.

Table 1 Characteristic features of the HDRB

Stiffness	Vertical Stiffness (kN/m)	Initial Stiffness (kN/m)	Effective Stiffness (kN/m)	Yielding Force (kN)	Post Yield Stiffness Ratio	Mass (kg)
Low	1751.10^3	1.751	263	22,24	0,2	175,5
Mid	$1373.\ 10^3$	7.786	1.079	77,87	0,043	175,5

Table 2 Models formulated

H/B (B=12 m)	Number of Bays	Type of Structure	Model Name
1-2-3	2-3-4	Conventional Structure	A
		HDRB with low stiffness	B
		HDRB with middle stiffness	C
		HDRB with high stiffness	D

Table 3 Parameters investigated in the models

Base Shear Force	$V_{base\ max}$
Relative Displacements between the Top storey and Base Levels of the Structure	$(\delta_{top\ max} - \delta_{base\ max})$
Structure Period	T
Last Storey Acceleration of the Structure	$a_{top\ max}$

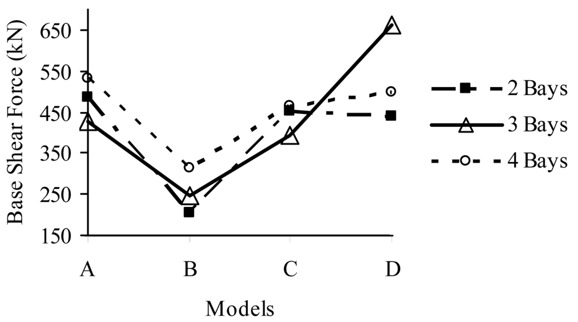

Fig. 5 The maximum base shear force for (*H/B*) = 1

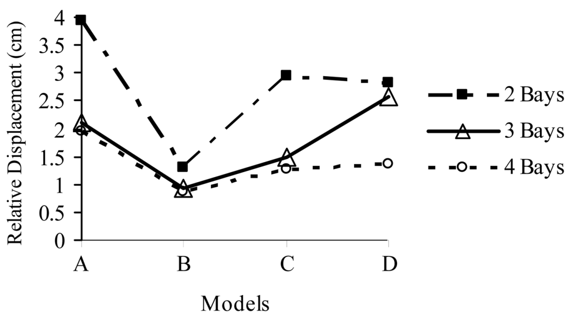

Fig. 6 The maximum relative displacement for (*H/B*) = 1

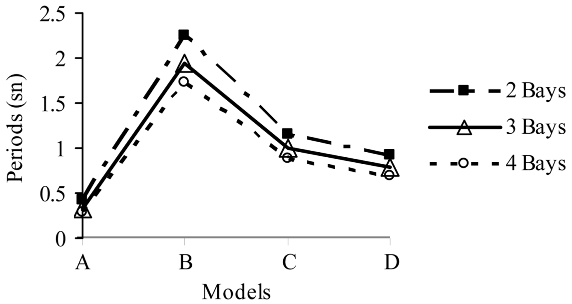

Fig. 7 Structure period for (*H/B*) = 1

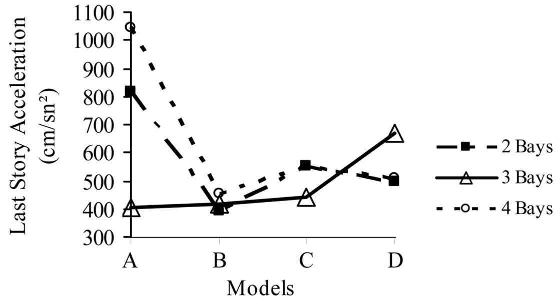

Fig. 8 The maximum last storey acceleration for (*H/B*) = 1

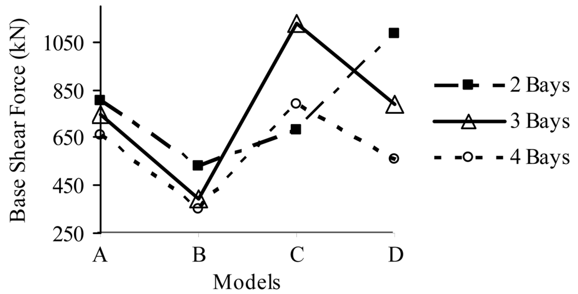

Fig. 9 The maximum base shear force for (*H/B*) = 2

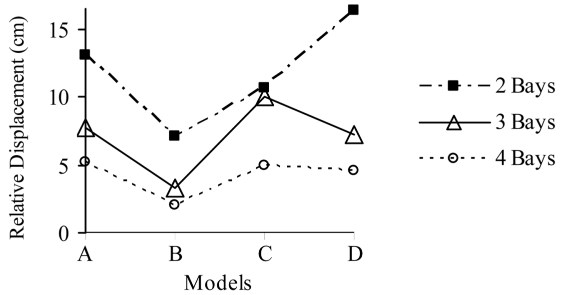

Fig. 10 The maximum relative displacement for (*H/B*) = 2

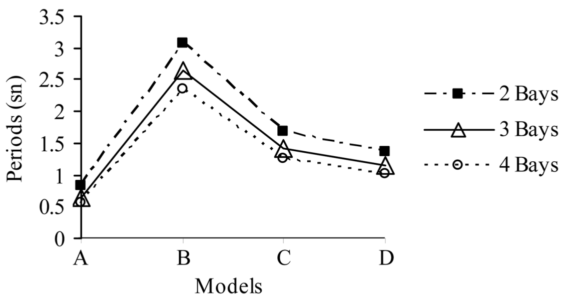

Fig. 11 Structure period for (*H/B*) = 2

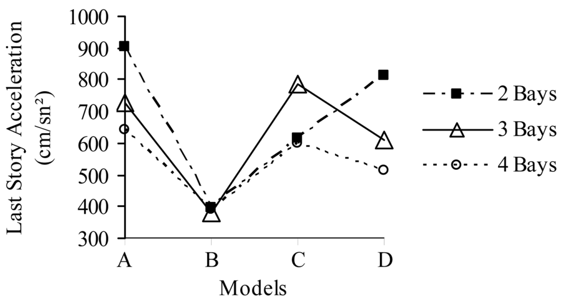

Fig. 12 The maximum last storey acceleration for (*H/B*) = 2

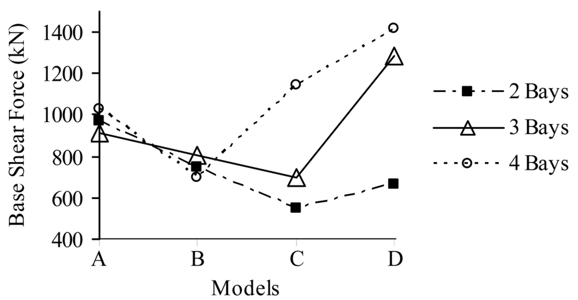

Fig. 13 The maximum base shear force for (*H/B*) = 3

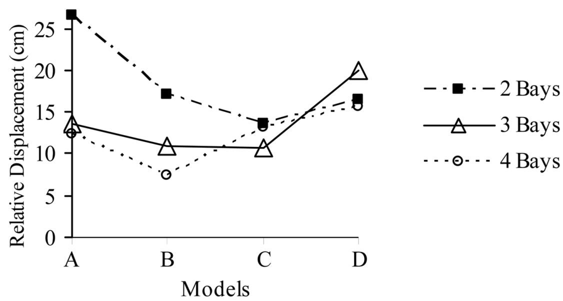

Fig. 14 The maximum relative displacement for (*H/B*) = 3

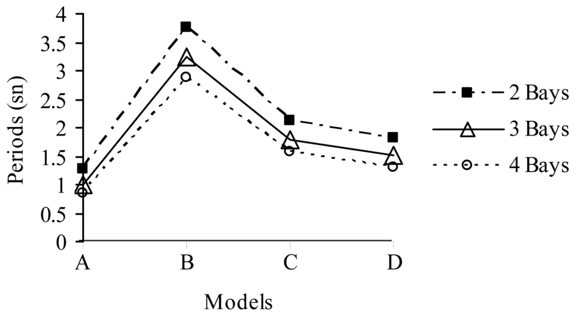

Fig. 15 Structure period for (*H/B*) = 3

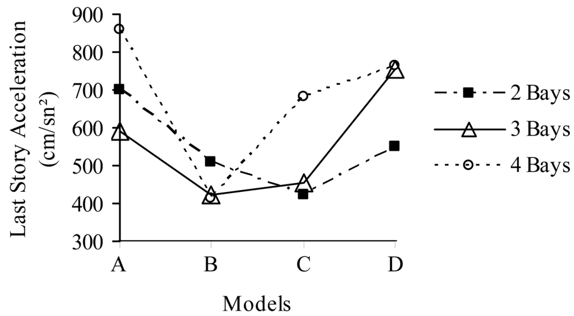

Fig. 16 The maximum last storey acceleration for (*H/B*) = 3

7. Conclusions

The appropriate isolation systems are searched for the 2 dimensional frame systems and by comparing them to the conventional structural behaviors, the following conclusions are obtained:

1. For the low-rise structure, depending on the increase of number of bays, the period and therefore relative displacements also increase. This situation is not related to the base isolation system used. For the short bay models, the rubber bearings with high stiffness are effective while for the high bay models, the rubber bearings with soft stiffness are not effective.
2. For 2 dimensional frame systems with average height, rubber bearings with low stiffness are effective and independent of the number of bays. For the usage of rubber bearings with high stiffness, the effectiveness of the base isolation system compared to the conventional structural design disappears.
3. For 2 dimensional high frame systems, while rubber bearings with average stiffness are effective for 2 and 3 bay systems, the effectiveness of the base isolation system for 4 bay systems diminishes. In this case, additional dampers should be used.

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