

Limiting the sway on multi-storey un-braced steel frames bending on weak axis with partial strength connections

Mahmood Md. Tahir* and Shek Poi Ngian

Steel Technology Center, Universiti Teknologi Malaysia, 81310 UTM Skudai, Johor, Malaysia

(Received October 23, 2008, Accepted April 22, 2011)

Abstract. This paper investigates the design using wind-moment method for semi-rigid un-braced steel frames bending on weak axis. A limiting sway method has been proposed to reduce the frame sway. Allowance for steel section optimization between moment of inertia on minor axis column and major axis beam was used in conjunction with slope-deflection analysis to derive equations for optimum design in the proposed method. A series of un-braced steel frames comprised of two, four, and six bays ranging in height of two and four storey were studied on minor axis framing. The frames were designed for minimum gravity load in conjunction with maximum wind load and vice-versa. The accuracy of the design equation was found to be in good agreement with linear elastic computer analysis up to second order analysis. The study concluded that the adoption of wind-moment method and the proposed limiting sway method for semi-rigid steel frame bending on weak axis should be restricted to low-rise frames not more than four storey.

Keywords: limiting sway method; partial strength connection; un-braced frame; weak axis; wind-moment method

1. Introduction

Multi-storey steel frames are usually designed as braced frame where the horizontal load due to wind is catered by the bracing system. However, the use of bracing system has becoming less popular as the appearance of this bracing system is not aesthetically favourable in the views of architects. Other alternative system to stiffen the un-braced steel frames is to introduce cruciform column section using universal beam sections as proposed by Tahir *et al.* (2009) as shown in Fig. 1. However, this section will increase the fabrication cost. Generally, the fabrication cost for a multi-storey steel frame is in the range of 30% to 50% of the total cost (SCI P207 1996). Other than high fabrication cost, this alternative section is not popular as the performance of the connection has not yet been established. Therefore, there is a need to study the structural behaviour of the un-braced steel frame which takes into account the effect of sway frame with column bending on minor axis. As the frames need to design on minor axis, the stiffening of the frames relies on the stiffness of the connections, columns and beams. An optimum design method is introduced in this paper to limit the

*Corresponding author, Ph.D., E-mail: stcutm@gmail.com

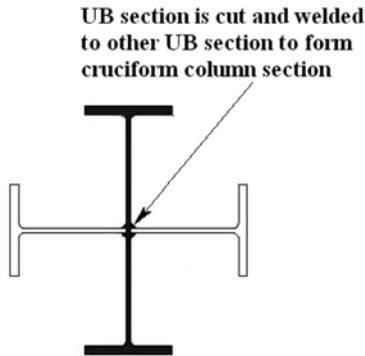


Fig. 1 Cruciform column section

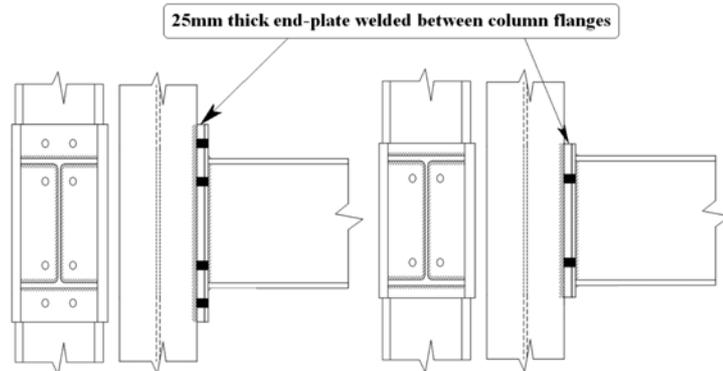


Fig. 2 Flush and extended end-plate connection for minor axis joint

sway of the frames by assuming the connections are partial strength, as full strength or rigid connection is unlikely to be achieved in minor axis connection (Irwan 2001). An experimental study conducted by Irwan (2001) on five flush end-plate connections and five extended end-plate connections connected between column flanges with thick end-plate (25 mm) (as shown in Fig. 2) has developed deformation on the thick end-plate, and the tested connections are classified as partial strength based on the moment resistance of the connections. On the other hand, beam connected directly to the column web using typical flush and extended end-plate connection has resulted large deformation on the column web and showed relatively low moment resistance (Irwan 2001, Kim 1988).

The basic principle of multi-storey steel frame design is to classify the frame as braced or un-braced before carrying out the analysis and design. For a frame to be classified as braced, the bracing system provided should be at least five times stiffer than the stiffness of the frame itself (European Steel Design Education Programme 1994). In Eurocode 3 (BSI 2005), the frame is classified as braced when the bracing system reduces the horizontal displacement by at least 80%. A steel frame which does not satisfied the criterion for a braced frame is classified as un-braced. Although an un-braced frame may be treated as a three-dimensional entity, it is usually idealised as a series of two-dimensional frames that resist loading (horizontal and vertical) in each plane primarily by bending action. In practice, frames are usually braced against horizontal displacements in one direction to simplify the behaviour and to avoid the bending action about the minor axes of the column. However, if no bracing system is installed on the minor axis plane, the design of un-braced frame bending on minor axis is inevitable.

In un-braced frames, it is important to note that limitation on sway under service loading must be satisfied, as well as the ultimate strength. These concerned both the inter-storey drifts and the structure as a whole. For example, the limits recommended by BS 5950: Part 1 (BSI 2000) are $h/300$ for inter-storey drift and $h_o/300$ for structure as a whole, where h is the storey height and h_o is the overall height of the building. Details of the measurement of h and h_o are shown in Fig. 3. The δ_o shown in Fig. 3 is the sway distance on the tip of the frame due to the applied wind load. The inter-storey drift of the frame is defined as the difference between the sway distances on each floor level. For example, the inter-storey drift on level 3 is taken as the difference between the sway of the frame ($\delta_3 - \delta_2$) at level 2 and level 3.

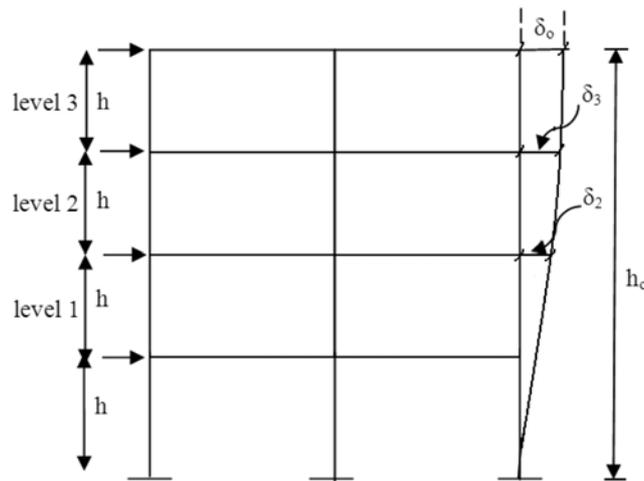


Fig. 3 Position of h and h_o for un-braced frame

2. Design approach for un-braced frame

The loading on un-braced frame is resisted solely by the bending capacity of the frame's members without any bracing system, therefore the most practical design approach is to apply rigid or semi-rigid joints. For un-braced frames, the main design consideration is to limit the sway, to avoid unacceptable deflections under service loads and to avoid premature collapse due to frame instability (Anderson and Lok 1983). Frame stability can be achieved by using stiff joints and appropriate member sections. Fully welded beam-to-column connections are the closest approach to a truly-rigid joint but induce expensive fabrication cost; while for most major axis connection, beam welded to an extended end plate and bolted to the column flange provides reasonable strength and stiffness. However, both welded and bolted joints are likely to require stiffening in the tension and compression zones of the column web, and possibly in shear. For weak axis connection, fully welded connection cannot be considered as rigid connection, as a conclusion drawn from the previous study by Irwan (2001), in which the column web connection is insufficient to be classified as full strength or rigid.

A study on the effective length factor of columns in un-braced steel frames was carried out by Kishi *et al.* (1997). The study presented the governing equations for determining the K-factor of column for both semi-rigid and rigid connections in un-braced steel frames under various boundary conditions. Dhillon and O'Malley (1999) have developed a computer based analysis and design method for semi-rigid steel frame with the use of second-order non-linear analysis, which also include the effects of the flexibility of the connections and the member's geometric non-linearity. The study concluded that frames designed with semi-rigid connections offer greater economy by a variation of connection stiffness that balances the span and the end moments in a beam. Van Keulan *et al.* (2003) proposed a half initial secant stiffness method to predict the full moment-rotation characteristic of a connection. The method was used to compare the results calculated from an "exact" reference analysis, and the overall frame behaviour according to the proposed method was very close to that of the reference analysis. The study has concluded that the half initial secant stiffness method described in Eurocode 3 is an appropriate method to predict frame

force distribution.

Degertekin and Hayalioglu (2004) conducted a series of analysis and design for steel frames with semi-rigid connections and column bases. The analysis and design refer to Frye and Morris polynomial model and the specifications of American Institute of Steel Construction (AISC). Degertekin and Hayalioglu concluded that semi-rigid connection was more economical than rigid connection as the semi-rigid column base has resulted to lighter frames. Changes in the connection stiffness could contribute to more economical solutions and the alteration of the frame sways. Kemp (2004) proposed a flexibility-based method for pin-jointed frame analysis, which can be used as manual method of linear or nonlinear analysis for sway frames. This method provides a basis in developing the understanding of bending moment diagrams and deflected shapes, and the influence of sway translations, loads, temperature and settlement action. Mageirou and Gantes (2006) proposed a simplified approach to evaluate the critical buckling load of multi-storey frames with semi-rigid connections. Analysis of sway, non-sway and partially-sway structures with semi-rigid connections were compared with finite element analysis and the results showed high accuracy. The analysis concluded that rotational stiffness of the columns and beams converged at bottom and top ends of the column with semi-rigid connections which depend heavily on the boundary conditions at their far end and the existence of the axial force. Maquoi *et al.* (2006) conducted a series of numerical and analytical investigations on composite sway frames at Liege University. Two simplified design analytical methods were performed in the analysis have concluded that “amplified sway moment method” provide high accuracy, except the “Merchant-Rankine” approach needs further improvements. Xu and Wang (2007) proposed a linear programming method to evaluate the elastic buckling loads of multi-storey un-braced steel frames subjected to variable loading or non-proportional loading. The study concluded that maximum and minimum frame-buckling loads and the associated load patterns can be obtained by solving the maximization and minimization problems using the proposed method. Lu *et al.* (2008) proposed the use of direct second-order elastic analysis for steel frames, which can directly calculate the load distributions and deformations with the inclusion of both $P-\Delta$ and $P-\delta$ effects on a structural system. Lu concluded that second-order elastic analysis approach is convenient, versatile and adaptable for different structural forms, and led to an efficient and competitive design process, non-labour- intensive, and reduce the cost of the design subsequently, even the complicated large-scale structures.

Schueller (1977) proposed an equation to predict the multi-storey frame sway based on the Portal Method. The method takes into account the relationships between beam rotation-deformation and column rotation-deformation. Mahmud Ashraf *et al.* (2004) adopted Schueller’s equation by proposing a number of modifications to the equation for rigid multi-storey steel frames. A design chart has been developed to improve further the Schueller’s equations. The results from the proposed equation are almost identical to those predicted using finite element analysis. The proposed equation has been applied for both regular and irregular steel frames. The proposed equation has been concluded as a useful tool in predicting the overall sway of a multi-storey frame other than numerical modelling. The study is then further extended to investigate the sway behaviour of semi-rigid regular steel frames having the same arrangement of beam and column sections at all levels. Design charts to predict the frame sway have been introduced to eliminate the needs of numerical modelling. Schueller’s equation for rigid frames has been adopted and modified to incorporate the flexibility of semi-rigid connection. The sway prediction using modified Schueller’s equation is given as:

$$\Delta_{semi-rigid} = \left(\frac{HV_c h^2}{12EI_c} + \frac{HV_g L^2}{12EI_g} + \frac{2N_c H^2}{3EA_c B} \right) + \frac{V_g LH}{2K_j}$$

where, N_c is axial force in exterior column at the base due to wind; V_c is shear force in exterior column due to wind at specified level; I_c is moment of inertia of column at the same level as V_c about axis of bending; A_c is area of the exterior column at the base; V_g is shear force in girders due to wind at the same level as V_c ; I_g is moment of inertia of girder about x -axis at the same level as V_c ; H is total height of the frame; B is total base width of the frame; h is typical storey height; L is girder span; E is modulus of elasticity and K_j is the connection stiffness. The proposed equation has overestimated the sway by nearly 10% as compared to the results from finite element analysis. The study also concluded that the proposed design charts accurately predicted the semi-rigid frame sway without using finite element analysis. For irregular frames, Mahmud Ashraf *et al.* (2006) has investigated the sway behaviour of building frames with stepped configuration using numerical technique. Relationships are established to compare the maximum lateral sway for a stepped frame and its' corresponding regular frame so as to utilise the proposed design charts for regular frames. The predicted sway values are compared with those obtained numerically and the results showed good agreement.

This paper introduces the investigation of semi-rigid steel frames using wind-moment method at the initial stage of design. Secondly, the study proposed a simplified method to predict the frame sway in conjunction to the sway limit as suggested in BS 5950: Part 1 (BSI 2000). The proposed method has integrated the optimization of the beam and column sizes and takes into account the flexibility of the connection. The stability of the frame is checked using (Kavianpour 1990) method which takes into the consideration of first order and second order analysis. The study concluded that the sections of columns and beams for the final frames have met the requirements for both the serviceability and ultimate limit state design.

2.1 Wind-moment approach

In un-braced frame, the resistance to lateral forces usually rely on the frame action involving interaction between the bending stiffness of the connected member and rigidity of the connection. A simple design method known as the wind-moment or wind-connection method has been introduced to design an un-braced frame. The advantage of the method is simplicity. The frame is statically determinate thus the internal moments and forces are not dependent on the relative stiffness of the frame members. The need of iterative analysis correspond to the change of section size is therefore avoided. The beam sections generally having the same size at all floor level as the mid-span moment due to gravity load usually controls the design, thus avoid repetition in fabrication and the cost of splicing. The use of wind-moment method in conjunction with BS 5950: Part 1 (BSI 2000) is well-established for major axis framing (Anderson and Kavianpour 1991, Anderson *et al.* 1991). This method effectively employs semi-rigid connections which are also "partial strength" relative to the connected beams; a range of standard joints of this type are also available (SCI 1996). The existing rules for wind-moment design have been shown to provide adequate resistance to frames with major axis sway, but have not been established for minor axis sway. This study emphasis on the frames which sway about the minor axis, with beam-to-column connections assumed as partial strength. For partial strength connection, the moment resistance of the connection should be at least equal to 25% but not greater than 100% of the moment resistance of the connected beam (SCI

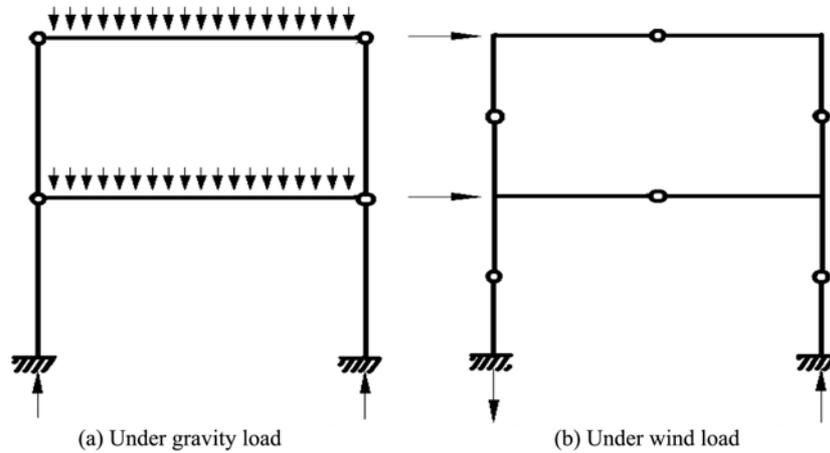


Fig. 4 Frame under gravity and wind load with assumed pin joint and point of contra flexure

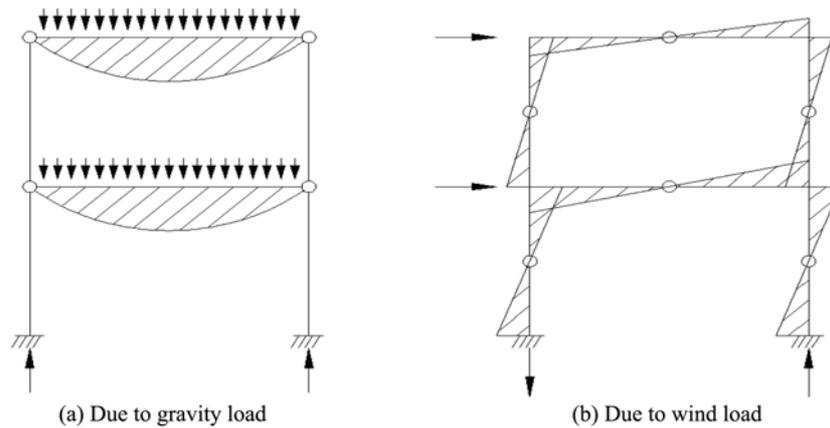


Fig. 5 Internal forces and moments

1996). These existing rules are adopted in this study to design multi-storey steel frame bending on minor axis. In its simplest form the wind-moment method assumes (Tahir 1997):

- i) Under gravity load, the connections act as pins (see Fig. 4(a)); this means that the beam members are designed as simply supported with no moments transferred to the column, other than nominal “eccentricity” moments. The connection is assumed as pins even though the actual connection is a partial strength connection. The purpose is to simplify the design process for beam. The beam-to-column connections will generally be under-designed. As beams are usually governed by mid-span moment, whilst connections are sized only for end moment, the connections will behave as partial-strength with respect to the beams. Therefore, the use of partial strength connection is suggested so as to increase the stiffness of the frame.
- ii) Under wind load, the connections behave as rigid joints, with points of contra flexure at the mid-height of columns and mid-length of beams (see Fig. 4(b)). The purpose of assuming the connections as rigid is to predict the initial section size which will then be checked for ultimate limit state and service limit state.

- iii) The internal forces and moments due to the gravity and wind loads (see Fig. 5(a) and Fig. 5(b)) are superimposed according to different load cases as described in BS 5950: Part 1 (2000). The design at ultimate limit state is completed by amending the initial section sizes and other details for the members and connections, to sustain the combined load effects.

2.1.1 Design of beams

The design of beam section should be classified as plastic or compact section (class 1 or class 2) of universal beam sections (Tahir 1997). The beam design refers to the required moments and shears capacity due to the combination of vertical shear, wind loads or notional horizontal forces. The design moment capacity is limited to 90% of the plastic moment resistance (M) of the section to ensure that the beams provide directional restraint to the column in accordance to BS 5950: Part 1 (BSI 2000). Therefore, a nominal allowance of 10% is made for partial fixity of the beam to column connection as shown in Fig. 6. The maximum sagging moment is then reduced to 10% to allow for connection restrained as a result of using partial strength connection. The following relationship should therefore be satisfied as $0.9M \leq 0.9p_y S_x$ or alternatively $M \leq p_y S_x$, where M is the applied moment due to vertical loading by assuming that the beam is simply supported, p_y is the design strength of the steel and S_x is the plastic modulus of the beam about the major axis. As part of the beam that is not effectively restrained, a lateral torsional buckling is checked to satisfy a requirement of BS 5950: Part 1 (2000). For the arrangement of frame layout as suggested in Fig. 7, the beam spanning on column bending on minor axis is checked for lateral torsional buckling. However, for beam spanning on column bending on major axis, no check on lateral torsional buckling is needed as the beam is considered fully restrained by the slab (Tahir 1997, Salter *et al.* 1999).

For a beam layout as shown in Fig. 7, however, the gravity loading on the beams spanning on minor axis column will be negligible and horizontal loading will govern the required strength of the minor axis beams. In such case, the stiffness requirements of beams are likely to dictate the final choice of section size. As horizontal loads due to wind is becoming more significant, the hogging moment developed at the connected beam end tend to be greater than the sagging moment at the mid-span of the beam. The term wind-moment refers to the moment developed from the horizontal wind forces. This wind moment is accumulated as a result of the accumulation of wind forces from the top storey to the bottom storey which resulted to the largest moment developed at the bottom floor level as shown in Fig. 8. As a result, this wind moment formed hogging moment in the

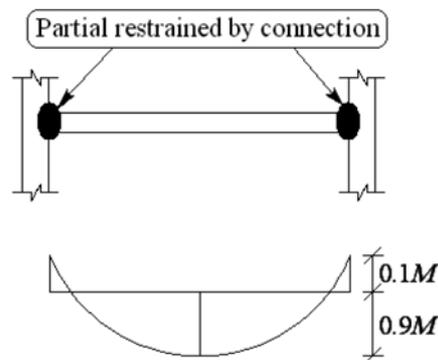


Fig. 6 Moment capacity of the beam is limited to 90% of the plastic moment resistance of the section

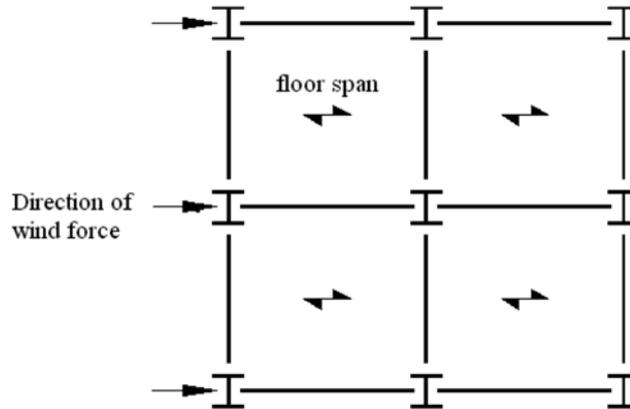


Fig. 7 Beam layout with floor spanning to major axis beams

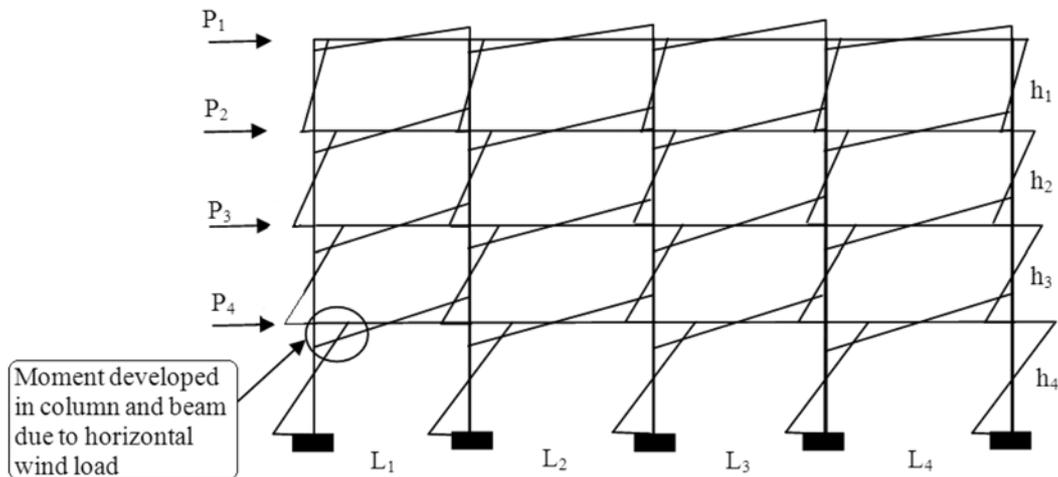


Fig. 8 Accumulation of moment in the column and beam due to horizontal wind load

connected beam which is larger than the sagging moment developed from the gravity load at the centre of the beam span. In this case, the design of beam should be based on the hogging moment instead of sagging moment due to gravity load.

2.1.2 Design of column

The design of column is based on the simple construction method where the column is assumed to carry axial load and nominal moment. However, in actual situation, the moments transferred to the column is much higher than the nominal moment, leading to the selected column section is under-designed, due to the detrimental effect on such member of the hogging moments developed in the beams. These moments particularly affect external columns subject to unbalanced loading. To encounter the under-designed problem, the slenderness used to calculate the compressive resistance (P_{cx} or P_{cy}) about the major and minor axes of the columns is based on the effective length of $1.5L$. This is to ensure that the selected column will improve the stiffness of the frame which resulted to

minimize the sway of the frame. No calculation is made for second-order moments due to the “ $P-\Delta$ ” effect. It is assumed that this can be accounted for by using effective column length of $1.5L$ greater than the true length for frame bending on both axes. However, the frames will be improved further by the proposed method to cater for sway limitation. All columns are checked to satisfy the local capacity check and overall buckling check in accordance to BS 5950: Part 1 (BSI 2000).

2.2 Range of applications

The proposed wind moment method has been validated by a comprehensive non-linear finite element parametric study for a broad range of low rise frames (Anderson and Kavianpour 1991, Brown 1995, Salter *et al.* 1999, Reading 1989). Theoretically the method could be used for a wider range of unbraced steel frames. However, to simplify the procedures and remain within the bounds of the validation study, the method described in this paper only limited to unbraced steel frames that satisfy with the following requirements (Tahir 1997, Tahir and Anderson 1999, Salter *et al.* 1999):

- i) The geometrical configuration of the frame should be within the range as shown in Table 1
- ii) The width of each bay should be constant over the height of the frame
- iii) The structure should comprised of regular arrangement of orthogonal beams and columns
- iv) The beam layout should have similar arrangement as shown in Fig. 7.

If the frames comply with the limits as stated in Table 1, the adoption of wind-moment method for un-braced steel frame bending on minor axis should not be led to weaker sections than those studies (Anderson and Kavianpour 1991, Brown 1999, Reading 1989) established for un-braced steel frame bending on major axis. As for the unbraced steel frames bending on minor axis, the range of the study is for two, four and six bays with height varying from two and four storeys. In recognition of the unlikelihood of the frame consisting of only one longitudinal bay, the minimum number of bays in the minor axis framing is taken as two. Each longitudinal bay is assumed to be six meter in length. The limitations on frame dimensions conformed to those specified in the existing guide (Anderson *et al.* 1991) for wind-moment design. In view of possible difficulty in ensuring adequate stability and stiffness, the study assumed S275 steel, rather than higher grade material used in some of the earlier studies (Brown 1995). The arrangement of floor grids is shown in Fig. 7. The floor slabs are assumed to span six meter between the major-axis frames; which results in the minor-axis beams being free of significant gravity forces, thus the main loading come from wind-moments. Notional horizontal force is considered in the design to represent the initial sway imperfection due to lack of verticality BS 5950: Part 1 (BSI 2000). The load is taken as a

Table 1 Proportions of frames suitable for use of the wind-moment method

Relative dimensions	Minimum	Maximum
Number of storeys	2	4
Number of bays	2	4
Bay width (m)	4.5	12
Bottom storey height (m)	4.5	6
Storey height elsewhere (m)	3.5	5
Bay width : storey height (bottom storey)	0.75	2.5
Bay width : storey height (above bottom storey)	0.9	3
Greatest bay width : smallest bay width	1	2

minimum of 0.5% of the factored vertical dead and imposed loads applied at the same floor level (BS 5950: Part 1:2000). However, the load is not governing the design of the frame as it is smaller than the horizontal wind force.

2.3 Determination of wind forces

As suggested by the Steel Construction Institute (Anderson *et al.* 1991, Salter *et al.* 1999, Hensman and Way 2000), the wind loads may be derived using either CP 3: Chapter V: Part 2 (BSI 1972) or BS 6399: Part 2 (BSI 1997). The resulting (unfactored) horizontal force at each floor level should be within the range of 10-40 kN. This level of loading corresponds to a minimum wind speed of 37 m/s according to CP 3 or 20 m/s according to BS 6399. The minimum wind force is taken in conjunction to the maximum gravity load and vice-versa. The combinations of wind load and gravity load are in accordance to BS 5950: Part 1 (BSI 2000) and suggested by the SCI to develop critical load combination to the designed frame. These loads combinations are applied to the frame to satisfy the requirement of ultimate limit states (ULS) and serviceability limit states (SLS).

2.4 Load combinations

For serviceability limit states, loads were taken as un-factored. For the load case that take into account the dead load, imposed load and wind load, only 80% of the imposed load and wind load need to be considered. Frames were analysed under three load combinations as follows:

- i) 1.0 dead load + 1.0 imposed load + un-factored notional horizontal force
- ii) 1.0 dead load + 0.8 imposed load + 0.8 wind load
- iii) 1.0 dead load + 1.0 wind load

For ultimate limit states, loads were taken as factored. Frames were analysed under three load combinations as follows:

- i) 1.4 dead load + 1.6 imposed load + factored notional horizontal force
- ii) 1.2 dead load + 1.2 imposed load + 1.2 wind load
- iii) 1.4 dead load + 1.4 wind load

3. Frame design on minor axis bending

Frames design using wind moment method is checked on both ULS and SLS. The authors have developed a software to design the columns and beams for frame bending about minor axis to cater both SLS and ULS. The checking of beams and columns for ULS has satisfied the requirements in accordance to BS 5950: Part 1 (BSI 2000). However, the checking for SLS for sway deflections of the frames need to be improved further. As a result, the frames designed by wind moment method should be revised to limit the sway of the frames to sway index of 1/300. The rules to design the individual members to limit the sway are described below.

3.1 Limiting sway by optimization of beam and column

Anderson and Islam (1987), Anderson (1987) had established a method to limit the sway of multi-storey steel frame bending on major axis by limiting the inter-storey sway. In this paper, the authors

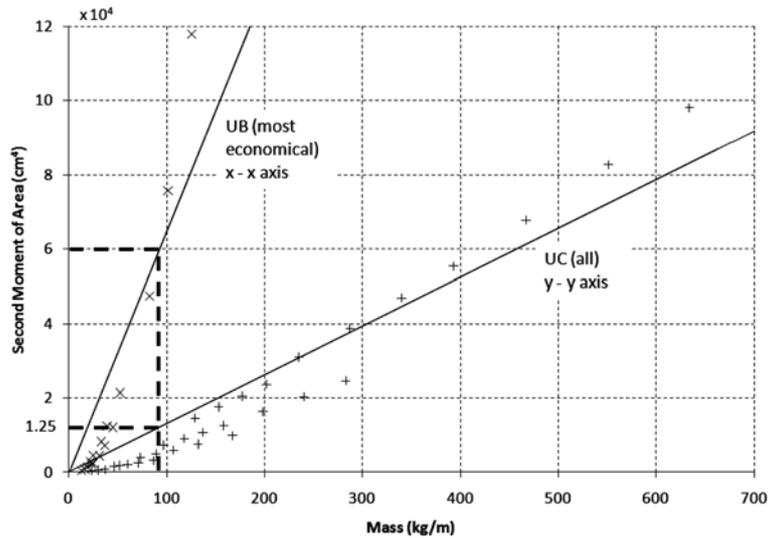


Fig. 9 Optimization of second moment of area between beams and columns sections

had adopted the same method to optimise the size of beam bending on major axis and column bending on minor axis. An element optimisation is included, which permits account to be taken of the differing efficiencies of various section shapes in providing flexural rigidity. This proposed method is used to derive the difference of second moment of area of beam on major axis I_{x-x} and second moment of area of column on minor axis I_{y-y} in the design process to optimize the size of beams and columns. Comparison of the more efficient universal beams in major-axis bending for the most economical sections with the minor axis properties of universal columns for all sections showed that the former are approximately five times more efficient in providing flexural rigidity as shown in Fig. 9 (Tahir 1997). Account is taken of this difference when using the formulae given in limiting sway method (Anderson and Islam 1987) where factor k_3 was accounted for such difference; the value is taken as 4.8 (see Fig. 9). The effect of this factor is to encourage the use of deeper beams to provide overall sway stiffness. If however the formulae predicted that the optimum design required smaller columns than the calculated section of wind-moment, the formulae were then used the section designed by the wind moment method. This enables beam sections to be selected to meet the deflection limit, taking into account the rigidity of the already-chosen columns. To avoid an undue number of splices, column sections were only changed every two storeys for two and four storey frames. All frames are checked for overall buckling checks and local capacity checks as suggested in BS 5950: Part 1 (BSI 2000) in clause 4.8.3.

3.1.1 Design equations for the proposed limiting sway method

The design equations proposed are to limit the values of horizontal sway deflection. The frame is divided into statically determinate sub-frames by assuming points of contra flexure. Allowance for section optimization is then used, together with slope-deflection analysis, to derive equations for optimum design. This method is suitable for hand calculation.

For the top storey, the sub-assembly shown in Fig. 10(a) is used to derive the design equations as stated below

$$I_{1,2} = \frac{P_1 h_1 I_{2,2}}{P_1 h_1 + P_2 h_2} \tag{1}$$

$$I_{2,2} = \frac{(P_1 h_1 + P_2 h_2) h_1 L_2^2 I_{3,2}}{24 E \Delta B I_{3,2} - P_1 h_1^3 (L_1 + L_2)} \tag{2}$$

$$I_{3,2} = \frac{\left[P_1 h_1^3 (L_1 + L_2) + h_1 L_2 \sqrt{\frac{P_1 h_1 (L_1 + L_2) [2 W_1 P_1 h_1 + W_2 (P_1 h_1 + P_2 h_2)]}{W_3}} \right]}{24 E \Delta B} \tag{3}$$

$$I_{3,2} = \frac{I_{3,2}}{2} \tag{4}$$

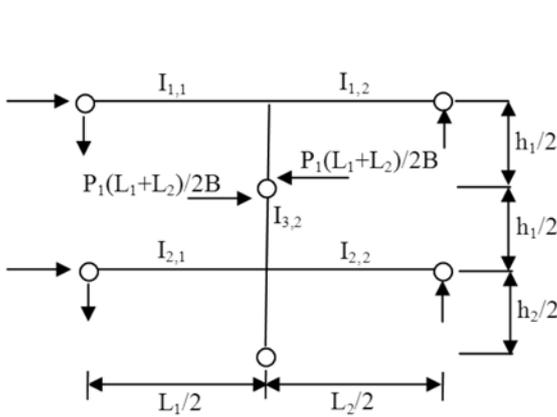


Fig. 10(a) Top storey sub-assembly

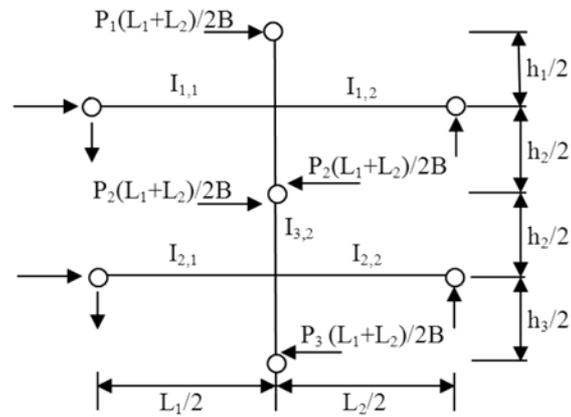


Fig. 10(b) Intermediate storey sub-assembly

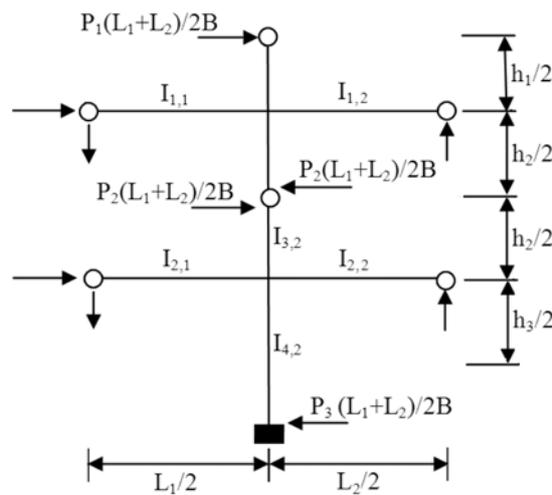


Fig. 10(c) Sub-assembly for bottom 2 storeys

For intermediate storey, the sub-assembly shown in Fig. 10(b) is used to derive the design equations as stated below. It is assumed that the total horizontal shear is divided between the bays in proportion to the widths.

$$I_{1,2} = \frac{(P_1 h_1 + P_2 h_2) I_{2,2}}{P_2 h_2 + P_3 h_3} \quad (5)$$

$$I_{2,2} = \frac{(P_2 h_2 + P_3 h_3) h_2 L_2^2 I_{3,2}}{24 E \Delta B I_{3,2} - P_2 h_2^3 (L_1 + L_2)} \quad (6)$$

$$I_{3,2} = \frac{\left[P_2 h_2^3 (L_1 + L_2) + h_2 L_2 \sqrt{\frac{P_2 h_2 (L_1 + L_2) [W_1 (P_1 h_1 + P_2 h_2) + W_2 (P_2 h_2 + P_3 h_3)]}{W_3}} \right]}{24 E \Delta B} \quad (7)$$

$$I_{3,2} = \frac{I_{3,2}}{2} \quad (8)$$

For bottom two storeys of a fixed base frame, the sub-assembly shown in Fig. 10(c) is used to derive the design equations stated below. The fixity of the base attracts more moment than the upper column. As a result, the design may be governed by the permissible deflection Δ of the upper storey. The effect of fixed base is more pronounced when $h_2 = h_3$, and the bottom storey column inertia than has to be made equal to $I_{3,2}$ to avoid reverse taper.

$$I_{3,2} = \frac{(P_2 h_2 + L_2 Y) h_2^2 (L_1 + L_2)}{24 E \Delta_2 B} \quad (9)$$

$$Y = \sqrt{\frac{3 P_2 h_3 [W_1 (P_1 h_1 + P_2 h_2) + 2 W_2 (P_2 h_2 + P_3 h_3)]}{(3 W_3 h_3 (L_1 + L_2) (h_2 + h_3) - W_2 L_2^2)}} \quad (10)$$

$$I_{1,2} = \frac{(P_1 h_1 + P_2 h_2) h_2 L_2^2 I_{3,2}}{24 E \Delta_2 B I_{3,2} - P_2 h_2^3 (L_1 + L_2)} \quad (11)$$

$$I_{2,2} = \frac{(P_2 h_2 + P_3 h_3) h_2 L_2^2 I_{3,2}}{24 E \Delta_2 B I_{3,2} - P_2 h_2^3 (L_1 + L_2)} - \frac{L_2^2 I_{3,2}}{(6 h_3 (L_1 + L_2))} \quad (12)$$

$$W_1 = \frac{k_{1,1} L_1^3 + k_{1,2} L_2^3 + \dots + k_{1,m} L_m^3}{2 L_2^2} \quad (13)$$

$$W_2 = \frac{k_{2,1} L_1^3 + k_{2,2} L_2^3 + \dots + k_{2,m} L_m^3}{2 L_2^2} \quad (14)$$

$$W_3 = \frac{(k_{3,1} + k_{3,2}) L_1 + (k_{3,2} + k_{3,3}) L_2 + \dots + (k_{3,m} + k_{3,m+1}) L_m}{(L_1 + L_2)} \quad (15)$$

where

P_1 is the total horizontal shear in the top storey columns,

P_2 is the total horizontal shear in all the columns of the storey being designed,

P_3 are the total horizontal shear in the columns of the storeys immediately above and below,

L_1 and L_2 are the span of the beams; h_1 , h_2 and h_3 are the height of the columns,

Δ equals the allowable sway over the storey height h_2 ,

B is the total width of the frame; E is Young's modulus,

$I_{1,2}$ is the inertia of the upper beams in the storey,

$I_{2,2}$ is the inertia of the lower beams in the storey,

$I_{3,2}$ is the inertia of the internal designed column in the storey,

$I_{3,2'}$ is the inertia of the external designed column in the storey.

W_1 , W_2 , and W_3 are the cost factors for a member of inertia $I_{i,j}$.

For frames with the grid of Fig. 7, the formulae are used in conjunction with a deflection limit of height/300 and the full un-factored wind load. The formulae are based on an assumed first-order elastic response. This is partly to check that the formulae have generated a reasonable stiff design, but also permitted account to be taken of second-order effects. When these caused the limiting index of 1/300 to be exceeded, beam sections are further increased until the second-order analysis showed this limit had been satisfied.

3.2 Limiting sway by the proposed Tahir formulae

Frames designed by optimization that did not satisfied the sway index of 1/300 are further improved by increasing the size of beams based on a method proposed by the authors. In order to ensure that the minor axis beams have adequate stiffness for frame stability, the relative beam to column stiffness suggested in Fig. 11 should satisfy the following criteria (Tahir 1997)

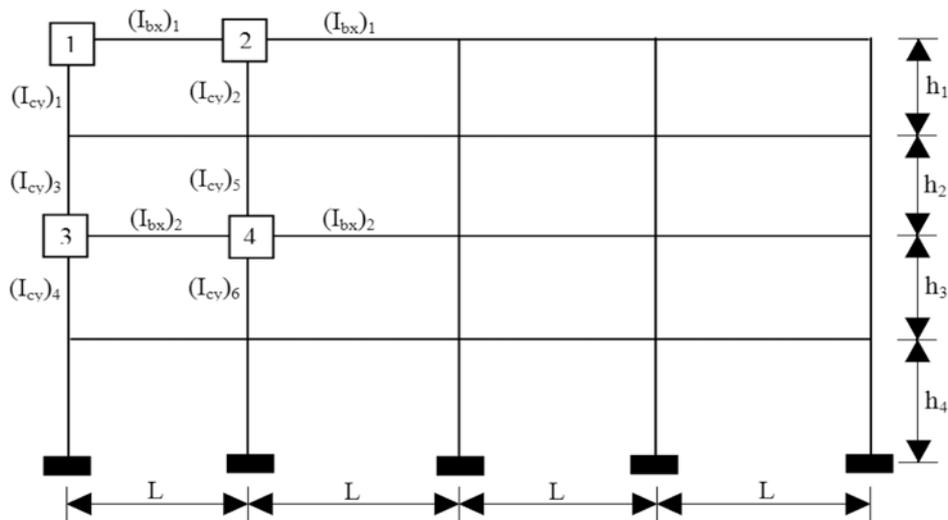


Fig. 11 Notations used to define the relative beam-to-column stiffness requirements

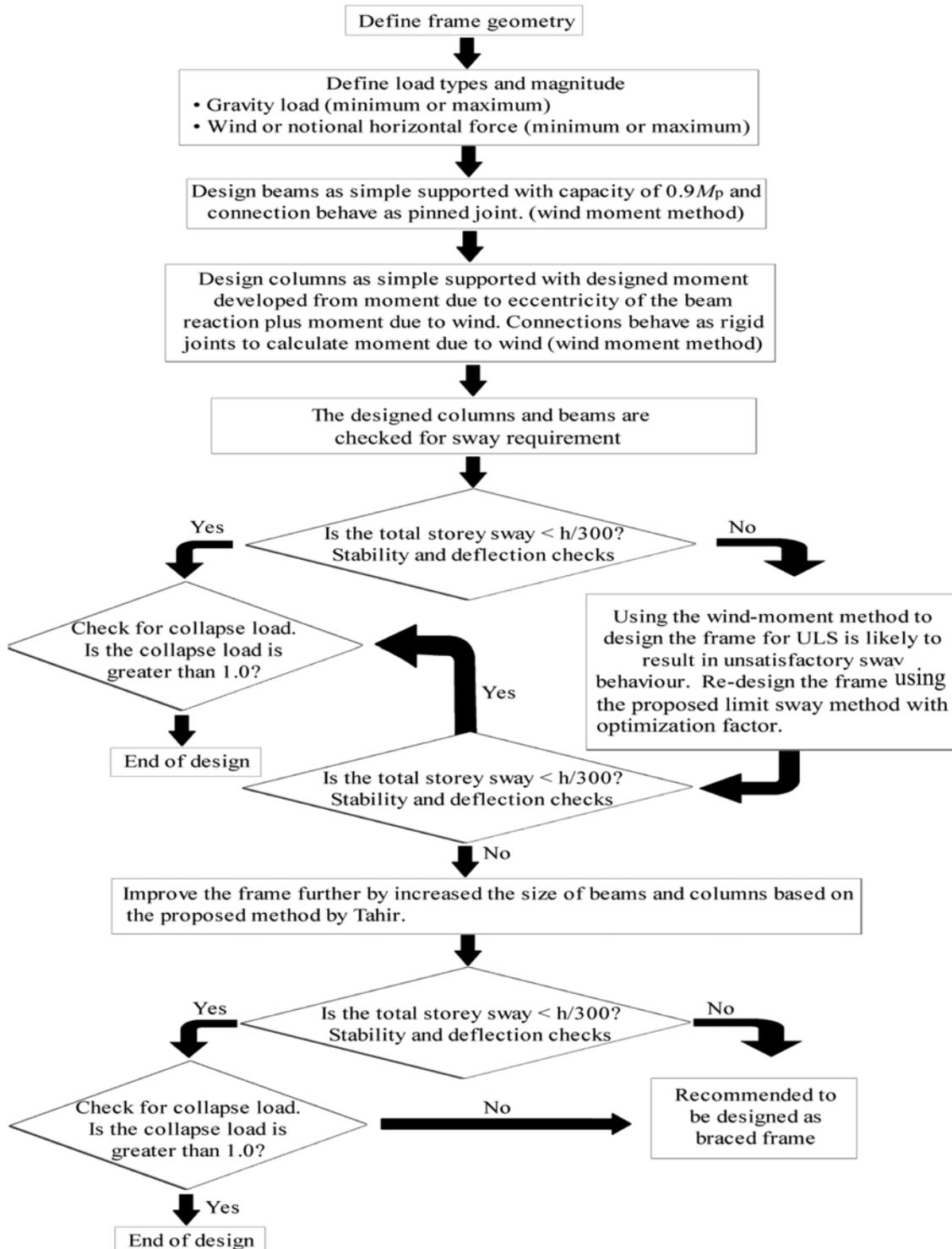


Fig. 12 Flow chart to determine the suitability of the proposed method for a given minor axis frame

$$\text{At node 1: } (I_{bx})_1/L \geq (I_{cy})_1/h_1 \quad (16)$$

$$\text{At node 2: } (2I_{bx})_1/L \geq (I_{cy})_2/h_1 \quad (17)$$

$$\text{At node 3: } (I_{bx})_2/L \geq (I_{cy})_3/h_2 + (I_{cy})_4/h_3 \quad (18)$$

$$\text{At node 4: } (2I_{bx})_2/L \geq (I_{cy})_5/h_2 + (I_{cy})_6/h_3 \quad (19)$$

The above criteria are adopted for the rest of the frame. The proposed Tahir formulae (Eqs. (16) to (19)) allow an increment to the size of beam instead of the size of column by selecting the next higher weight. The increased in the beam size instead of column is due to the relative stiffness of the beam in major axis to the column in minor axis is about 5 times higher as mentioned earlier. Details on the procedures to design frame on minor axis bending is summarised as flowchart in Fig. 12.

4. Parametric study

The geometrical arrangement and loadings are listed in Table 2. Table 2 concerns on minimum wind combined with maximum gravity load and maximum wind combined with minimum gravity load as suggested by wind-moment publication (Anderson *et al.* 1991). The wind-moment design is given in Table 3 for minimum wind combined with maximum gravity load and Table 4 for maximum wind combined with minimum gravity load. To improve further the frames stiffness and satisfy the deflection limits, frames are designed with the proposed limiting sway formulae by including the optimization factor and the proposed Tahir formulae are listed in Table 5 and Table 6 respectively. Moment connections' requirements are tabulated in Table 3 and Table 4. The moment resistance of selected partial strength connection for the designated frame should be more than the required design moment of the connection. The load-deflection (sway) behaviour for each of the frame up to the point of collapse is examined for first and second-order analysis at ULS using the software developed by Kavianpour (1990) and modified by the authors. The results for deflection check on first and second order analysis with collapse load factor are tabulated in Table 7 for frame with minimum wind combined with maximum gravity load and in Table 8 for frame with maximum wind combined with minimum gravity load.

4.1 Verification

To justify the design recommendations which include proposed rules to limit sway, the frames are subjected to second-order analysis accounted for the semi-rigid nature of the joints. Software is first developed for this purpose by Kavianpour (1990) for major axis and modified by the authors to suit with the minor axis framing to carry out the first and second-order analysis. The software is based on elasto-plastic analysis for frames with rigid connections using matrix displacement method. It is capable of analysing any combination of pinned connections, fully-rigid joints and semi-rigid joints with known $M-\phi$ behaviour. Frames are analysed up to the collapse load for the plane frame. This software allows discrete hinges to form in members and second order effects to be accounted for. This software is used to carry out the analysis on the minor axis framing. Generally, when the overall sway deflections are calculated, both first-order and second-order values were obtained. The

Table 2 Frames design for minimum wind in conjunction with maximum gravity load

Basic Frame Type	Width of Bay (m)	Height of Column		Width of Longitudinal Bays (m)	Maximum Gravity Load (kN/m ²)				Min. Wind Speed (m/s)	Minimum Gravity Load (kN/m ²)				Max. Wind Speed (m/s)
		Ground Elevated (m)			Floor		Roof			Floor		Roof		
					L.L	D.L	L.L	D.L		L.L	D.L	L.L	D.L	
2 Storey 2 Bay	6m precast floor	6	5	6.0	5.0	7.5	3.75	1.5	20	3.5	4.0	3.75	1.5	40
4 Storey 2 Bay	6m precast floor	6	5	6.0	5.0	7.5	3.75	1.5	20	3.5	4.0	3.75	1.5	40
4 Storey 4 Bay	6m precast floor	6	5	6.0	5.0	7.5	3.75	1.5	20	3.5	4.0	3.75	1.5	40
4 Storey 6 Bay	6m precast floor	6	5	6.0	5.0	7.5	3.75	1.5	20	3.5	4.0	3.75	1.5	40

Table 3 Frames design for 2, 4 and 6 bays with minimum wind in conjunction with maximum gravity load

Basic Frame Type	Wind moment method					Connection Requirements			
	Universal Beam			Universal Column		Bending moment (kN.m)		Shear force (kN)	
	Floor	Roof		External	Internal	Floor	Roof	Floor	Roof
2 Storey 2 Bay	1st 203×133×25	203×133×25	Up to 2nd Storey	203×203×71	203×203×71	1st 24	7	1st 8	2
4 Storey 2 Bay	1st 305×102×25	203×133×25	Up to 2nd Storey	305×305×97	356×368×129	1st 79	11	1st 26	4
	2nd 203×133×25		2nd to 4th Storey	203×203×60	254×254×73	2nd 54		2nd 18	
	3rd 203×133×25					3rd 33		3rd 11	
4 Storey 4 Bay	1st 203×133×25	203×133×25	Up to 2nd Storey	305×305×97	356×368×129	1st 39	5	1st 13	2
	2nd 203×133×25		2nd to 4th Storey	203×203×60	254×254×89	2nd 27		2nd 9	
	3rd 203×133×25					3rd 16		3rd 5	
4 Storey 6 Bay	1st 203×133×25	203×133×25	Up to 2nd Storey	305×305×97	356×368×129	1st 26	4	1st 9	1
	2nd 203×133×25		2nd to 4th Storey	203×203×60	254×254×89	2nd 18		2nd 6	
	3rd 203×133×25					3rd 11		3rd 4	

Table 4 Frames design for 2, 4 and 6 bays with maximum wind in conjunction with minimum gravity load

Basic Frame Type	Wind moment method					Connection Requirements			
	Universal Beam			Universal Column		Bending moment (kN-m)		Shear force (kN)	
	Floor	Roof		External	Internal	Floor	Roof	Floor	Roof
2 Storey 2 Bay	1st 356×127×33	203×133×25	Up to 2 nd Storey	254×254×73	305×305×118	1st 107	30	1st 36	10
4 Storey 2 Bay	1st 457×191×67 2nd 406×140×46 3rd 356×127×33	203×133×25	Up to 2nd Storey 2nd to 4th Storey	356×368×153 254×254×73	356×406×235 305×305×118	1st 318 2nd 207 3rd 120	38	1st 106 2nd 69 3rd 40	13
4 Storey 4 Bay	1st 406×140×39 2nd 356×127×33 3rd 203×133×25	203×133×25	Up to 2nd Storey 2nd to 4th Storey	305×305×97 203×203×52	356×368×153 254×254×73	1st 159 2nd 104 3rd 60	19	1st 53 2nd 35 3rd 20	6
4 Storey 6 Bay	1st 356×127×33 2nd 305×102×25 3rd 203×133×25	203×133×25	Up to 2nd Storey 2nd to 4th Storey	254×254×89 203×203×46	305×305×118 254×254×73	1st 106 2nd 69 3rd 40	13	1st 35 2nd 23 3rd 13	4

Table 5 Limiting sway formulae by optimization factor and the proposed Tahir formulae for minimum wind in conjunction with maximum gravity load

Basic Frame Type	Limiting sway formulae with optimization factor					Proposed Tahir formulae				
	Universal Beam			Universal Column		Universal Beam			Universal Column	
	Floor	Roof		External	Internal	Floor	Roof		External	Internal
2 Storey 2 Bay	1st 203×133×25	203×133×25	Up to 2nd Storey	203×203×71	305×305×97	1st 254×102×25	203×133×25	Up to 2nd Storey	203×203×71	305×305×97
4 Storey 2 Bay	1st 406×140×39 2nd 406×140×39 3rd 356×127×33	203×133×25	Up to 2nd Storey 2nd to 4th Storey	305×305×97 254×254×73	356×368×129 305×305×97	1st 406×140×39 2nd 406×140×39 3rd 356×127×33	203×133×25	Up to 2nd Storey 2nd to 4th Storey	305×305×97 254×254×73	356×368×129 305×305×97
4 Storey 4 Bay	1st 305×102×25 2nd 254×102×25 3rd 203×133×25	203×133×25	Up to 2nd Storey 2nd to 4th Storey	305×305×97 203×203×60	356×368×129 254×254×89	1st 356×127×33 2nd 356×127×33 3rd 356×127×33	305×102×25	Up to 2nd Storey 2nd to 4th Storey	305×305×97 203×203×60	356×368×129 254×254×89
4 Storey 6 Bay	1st 254×102×25 2nd 203×133×25 3rd 203×133×25	203×133×25	Up to 2nd Storey 2nd to 4th Storey	305×305×97 203×203×60	356×368×129 254×254×89	1st 356×127×33 2nd 356×127×33 3rd 356×127×33	305×102×25	Up to 2nd Storey 2nd to 4th Storey	305×305×97 203×203×60	356×368×129 254×254×89

Table 6 Limiting sway formulae and the proposed Tahir method for maximum wind in conjunction with minimum gravity load

Basic Frame Type	Limiting sway formulae with optimization factor				Proposed Tahir formulae					
	Universal Beam		Universal Column		Universal Beam		Universal Column			
	Floor	Roof	External	Internal	Floor	Roof	External	Internal		
2 Storey 2 Bay	1st 406×140×46	305×102×33	Up to 2nd Storey	305×305×97	356×368×129	1st 457×191×82	406×140×39	Up to 2nd Storey	305×305×97	356×368×129
4 Storey 2 Bay	1st 610×229×101	305×102×33	Up to 2nd Storey	356×368×153	356×406×287	1st 610×229×113	356×127×33	Up to 2nd Storey	356×368×153	356×406×287
	2nd 533×210×92		2nd to 4th Storey	356×368×129	356×368×202	2nd 610×229×101		2nd to 4th Storey	356×368×129	356×368×202
4 Storey 4 Bay	1st 533×210×82	254×102×25	Up to 2nd Storey	305×305×118	356×368×153	1st 533×210×92	356×127×33	Up to 2nd Storey	305×305×118	356×368×153
	2nd 457×152×67		2nd to 4th Storey	305×305×97	356×368×129	2nd 457×191×74		2nd to 4th Storey	305×305×97	356×368×129
	3rd 406×140×46		2nd to 4th Storey	305×305×97	356×368×129	3rd 457×152×52		2nd to 4th Storey	305×305×97	356×368×129
4 Storey 6 Bay	1st 457×152×52	203×133×25	Up to 2nd Storey	305×305×97	356×368×129	1st 457×152×60	356×127×33	Up to 2nd Storey	305×305×97	305×305×129
	2nd 406×140×46		2nd to 4th Storey	254×254×73	305×305×118	2nd 457×152×52		2nd to 4th Storey	254×254×73	305×305×118
	3rd 356×127×39		2nd to 4th Storey	254×254×73	305×305×118	3rd 406×140×39		2nd to 4th Storey	254×254×73	305×305×118

Table 7 ULS collapse load factor and deflection at SLS with partial strength connection for 2, 4, and 6 bays frames (Frames design for minimum wind and maximum gravity load)

Basic Frame Type	Load Case	Limiting sway formulae with optimization factor			Proposed Tahir formulae		
		Collapse Load Factor (2nd order)	Deflection Check		Collapse Load Factor (2nd order)	Deflection Check	
			1st order	2nd order		1st order	2nd order
2 Storey 2 Bay	Load case 1	1.31	1/840	1/733	N/A	N/A	N/A
	Load case 2	1.59	1/537	1/474			
	Load case 3	1.89	1/512	1/476			
4 Storey 2 Bay	Load case 1	1.41	1/1250	1/1019	N/A	N/A	N/A
	Load case 2	1.46	1/476	1/398			
	Load case 3	1.68	1/400	1/370			
4 Storey 4 Bay	Load case 1	0.83	To be improved	To be improved	1.27	1/693	1/471
	Load case 2	0.87			1.26	1/436	1/350
	Load case 3	1.07			1.38	1/367	1/310
4 Storey 6 Bay	Load case 1	0.56	To be improved	To be improved	1.04	1/634	1/505
	Load case 2	0.68			1.05	1/527	1/430
	Load case 3	0.93			1.18	1/469	1/356

Table 8 ULS collapse load factor and deflection at SLS with partial strength connection for 2, 4, and 6 bays frames (Frames design for maximum wind and minimum gravity load)

Basic Frame Type	Load Case	Limiting sway formulae with optimization factor			Proposed Tahir formulae		
		Collapse Load Factor (2nd order)	Deflection Check		Collapse Load Factor (2nd order)	Deflection Check	
			1st order	2nd order		1st order	2nd order
2 Storey 2 Bay	Load case 1	1.23	1/1429	1/1341	1.31	1/747	1/686
	Load case 2	1.09	1/248	1/234	1.38	1/393	1/365
	Load case 3	1.18	1/208	1/201	1.28	1/314	1/301
4 Storey 2 Bay	Load case 1	1.65	1/5833	1/1707	1.43	1/1193	1/1083
	Load case 2	1.03	1/366	1/226	1.31	1/370	1/335
	Load case 3	1.02	1/292	1/191	1.42	1/318	1/300
4 Storey 4 Bay	Load case 1	0.83	To be	To be	1.27	1/693	1/471
	Load case 2	0.87	improved	improved	1.16	1/436	1/370
	Load case 3	1.07			1.08	1/367	1/310
4 Storey 6 Bay	Load case 1	0.56	To be	To be	1.20	1/734	1/405
	Load case 2	0.68	improved	improved	1.25	1/627	1/380
	Load case 3	0.93			1.15	1/469	1/320

resistance moment of the column sections is taken as the plastic moment about the minor axis, reduced to take account of co-existent axial force, in accordance with the usual formulae given in British tables for steel sections. It should be noted that because of the shape factor about the minor axis, the attainment of the plastic moment at the end of a column will be accompanied by plastic zones of significant length away from the theoretical plastic hinge. The computer analysis does not account for the loss of stiffness resulting from partially-plastic regions. This however, does not invalidate the conclusions from the study because subsequent checks are made on the local behaviour of each column length.

To ensure local column stability, checks on overall buckling and local capacity are made in accordance with Clause 4.8.3.2 and Clause 4.8.3.3 of BS 5950: Part 1 (BSI 2000) for all frames. The moments and forces are those given by the analysis at the design load levels for ULS. The equivalent uniform moment factors are calculated from the distribution of bending moments revealed in the columns. The resulting comparisons against unity are termed as "Stability Factors". For the overall buckling check, the minor axis moment of resistance is taken as the yield moment $p_y Z_y$, not the plastic moment. For the local capacity check, the moment is taken as the lesser of the minor axis plastic moment and $1.2p_y Z_y$.

4.2 Assessment of results

The use of wind moment method has resulted to heavier section in the minor axis framing than the major axis framing. These results are as expected as the second moment of area which contributed to stiffening the frame in weak axis is smaller than the major axis. The use of effective length from typical $1.5L$ for minor axis has not satisfied the deflection limit index of 1/300 of the frames. Further improvement of the frames by introducing the optimization factor has contributed

significantly to the stiffening of the frames. However, some of the frames are still not able to satisfy the limit sway index of 1/300. Therefore, to improve the frames further formulae proposed by Tahir (1997) have been introduced by further increase the size of the beam by selecting the next higher steel weight of the beam. As a result, the frames have satisfied both the stability checks and the limit sway index of 1/300. For frames where 'N/A' is shown, the frames have satisfied both the stability checks and the limit sway index of 1/300 without any changes needed to improve the frames. However, for frame indicated as 'To be improved' is due to failure on collapse load less than 1.0, the frames need to be improved further by the proposed Tahir formulae. Frames that do not satisfy the checking on the sway index of 1/300 as in Table 8 should also be further improved by the proposed Tahir formulae.

For frames with minimum wind combined with maximum gravity load, frames designed for 2 bay 2 storey and 2 bay 4 storey have satisfied both the ULS and SLS with collapse load greater than 1.0. However, for 4 bay 4 storey frame and 6 bay 4 storey frames, the frames need to be stiffen further by including the optimization factor in order to satisfy the required checks. All frames are checked to satisfy the collapse load factor to be at least equal to 1.0 and to satisfy the deflection checks for 1st and 2nd order analysis. The deflection checks for these frames are slightly higher than the sway index of 1/300.

5. Conclusions

Despite the assumption of relatively stiff minor-axis connections in which the joints are considered partial strength, a straightforward extension of the previous rules for wind moment design did not always result in frames of adequate overall stability. This is particularly true of frames in which floor units span between major-axis beams. The improvement of frame by introducing the optimization factor has resulted to the design of the frames that can satisfy both the stability checks and the sway index of 1/300 for first order and second order analysis in minimum wind in conjunction with maximum wind load for 2 bay 2 storey and 2 bay 4 storey frames. However, for 4 bay 4 storey and 6 bay 4 storey the frames need to be improved further by increasing the size of beam using the proposed Tahir method to satisfy stability checks and the sway index of 1/300. The use of the proposed limiting sway formula with optimization factor has resulted to a better frames design which is close to satisfying the deflection and the stability checks for all frames. The study also concluded that the maximum wind load in conjunction with minimum gravity load governed the design of all the frames bending on minor axis. In view on the scope of the studies and the problems revealed in providing a frame of adequate resistance, it is concluded that the use of the wind-moment method together with the proposed methods in minor axis bending should be restricted to low rise frames not more than four storeys with partial strength joints.

References

- Anderson, D. (1987), "Design of multi-storey frames to sway deflection limitations steel-framed structures", *Stability & Strength*, Edition Narayanan, R., Elsevier Applied Science, 665-673.
- Anderson, D. and Kavianpour, K. (1991), "Analysis of steel frames with semi-rigid connections", *Struct. Eng. Rev.*, **3**, 79-87.

- Anderson, D. and Islam, M.A. (1987), "Design of multi-storey frames to sway deflection limitations", *Struct. Eng.*, **57B**, 11-17.
- Anderson, D., Reading, S.J. and Kavianpour, K. (1991), "Wind-moment design for unbraced frames", Steel Construction Institute Publication No. 082.
- Ashraf, M., Nethercot, D.A. and Ahmed, B. (2004), "Sway of semi-rigid steel frames Part 1: Regular frames", *Eng. Struct.*, **26**, 1809-1819.
- Ashraf, M., Nethercot, D.A. and Ahmed, B. (2006), "Sway of semi-rigid steel frames Part 2: Irregular frames", *Eng. Struct.*, **26**, 1809-1819.
- Ashraf, M., Nethercot, D.A. and Ahmed, B. (2004), "A simplified method to predict sway of rigid multi-storey steel frames", *Steel Compos. Struct.*, **4**(4), 317-331.
- British Standards Institute BS 5950-Part 1 (2000), Structural Use of Steelwork in Building Part 1: Code of Practice for Design – Rolled and Welded Sections, British Standards Institution, London.
- British Standards Institute BS 6399 (1997), Code of Basic Data for the Design of Buildings: Part 2: Wind Loads, British Standards Institution.
- Brown, N.D. (1995), "Aspects of sway frame design and ductility of composite end plate connections", Ph.D Thesis, University of Warwick.
- Degertekin, S.O. and Hayalioglu, M.S. (2004), "Design of non-linear semi-rigid steel frames with semi-rigid column bases", *E. J. Struct. Eng.*, **4**, 1-16.
- Dhillon B.S. and O'Malley, J.W. (1999), "Interactive design of semi-rigid steel frames", *J. Struct. Eng.*, **125**(5), 556-564.
- Eurocode 3 (2005), Design of Steel Structures: ENV 1993-1-1: Part 1.1: General Rules and Rules for Buildings, CEN, Brussels.
- European Steel Design Education Programme (1994) "Structural System: Building II", Group 14, Volume 21, The Steel Construction Institute, Silwood Park, Ascot, United Kingdom.
- Irwan, J. (2001), "Performance of connections on Minor Axis using Local Sections", M.Phil, Universiti Teknologi Malaysia, Skudai, Johor, Malaysia.
- Kavianpour, K. (1990), "Design and analysis of unbraced steel frames", Ph.D Thesis, University of Warwick, 1990.
- Kemp, A.R. (2004), "Use of pin-jointed structures in teaching frame analysis", *Eng. Struct.*, **26**, 883-894.
- Kim, Y.W. (1988), "The behaviour of beam-to-column web connections with flush end-plates", M.Sc Thesis, University of Warwick.
- Kishi, N., Chen, W.F. and Goto, Y. (1997), "Effective length factor of column in semi-rigid and unbraced frames", *J. Struct. Eng.*, **123**(3), 313-320.
- Lu, C.K., Chen, W.F., Chan, S.L. and Ma, T.W. (2008), "Direct second-order elastic analysis for steel frame design", *Struct. Eng.*, **12**(6), 379-389.
- Mageirou, G.E. and Gantes, C.J. (2006), "buckling strength of multi-storey sway, non-sway and partially-sway frames with semi-rigid connections", *J. Constr. Steel Res.*, **62**, 893-905.
- Maquoi, R., Jaspard, J.P. and Demonceau, J.F. (2006), "Contribution to the design of sway composite frames", *Composie Construction in Steel and Concrete V*, **186**(61), 653-663.
- Reading, S.J. (1989), "Investigation of the wind connection method", M.Sc Thesis, University of Warwick.
- Salter, P.R., Couchman, G.H. and Anderson, D. (1999), "Wind-moment design of low rise frames", SCI Publication P263, Silwood Park, Ascot, Berkshire, SL 5 7QN.
- Schueller, W. (1977), *High-Rise Building Structures*, John Wiley and Sons.
- Steel Construction Institute and British Constructional Steelwork Association (1992), SCI P206, Joints in Simple Construction, Volume 2: Practical Application, London.
- Steel Construction Institute and British Constructional Steelwork Association, SCI P207 (1996), *Joints in Steel Construction. Moment Connections*, Silwood Park, Ascot, Berks SL 7QN, London.
- Sulaiman, A. (2007), "Behaviour of partial-strength connection in semi-continuous construction for multi-storey braced steel frame using TWP sections", Ph.D Thesis, Faculty of Civil Engineering, Universiti Teknologi Malaysia, Skudai, Johor, Malaysia.
- Tahir, M.M. (1997), "Structural and economic aspects of the use of semi-rigid joints in steel frames", Ph.D Thesis, University of Warwick.

- Tahir, M.M., Shek, P.N., Sulaiman, A. and Tan, C.S. (2009), "Experimental test on cruciform column sections using universal beam section", *Int. J. Const. Build. Mater.*, **23**, 1354-1364.
- Van Keulen, D.C., Nethercot, D.A., Snijder, H.H. and Bakker, M.C.M. (2003), "Frame analysis incorporating semi-rigid joint action: applicability of the half initial secant stiffness approach", *J. Constr. Steel Res.*, **59**, 1083-1100.
- Xu, L. and Wang, X.H. (2007), "Stability of multi-storey unbraced steel frames subjected to variable loading", *J. Constr. Steel Res.*, **63**, 1506-1514.