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Theoretical research on the identification method of bridge dynamic parameters using free decay response

Guo-jin Tan[a], Yong-chun Cheng[b], Han-bing Liu* and Long-lin Wang

College of Transportation, Jilin University, Changchun, China

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Abstract. Input excitation and output response of structure are needed in conventional modal analysis methods. However, input excitation is often difficult to be obtained in the dynamic load test of bridge structures. Therefore, what attracts engineers' attention is how to get dynamic parameters from the output response. In this paper, a structural experimental modal analysis method is introduced, which can be used to conveniently obtain dynamic parameters of the structure from the free decay response. With known damping coefficients, this analysis method can be used to identify the natural frequencies and the mode shapes of MDOF structures. Based on the modal analysis theory, the mathematical relationship of damping ratio and frequency is obtained. By using this mathematical relationship to improve the previous method, an improved experimental modal analysis method is proposed in this paper. This improved method can overcome the deficiencies of the previous method, which can not identify damping ratios and requires damping coefficients in advance. Additionally, this improved method can also identify the natural frequencies, mode shapes and damping ratios of the bridge only from the free decay response, and ensure the stability of identification process by using modern mathematical means. Finally, the feasibility and effectiveness of this method are demonstrated by a numerical example of a simply supported reinforced concrete beam.

Keywords: dynamic load test; dynamic parameters; free decay response; modal analysis method; modal damping ratio.

1. Introduction

Bridge condition estimation based on measured dynamic parameters is an important method of bridge nondestructive detection. The accuracy of the estimation is directly affected by the precision of the identified bridge dynamic parameters. Therefore, precise identification of dynamic parameters is the key to bridge nondestructive detection.

Frequency response function (FRF) or impulse response function (IRF) is needed in traditional modal analysis methods (Forment and Richardson 2002). So input excitation and output response are needed to be collected for forming the FRF or IFR in the dynamic test of bridge structures.

*Corresponding author, Professor, E-mail: tgj@jlu.edu.cn
[a]Lecturer, E-mail: ttggjj@163.com
[b]Professor, E-mail: chengyc@jlu.edu.cn

However, output response may be collected only in the dynamic test of bridge structures, which can be divided into free decay response and random response under ambient excitation. Therefore, the modal analysis methods of bridge structures based on only output response have been studied. These methods include two types, one is based on random response and the other is based on free decay response.

The identification methods of bridge dynamic parameters based on random response mainly contain Peak Picking Method (PP) (Ren and Harik 2002), Frequency Domain Decomposition Method (FDD) (Brincker and Zhang 2000), Series Analysis Method (Lardies and Larbi 2001) and Stochastic Subspace Method (Jiang and Xin 2005) etc. PP method and FDD method in frequency domain are applicable to the case with small damping and white noise input. There are no mature methods reported to determine the order of identification model or system of Series Analysis Method and Stochastic Subspace Method at present.

The free decay response of measuring points can be obtained in two ways. One is obtained from the random response by using random decrement techniques, and the other is directly collected in the dynamic test. Therefore, what attracts engineers' attention is how to get the structure dynamic parameters from the free decay response. Feeny and Kappaggantu may be the first to discuss the method of extracting dynamic parameters from free vibration response. They have gotten modes of structures using the proper orthogonal decomposition method (Feeny and Kappagantu 1998), however, the identification work could not be carried on if the mass matrix of structure is unknown, which limits its application. Subsequently, Kerschen and Arunasis have made a further research on the method by extracting dynamic parameters from the free vibration response (Kerschen and Golinval 2004, Arunasis *et al.* 2006), but, the computation process of this method is complicated and suitable for small damping system. The Eigensystem Realization Algorithm (ERA) was utilized to extract a twin-deck bridge modal parameters from the coupled free-decay responses (Qin *et al.* 2007). In oeder to examine the effects of torsional-to-vertical natural frequency ratio of 2DOF bridge dynamic systems on the aerodynamic and dynamic properties of bridge decks, a series of wind tunnel free-decay sectional model dynamic tests were conducted (Qin *et al.* 2009). Recently, a more convenient modal analysis method based on the displacement response of free vibration has been proposed (Wang and Cheng 2008). The premise of this method is that the structure damping coefficients are known, and this makes the determination of the damping coefficients according to the measured data become very important. Therefore, the application of this approach is limited, due to the fact that damping coefficients in the bridge dynamic tests are usually not available in advance.

On the basis of the method presented by Wang and Cheng (2008), in this paper, an identification algorithm of dynamic parameters (frequencies, mode shapes and damping ratios) of the bridge is proposed, which is based on free decay response without the necessity of knowing damping coefficients in advance. Modern mathematical means are adopted to ensure identification accuracy of the bridge dynamic parameters and solution stability of identification algorithm.

2. Modal analysis theory of MDOF system

In this paper, the bridge is discretized to a MDOF system. Therefore, the identification algorithm for dynamic parameter is evolved from the modal analysis theory of MDOF system. In the forming process of this algorithm, two types of basic theories are used: one is the mathematical relationship between the modal damping ratio and frequency, the other is the theory on identifying frequency and mode shape from free decay response. The modal analysis theory of MDOF system in the both two aspects is introduced in the next section.

2.1 Mathematical relationship between modal damping ratio and natural frequency

The free vibration equation of MODF system with proportional viscous damping can be expressed as

$$[M]\cdot\ddot{x}+[C]\cdot\dot{x}+[K]\cdot x=0 \tag{1}$$

where

$$[C]=\alpha\cdot[M]+\beta\cdot[K]$$

α, β are damping coefficients of the bridge structure.

Let

$$x=[X]\cdot e^{i\omega t}$$

according to the modal analysis theory of MDOF system, it can be concluded that (Li and Lu 2001)

$$\xi_r=\frac{\alpha}{2\omega_r}+\frac{\beta\omega_r}{2} \tag{2}$$

$$[M]^{-1}\cdot[K]\cdot[X]=\omega^2\cdot[X] \tag{3}$$

where, ξ_r is the rth modal damping ratio, ω_r is the rth frequency. Eq. (2) presents the relationship among the damping ratio, damping coefficients and natural frequency. If any two parameters of these are known, the third can be solved. Eq. (3) shows that solving the n modes of MDOF system is namely to solve the n-pairs eigenvalues and eigenvectors of the $[M]^{-1}[K]$ matrix.

2.2 Identification algorithm of mode shapes and frequencies

Rearranging the free vibration equation of MDOF system (Eq. (1)), it can be formulated that (Wang and Cheng 2008)

$$[M]\cdot(\ddot{x}+\alpha\dot{x})=-[K]\cdot(x+\beta\dot{x}) \tag{4}$$

$x_{r,k}, \dot{x}_{r,k}$, and $\ddot{x}_{r,k}$ denote the displacement, velocity and acceleration of the rth degree of freedom at time t_k, respectively.

$$[X]=[X]_{M\times n}=\begin{bmatrix}\begin{Bmatrix}x_{1,k}\\x_{2,k}\\\vdots\\x_{n,k}\end{Bmatrix} & \begin{Bmatrix}x_{1,k+1}\\x_{2,k+1}\\\vdots\\x_{n,k+1}\end{Bmatrix} & \cdots & \begin{Bmatrix}x_{1,k+M-1}\\x_{2,k+M-1}\\\vdots\\x_{n,k+M-1}\end{Bmatrix}\end{bmatrix}^T=\begin{bmatrix}x_{1,k} & x_{2,k} & \dots & x_{n,k}\\x_{1,k+1} & x_{2,k+1} & \dots & x_{n,k+1}\\\vdots & \vdots & \ddots & \vdots\\x_{1,k+M-1} & x_{2,k+M-1} & \dots & x_{n,k+M-1}\end{bmatrix} \tag{5}$$

Similarly

$$[\dot{X}] = \begin{bmatrix} \dot{x}_{1,k} & \dot{x}_{2,k} & \ldots & \dot{x}_{n,k} \\ \dot{x}_{1,k+1} & \dot{x}_{2,k+1} & \ldots & \dot{x}_{n,k+1} \\ \vdots & \vdots & \ddots & \vdots \\ \dot{x}_{1,k+M-1} & \dot{x}_{2,k+M-1} & \cdots & \dot{x}_{n,k+M-1} \end{bmatrix} \tag{6}$$

$$[\dot{X}] = \begin{bmatrix} \ddot{x}_{1,k} & \ddot{x}_{2,k} & \ldots & \ddot{x}_{n,k} \\ \ddot{x}_{1,k+1} & \ddot{x}_{2,k+1} & \ldots & \ddot{x}_{n,k+1} \\ \vdots & \vdots & \ddots & \vdots \\ \ddot{x}_{1,k+M-1} & \ddot{x}_{2,k+M-1} & \cdots & \ddot{x}_{n,k+M-1} \end{bmatrix} \tag{7}$$

Discretizing x, $\dot{x}$ and $\ddot{x}$ in Eq. (4) at M moments, and using $[X], [\dot{X}]$ and $[\ddot{X}]$ to represent the displacement, velocity and acceleration matrix respectively, combining Eqs. (4)-(7), Eq. (4) can be expressed as

$$[M]\cdot([\ddot{X}]^T+\alpha[\dot{X}]^T) = -[K]\cdot([X]^T+\beta[\dot{X}]^T) \tag{8}$$

Left multiply both sides of Eq. (8) by $[M]^{-1}$, then

$$([\ddot{X}]^T+\alpha[\dot{X}]^T) = -[M]^{-1}[K]\cdot([X]^T+\beta[\dot{X}]^T) \tag{9}$$

Right multiply both sides of Eq. (9) by $([X]^T+\beta[\dot{X}]^T)^T$, then

$$([\ddot{X}]^T+\alpha[\dot{X}]^T)\cdot([X]^T+\beta[\dot{X}]^T)^T = -[M]^{-1}[K]\cdot([X]^T+\beta[\dot{X}]^T)\cdot([X]^T+\beta[\dot{X}]^T)^T \tag{10}$$

Right multiply both sides of Eq. (10) by $(([X]^T+\beta[\dot{X}]^T)\cdot([X]^T+\beta[\dot{X}]^T)^T)^{-1}$, then

$$[M]^{-1}[K] = -([\ddot{X}]^T+\alpha[\dot{X}]^T)\cdot([X]^T+\beta[\dot{X}]^T)^T\cdot(([X]^T+\beta[\dot{X}]^T)\cdot([X]^T+\beta[\dot{X}]^T)^T)^{-1} \tag{11}$$

Comparing Eq. (3) with Eq. (11), it can conclude that the process of getting n frequencies and mode shapes of bridge structures is namely to solve n-pairs eigenvalues and eigenvectors of the $[M]^{-1}[K]$ matrix in Eq. (11).

3. Identification algorithm of bridge dynamic parameters

3.1 Basic principle of the identification algorithm

Based on the modal analysis theory of MDOF system, the identification algorithm of bridge dynamic parameters is formed, using iterative algorithm and cubic spline curve approximation method. The basic principle of this identification algorithm is as follows:

When displacement response is known, the velocity and acceleration response can be gotten by using the finite difference method. Based on Eq. (11), it can be concluded that if the damping coefficients α, β can both be determined, then the $[M]^{-1}[K]$ matrix can be formulated, and the natural frequencies and mode shapes of the bridge can be identified . Before carrying out dynamic parameters identification, the damping coefficients α and β are unknown. Therefore, the displacement response can not be used directly to formulate the $[M]^{-1}[K]$ matrix by Eq. (11).

Based on the modal analysis theory of MDOF system, Eq. (2) illustrates the relationship among the damping ratio, damping coefficient and frequency. If the damping ratio and frequency are known, the damping coefficients can be calculated. Combing Eq. (2) with Eq. (11), an iterative algorithm can be constituted. As long as the iterative initial value of damping ratio and frequency are given, the natural frequencies, mode shapes and damping ratios of the bridge could be identified from the displacement response.

In order to guarantee the stability and precision of the iterative algorithm, a cubic spline curve is used to approximate the free decay response of displacement before the iteration.

3.2 Identification process of dynamic parameters

With the aid of iterative algorithm and cubic spline interpolation theory, the identification process of bridge dynamic parameters is as follows:

(1) Making the displacement response pass the low-pass or band-pass filter, the displacement response which contains only the first n-order frequency components can be gotten.

(2) The velocity and acceleration response are calculated.

Because the first order derivative of displacement with respect to time is velocity and the first order derivative of velocity with respect to time is acceleration, the velocity response can be obtained by using the finite difference method from the displacement response, as that of the acceleration response can be obtained in the same way from the velocity response. In this paper, the second-order central difference method is used to calculate the velocity and acceleration response.

When the displacement response has a random error, using finite difference method to form the velocity response and acceleration response will magnify the error, which makes the formulated $[M]^{-1}[K]$ matrix have a greater error. So the identification algorithm is instable and have a low precision. The ways to resolve this problem are: a) increasing the sampling frequency (namely shortening time interval); b) using cubic spline curve to approximate the displacement response. It is known that cubic spline curve have a second order continuous derivative in each interval. Because the sampling frequency can't be increased unlimitedly, therefore cubic spline curve approximation of the displacement response is adopted to guarantee the identification precision of dynamic parameters in this paper.

The solving process of the velocity and acceleration response is to approximate the measured displacement response by the cubic spline curve firstly, then to adopt the second-order central difference method to calculate the velocity and acceleration response.

(3) Based on the filtered displacement response, the iterative initial value of first n-order frequencies and damping ratios of the bridge can be obtained by other methods.

(4) The damping coefficients α_0, β_0 are calculated.

Let

$$\{\xi_0\} = \{\xi_{r,0} \ \ \xi_{r+1,0} \ \cdots \ \xi_{r+n-1,0}\}^T$$

$$\{\omega_0\} = \{\omega_{r,0} \ \ \omega_{r+1,0} \ \cdots \ \omega_{r+n-1,0}\}^T$$

Where, $\xi_{r,0}$ is the rth modal damping ratio preliminarily determined in the ste step(3), and $\omega_{r,0}$ is the rth frequency preliminarily determined in the step(3). Based on the least squares principle, and by using the relationship equation of the modal damping ratio and frequency of MDOF system (Eq. (2)), it can be obtained that

$$\begin{Bmatrix} \alpha_0 \\ \beta_0 \end{Bmatrix} = ([Q]^T[Q])^{-1}[Q]^T\{\xi_0\} \tag{12}$$

where

$$[Q] = \begin{bmatrix} \dfrac{1}{2\omega_{r,0}} & \dfrac{\omega_{r,0}}{2} \\ \dfrac{1}{2\omega_{r+1,0}} & \dfrac{\omega_{r+1,0}}{2} \\ \vdots & \vdots \\ \dfrac{1}{2\omega_{r+n-1,0}} & \dfrac{2\omega_{r+n-1,0}}{2} \end{bmatrix} \tag{13}$$

(5) The frequency $\omega_{r,1}$ and mode shape $\phi_{r,1}$ are solved.

Substituting the damping coefficients α_0, β_0 and the response matrix of displacement, velocity and acceleration to Eq. (11), $[M]^{-1}[K]$ can be formulated. The rth frequency $\omega_{r,1}$ and mode shape $\phi_{r,1}$ are obtained by solving the eigenvalues and eigenvectors of the $[M]^{-1}[K]$ matrix. The rth damping ratio $\xi_{r,1}$ can be solved by substituting α_0, β_0 and $\omega_{r,1}$ to Eq. (2).

(6) The damping coefficients α_1, β_1 are solved.

Let $\{\Delta\xi\} = \{\xi_1\} - \{\xi_0\}$, $\Delta\alpha = \alpha_1 - \alpha_0$, $\Delta\beta = \beta_1 - \beta_0$ it can be obtained from Eq. (12) that

$$\begin{Bmatrix} \Delta\alpha \\ \Delta\beta \end{Bmatrix} = ([Q]^T[Q])^{-1}[Q]^T\{\Delta\xi\} \tag{14}$$

Where, $\omega_{r,0}$ in $[Q]$ is replaced by $\omega_{r,1}$ at this time, $\{\xi_1\} = \{\xi_{r,1} \ \ \xi_{r+1,1} \ \cdots \ \xi_{r+n-1,1}\}^T$.

As long as $\Delta\alpha$ and $\Delta\beta$ are solved by Eq. (14), α_1, β_1 can be calculated by Eq. (15).

$$\alpha_1 = \Delta\alpha + \alpha_0, \quad \beta_1 = \Delta\beta + \beta_0 \tag{15}$$

(7) The frequency $\omega_{r,2}$ and mode shape $\phi_{r,2}$ are solved.

Substituting the determined damping coefficients α_1, β_1 and the response matrix of displacement, velocity and acceleration to Eq. (11), $[M]^{-1}[K]$ can be formulated again. The rth frequency $\omega_{r,2}$ and mode shape $\phi_{r,2}$ are obtained by solving the eigenvalues and eigenvectors of the $[M]^{-1}[K]$ matrix again.the rth damping ratio $\xi_{r,2}$ can be calculated by substituting α_1, β_1 and $\omega_{r,2}$ to Eq. (2).

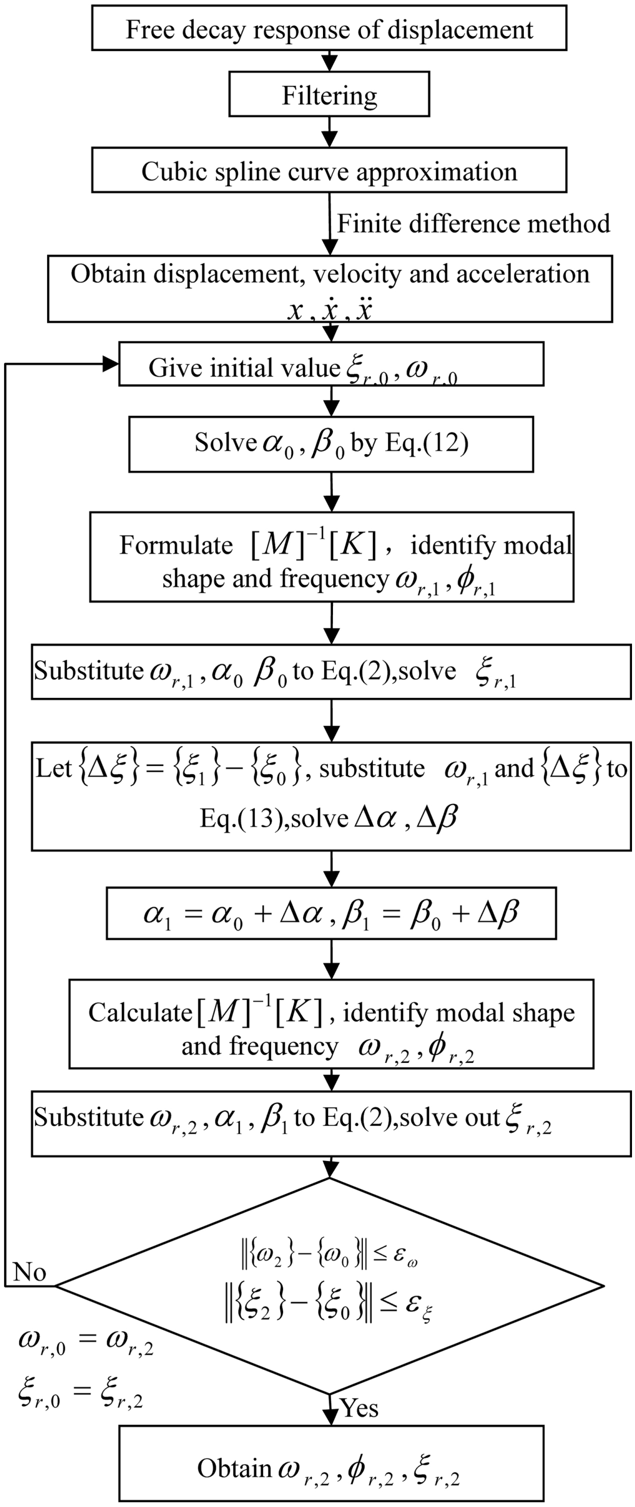


Fig. 1 Identification process of dynamic parameters

(8) The conditions of ending iteration are determined.

The error of frequency ε_ω and the error of damping ratio ε_ξ of the iterative algorithm are given. Checking whether the next two equations are satisfied stimultaneously

$$\|\{\omega_2\} - \{\omega_0\}\| \le \varepsilon_\omega \tag{16a}$$

$$\|\{\xi_2\} - \{\xi_0\}\| \le \varepsilon_\xi \tag{16b}$$

if yes, the iterative can be ended, and $\omega_{r,2}$, $\phi_{r,2}$ and $\xi_{r,2}$ can be deemed as the identified rth frequency, mode shape and damping ratio of the bridge respectively. If no, let $\{\omega_0\} = \{\omega_2\}$ and $\{\xi_0\} = \{\xi_2\}$, repeat the step(4)-(8) until the errors satisfy the requirement. $\|\ \|$ in Eq. (16) is 2-norm of a vector.

After preprocessing the displacement responses (filtering and cubic spline curve approximation), the iterative process of the dynamic parameters identification algorithm could begin by giving initial values of the damping ratio and frequency determined by other methods. The dynamic parameters identification process is shown in Fig. 1.

4. Numerical example

4.1 Beam profile and the calculation of dynamic response

A simply supported reinforced concrete beam is taken as an example. The cross-section of this

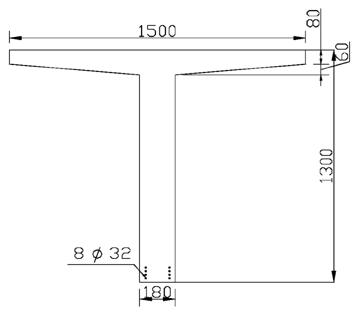


Fig. 2 Transverse section of reinforced concrete beam (mm)

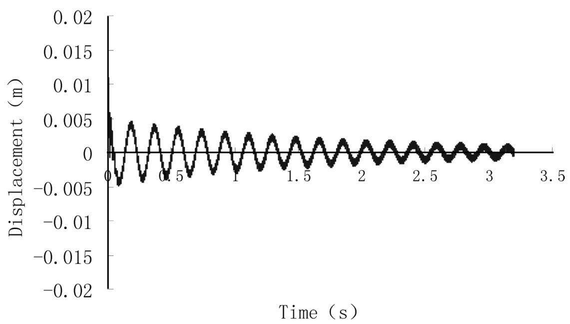


Fig. 3 Mid span free vibration response with 5% random error

beam is shown in Fig. 2. The beam is 20 m long with the density of 3101 kg m^{-3}, and its elastic modulus E is 28.5 KN/mm^2. There are 21 dynamic measuring points on the beam totally (at intervals of 1 m). The initial displacement is imposed to the position where is 3 m far away from one of the bearings. The displacement response of the measuring points of the beam is Calculated. The free decay response of displacement at the mid-span with 5% random error (namely the 11th dynamic measuring point) is shown in Fig. 3.

4.2 Identfication results of dynamic parameters

The cubic spline curve is used to approximate the displacement response. The velocity and acceleration response can be obtained using the second-order central difference method. If the initial value of damping ratios and the initial value of frequencies are given, frequencies, mode shapes and damping ratios, can be obtained with required accuracy by using the identification algorithm presented in this paper. Only the displacement response with 5% random error and the corresponding identification results are listed in this paper.

(1) Identification results of frequencies

The identification results of frequencies are shown in Table 1. From Table 1, it indicates that the frequencies identified by this method have high precision.

(2) Identification results of mode shapes

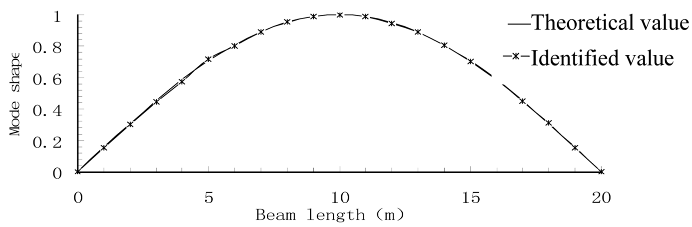
The precision of mode shapes identified can be judged by Modal Assurance Criteria (MAC) value, and the MAC value of the first 5 order mode shapes identified are listed in Table 2. The graphics of first 2 order mode shapes are shown in Fig. 4. From Table 2 and Fig. 4, it shows that the mode shapes identified by this method also have high precision.

Table 1 Identification results of frequencies (Hz)

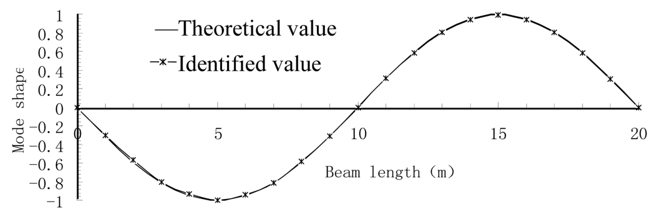
Mode	Theoretical value (Hz)	Identified value (Hz)	Error
1	5.403	5.447	0.82%
2	21.612	21.689	0.36%
3	48.626	49.034	0.84%
4	86.440	86.903	0.54%
5	135.040	130.300	3.51%

Table 2 MAC of mode shapes

Mode	1	2	3	4	5
1	1.00	0.00	0.00	-0.02	0.00
2	0.00	1.00	0.00	-0.01	-0.03
3	0.00	0.00	1.00	-0.04	-0.04
4	-0.02	-0.01	-0.04	1.00	0.04
5	0.00	-0.03	-0.04	0.04	1.00



(a) Identification result of first mode shape



(b) Identification result of second mode shape

Fig. 4 Identification result of mode shapes

Table 3 Identification results of modal damp ratio (%)

Mode	Theoretical value	Identified value	Error
1	1.70	1.68	1.11
2	6.79	6.57	3.24
3	15.28	14.79	3.18
4	27.16	26.12	3.82
5	42.42	45.31	6.80

(3) Identification results of damping ratios

The identification results of damping ratios are presented in Table 3. It shows that the precision of the damping ratios identified by this method is lower than that of the frequency identified by this method, but it can meet the engineering requirement to some extent.

5. Conclusions

An improved experimental modal analysis method is formed by using this mathematical relationship of damping ratio and frequency to improve the method presented by Wang and Cheng (2008). This method can identify the three major dynamic parameters of the bridge structures (frequencies, mode shapes and damping ratios) only by using the free decay response of displacement at measuring points. The usage of cubic spline curve fitting to the displacement response in solving process can ensure the stability of the solution process and improve the identification precision of the bridge dynamic parameters. Some conclusions and recommendations are summarized as follows:

1. Numerical example results show that the frequencies, mode shapes and damping ratios can be identified from the free decay response of displacement of the bridge structures by using the dynamic parameters identification method proposed in this paper.
2. The frequencies and mode shapes identified have high precision.
3. The precision of the identified damping ratios are lower than that of the frequencies identified by this method, but, to some extent, it can meet the engineering requirement.
4. The usage of cubic spline curve fitting to the displacement response in solving process can ensure the stability of the solution process and improve the identification accuracy of bridge dynamic parameters with a certain sampling frequency.
5. Thus the work of this paper can provide technical support to the dynamic load test of bridges, and provide theoretical support to other dynamic parameters identification method of bridges.

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