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Geodesic shape finding of membrane structure with geodesic string by the dynamic relaxation method

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Abstract. The explicit nonlinear dynamic relaxation method (DRM) is applied to the nonlinear geodesic shape finding analysis by introducing fictional tensioned 'strings' along the desired seams with a three or four-node membrane element. A number of results from the numerical example for the nonlinear geodesic shape finding and patterning analysis are obtained by the proposed method to demonstrate the accuracy and efficiency of the developed method. Therefore, the proposed geodesic shape finding algorithm may improve the applicability of a four-node membrane element to membrane structural engineering and design analysis simultaneously for the shape finding, stress, and patterning analysis.

Keywords: tension membrane structures; patterning; geodesic element; shape finding algorithm; dynamic relaxation method; kinetic damping

1. Introduction

The curved 3D surfaces of architectural tension membrane structures are determined by shape finding analysis with an introduced prestress. The results of shape finding analysis are used as the basic configuration for static load analysis and the pattern generation process. The distinctive feature of tension membrane structures in comparison with other general structural systems is the shape finding analysis with the introduced prestress.

Linear and nonlinear shape finding analysis methods have been proposed from sophisticated researches which have been carried out over the past several decades. As a result, for the nonlinear shape finding methods, the Dynamic Relaxation method (DRM) and the Newton Raphson method(NR) are used as basic approaches to the overall problem of shape finding and load analysis. The vector method (Barnes 1988, Wakefield 1999, Topping and Iványi 2007) (DRM) is considered to be the preferred for tension membrane structures in comparison to the standard matrix method (NR) because of the simple and non-singularity in the vector's iterative analysis process. The DRM could overcome the local instability or the nearly singular problem with an iterative vector analysis, which uses the most effective kinetic damping technique (Cundall 1976, Barnes 1988). The DRM has been applied to the highly nonlinear problems in various fields, such as shape-finding and load

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analysis of prestressed cable-membrane tension structures (Barnes 1988, Wakefield 1999, Topping and Iványi 2007, Lewis 1989, Gosling and Lewis 1996, Wood 2002, Han and Lee 2003, Brew and Lewis 2003), post buckling analysis (Papadrakakis 1981, Ramesh and Krishnamoorthy 1993, Kadkhodayan *et al*. 1997, Zhang *et al*. 2000, Pasqualino and Estefen 2001, Salehi and Aghaei 2005) and hydro dynamic FEM codes (Hallquist 1998). The effective multi-core parallel solution algorithm can be applied to the DRM procedures (Topping *et al*. 1994, Topping and Khan 1996, Topping and Iványi 2007).

For linear shape finding methods (Linkwitz 1999, Sheck 1974, Grundig and Bahndorf 1988, Grundig *et al*. 1996), the force density method (FDM) (Sheck 1974, Grundig and Bahndorf 1988, Grundig *et al*. 1996) is the most effective and popular method. The advantage of this method is that it provides a linear solution to the equilibrium shape finding problem by controlling the element's force density. However, the main drawback of the linear force density method is the difficulty involved in determining the distributed member's stress level from the applied loads. As a result, once a form has been found, a vector or stiffness matrix based NR method must be used to analyze its response under load (Wakefield 1999).

To fabricate the 3D surface membrane structures, which are found by the linear or nonlinear shape finding methods, the surface is converted into a set of planar cloths. This process is called cutting pattern generation (Barnes 1988, Wakefield 1982, Moncrief and Topping 1990, Wakefield 1999, Topping and Iványi 2007, Ong *et al.* 1991, Levy and Spiller 2003, Ishii 1972) and is the basic drawing for fabrication. The real 3D surface is installed and achieved by the adhesion of the individual cloths.

To determine the cutting pattern for the 3D surface of the membrane, a seam line is needed across the 3D geometry. The seam line creation method is a major aspect of all cutting pattern generations. The quality of the seam geometry significantly affects both the technical and aesthetic performance of textile roofs. The geodesic line is considered to be the most effective seam line. As geodesic lines pass over a surface, they do not curve in the tangential plane. Consequently, the clothes of a surface that are properly patterned on the basis of geodesic lines can minimize cloth usage as well as reduce the angles between the textile's weave and the surface's principle stresses (Grundig *et al*. 1996).

There are fundamentally different approaches to converting the 3D surface data into individual cloth data sets. The numerical algorithm that is used to determine the geodesic line for membrane structures is classified, in general, as having two approaches. One approach involves the nonlinear geodesic shape finding by using a fictitious geodesic string element with an initial prestress (Barnes 1988, Wakefield 1982, Moncrief and Topping 1990, Wakefield 1999). The Newton Raphson method or the dynamic relaxation method can then be used as nonlinear shape finding algorithms, though the later is more efficient and stable than usual. The other approach involves a geodesic line cutting or searching algorithm (Grundig *et al*. 1996, Ishii 1972) for arbitrarily curved 3D surface shapes that are determined by any other shape finding linear or nonlinear algorithm. However, the linear shape finding algorithm, i.e., the FDM, is preferable to the nonlinear methods as a basic surface for the geodesic line cutting analysis because of its simplicity and effectiveness (Moncrief and Topping 1990).

Recently, Topping and Iványi (2007) have described a wide range detailed review of computational methods for cable-membrane structures such as mesh and model generation, finite element formulations for cables, membrane and geodesic strings, shape finding, load analysis and cutting pattern generation. And also explains how new computational technology, such as parallel

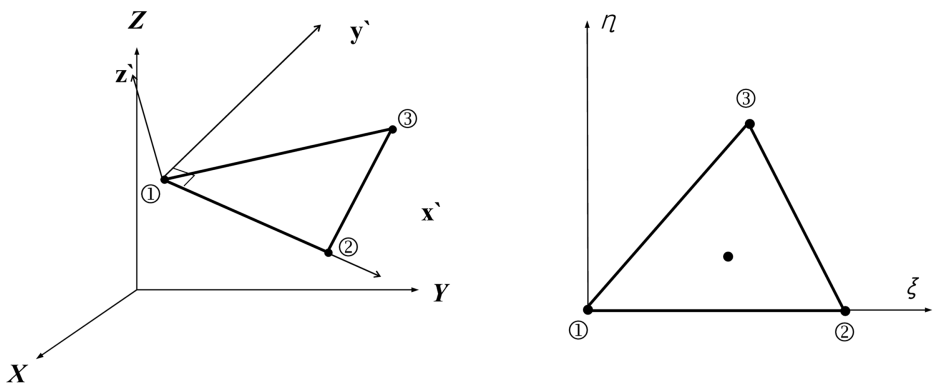


Fig. 1 Three-node membrane element

(Topping *et al*. 1994, Topping and Khan 1996) and distributed computing and genetic algorithms may be used to speed up both analysis and design. The additional information may be available from that reference (Topping and Iványi 2007).

In many research papers and practical projects, the three-node triangle CST element, which is illustrated in Fig. 1, has been used for shape finding, load and pattern generation analysis. The general reason for the infrequent use of the higher-order element (Gosling and Lewis 1996, Brew and Lewis 2003) or quadrilateral elements is as follows. The pattern analysis should be performed with three-node element for geodesic shape finding (Barnes 1988, Wakefield 1982, Moncrief and Topping 1990, Wakefield 1999, Topping and Iványi 2007, Ong *et al*. 1991, Levy and Spiller 2003) or nearest surface searching algorithm (Grundig *et al*. 1996, Ishii 1972). And the thickness of the membrane element is less than 2 mm and cannot resist the compression and bending moments. Therefore, the structural behavior of the membrane element is simpler, and the error when using the lower-order finite element rather than the shell or plate finite element is somewhat insignificant. Consequently, the CST element is the most frequently used finite element in architectural fabric tension membrane structures.

However, the quadrilateral element in other structural engineering fields has generally many advantages in design and analysis for this type of architectural fabric membrane structure. The quadrilateral element behaves orthogonally in its local principle direction, and its shear resistance capacity is negligible. Moreover, the visual image of the graphic mesh is more attractive and comprehensive than that of the triangular mesh. However, the lack of a pattern generation algorithm for the quadrilateral element is the only obstacle.

In this paper, we bring together the geodesic shape finding algorithm of Wakefield and Barnes, which uses the fictitious geodesic string element in the kinetic damped DRM process with the CST element (Levy and Spiller 2003, Ishii 1989) and the quadrilateral finite element (Ishii 1990). The overall theory of these papers has been well defined and verified by excellent previous research work. The only trial of our research is the use of the quadrilateral finite element with the geodesic string element in a kinetic damping DRM process, the results of which have not been presented. A numerical example will show the effectiveness and possibility of the present methodology. As a result, the use of a quadrilateral element (Ishii 1990) in pattern generation analysis will increase the effectiveness and applicability of both the design and analysis in membrane structure engineering, as shown in Fig. 2.

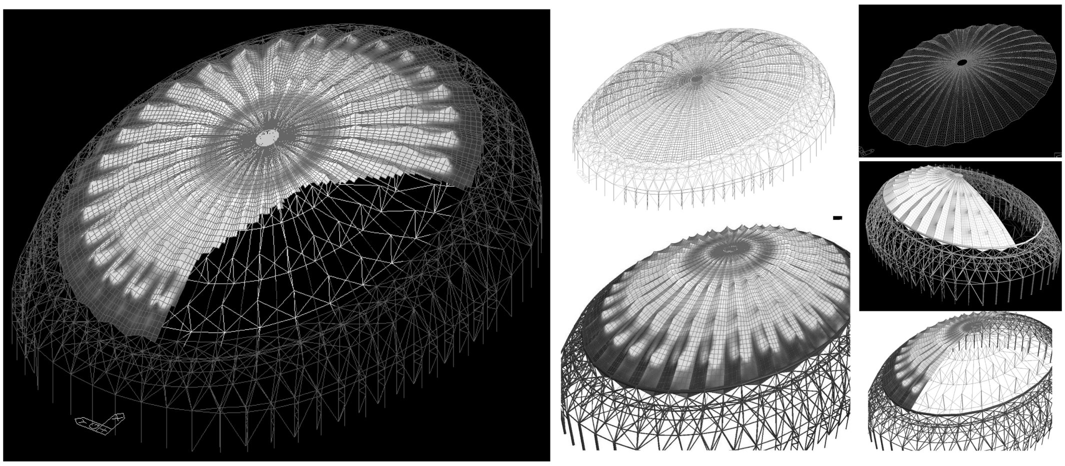


Fig. 2 Visual image of the membrane structure analysis with the quadrilateral element

2. Governing equation of the nonlinear shape finding analysis

The basic governing equation for the nonlinear shape finding algorithm is the same as the general total potential energy formulation, but the external applied load is not presented. The static equilibrium state of the structure is the state of minimum potential energy, the characteristic equation of static equilibrium can be expressed as follows.

$$\frac{\partial \mathbf{\Pi}}{\partial \mathbf{d}} = 0 \tag{1}$$

Generally, Eq. (1) cannot be satisfied with a value that is exactly zero. Therefore, the static equilibrium state is achieved only when the convergence criteria is satisfied with a value close to zero.

In the case of the shape finding analysis with the external load vector set to zero, the initially introduced prestress, $\boldsymbol{\sigma}_0$, is the only influencing factor for the initial shape. Hence, the unbalanced load vector is simplified only by the internal force vector as follows.

$$\mathbf{g} = -\int_V \mathbf{B}^T \boldsymbol{\sigma}_0 dV \tag{2}$$

With a nonlinear iterative process, the nodes of the initial surface mesh may move to the balanced unknown configuration when the unbalanced force converges to zero.

The minimum or equally stressed surface that is frequently used as the initial surface shape of the membrane is determined by an iterative shape finding analysis until the norm of the residual vector in Eq. (2) converges to nearly zero with the initially introduced prestress, $\boldsymbol{\sigma}_0$, which should be fixed as a continually constant value. Usually the standard NR with the matrix equation or the different DRM with the vector equation can be used to minimize the unbalanced force with an iterative procedure. However, since it is a nonlinear shape finding or initial stable state determination method for tension or unstable structures, the DRM is more preferable than the NR due to its stable, fast and economical vector equation.

3. DRM with kinetic damping

DRM, which was proposed by Day (1960), has been applied to static nonlinear problems in various fields. Among them, the architectural fabric membrane structure can be called the most successful structural system to be applied (Barnes 1988, Wakefield 1999, Topping and Iványi 2007). The vector equation for the DRM with the kinetic damping technique, which was formulated by many researchers, has been applied in many important projects world-wide. Moreover, this kind of structural system may exhibit fundamentally unstable phenomena because, to remove the unstable state or to impose the stable state to withstand external environmental loads, we introduce an initial prestress. The process of introducing a prestress into an unstable structure may be called a stabilizing process for unstable structures (Han and Lee 2003). As mentioned above, the main structural component of architectural fabric membrane structures (a prestressed 3D plane stress membrane element and a boundary or hanging cable element) behaves unstably in many cases and in unexpected situations. Such problems indicate that the DRM would be favorable over the standard NR. To use the DRM, the locally unstable state could be passed over to the stable state with a non-using matrix equation that may exhibit a singularity in such cases. The shape finding analysis for membrane structures would be a similar case since the unstable structures of the objected final configuration could be obtained only with a prestress. The standard DRM procedure routine can be applied, similar to NR, to many highly nonlinear problems, such as shape finding analysis, load analysis, stabilizing processes, buckling analysis, etc.

Comprehensive detailed descriptions of the DRM with kinetic damping are given in the literatures (Barens 1988, Wakefield 1999, Lewis 2003, Topping and Iványi 2007). We briefly introduce the simple and effective kinetic damping technique that was used in this study.

By considering the DRM without viscous damping, the analysis only with kinetic damping

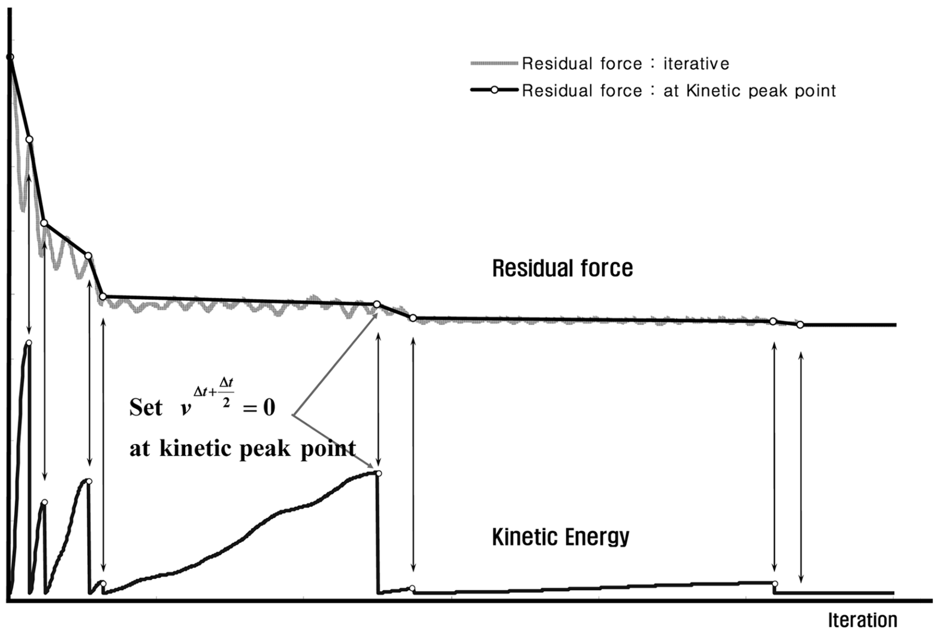


Fig. 3 Residual and kinetic energy convergence process for the simple shape finding example

becomes very simple because the number of parameters can be reduced. In other words, the numerical analysis in the DRM can be controlled only with time increments and nodal mass terms, which is effective in large deformation analysis. Without viscous damping terms, the behavior of structures yields a harmonic free vibration. The behavior of the structure can be re-established by setting the nodal velocity at the maximum kinetic energy state to zero at each time increment. The iterative calculation is repeated until the appropriate convergence criterion is satisfied.

Fig. 3 shows the residual and kinetic energy convergence process for the simple shape finding example. The kinetic energy at time increment $t+\Delta t/2$ can be expressed in terms of the velocity: $\mathbf{M}(\mathbf{v}^{t+\Delta t/2})^2/2$. If the current kinetic energy is less than the kinetic energy at the previous time increment, we assume that the kinetic energy's peak is achieved. All of the nodal velocities are set to zero, and then we continue to the next DRM process. This simple process is called the kinetic damping technique. The preparation or calculation of any other viscous damping terms is unnecessary.

In the DRM process with kinetic damping, the velocity vector at the next time step $t+\Delta t/2$, $\mathbf{v}^{t+\Delta t/2}$ can be expressed explicitly by the velocity vector at the previous time step $t+\Delta t/2$, $\mathbf{v}^{t-\Delta t/2}$. Thus, the current static residual, $\mathbf{g}^t$, with the assumed fictitious constant diagonal mass matrix, $\mathbf{M}$, is as follows

$$\mathbf{v}^{t+\Delta t/2} = \mathbf{v}^{t-\Delta t/2} + \frac{\Delta t \mathbf{g}^t}{\mathbf{M}} \tag{3}$$

The explicit form of Eq. (3) implies that the overall analysis could be performed with a vector equation rather than with a matrix. The mass matrix could be a simpler diagonal rather than a consistent mass. The storage and numerical computational work can thus be reduced remarkably. However, the iteration number and computing time for convergence to static equilibrium may be moderately larger than in the NR process for general cases. Nevertheless, in the case of highly nonlinear or nearly singular problems, such as a shape finding analysis for an equally stressed membrane surface or the stabilization process in tension membrane structures, the DRM is more stable in a current computer hardware machine.

The most important and maybe unique term to control or influence the convergence of the DRM process in Eq. (3) is the diagonal mass matrix.

In Eq. (3), the diagonal mass matrix may be the most important and most unique term that is used to control the DRM's convergence process with the kinetic damping technique. To determine static equilibrium with the DRM, the mass term in Eq. (3) need not be the true structural mass. However, the fictitious mass that is proportional to the stiffness of the system would increase the convergence's stability and speed.

Barnes (1988) proposed that the incremental time step should be less in the following equation to maintain stability in the DRM

$$\Delta t \leq \sqrt{2\frac{\mathbf{M}}{\mathbf{K}}} \tag{4}$$

In this study, the optimal mass matrix can be expressed by the diagonal tangent stiffness matrix, $\mathbf{K}_{ii}$, from Eq. (4)

$$\mathbf{M} = \lambda \frac{\Delta t^2}{2} \mathbf{K}_{ii} \tag{5}$$

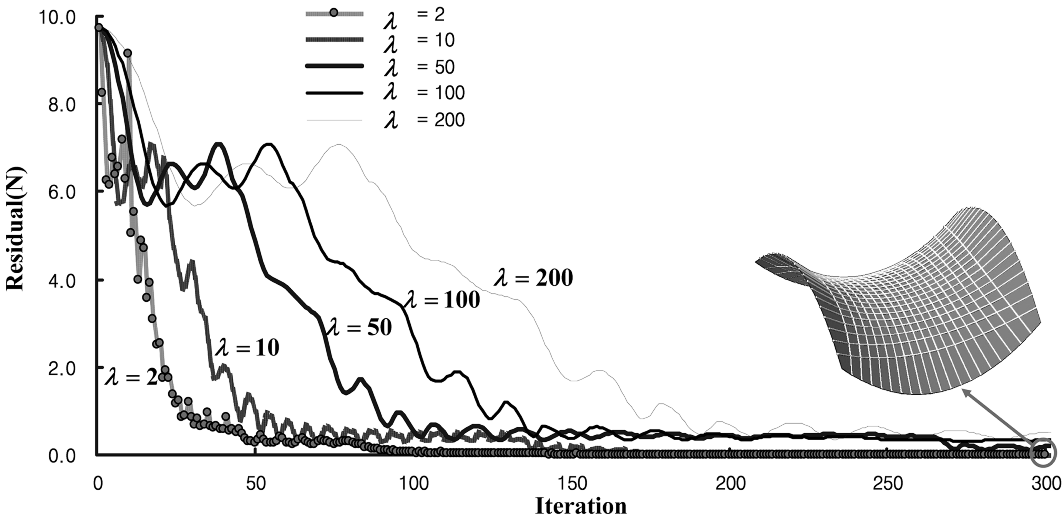


Fig. 4 Convergence process according to λ

At the initial stage of preparing the DRM process, the diagonal tangent stiffness matrix, $\mathbf{K}_{ii}$, could be assembled only with a minimally modified NR routine in a vector storage array rather than in a banded matrix. In Eq. (5), λ is the scale parameter for convergence to control the DRM process. In the case where the scale parameter, λ, has a larger value, the natural period of the system and the total analysis speed increase. As a result, we need to secure the numerical stability as mentioned by Barnes (1988) and shown in Eq. (4).

Fig. 4 shows the effect of the scale parameter, λ, from Eq. (5) on the convergence process of the shape finding example in chapter 6.2. When λ increases, the mass matrix is scaled up and the system's natural period is extended by the assumed mass matrix. Such a relationship between the mass and the natural period is the same phenomenon as in structural dynamic theory.

Since the DRM and the assumed mass are fictitious, the calculated system's period is imaginary rather than real. However, we can obtain numerical stability in an explicit DRM process by increasing the natural period because the slow motion of the domain will prevent an unexpected or unstable behavior. Since such a controlling parameter is insignificant in the explicit dynamic finite differential method for acquiring numerical stability, it should use the short value of the incremental time step.

As λ decreases to 1.0, the motion of the system may fail to take the stable convergence path, as shown in Fig. 5. Therefore, to find the equilibrium path, the scale parameter, λ, should be increased.

By substituting Eq. (5) into Eq. (3), the nodal velocity at time $t+\Delta t/2$ can be written as follows

$$\mathbf{v}^{t+\Delta t/2} = \mathbf{v}^{t-\Delta t/2} + \frac{2\mathbf{g}^t}{\lambda \Delta t \mathbf{K}_{ii}} \tag{6}$$

From Eq. (6), the structural behavior may be independent of the mass since the mass matrix is only related to the convergence rate, according to the scale parameter, λ.

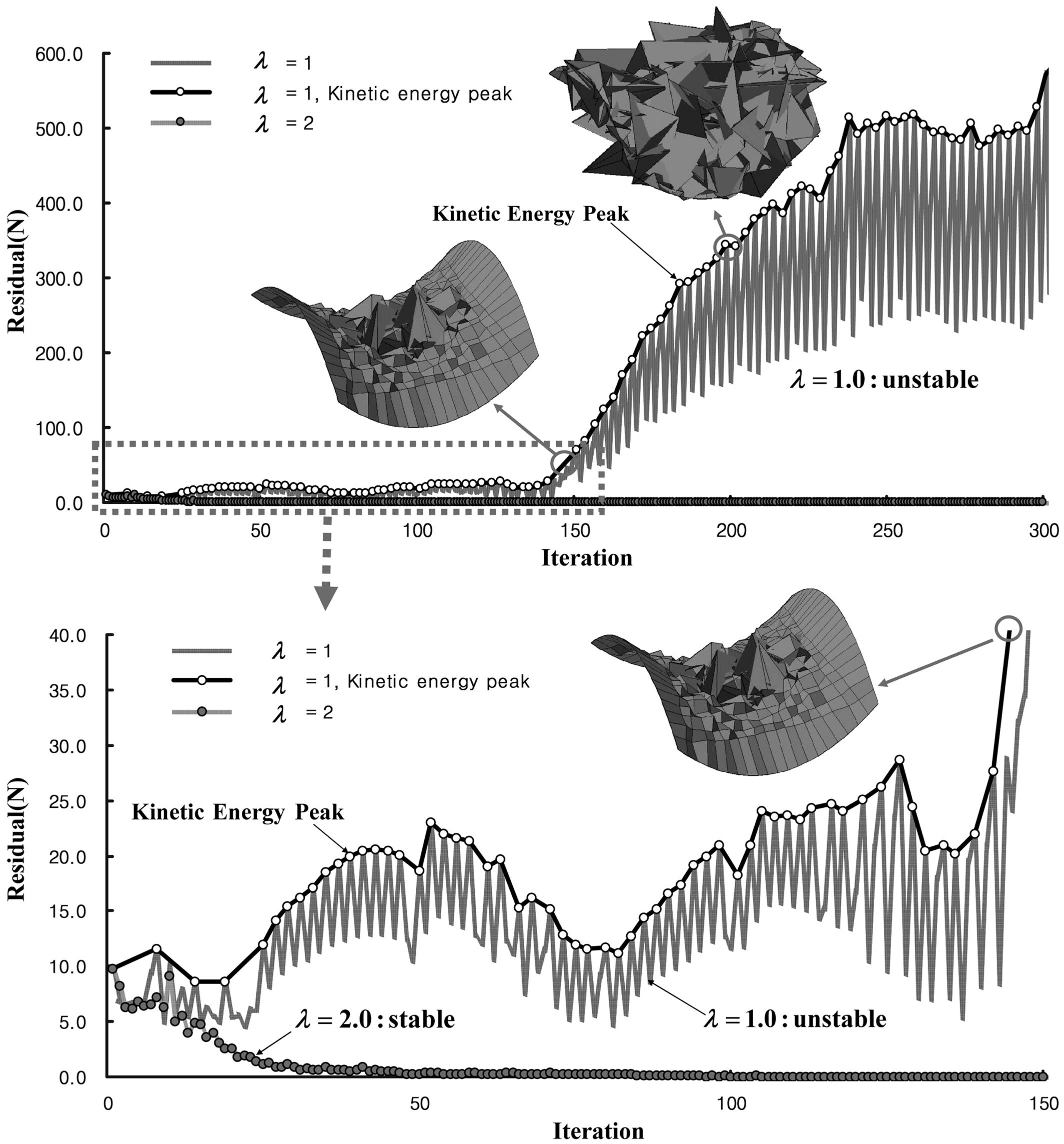


Fig. 5 Convergence and divergence process according to λ

The iterative displacement at time $t+\Delta t$ can be obtained by linearly interpolating the velocity at time increment Δt

$$\delta^{t+\Delta t} = \Delta t \mathbf{v}^{t+\Delta t/2} \tag{7}$$

4. 3D four-node quadrilateral membrane element

A comparison between the different DRM formulations from the elementary theories and the

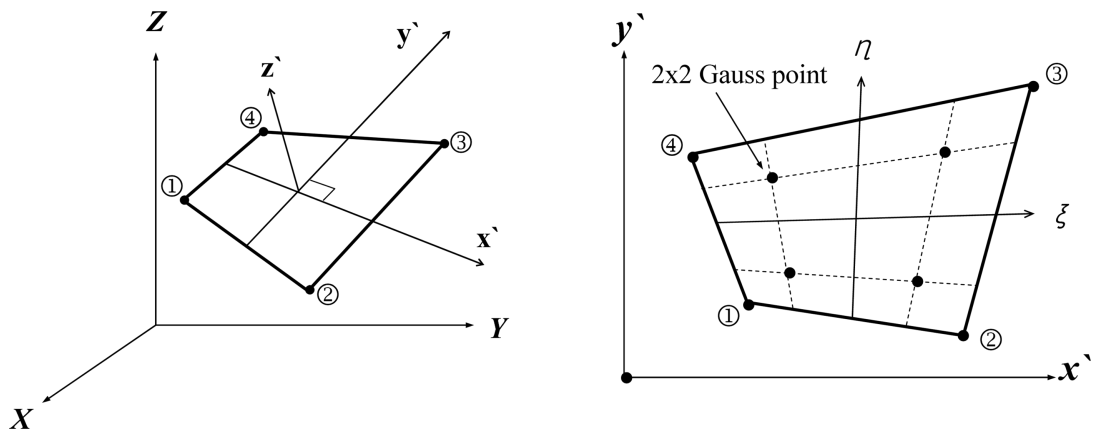


Fig. 6 Quadrilateral plane stress membrane element

NR is only made to determine the global stiffness for the tangent stiffness matrix and to convert it to vector equations. The constitutive and equilibrium equation for each element type could be the same. Therefore, in our study, the developed NR routine can be modified to the DRM very easily.

The general plane stress membrane element can be derived easily from well defined elementary theory. Basically, the 3D quadrilateral four-node membrane element, which is shown in Fig. 6, can be expressed as the transformed 2D isoparametric plane stress finite element. The detailed formulations have been presented by Ishii (1989, 1990). These formulations were used in our research for the geodesic shape finding algorithm with a geodesic string element to find the geodesic line of the surface mesh.

The stiffness of the element that is used in architectural fabric membrane structures can usually be assured by the introduction of prestress because of its material characteristics. The external loads that are applied to the membrane's surface were resisted by the geometrically large deformation. The material's nonlinear effects, such as the orthogonal properties of the fabric fiber, may be less important than the governing geometrically nonlinear effect of large deformation.

5. Analysis of the equally stressed surface with the geodesic string element

The shortest path between two points over a 3D surface is a geodesic line. The geodesic line does not curve and is straight in the tangential plane. It is therefore considerably suitable for seam generation in textile architectural applications. Consequently, the cloths in a surface that is properly patterned on the basis of geodesic lines can minimize cloth usage and reduce the angle between the textile's weave and the surface's principle stresses (Grundig *et al.* 1996).

From the theoretical definition, the geodesic line is determined by the "Length" or "Angular" based approach (Grundig *et al.* 1996). Grundig (1996) and Ishii (1972) have used the "Angular" approaches. Grundig (1996) proposed that the geodesic line point can be adjusted for the position of the nodes according to observations of the normal plane's angles themselves. In such an approach the surface's tangential planes in the neighborhood of each line point are determined to define multiple local coordinate systems. Ishii (1972) proposed that the geodesic line can be defined as a poly-line in terms of the intersection points with the same side angle of the neighboring triangles.

Wakefield (1982, 1999) and Barnes (1988) introduced the geodesic string element or links into the shape finding mesh and used it to pull the seams into geodesic paths so that the total length of the line was minimized in the shape finding process with the DRM. This is defined as the "Length" control approach in comparison. Practical applications are demonstrated in the literature (Ong *et al*. 1991).

The geodesic line configurations are determined by introducing elastically or tensioned controlled strings into the membrane's surface. By introducing these strings with high tensions in such a way that they only control the in-plane movement of the surface nodes, the strings take up geodesic lines whose orientation is governed by their boundary end positions. An elastic control can be used to control the spacing of the nodes along the string, while the residual force components of their out-of plane and string ends are suppressed so as not to influence the surface's shape and boundary or edge cable geometries.

In this paper, we use the geodesic shape finding technique to introduce a fictional tensioned 'string' along the desired seams, as proposed by Wakefield and Barnes. The surface mesh that was used in this study is the quadrilateral or triangle CST membrane element. We can apply the quadrilateral membrane element with a fictitious geodesic string. This is the only difference from previous approaches.

6. Geodesic shape finding example

During the last decades, researchers have focused on the geodesic shape finding problem in architectural membrane structures. Nevertheless, there are geodesic shape finding examples that use only a three-node CST membrane element with a geodesic string element in DRM or NR. Therefore, in this chapter, we present a number of examples of geodesic shape finding with four-node quadrilateral membrane elements that have not been shown before. Using a four-node element in geodesic shape finding is easier to understand, more visually attractive, and is more appropriate when considering the orthogonal or anisotropic material effects.

6.1 Geodesic shape finding analysis with the three-node CST element

To illustrate the effect of using the geodesic string element, artificial string elements were only applied on the left portion of the mesh with a general three-node CST membrane element for an equally stressed surface.

The resulting equally stressed shapes are shown in Fig. 7. The left half of the meshes, in which the geodesic string elements are applied, is arranged with a minimum surface length that connects the start and end of the two points. This continuous string line can be the geodesic line. However, the right half of the meshes is shown as irregular and distorted with arbitrary equally stressed surface meshes. Fig. 8 shows the convergence process of the total length of the geodesic line. Therefore, this example shows that the geodesic shape finding algorithm with the geodesic string in the DRM procedure creates a successful geodesic surface as a mesh.

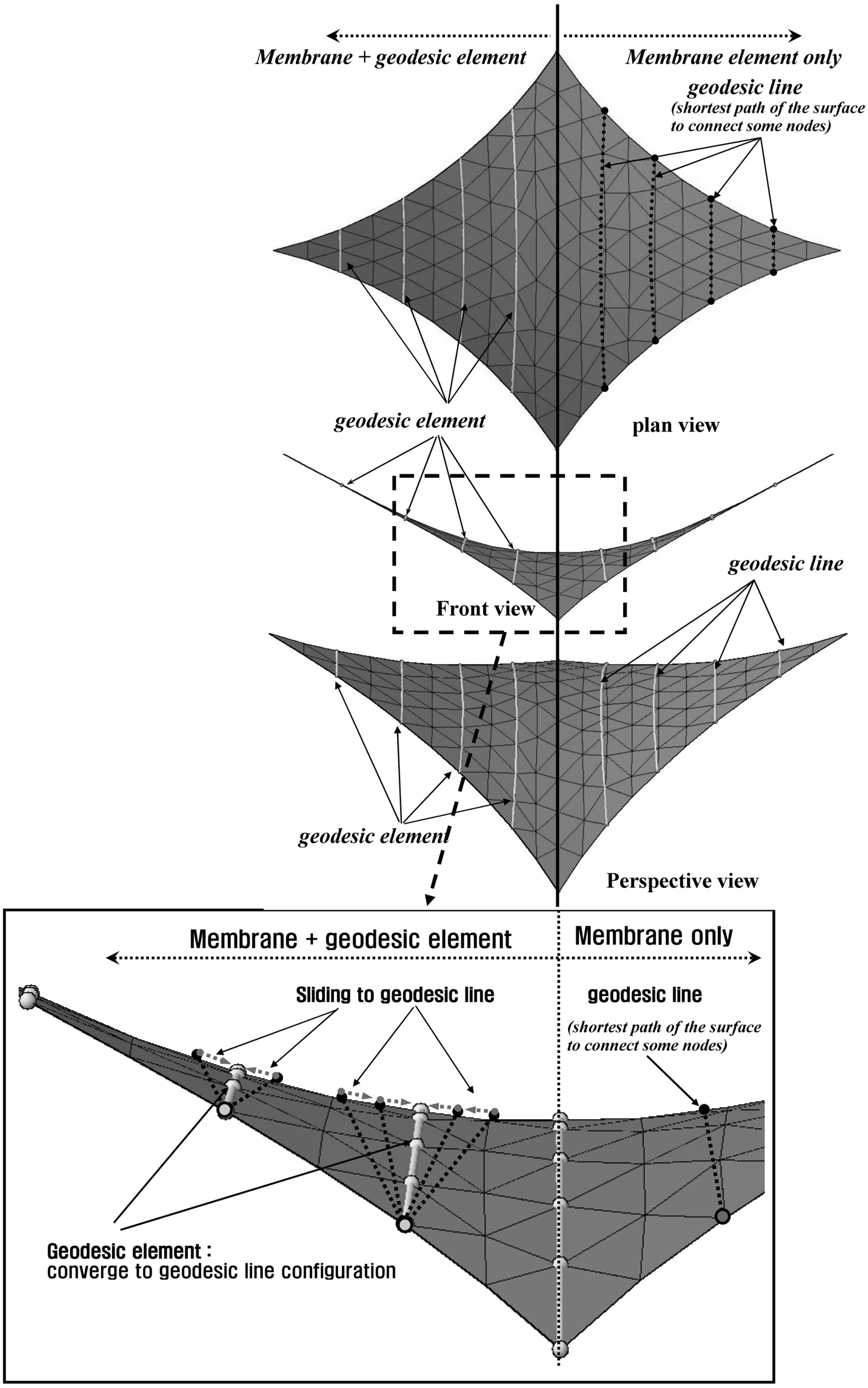


Fig. 7 Example of a geodesic shape finding analysis with the three-node CST element

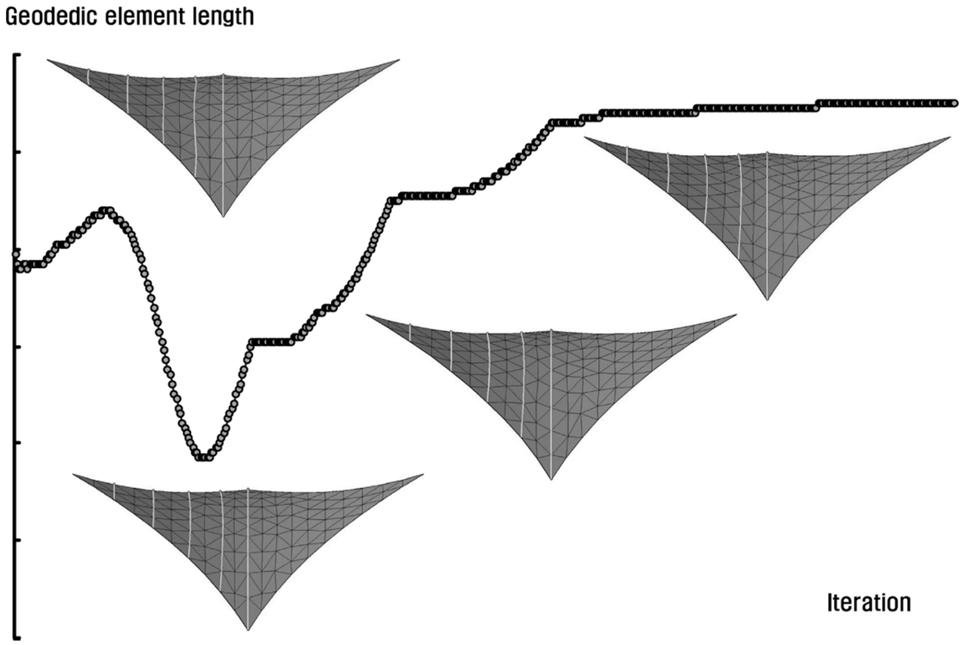


Fig. 8 Convergence process of the total length of the geodesic line

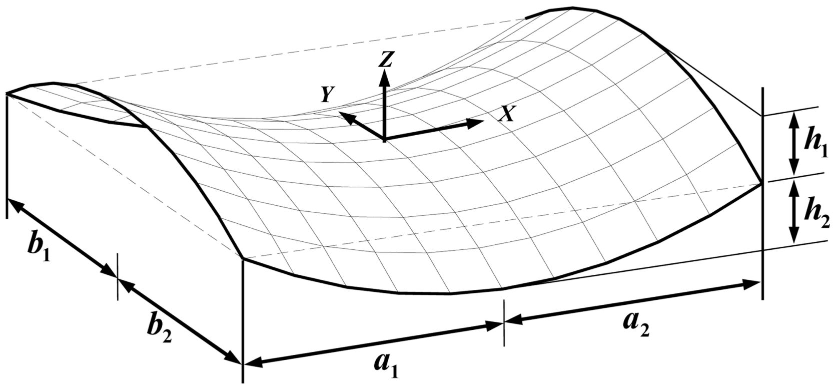


Fig. 9 Geometric dimensions of the saddle's surface

6.2 Geodesic shape finding analysis with a four-node quadrilateral element of a saddle-1 surface

The theoretical surface equation of a saddle-like HP shell is as follows

$$z = \frac{h_1 x^2}{a^2} - \frac{h_2 y^2}{b^2} \tag{8}$$

The convergence processes of saddle-1 are given in Fig. 10 and Fig. 11 with scale parameter $\lambda = 2$. The numerical effect, according to arbitrary values, has also been previously given in Fig. 4

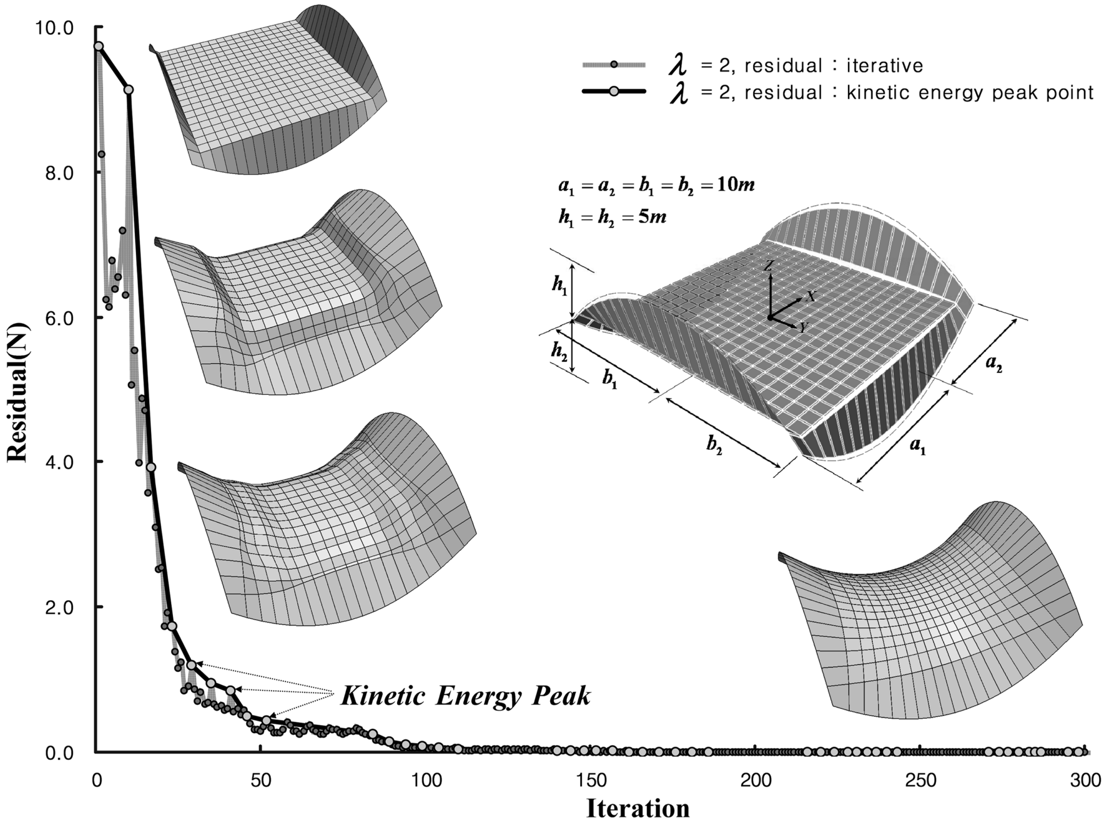


Fig. 10 Converge process of the saddle-1 surface with the scale parameter

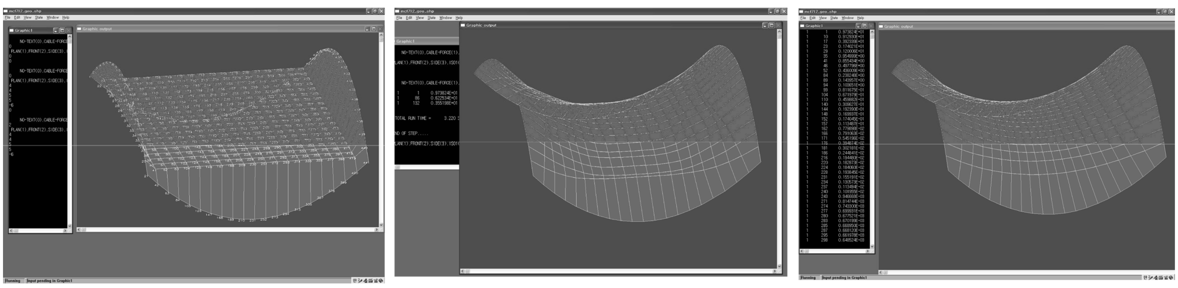
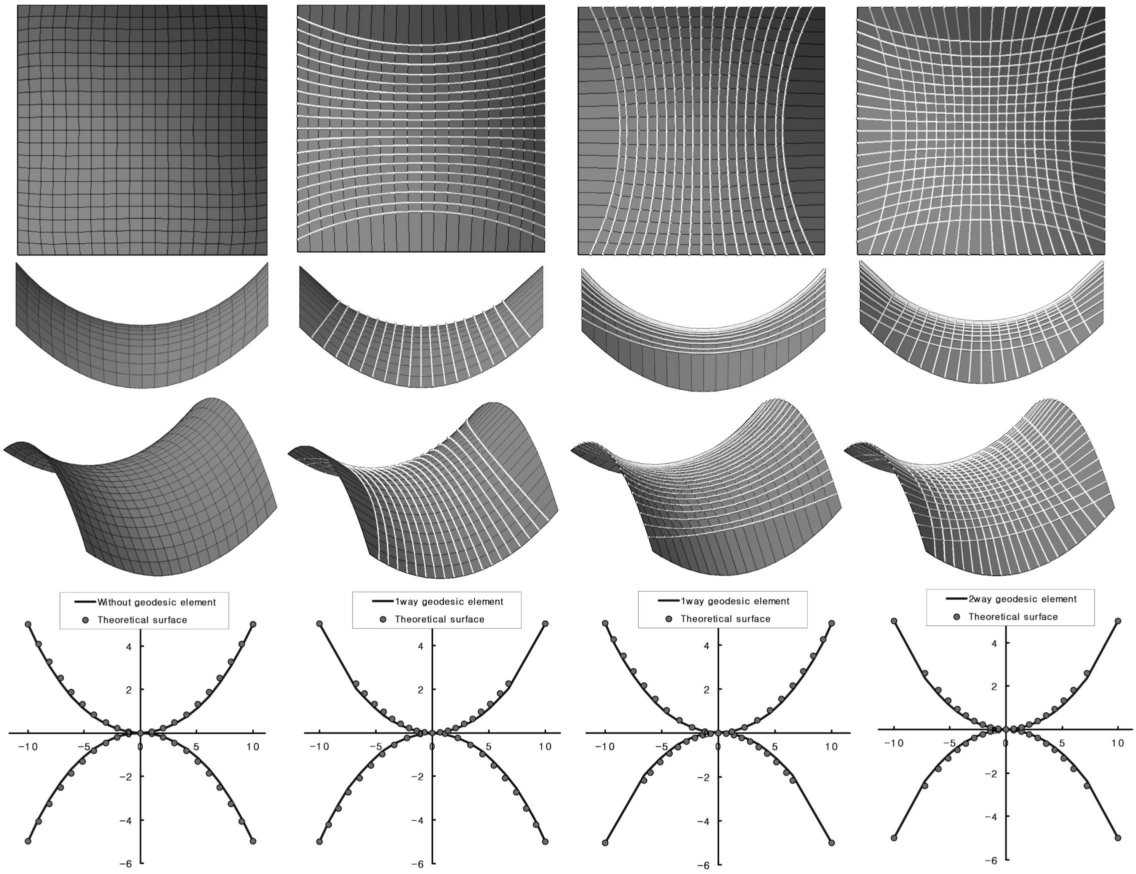


Fig. 11 Shape finding process for the saddle-1 capture image with the developed analysis program and the DRM routine

and Fig. 5. In Fig. 12, the shape finding result with various geodesic string layouts for an equally stressed saddle surface is given, and the resulting shape is compared with the theoretical shape of Eq. (8). From the result of the geodesic shape finding analysis that is shown, the geodesic string element was found to have little effect on the theoretical surface.



without geodesic element **1-way geodesic element** **1-way geodesic element** **2-way geodesic element**

Fig. 12 Result of the shape finding analysis with various layouts of the geodesic string elements for the equally stressed surface of saddle-1

6.3 Geodesic shape finding analysis with the four-node quadrilateral element for the saddle-2 surface

The theoretical surface equation of this saddle example is the same as the previous example of Eq. (7) but the geometric boundary differs. To process the geodesic shape finding algorithm for an equally stressed surface, the Young's modulus is set to zero and the membrane's prestress is set at a unit value without a shear prestress. The analysis model and the convergence process for the example are shown in Fig. 13. Fig. 14 shows the convergence process for the geodesic line's total length. Fig. 15 and Fig. 16 show the geodesic shape finding results according to the geodesic string layout types and the membrane element types.

From the shape finding results of Fig. 15 and Fig. 16, we can successfully obtain the geodesic shape of the surface with respect to the geodesic string element's layout types in various ways. The membrane element type had little effect on the final geodesic shape of the membrane.

However the required number of element to model the surface is nearly halves when four-node

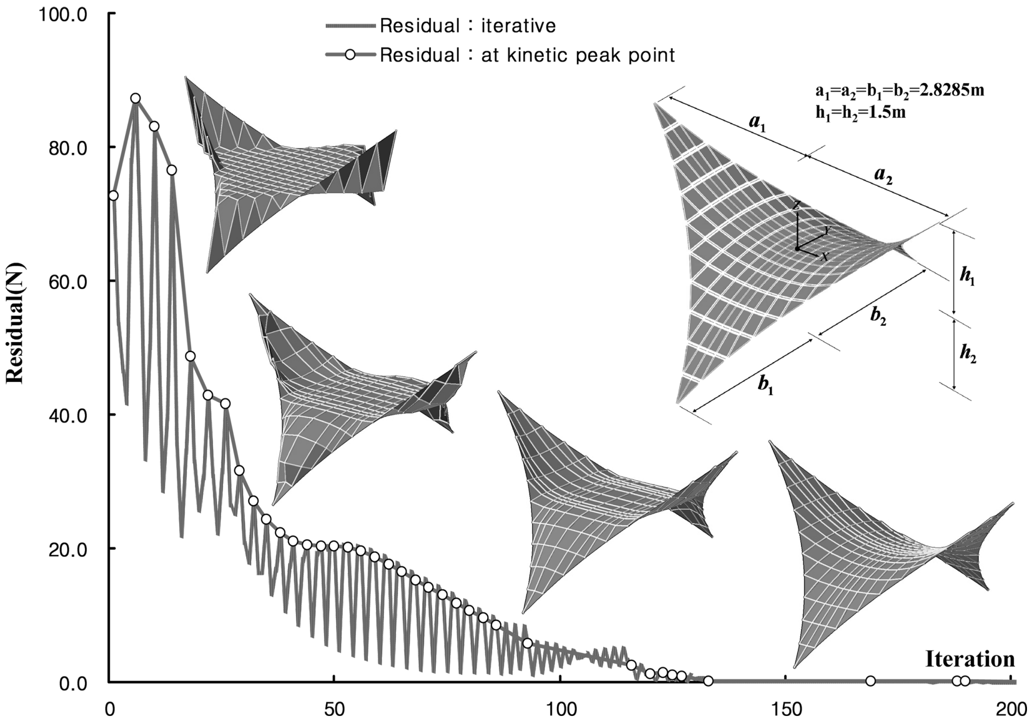


Fig. 13 Convergence process for the saddle-2 surface

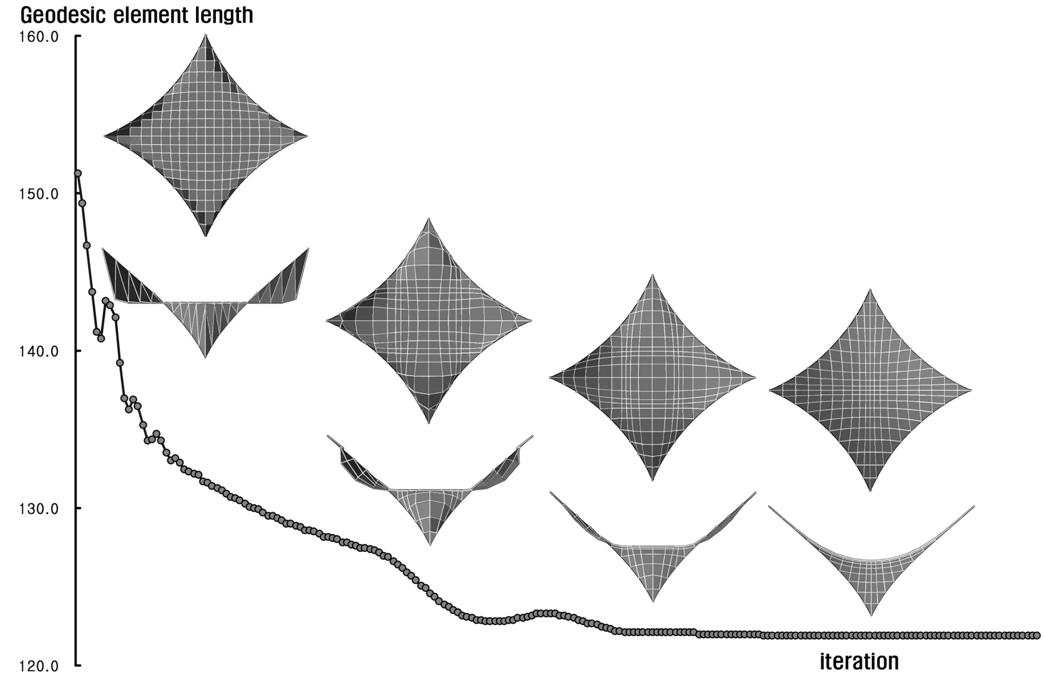


Fig. 14 Convergence process for the total length of the geodesic line

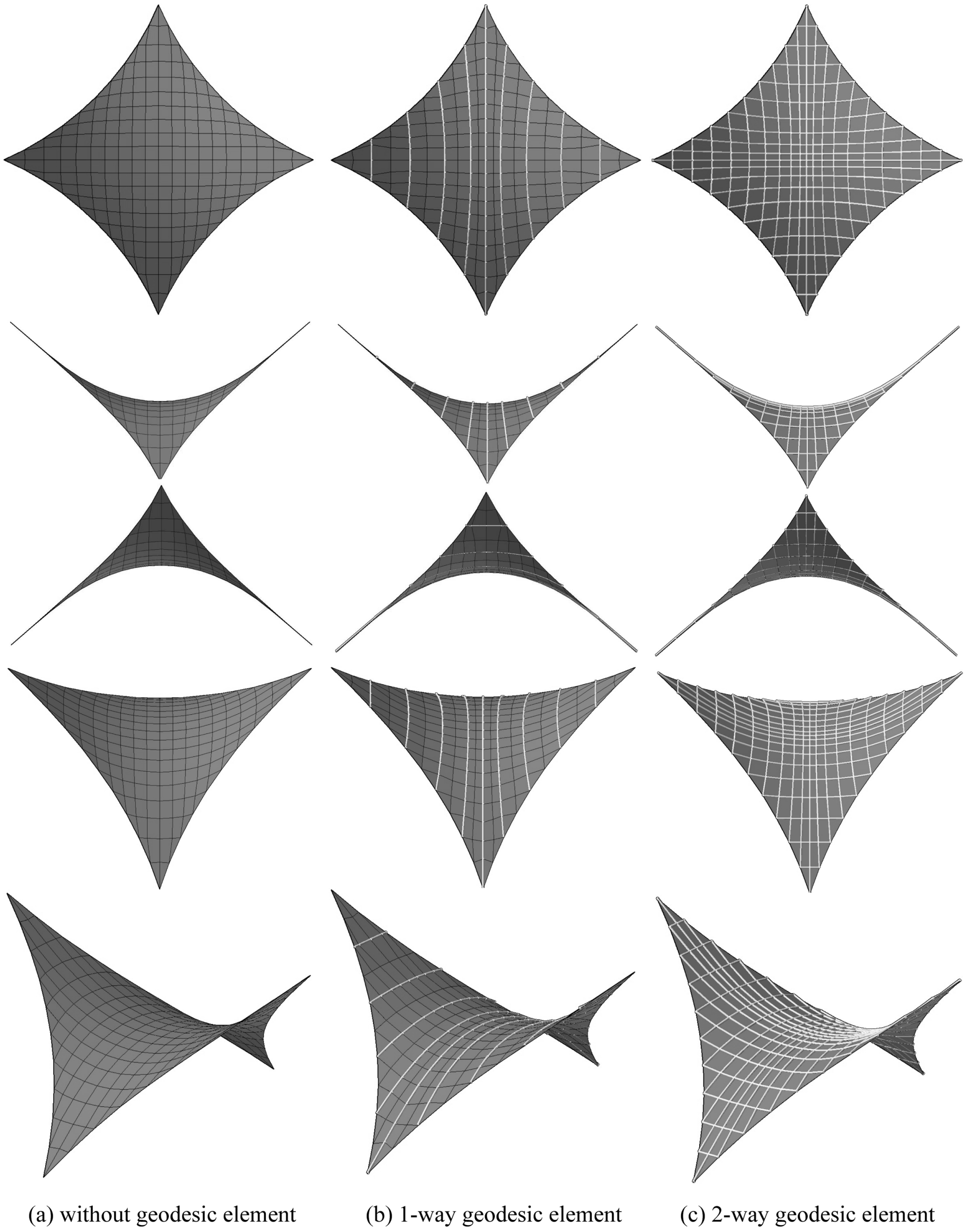


Fig. 15 Results of the geodesic shape finding analysis according to the geodesic string layout types

quadrilateral element is used. And many kine of fabric membrane structure have this type of shape. The visual image of using 4node element is neat and clear to understand without triangulate meshed line for orthogonal nature of fabric material.

Fig. 17 shows the results of the pattern generation for the model in Fig. 15, which shows the pattern of the shape when the geodesic string element is used in the model and when it is not used. The pattern of the shape without a geodesic string model is called the planar pattern, while the

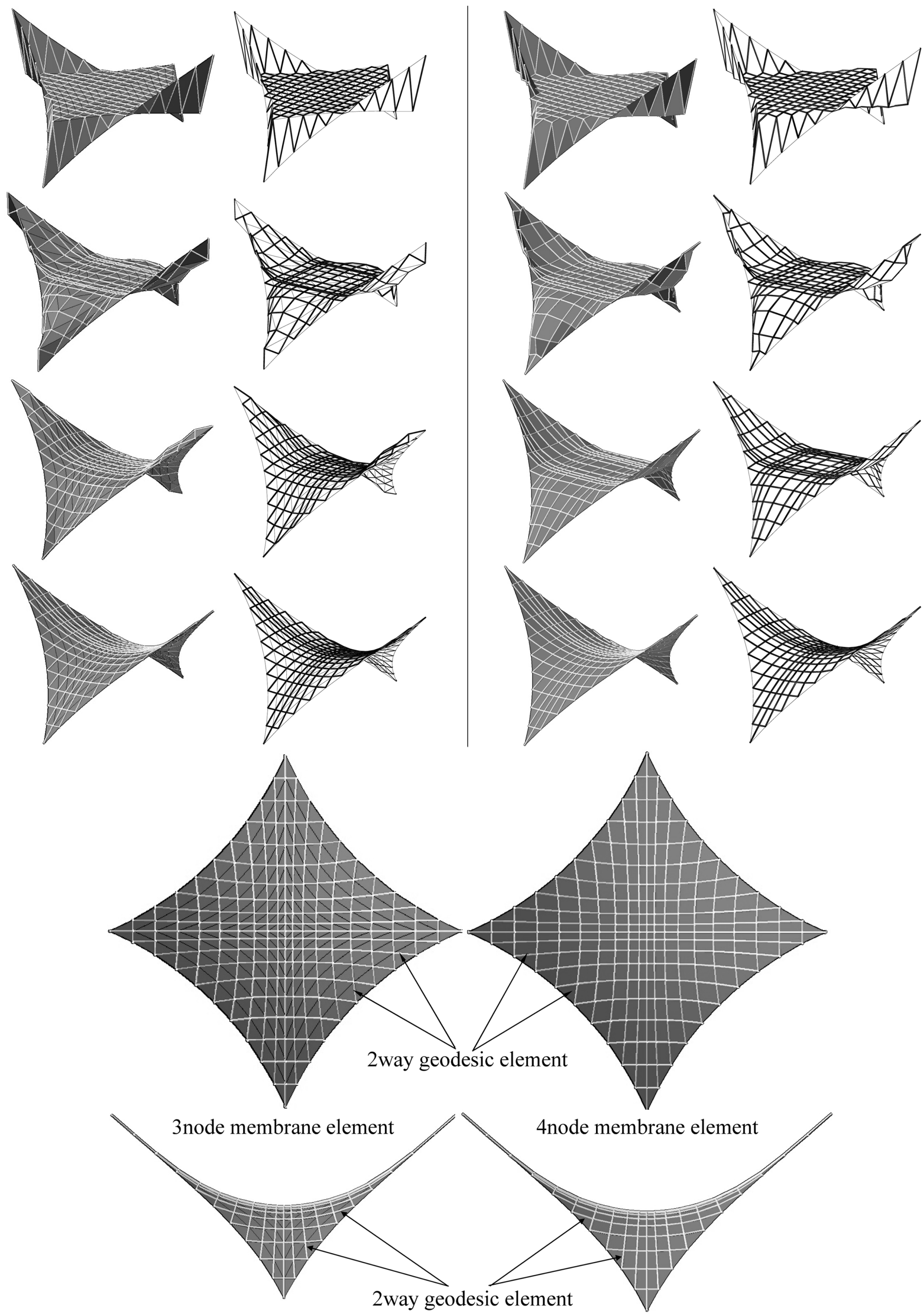


Fig. 16 Geodesic shape finding analysis according to the membrane element types

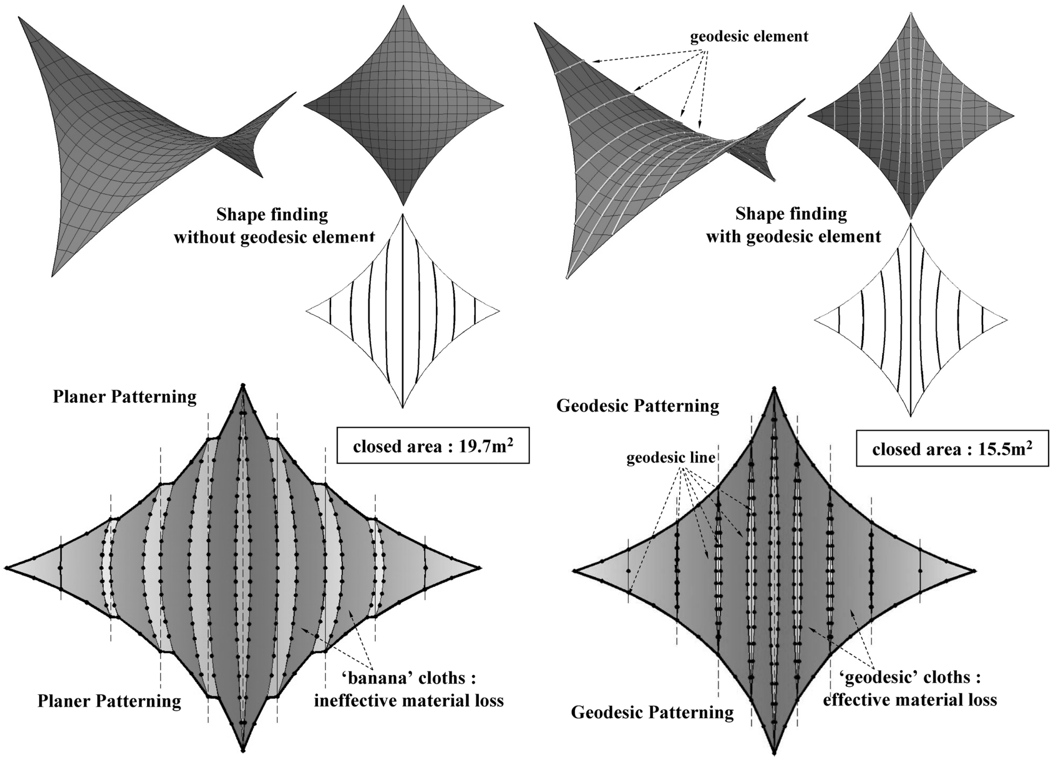


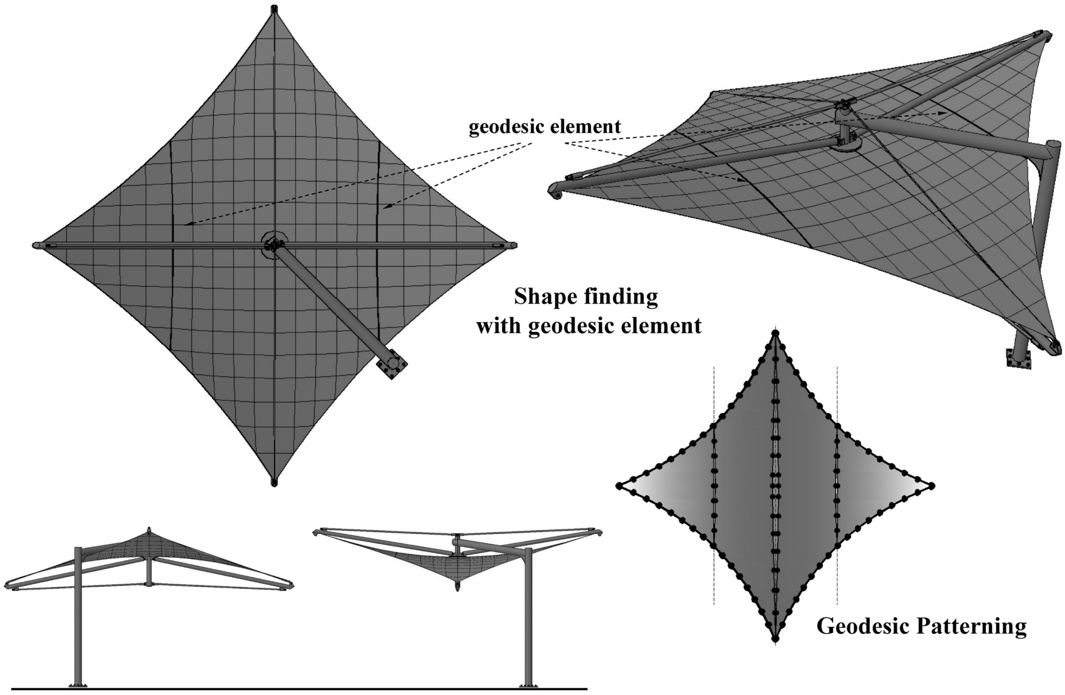
Fig. 17 Results of pattern generation according to the shape finding result types

pattern of the shape with a geodesic string model is called the geodesic pattern. By using the geodesic string, the geodesic pattern can minimize cloth usage and reduce the angles between the textile's weave and the surface's principle stresses.

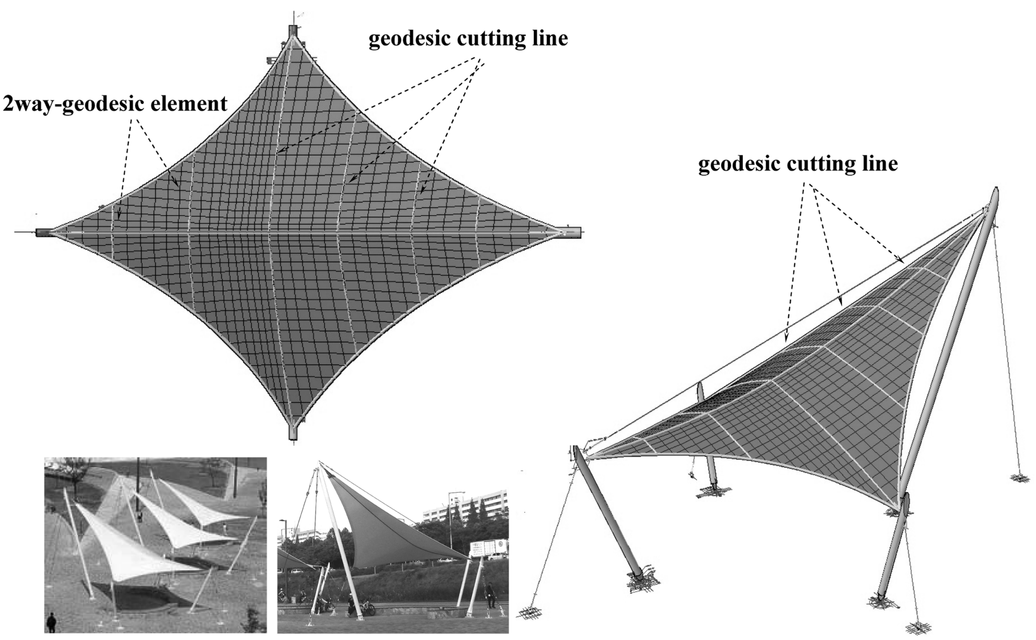
The patterning of fabric membrane structure is the cutting detail or shop-drawing for manufacture. The banana shaped planer patterning or geodesic patterning may be used. The choice is not deterministic but optional. The planer patterning is simple and easy to determine the shape of surface rather than the geodesic patterning. The geodesic patterning is economical and practical but need some more numerical effort to obtain the geodesic shape of surface. The geodesic shape finding process with geodesic element must be performed by using the method of this paper. And there is another approach involves a geodesic line cutting or searching algorithm (Grundig *et al.* 1996, Ishii 1972) for arbitrarily curved 3D surface shapes.

Until now, there is not reported the proper and well developed algorithm for patterning by using 4node quadrilateral element though this element type are more exact than 3node CST element in FE formulation when load and stress analysis. The applicability of 4node quadrilateral element is not limited but the choice of engineer and designer in shape finding and stress analysis problem.

Fig. 18 shows other simple examples of application of this paper. The more complicated large scale examples is also possible. In Fig. 18(a), only two lines of geodesic strings are used for geodesic shape finding. The more lines of geodesic string are not necessary. These two lines of geodesic strings affect the overall shape of surface to meet the constraint. More complicated 2-way



(a) Example of 1-way geodesic element



(b) Example of 2-way geodesic element

Fig. 18 Practical applications of geodesic shape finding with 4node quadrilateral element geodesic string elements are used in Fig. 18(b) to show the applicability of this paper. The usage of geodesic string element in shape finding analysis is not deterministic but the choice of possible approach. The geodesic shape finding problems have many choice of application. 3node or 4node element can be used to obtain the resulting shape of 3D surface. However the 4node element is more appropriate for the shape finding of fabric membrane structure for the orthogonal nature of membrane material.

7. Conclusions

In this study, we proposed the numerical techniques for generating a geodesic line that could be applied to three- or four-node elements at the same time. The explicit nonlinear static DRM is applied to the nonlinear geodesic shape finding analysis by introducing the fictional tensioned 'strings' along the desired seams with a three- or four-node membrane element. Some results of the numerical example for the nonlinear geodesic shape finding and patterning analysis are carried out with the proposed method to demonstrate the developed method's accuracy and efficiency. Therefore, the proposed geodesic shape finding algorithm may improve the applicability of the four-node membrane element to membrane structural engineering and design analysis simultaneously for shape finding, stress, and patterning analysis.

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