Structural Engineering and Mechanics, Vol. 38, No. 2 (2011) 231-247
DOI: http://dx.doi.org/10.12989/sem.2011.38.2.231

A load increment method for ductile reinforced concrete (RC) frame structures considering strain hardening effects

M. Günhan Aksoylu* and Konuralp Girgin[a]

Department of Civil Engineering, Istanbul Technical University, 34469, Maslak, Istanbul, Turkey

(Received February 26, 2010, Accepted December 12, 2010)

Abstract. This study introduces a new load increment method for the ductile reinforced concrete (RC) frame structures by including strain-hardening effects. The proposed method is a nonlinear static analysis technique employed for RC frame structures subjected to constant gravity loads and monotonically increasing lateral loads. The material nonlinearity in RC structural elements is considered by adopting plastic hinge concept which is extended by including the strain hardening as well as interaction between bending moment and axial force. Geometric non-linearity, known as second order effect, is implemented to the method as well.

Keywords: RC frames; strain-hardening; load increment method; seismic evaluation; non-linear static analysis; second-order effect.

1. Introduction

The non-linear analysis methods result both more realistic and more economical solutions in design of RC structures. Non linear response of these structures can be easily predicted by using these methods. Furthermore, it can be employed to evaluate seismic performance of RC structures.

This study introduces a new effective load increment method for the materially and geometrically nonlinear analysis of ductile RC structures by including the strain-hardening effects. Material nonlinearity exists when there is a nonlinear relationship between force and displacement. Geometrical nonlinearity is called second order effect as well.

2. Literature survey

Many papers about the nonlinear analysis of RC structures are available in the current literature, i.e., Kunnath *et al.* (1992), Elnashai (2001), Chopra and Goel (2002) and Aydinoglu (2003).

A load increment method for the non-linear analysis of plane steel frames was proposed by Özer

*Corresponding author, Ph.D., E-mail: aksoylug@itu.edu.tr
[a]Associate Professor

(1987); the method has been applied and further extended to 3D steel and RC frame structures by İrtem (1991) and Girgin (1996) in their PhD theses, respectively. By including the lumped plasticity approach and the second order effects, Türker (2005) presented an effective multi modal adaptive pushover analysis for the RC plane frames in his PhD thesis and the findings are available in Türker and İrtem (2007). Aksoylu (2006) introduced a new load increment method by including the strain-hardening effects for the materially and geometrically non-linear analysis of RC structures.

Mwafy and Elnashai (2001) investigated the applicability and accuracy of the inelastic pushover analysis in evaluation seismic responses of RC buildings. Liew *et al.* (2000) proposed a methodology for the adaptive nonlinear analysis of space frame structures by adopting the plastic hinge model and the detailed beam-column formulation. Their method takes into consideration the second order effect, the bowing effect and the elastic coupling effects between axial, flexural and torsional displacements. It is particularly suitable for space frame structures where slender members are subjected to high axial forces.

Long and Hung (2008) presented an algorithm by considering strain hardening effects for the plastic-hinge assumption in the analysis of 3D steel frames by adopting elasto-plastic incremental relationship between the modal forces and the corresponding displacements. Dutta *et al.* (2009) have investigated the efficiency of pushover analysis methodologies. Yukio and Mashiko (1992) developed a general theory for the elasto-plastic analysis of structures by extending the basic theory of the plastic node method (PNM) and by considering strain-hardening effects. Sebastian (2007) defined an iterative computational procedure where secant flexural stiffness is used in the analysis, to predict ultimate loading in indeterminate RC structures by including strain hardening effects.

Iu *et al.* (2009) presented a numerical procedure for composite steel-concrete structures by taking the geometric and material nonlinearities into consideration. They employed a refined plastic hinge approach where the elasto-gradual-plastic material nonlinearity including strain-hardening under the interaction of bending moment and axial force is used. Ziemian and McGuire (2002) proposed a modified tangent modulus approach in the application of the second-order inelastic hinge method that can produce more accurate results in the analysis of in-plane behaviour of frames having compact doubly symmetric-section.

3. Descriptions of the load increment method

3.1 Assumptions

Basic assumptions made in the RC section subjected to bending moment and axial force are as follows:

a. plane sections remain plane after bending,
b. full bond exists between concrete and reinforcement steel,
c. tensile strength of concrete after cracking is negligible.

The following assumptions and limitations are imposed throughout this study:

a. The relationships between bending moment and curvature in RC sections subjected to constant axial force can be idealized to be bilinear.
b. Non-linear deformations can be assumed to be accumulated at plastic sections while the remaining part of structure remains linear-elastic. In this study, the classical plastic hinge concept is extended

by taking into account axial deformations in addition to bending deformations.

c. Yield and failure conditions of a cross section may be expressed in terms of bending moment and axial force. Effect of shear forces on these conditions is neglected.
d. Non-linear yield and failure conditions can be linearized.
e. Plastic deformations due to bending moments and axial forces are governed by the normality criterion.
f. Second order effects are taken into consideration in slender elements subjected to high axial forces.

3.2 Principles of the method

In the proposed method, the structure is analyzed under anticipated constant gravity loads and proportionally increasing lateral loads. When the gravity loads are known, the member axial forces can be easily estimated by using equilibrium equations only. The second-order effects of the estimated axial forces are considered by modifying the elements of the stiffness and loading matrices. When the axial forces obtained at the end of the analysis are not close to the estimated ones, the analysis is repeated. However, in most practical cases such a repetition is not required.

In this method, the structure is analyzed by considering load increments. At the end of each load increment, it is controlled whether a new plastic section occurs by checking internal forces at any critical section reaches to the state defined by the yield condition. The plastic rotation of the section θ_p is introduced as a new unknown for the next load increment. At the same time, an equation is added to the system of equations to express the relationship between the increments in the internal forces and the plastic deformations developed in the last formed plastic section. Because the yield condition is idealized by linear segments, this new equation is linear. Since the system of equations corresponding to the previous step of the load increment has already been solved, the solution for the current load increment is obtained by eliminating of the new unknown.

In the elasto-plastic theory including the second order effects, the structures generally collapse at the second-order limit load due to the lack of stability of the structural system. This situation is evaluated by checking the determinant of the extended system of equations. When the determinant is less than or equal to zero, the system reaches to its second-order limit load; hence, the computational procedure is terminated.

4. Moment-curvature relationship for reinforced concrete sections and linearization

The relationship between the bending moment M and the curvature χ for a RC section subjected to a constant axial force N_o is given in Fig. 1. The strain-hardening effect is considered by assuming a bilinear relationships between M and χ. Herein, M_{L1} represents the first plastic deformations occurred in the section and M_{L2} is the ultimate moment carrying capacity. The curvatures of χ_{L1} and χ_{L2} correspond to the bending moments M_{L1} and M_{L2}, respectively.

The materially non-linear analysis is performed due to the nonlinearity in the constitutive equation. It means that large plastic deformations occur in addition to elastic ones. In this analysis the plastic hinge approach is adopted which means that plastic deformations are accumulated at certain sections, called as the plastic hinges, while other parts of the structure remain linear-elastic. In this study the classic plastic hinge concept is generalized by including the strain hardening and

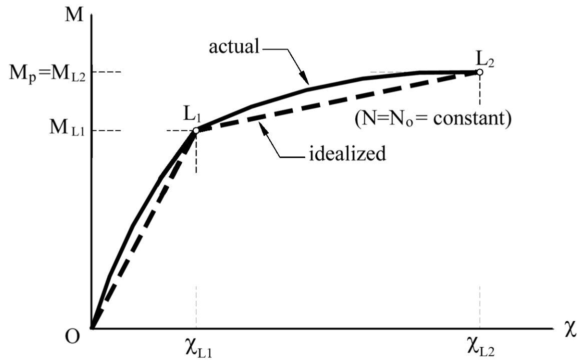

Fig. 1 The actual and the idealized relationships between M and χ for a RC section subjected to a constant axial force

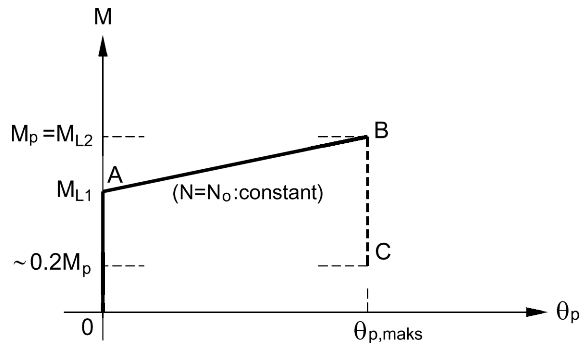

Fig. 2 The idealized relationship between M and θ_p for constant N_o in a plastic section

by considering combined effects of axial force and bending moment as well.

In the plastic hinge concept, an idealized relationship between the bending moment M and the plastic rotation θ_p is necessary for the non-linear analysis of structures. This idealized relationship (Fig. 2) for constant axial force (N_o) is as follows

$$M = M_{L1} + \frac{\theta_p}{\theta_{p,\max}}(M_{L2} - M_{L1}) \quad \text{for} \quad M > M_{L1} \tag{1}$$

where

$$\theta_p = (\chi - \chi_{L1})l_p = \chi_p l_p \quad (\chi > \chi_{L1}) \tag{2}$$

$$\theta_{p,\max} = (\chi_{L2} - \chi_{L1})l_p = \chi_{p,\max} l_p \tag{3}$$

The l_p length of plastic section is assumed to be $h/2$ where h is the height of the cross section in the bending direction.

5. Interaction of bending moment and axial force (yield/failure surfaces) and idealization

The yield and failure conditions in RC sections subjected to bending moment and an axial force are idealized by linear segments, as given in Fig. 3.

In Fig. 3, the points 2 and 2' denote to the balance points where the ultimate bending moments occur, and the points 3 and 3' signify the pure bending case. The points 1,1' and 4, 4' are the pure axial load case of compression and tension force only, respectively.

6. Yield, failure conditions and plastic deformation vector

The yield condition can be defined as a boundary where plastic deformations start to occur and it

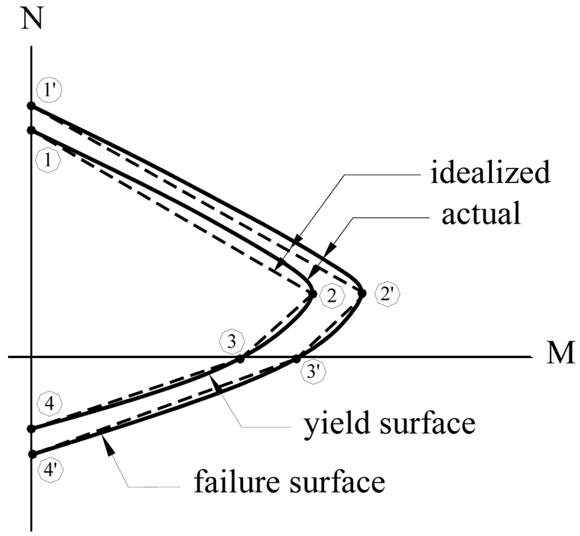

Fig. 3 Interaction of bending moment and axial force (The actual and idealized surfaces)

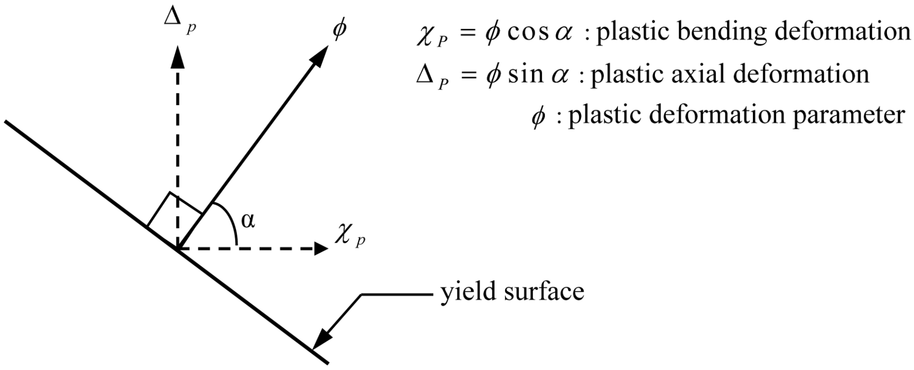

Fig. 4 Plastic deformation vector

is represented on the bending moment-axial force plane (Fig. 3). While the internal forces are in the inner region of the yield curve, the deformations are linearly elastic. Beyond the linear-elastic limit, internal forces are between the yield and the failure curves, thus plastic deformations develop in the RC section. Internal forces are limited by the failure curve governed by the failure conditions.

The yield vector represents the plastic deformation increments due to increments of the bending moment and the axial force. This vector is assumed to be normal to the yield surface (Fig. 4). The normality rule is not proven but its accuracy has been verified theoretically by Çakiroglu *et al.* (1999). Karabinis and Kiousis (2001) employed this rule in their study as well.

By neglecting the effects of shear forces, the yield condition can be expressed as follows

$$K(M, N) = 0 \tag{4}$$

An increment of the internal forces between the yield and failure surface results a K increment in the potential function K, which can be expressed as

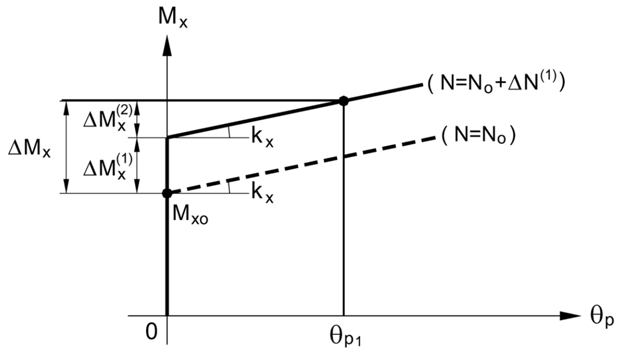

Fig. 5 The M_x-θ_p relationships in a plastic section for two axial forces

$$\Delta K = \frac{\partial K}{\partial M_x}\Delta M_x + \frac{\partial K}{\partial N}\Delta N \tag{5}$$

where the combined effect of the bending moment and the axial force can be seen. If rigid-plastic behaviour is considered in plastic sections, the yield surface and the failure surface are the same, this means that any increment in the potential is not under consideration i.e., ($\Delta K = 0$).

When strain hardening property is present, the relationship between the bending moment M_x and the plastic rotation θ_p is schematically displayed in Fig. 5. The dashed line segment displays the relationship between the bending moment and the plastic rotation after plastic section is formed. On the other hand, solid line segment denotes the same relation when the point on the intersection point moves beyond the yield surface where k_x is the slope of the idealized M_x-θ_p relationship in the strain-hardening region. Numerical analyses indicated that the slope k_x remains constant for the different levels of axial load.

If the strain-hardening on the rigid-plastic behaviour is under consideration and k_x is assumed to be independent from the effect of axial force as well, the linear relationship between the bending moment and plastic rotation can be written as

$$\Delta M_x = \Delta M_x^{(1)} + M_x^{(2)} = \Delta M_x^{(1)} - k_x\theta_x \tag{6a}$$

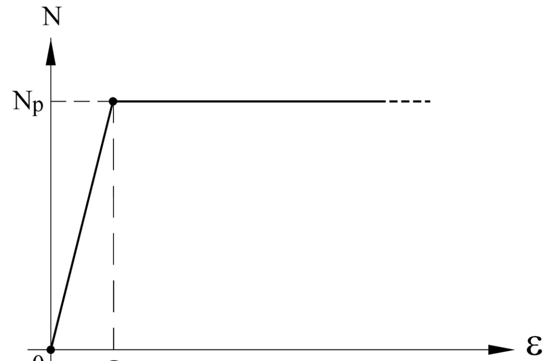

Fig. 6 The idealized N-ε relationship

where the minus sign in front of k_x signifies the internal forces and plastic deformations in opposite directions. On the other hand, by assuming axial force-axial deformation relationship to be ideal elasto-plastic (Fig. 6), the axial force increment ΔN in any step of the load increment can be expressed as

$$\Delta N = \Delta N^{(1)} + \Delta N^{(2)} = \Delta N^{(1)} \tag{6b}$$

By substituting 6a-6b into Eq. (5), the increment in internal forces is found to be

$$\Delta K = -\left(\frac{\partial K}{\partial M_x} k_x \theta_x\right) \tag{5a}$$

Since the yield vector is assumed to be normal to the yield surface, the plastic deformations can be expressed in terms of a single parameter θ

$$\theta_x = \theta \frac{\partial K}{\partial M_x}, \quad \Delta = \theta \frac{\partial K}{\partial N} \tag{7}$$

Since the yield surface is expressed by linear segments, the linear yield condition can be written as

$$K(M_x, N) \cong A_1 M_x + A_2 N + B = 0 \tag{8}$$

The plastic deformations θ_x and Δ corresponding to the internal forces M_x and N

$$\theta_x = \theta \frac{\partial K}{\partial M_x} = \theta A_1, \quad \Delta = \theta \frac{\partial K}{\partial N} = \theta A_2 \tag{9}$$

In case of strain hardening, the plastic behavior of the section between the yield and the failure surfaces is expressed as follows

$$\frac{\partial K}{\partial M_x} \Delta M_x + \frac{\partial K}{\partial N} \Delta N + A_1^2 k_x \theta = 0 \tag{10}$$

7. Mathematical formulation

Plane frame structures having plastic sections are analyzed in the each step of load increments. The unknowns are the displacement and plastic rotation components of nodal points. Equations for these unknowns are:

a. equilibrium equations written in the directions of nodal displacements
b. relationships between the internal forces and plastic rotations in plastic sections.

These equations can be combined and written as

$$[S_{dd}]_{3n \times 3n}[d]_{3n} + [S_{d\theta}]_{3n \times m}[\theta_p]_m = [q] \tag{11}$$

where n denotes the number of the nodal points, m is the number of the plastic sections.

$[S_{dd}]$ is the elastic stiffness matrix of the system without taking into account plastic hinges. If the second order theory is under consideration, the related terms of element stiffness and loading matrices for the members under compression should be obtained according to this theory.

$[S_{d\theta}]$ is a rectangular matrix which represents the effect of unit plastic deformations in the plastic sections on the equilibrium equations. Its k^{th} column consists of the end-forces due to the unit plastic rotation $\theta_k = 1$, while all other plastic rotations and all nodal displacements are zero.

$[d]$ and $[\theta_p]$ are the column matrices respectively including nodal displacements and the plastic rotations in plastic sections.

$[q]$ is the column matrix of the nodal external loads.

In each incremental step, the relationships between the internal forces and the plastic deformations should satisfy Eq. (10). These relationships can be rewritten in matrix form, as follows

$$[S_{\theta d}][d] + [\bar{S}_{\theta\theta}][\theta_p] = [0] \tag{12}$$

By using the Betti's reciprocal theorem and by assuming the incremental plastic deformation vector to be normal to the yield surface $[S_{\theta d}] = [S_{d\theta}]^T$.

$$[\bar{S}_{\theta\theta}]_{m\times m} = [S_{\theta\theta}] + [R]$$

where $[S_{\theta\theta}]_{m\times m}$ is a square diagonal matrix having positive diagonal terms and its k^{th} column represents the increment in the internal forces due to the only unit plastic rotation in plastic section k while all nodal displacements and the external forces are zero.

The increment of the internal forces in the plastic sections are defined as

$$A_1\Delta M_x + A_2\Delta N \tag{13}$$

$[R]_{m\times m}$ is a diagonal matrix where its k^{th} diagonal term corresponds to the increment of the internal force defined as

$$A_1^2 k_x \tag{14}$$

due to the unit plastic rotation in the plastic section k while all nodal displacements and external forces are zero. The generalized equation system of a symmetric coefficient matrix can be rewritten as follows

$$\begin{bmatrix} [S_{dd}] & [S_{d\theta}] \\ [S_{\theta d}] & [\bar{S}_{\theta\theta}] \end{bmatrix} \begin{bmatrix} [d] \\ [\theta_p] \end{bmatrix} = \begin{bmatrix} [q] \\ [0] \end{bmatrix} \tag{15}$$

8. Analysis steps of the proposed method

By including the second order effect, the steps of the analysis can be given as:

i. Factored vertical loads are determined by increasing the vertical service loads by anticipated load factors.

ii. Axial forces due to the factored vertical loads can be obtained by using force equilibrium conditions only. (If axial forces can not be easily estimated, the system should initially analyzed according to the first order theory).

iii. Relationships between the bending moments and the plastic rotations under constant axial forces are obtained in the potential plastic sections.

iv. System is analyzed under the factored vertical loads. If any plastic sections occur in the system, vi.viiith steps are repeated for each plastic section.

v. System is analyzed for unit lateral load and the plastic section is determined by employing the yield conditions for all critical sections.

vi. Plastic rotation θ_p of the newly formed plastic section is taken an new unknown and is added to the current equation system. The new equation represents the incremental relationship between the bending moment and the axial force. Only new added equation is solved.

vii. By solving the extended equation system, all unknowns are obtained.

viii. Load is increased and the analysis is completed when the internal forces in any section correponds to the ultimate rotation capacity $\theta_{p,\max}$, otherwise the steps from v to viii[th] are repeated.

9. Numerical examples

Two RC plane frames are presented for the applicability and the verification of the method. The frames are analyzed under constant gravity loads and proportionally increasing lateral loads. Both geometric and material non-linearities are taken into consideration and all the results are verified with SAP2000 as well.

Example 1

Geometrical properties, factored vertical and service lateral loads of an one-storey RC plane frame structure are given Fig. 7. Dimensions of cross sections and areas of steel reinforcement are given in Tables 1 and 2, respectively.

Material properties are summarized below.

Concrete (C25): f_c = 25 MPa, ε_{co} = 0.002, ε_{cu} = 0.010, E_c = 30.25 × 10^3 MPa
Reinforcement steel (S420): f_{sy} = 420 MPa, f_{su} = 500 MPa, E_s = 2 × 10^5 MPa
ε_{sh} = 0.01, ε_{su} = 0.10

E_c, E_s are the elastic modulus of concrete and reinforcement steel, respectively. $\sigma-\varepsilon$ relationships of concrete and reinforcement are given in Fig. 8.

M-θ_p relationship for the beam sections and the M-N interaction diagram of the columns are presented in Fig. 9(a) and (b). It is clear that the M-N interaction diagram is obtained by using the

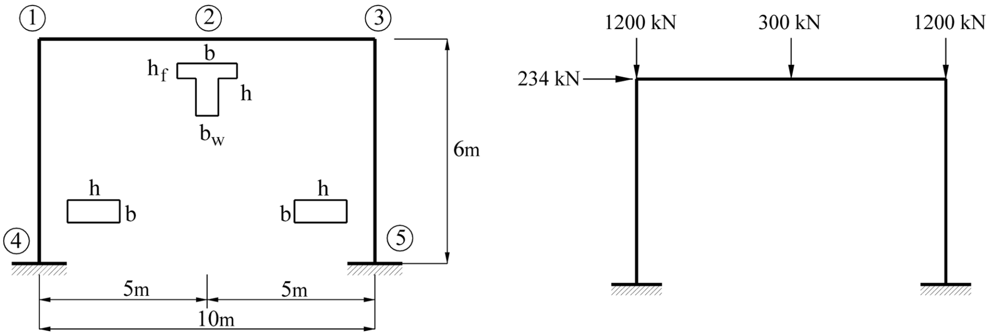

Fig. 7 Geometrical properties and loads of one-storey RC plane frame structure

Table 1 Dimensions of cross sections

Member	h (mm)	b (mm)	b_w (mm)	h_f (mm)
1-2, 2-3	800	2100	300	150
4-1, 3-5	800	300	---	---

Table 2 Reinforcement of beams and columns

Member	Location		Longitidunal reinforcement
1-2, 2-3	left and right ends of beam	top	4ϕ22+3ϕ16
		bottom	5ϕ22
	midspan of beam	top	3ϕ16
		bottom	5ϕ22
4-1, 3-5	along the column	-	10ϕ22

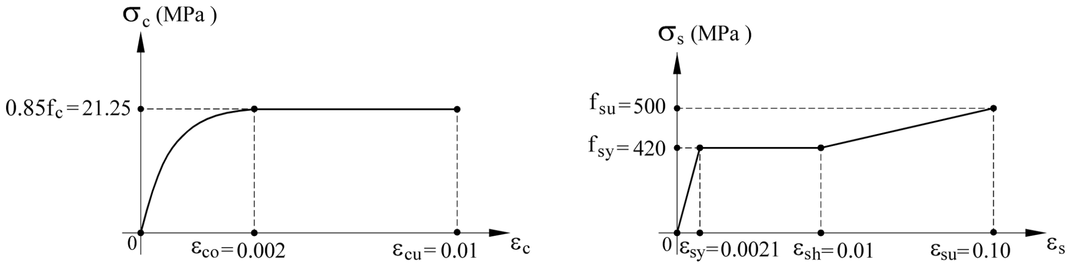

Fig. 8 $\sigma - \varepsilon$ relationships of concrete and reinforcement steel

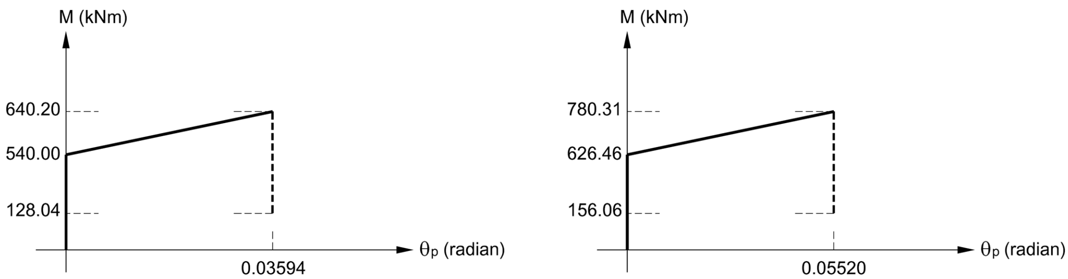

Fig. 9a M-θ_p relationships for sections at midspan and left/right side of beams (1-2, 2-3)

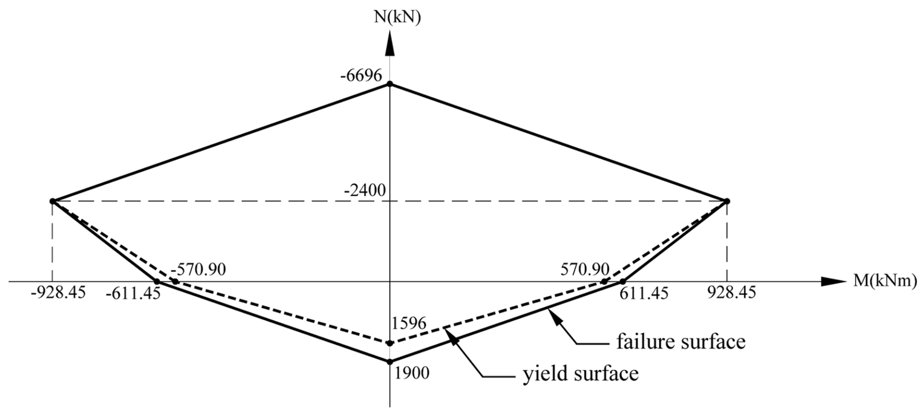

Fig. 9b Interaction of bending moment and axial force for columns (1-4, 3-5)

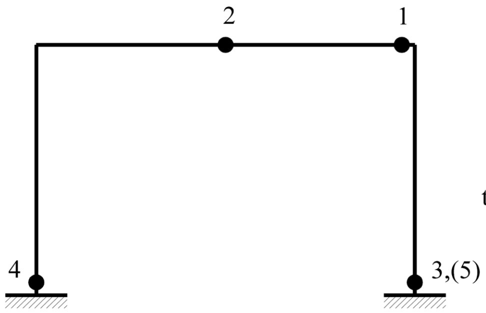

Fig. 10 Distribution and evolution of plastic sections

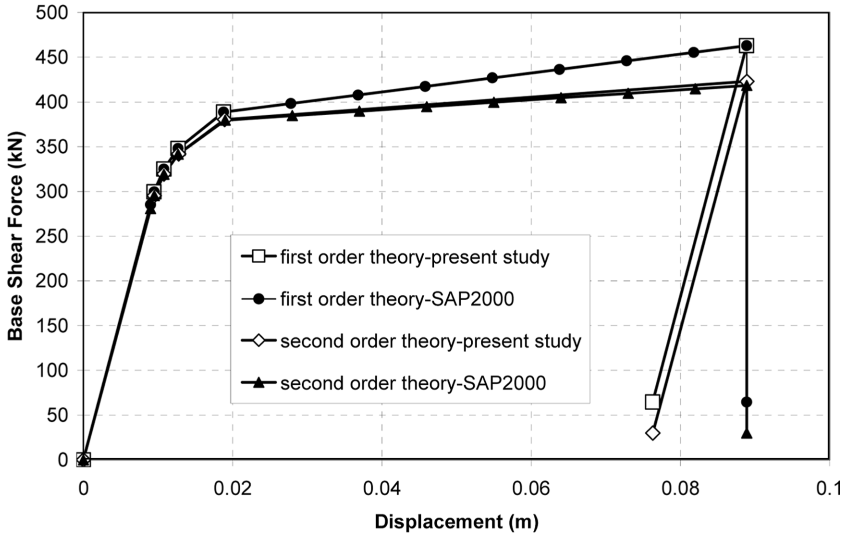

Fig. 11 Base shear-lateral top displacement diagram

Table 3 The locations of plastic sections and rotations at the final step

	Location of plastic section		Plastic rotations (radian)	
	Member	Distance from left end (m)	θ_p (Present Study)	θ_p (SAP2000)
First Order Theory	2-3	5.00	0.025834	0.025837
	1-2	5.00	0.024405	0.026374
	3-5	6.00	0.0123	0.0123
	1-4	6.00	0.011599	0.0116
Second Order Theory	2-3	5.00	0.025865	0.026053
	1-2	5.00	0.024446	0.026835
	3-5	6.00	0.0123	0.0123
	1-4	6.00	0.011590	0.011524

$\sigma-\varepsilon$ relationships of concrete and reinforcement steel.

Non-linear analysis of system is performed by the proposed load increment method. The steps of the load increment method are explained in the previous sections of the paper and the locations of the plastic hinges are displayed in Fig. 10, the base shear and the lateral top displacement diagram is illustrated in Fig. 11. The locations of the plastic sections and the rotations at the end of the load increments are shown in Table 3.

Example 2

Geometrical properties, factored vertical and service lateral loads of six-story RC plane frame structure are given Fig. 12. Dimensions of cross sections, steel reinforcement areas are presented in Tables 4 and 5, respectively.

Concrete (C30): f_c = 30 MPa, ε_{co} = 0.002, ε_{cu} = 0.010, E_c = 31.80 × 10^3 MPa
Reinforcement steel (S420): f_{sy} = 420 MPa, f_{su} = 500 MPa, E_s = 2 × 10^5 MPa,
ε_{sh} = 0.01, ε_{su} = 0.10

The $\sigma-\varepsilon$ relationships of concrete and reinforcement are illustrated in Fig. 13.

By employing the method presented herein, Example 2 is analyzed and base shear- lateral top displacement diagrams are evaluated according to the first and the second order theories in comparison with SAP 2000 (Fig. 14). The evolution and distribution of plastic sections are given in Fig. 15. For the second-order theory, the locations of the plastic sections and the rotations at the end of the loading are displayed in Table 6.

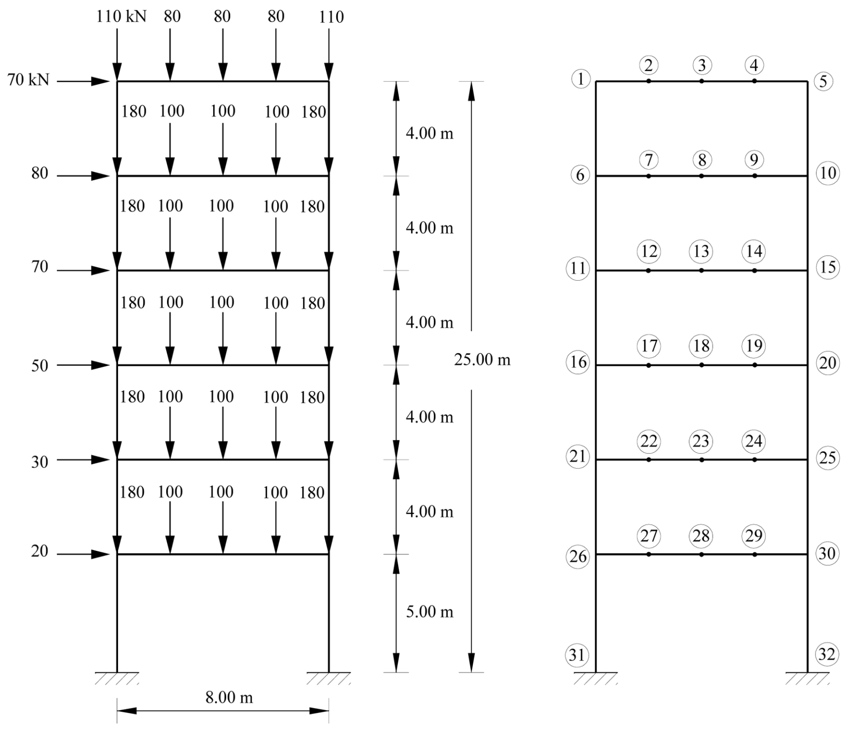

Fig. 12 Geometrical properties, loads and joint number of six-storey frame

Table 4 Dimensions of beams and columns

Member	h (mm)	b (mm)	b_w (mm)	h_f (mm)
1-5, 6-10, 11-15, 16-20, 21-25,26-30	800	1790	350	120
1-6, 5-10, 6-11, 10-15	350	350	----	----
11-16, 15-20, 16-21, 20-25	400	400	----	----
21-26, 25-30, 26-31, 30-32	500	500	----	----

Table 5 Reinforcement of beams and columns

Member	Location		Longitidunal reinforcement
1-5	left- right ends	top	$3\phi20+3\phi14$
		bottom	$4\phi28$
	midspan	top	$3\phi14$
		bottom	$4\phi28$
6-10	left- right ends	top	$3\phi20+3\phi14$
		bottom	$4\phi28$
	midspan	top	$3\phi14$
		bottom	$4\phi28$
11-15	left- right ends	top	$3\phi20+3\phi28$
		bottom	$4\phi28$
	midspan	top	$3\phi20$
		bottom	$4\phi28$
16-20	left- right ends	top	$3\phi20+3\phi28$
		bottom	$4\phi28$
	midspan	top	$3\phi20$
		bottom	$4\phi28$
21-25	left- right ends	top	$3\phi20+4\phi30$
		bottom	$4\phi28$
	midspan	top	$3\phi20$
		bottom	$4\phi28$
26-30	left- right ends	top	$3\phi20+4\phi30$
		bottom	$4\phi28$
	midspan	top	$3\phi20$
		bottom	$4\phi28$
1-6, 5-10	along the column		$8\phi22$
6-11, 10-15			$8\phi22$
11-16, 15-20			$8\phi28$
16-21, 20-25			$8\phi28$
21-26, 25-30			$8\phi28$
26-31, 30-32			$8\phi28$

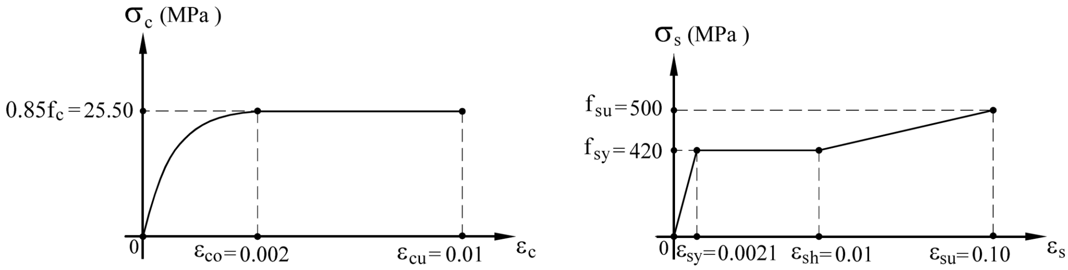

Fig. 13 $\sigma - \varepsilon$ relationships of concrete and reinforcement

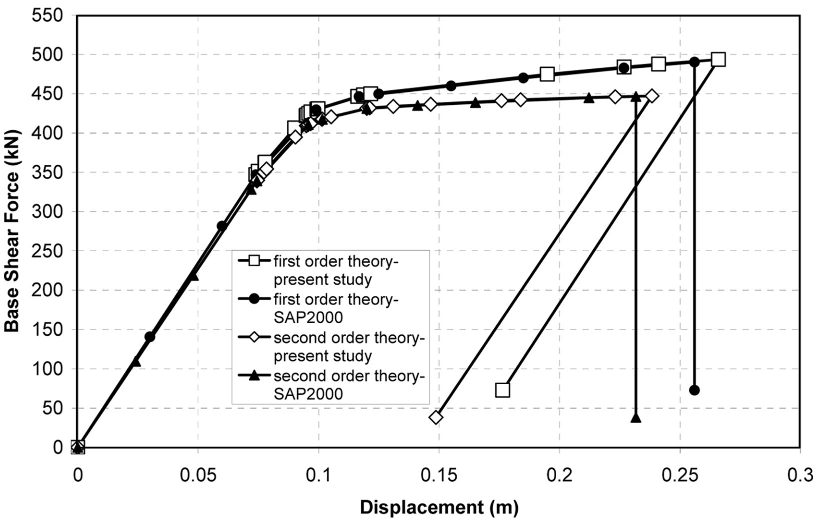

Fig. 14 Base shear-lateral top displacement diagram

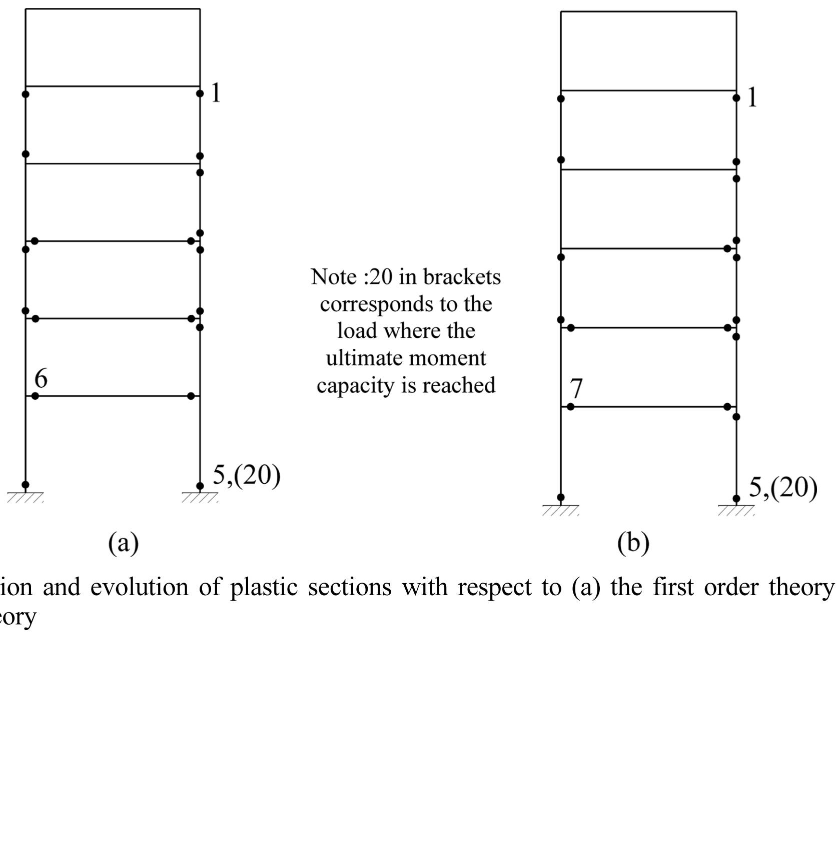

Fig. 15 Distribution and evolution of plastic sections with respect to (a) the first order theory (b) the second order theory

Table 6 Locations of plastic sections and rotations at the end of load increments including second-order effect

Location of plastic section		Plastic rotation (radian)	
Member	Distance from the left end (m)	θ_p (Present Study)	θ_p (SAP2000)
10-15	0.00	0.004861	0.004831
20-25	0.00	0.008643	0.009201
20-25	4.00	0.006318	0.009201
10-15	4.00	0.004517	0.004569
30-32	5.00	0.009180	0.008428
16-21	4.00	0.000697	0.004537
26-27	0.00	0.010319	0.009337
16-21	0.00	0.007134	0.009201
15-20	0.00	0.000702	0.000681
29-30	2.00	0.009865	0.008797
26-31	5.00	0.008733	0.007994
15-20	4.00	0.000058	0.000443
21-22	0.00	0.008747	0.007570
25-30	0.00	0.005495	0.004430
6-11	0.00	0.003079	0.003132
6-11	4.00	0.002827	0.002847
30-32	0.00	0.000086	0.000224
24-25	2.00	0.002893	0.002923
19-20	2.00	0.000475	0.000324

The unloading branches of base shear-lateral top displacement diagrams in Fig. 11 and Fig. 14 correspond to a sharp drop in the moment capacity at plastic sections.

The proposed method was applied to several numerical examples, for the sake of brevity only two of them are presented herein, and it is seen that all the results agree very well with the results from SAP2000.

10. Conclusions

This study introduces an efficient load increment method developed for the ductile reinforced concrete (RC) frame structures. The materially nonlinearity (nonlinearity due to the nonlinear constitutive conditions) in RC structural elements is represented by adopting plastic hinge concept and it is further extended by including the strain hardening effects as well as the interaction of bending moment and axial force. The method includes the geometric nonlinearity (second order effect) as well. Since plastic hinge locations and rotations are obtained in every steps of load increment method, seismic performance for existing RC structures can be efficiently evaluated, and suitable strengthening techniques can be developed. This method can be employed in the design of earthquake resisting structures as well. It should be noted that current status of the method is appropriate for regular and middle-rise RC buildings. The verification of method has been successfully performed on several examples. The method can be used to assess the seismic performance of steel structures as well.

Acknowledgements

The authors gratefully acknowledge to Professor Dr. Erkan Özer who is Ph.D. supervisor of both authors and also express sincere thanks to Associate Professor Dr. Z. Canan Girgin and Professor Dr. A. Erhan Aksoylu for editing the English of the paper.

References

Aksoylu, M.G. (2006), "An incremental method for non-linear static analysis of reinforced concrete structures", Ph.D. Thesis, Istanbul Technical University, Institute of Science, Istanbul.

Aydinoglu, M.N. (2003), "An incremental response spectrum analysis procedure based on inelastic spectral displacements for multi-mode seismic performance evaluation", *Bull. Earthq. Eng.*, **1**, 3-36.

Chopra, A.K. and Goel, R.K. (2002), "A modal pushover analysis procedure for estimating seismic demands for buildings", *Earthq. Eng. Struct. D.*, **31**, 561-582.

Çakiroglu, A., Özer, E. and Girgin, K. (1999), "Yield conditions and yield vector for combined biaxial bending of rectangular reinforced concrete sections", *Proceeding of the Ugur Ersoy Symposium on Structural Engineering*, Ankara.

Dutta, S.C., Chakroborty, S. and Raychaudhuri, A. (2009), "Efficacy of pushover analysis methodologies: A critical evaluation", *Struct. Eng. Mech.*, **31**(3), 265-276

Elnashai, A.S. (2001), "Advanced inelastic static (pushover) analysis for earthquake applications", *Struct. Eng. Mech.*, **12**, 51-69.

Girgin, K. (1996), "A method of load increments for the determination of second-order limit load and collapse safety of reinforced concrete framed structures", Ph.D. Thesis, Istanbul Technical University, Institute of Science, Istanbul.

Iu, C.K., Bradford, M.A. and Chen, W.F. (2009), "Second-order inelastic analysis of composite framed structures based on the refined plastic hinge method", *Eng. Struct.*, **31**, 799-813.

İrtem, E. (1991), "Determination of second-order limit load of framed space structures by a method of load increment", Ph.D. Thesis, Istanbul Technical University, Institute of Science, Istanbul.

Karabinis, A.I. and Kiousis, P.D. (2001), "Plasticity model for reinforced concrete elements subjected to overloads", *J. Struct. Eng.-ASCE*, **127**(11), 1251-1256.

Kunnath, S.K., Reinhorn, A.M. and Lobo, R.F. (1992), "IDARC Version 3.0 - a program for inelastic damage analysis of reinforced concrete structures", Tech. Rep. no. NCEER-92-0022, Nat. Centre for Earthq. Eng. Res., State University of New York at Buffalo.

Liew, J.Y.R., Chen, H., Shanmugam, N.E. and Chen, W.F. (2000), "Improved nonlinear plastic hinge analysis of space frame structures", *Eng. Struct.*, **22**, 1324-1338.

Mwafy, A.M. and Elnashai, A.S. (2001), "Static pushover versus dynamic collapse analysis of RC buildings", *Eng. Struct.*, **23**, 407-424.

Özer, E. (1987), "Determination of the second-order limit load by a method of load increments", *B. Tech. Univ. Istanbul*, **40**, 815-835.

Sebastian, V.M. (2007), "Secant stiffnesses for ultimate analysis of strain hardening indeterminate RC structures", *Eng. Struct.*, **29**, 3342-3351.

Türker, K. (2005), "Multi modal adaptive load increment method for determination of earthquake response of structures", Ph.D. Thesis, Balikesir University, Institute of Science, Balikesir.

Türker, K. and İrtem, E. (2007),"An effective load increment method for multi modal adaptive pushover analysis of buildings", *Struct. Eng. Mech*, **25**(1), 53-73.

Van Long, H. and Hung, N.D. (2008), "Second-order plastic-hinge analysis of 3-D steel frames including strain hardening effects", *Eng. Struct*., **30(**12), 3505-3512.

Yukio, U. and Masahiko, F. (1992), "Plastic node method considering strain-hardening effects", *Comput. Meth. Appl. Mech. Eng.*, **94**(3), 317-337.

Ziemian, R.D. and McGuire, W. (2002), "Modified tangent modulus approach, a contribution to plastic hinge analysis", *J. Struct. Eng.*, **128**(10), 1301-1307.

Wilson, E.L. (2003), *Three-dimensional Static and Dynamic Analysis of Structures*, 3rd Edition, Computers & Structures Inc., Berkeley.