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Time-dependent analysis of cable trusses
Part I. Closed-form computational model

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Abstract. In this paper the time-dependent closed-form static solution of the suspended pre-stressed biconcave and biconvex cable trusses with unmovable, movable and elastic or viscoelastic yielding supports subjected to various types of vertical load is presented. Irvine's forms of the deflections and the cable equations are modified because the effects of the rheological behaviour needed to be incorporated in them. The concrete cable equations in the form of the explicit relations are derived and presented. From a solution of a vertical equilibrium equation for a loaded cable truss with rheological properties, the additional vertical deflection as a time-function is determined. The time-dependent closed-form model serves to determine the time-dependent response, i.e., horizontal components of cable forces and deflection of the cable truss due to applied loading at the investigated time considering effects of elastic deformations, creep strains, temperature changes and elastic supports. Results obtained by the present closed-form solution are compared with those obtained by FEM. The derived time-dependent closed-form computational model is used for a time-dependent simulation-based reliability assessment of cable trusses as is described in the second part of this paper.

Keywords: cable truss; time-dependent closed-form computational model; creep of cable; elastic and viscoelastic yielding support; time-dependent cable equations.

1. Introduction

Cable trusses are composed of the top and bottom chords of the continuous pre-stressed cables, anchored at each end, between which numerous vertical light ties (biconcave cable truss) or rigid spacers (biconvex cable truss) are placed. Since the pre-stress is usually high, the geometry of truss is determined to a large part by the span and the lengths of the ties or spacers. The cable truss has several advantages as a means of supporting the roofs of large-span structures. Because of the structural efficiency of the truss, the roof is light and yet possesses considerable rigidity. Keeping the pre-stressing forces of the bottom and top cables for the relevant loading combinations in the required range during the entire service phase of structure, is a basic stiffness requirement of its serviceability.

In the time-dependent analysis of pre-stressed large-span cable structures, the input random variables such as structural configuration, modulus of elasticity and pre-stressing forces of the

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cables are functions of time. In contrast to the classical tension steel rods and bars, which operate in the linear elastic range, high-strength steel cables, and especially synthetic fibre ropes, have time-dependent non-linear viscoelastic properties. These properties depend on creep strain increments of the cables under long-term load, i.e., on the pre-stress level of the structure and on other loading effects during the service of structure. Creep of the cables cause a decrease of the Young's modulus of elasticity and of the pre-stressing forces of cables. To assess the structural safety and serviceability performance of pre-stressed cable structures during their entire service life, time-dependent structural and material effects due to rheological changes should be considered in the analytical model.

Most of the recent methods of analysis of cable structures are based on the discretisation of the equilibrium equations and solving the resulting non-linear algebraic equations by numerical methods (Saafan 1970, Jonatowski and Birnstiel 1970, Jayaraman and Knudson 1981, Kassimali and Parsi-Feraidoonian 1987, Kmet 1994, Talvik 2001, Gasparini and Gautam 2002, etc.). Kanno *et al.* (2002) derived a special method for friction and friction-less analysis of non-linear elastic cable structures based on second-order cone programming. Brew and Lewis (2003) propose an efficient numerical tool, which will allow a better integration of the design/analysis/manufacture of tension membrane structures.

The available literature on the closed-form analytical models of the pre-stressed cable trusses is not very extensive, and is mostly concerned with the engineering theory of shallow cables with relatively low sag (the ratio of cable sag to span is 1:8 or less), that are used for structural purposes. In its linearized form this approximate theory provides explicit consistent methods for finding static response of perfectly flexible and elastic, hinge-suspended cable truss to applied loads. Comprehensive analytical treatments on the cable trusses have been given by Schleyer (1969), Møllmann (1974) and by Irvine (1981). The approximate analytical methods for a linear static analysis of cable trusses are presented in the other works, e.g., Urelius and Fowler (1974), Moskalev (1980) and Buchholdt (1998), and are mostly concerned with the determination of displacement and change in tension in the cable members due to an external load. Their results lead to the simple manual methods for preliminary dimensioning of various types of cable beams. An interesting discussion of the relative merits of the truss configurations used to support large span roofs, as well as on the influence of various design parameters on their performance under working load conditions, can be found in the paper presented by Krishna *et al.* (1982). Sultan *et al.* (2001) present the general pre-stressability conditions for tensegrity structures, which can be analytically solved. These conditions are expressed as a set of non-linear equations and inequalities on the tendon tensions.

However, only little attention is paid to the rheological analysis of cable truss suspended on viscoelastic yielding supports. That is why the authors focus on these problems, and elaborating on them they start with the work of Irvine (1981), which has been further complemented. Kmet and Kokorudova (2006) presented a non-linear closed-form static computational model of a biconvex and biconcave suspended cable truss subjected to various types of static loads.

In this paper, the linearized closed-form solution of suspended cable truss with rheological properties subjected to static loads is examined, taking into consideration the non-linear effects of creep strain increments and temperature changes. The non-linear creep theory is adopted for rheological analysis. Irvine's convenient form of the cable equations is modified, because the effects of the creep strain increments need to be incorporated in them. All the geometrical and force quantities of a suspended cable, geometrical-deformational equations, physical and constitutive creep equations as well as all the cable and deflection equations are expressed as time and stress functions to exhibit non-linear creep and rheological parameters. In general, this approach gives an explicit, i.e., one-step method for determination of the static response of the suspended cable truss to specific load. The whole theory of rheological static analysis of a cable truss is fully formulated through a computer program. Serviceability criteria referring to deflections of the cable truss in the investigated time t i.e., the limits for its deflections during the entire service time of structure, will be analyzed in the second part of this paper.

An area, included improvement of theoretical approaches (which unlike of previous solutions, include creep) for predicting the time-dependent behaviour of pre-stressed cable trusses can be considered as distinct of this work. Attention is turned to the individual main steps in the probabilistic assessment procedure, i.e., to the definition and derivation of the closed-form analytical model suitable for the Monte Carlo technique application.

2. Closed-form analytical model

2.1 Initial assumptions for the calculation

The method of quasi-static linearized analysis of the cable truss with rheological properties, suspended on unmovable, movable, elastic, or viscoelastic supports, that is presented, is built upon the following assumptions:

- The cables are perfectly flexible, i.e., bending moments in all cable cross sections are equal to zero, they work only in tension, having zero stiffness in compression and bending. Profile geometries of the biconcave and biconvex cable trusses are shown in Fig. 1. The profiles of the bottom and top chords, respectively, are assumed to be parabolic and are given by the expressions

$$z_b(t_0) = 4(d_b - b_b)\frac{x}{l}\left(1 - \frac{x}{l}\right) + b_b$$

$$z_t(t_0) = 4(d_t - b_t)\frac{x}{l}\left(1 - \frac{x}{l}\right) + b_t \qquad (1)$$

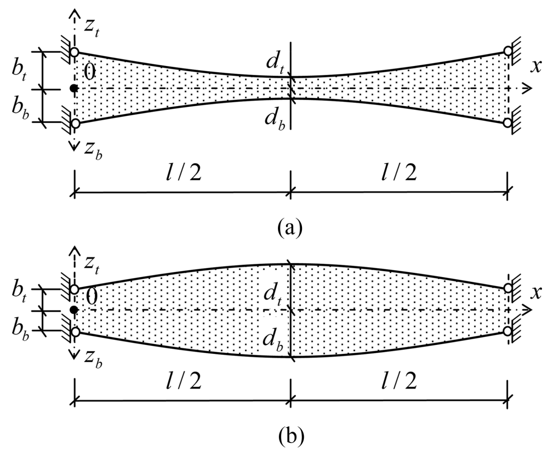


Fig. 1 Profile geometry for a biconcave (a) and biconvex (b) cable truss

where l is the span of the cable truss. In the case of biconcave truss, sag of the carrying cable is given as $s = b_t - d_t$ and camber of the stabilizing cable is $c = b_b - d_b$. For the biconvex truss, sag of the carrying cable is given as $s = d_b - b_b$ and camber of the stabilizing cable is $c = d_t - b_t$.

- Cable trusses with their biconcave or biconvex bottom and top chords are symmetric only about the vertical axis at mid-span. Only trusses with vertical ties or spacers will be considered. The use of inclined ties may stiffen the truss significantly, but analytical solutions are not really feasible, and it is best to resort directly to numerical methods. In the case of biconcave system, the chords are not allowed to be clamped together at mid-span, i.e., d_b and d_t never have zero values.
- The ties and spacers are inextensible and incompressible. In the analysis the ties and spacers are replaced by a continuous diaphragm, whose adjacent vertical elements may slide freely with respect to each other.
- Loading is arbitrarily vertical, being distributed over the plan projection of the cable truss.
- The expressions for a cable length are expanded into a binomial series, considering just its first two terms.
- The material of the cables and that of elastic or viscoelastic yielding anchorages is linearly elastic, following Hooke's law at the investigated time t. The non-linear creep theory is adopted for the description of rheological cable properties.
- For the time-dependent analysis of the cable truss, the time domain is divided into a discrete number of time steps. At each time step, the structure is analysed under the corresponding external applied load, geometric and material characteristics, support conditions and imposed deformations due to creep and temperature variations.
- In general, the additional horizontal components of cable tension in the bottom $\Delta H_b(t)$ and top chord $\Delta H_t(t)$, as well as the additional vertical deflection $w(x, t)$ of geometrically non-linear cable truss must be found from a pair of coupled cubic cable equations (Kmet and Kokorudova 2006), which can be separated only under the assumption of linear response. That assumption is plausible for normally encountered roof loads on trusses of acceptable curvature and we shall exploit it here and take it into consideration. We shall assume linearized cable equations, which means that the second order terms in the system of two general cable equations are neglected.

2.2 Deflection equations for the characteristic loads as a function of time

Consider a biconvex cable truss (see top part of Fig. 1). Under applied vertical load, the shear force at some cross section x along the span of the truss is Q. Following Irvine (1981), vertical equilibrium at a cross section of the biconvex cable truss at time t further requires that (see Fig. 2)

$$(H_b(t_0) + \Delta H_b(t))\frac{d(z_b(t_0) + w(t))}{dx} - (H_t(t_0) - \Delta H_t(t))\frac{d(z_t(t_0) - w(t))}{dx} = Q \tag{2}$$

where $H_b(t_0)$ and $H_t(t_0)$ are the initial horizontal components of the pretensions in the bottom and top chords, respectively, at time t_0; $\Delta H_b(t)$ and $\Delta H_t(t)$ are the additional horizontal components of cable tension owing to the applied load; $z_b(t_0)$ and $z_t(t_0)$ are the initial profiles of the chords given by expressions (1), and $w(t)$ is the additional vertical deflection at the time t. Consideration of the internal equilibrium of the unloaded truss, given by the expression $H_b(t_0)dz_b(t_0)/dx = H_t(t_0)dz_t(t_0)/dx$, allows Eq. (2) to be reduced to

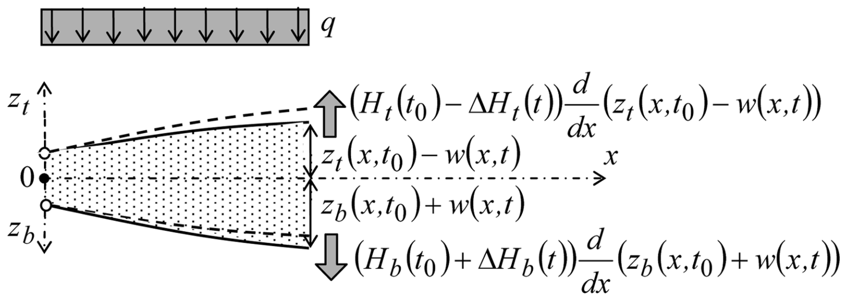
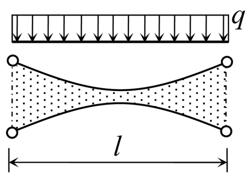
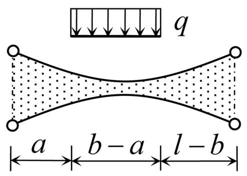
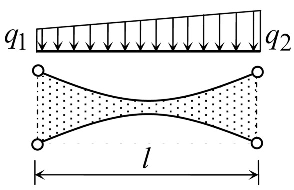
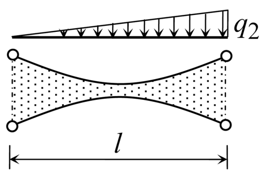
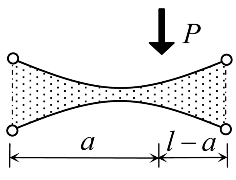


Fig. 2 Vertical equilibrium of biconvex cable truss

Table 1 Relationships for the values M of some loading types

Load		M
		$M = \frac{q.l}{2} x \left(1 - \frac{x}{l}\right)$
	for $x \in \langle 0, a \rangle$	$M = \frac{qx}{2l}(b-a)^2 + \frac{qx}{l}(b-a)(l-b)$
	for $x \in \langle a, b \rangle$	$M = \frac{qx}{2l}(b-a)^2 + \frac{qx}{l}(b-a)(l-b) - \frac{qx^2}{2} + qax - \frac{qa^2}{2}$
	for $x \in \langle b, l \rangle$	$M = \frac{qx}{2l}(b-a)^2 + \frac{qx}{l}(b-a)(l-b) - qx(b-a) + \frac{qb^2}{2} - \frac{qa^2}{2}$
	for $q_1 < q_2$	$M = \frac{lx}{6}(2q_1 + q_2) - \frac{q_1}{2}x^2 - \frac{x^3}{6l}(q_2 - q_1)$
		$M = \frac{q_2 x}{6l}(l^2 - x^2)$
	for $x \in \langle 0, a \rangle$	$M = Px\left(1 - \frac{a}{l}\right)$
	for $x \in \langle a, l \rangle$	$M = Pa\left(1 - \frac{x}{l}\right)$

$$(H_b(t_0)+H_t(t_0))\frac{dw(x,t)}{dx}+(\Delta H_b(t)-\Delta H_t(t))\frac{dw(x,t)}{dx}+\Delta H_b(t)\frac{dz_b(t_0)}{dx}+\Delta H_t(t)\frac{dz_t(t_0)}{dx} = Q \quad (3)$$

Let us confine our attention to the linearized solution. The problem is linearized by neglecting all second-order terms that appear in the differential equation of equilibrium (3). As a consequence, we obtain

$$(H_b(t_0)+H_t(t_0))\frac{dw(x,t)}{dx}+\Delta H_b(t)\frac{dz_b(t_0)}{dx}+\Delta H_t(t)\frac{dz_t(t_0)}{dx} = Q \quad (4)$$

The linearized differential Eq. (4) may be integrated directly, and after the boundary conditions have been applied, the equation for the vertical deflection of a biconvex cable truss at time t is obtained in the form

$$w(x,t) = \frac{1}{H_b(t_0)+H_t(t_0)}\left\{M+\Delta H_b(t)\frac{4}{l}(d_b-b_b)x\left[\frac{x}{l}-1\right]+\Delta H_t(t)\frac{4}{l}(d_t-b_t)x\left[\frac{x}{l}-1\right]\right\} \quad (5)$$

where M is a bending moment at some cross section x of horizontal simple beam of the same span and load as the cable truss. The relationships of M for some significant loads occurring in construction practice can be determined from Table 1.

2.3 Cable equations as time functions

To complete the solution, $\Delta H_b(t)$ and $\Delta H_t(t)$ must be evaluated at the time t. Use is made of the cable equations that incorporate Hooke's law, creep and temperature strains to provide closure conditions relating the changes in cables tensions to the changes in cables geometries at the time t when the bottom and top cables are displaced (in plane) from their original initial equilibrium profiles. The geometry of these displacements for a biconvex truss is shown in Fig. 3. If $ds_b(t_0)$ and $ds_t(t_0)$ are the original lengths of the bottom and top element, respectively, at the time t_0, and $ds_b(t)$ and $ds_t(t)$ are their new lengths at the time t, then $(ds_b(t_0))^2 = dx^2+(dz_b(x,t_0))^2$, $(ds_b(t))^2 = (dx+du_b(x,t))^2+(dz_b(t_0)+dw(x,t))^2$, $(ds_t(t_0))^2 = dx^2+(dz_t(x,t_0))^2$ and $(ds_t(t))^2 = (dx-du_t(x,t))^2+(dz_t(t_0)-dw(x,t))^2$, where $u_b(x,t)$, $u_t(x,t)$ and $w(x,t)$ are the longitudinal and vertical components of the displacements for the corresponding cables, respectively. If the profiles of the bottom and top cables are flat so that the ratio of their sag to span (and/or camber to span) is *1:8* or less, fractional changes in their lengths at time t, corrected to the first order (a geometrically linear model is applied), are

$$\varepsilon_b(t)=\frac{ds_b(t)-ds_b(t_0)}{ds_b(t_0)}=\frac{du_b(t)}{ds_b(t_0)}\frac{dx}{ds_b(t_0)}+\frac{dw(x,t)}{ds_b(t_0)}\frac{dz_b(x,t_0)}{ds_b(t_0)}$$

$$\varepsilon_t(t)=\frac{ds_t(t)-ds_t(t_0)}{ds_t(t_0)}=\frac{du_t(t)}{ds_t(t_0)}\frac{dx}{ds_t(t_0)}+\frac{dw(x,t)}{ds_t(t_0)}\frac{dz_t(x,t_0)}{ds_t(t_0)} \quad (6)$$

Elastic strains $\varepsilon_{eb}(t)$ and $\varepsilon_{et}(t)$ occur in the bottom and top chord of the truss at the time t, as the influence of cable force changes according to Hooke's law

$$\varepsilon_{eb}(t)=\frac{\Delta H_b(t)\frac{ds_b(t_0)}{dx}}{E_b(t)A_b}\ , \quad \varepsilon_{et}(t)=\frac{\Delta H_t(t)\frac{ds_t(t_0)}{dx}}{E_t(t)A_t} \quad (7)$$

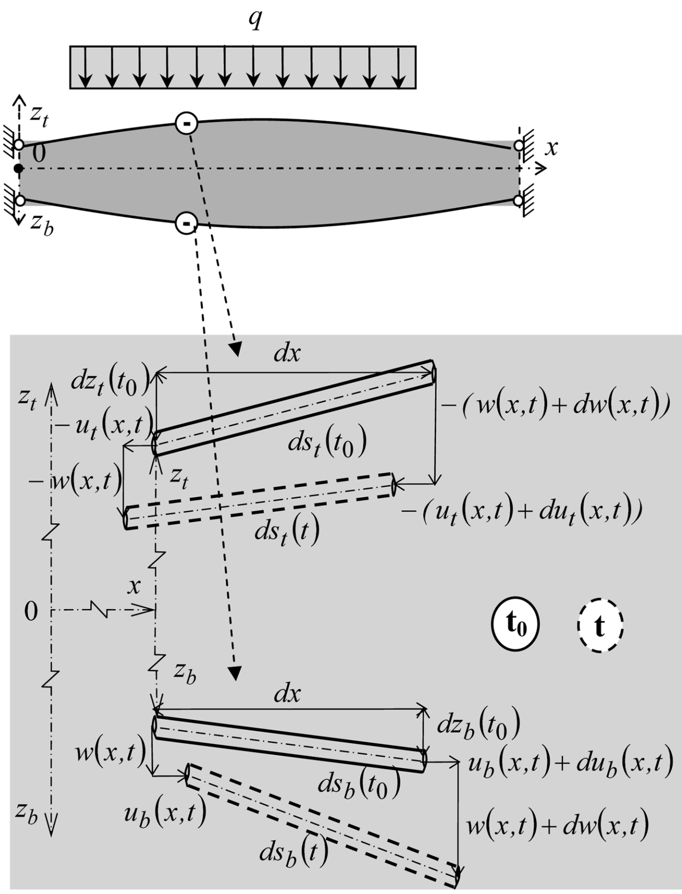


Fig. 3 Displacements of elements of bottom and top cables in biconvex truss

where $E_b(t)$ and $E_t(t)$ are the moduli of elasticity of the bottom and top chords, respectively, at the time t; and A_b and A_t are the cross-sectional areas of the bottom and top chords, respectively.

The strain increments $\varepsilon_{cb}(t)$ and $\varepsilon_{ct}(t)$ of the bottom and top chord of the truss, respectively, at time t and at the constant stress level σ, under non-linear creep, are characterized by the constitutive equations

$$\varepsilon_{cb}(t) = \left(\sum_{k=1}^{n} F_k(t)\sigma^k\right)_b, \quad \varepsilon_{ct}(t) = \left(\sum_{k=1}^{n} F_k(t)\sigma^k\right)_t \tag{8}$$

where $F_k(t)$ are the kernel functions which represent the time-dependent properties of the cables. Their values, as well as the forms of the constitutive equations, were obtained from creep tests (Kmet 2004, Kmet and Holickova 2004). By statistical investigation of the resulting creep curves, the optimal creep constitutive equations were obtained for steel cables, in the form of the logarithmic-exponential functions

$$\begin{aligned} \varepsilon_{cb}(t) &= (a + c\ln t)(1 - e^{-bt}) \\ \varepsilon_{ct}(t) &= (a + c\ln t)(1 - e^{-bt}) \end{aligned} \tag{9}$$

with coefficients a, b and c for the corresponding time t.

If the effects of a uniform temperature difference of $\Delta T_b(t) = T_b(t) - T_b(t_0)$ and/or $\Delta T_t(t) = T_t(t) - T_t(t_0)$ (where $T(t_0)$ and $T(t)$ are the initial and design temperatures, respectively) need to be incorporated, that occur in the bottom and top cable at the time t, the strains can be expressed as

$$\varepsilon_{Tb}(t) = \alpha \Delta T_b(t)$$
$$\varepsilon_{Tt}(t) = \alpha \Delta T_t(t) \tag{10}$$

where α is the coefficient of thermal expansion.

On the basis of Eqs. (6)-(10), cable equations for the bottom and top elements can be written as

$$\frac{\Delta H_b(t)\dfrac{ds_b(t_0)}{dx}}{E_b(t)A_b} + \varepsilon_{cb}(t) + \varepsilon_{Tb}(t) = \frac{du_b(x,t)}{ds_b(t_0)}\frac{dx}{ds_b(t_0)} + \frac{dw(x,t)}{ds_b(t_0)}\frac{dz_b(x,t_0)}{ds_b(t_0)}$$

$$\frac{\Delta H_t(t)\dfrac{ds_t(t_0)}{dx}}{E_t(t)A_t} + \varepsilon_{ct}(t) + \varepsilon_{Tt}(t) = \frac{du_t(x,t)}{ds_t(t_0)}\frac{dx}{ds_t(t_0)} + \frac{dw(x,t)}{ds_t(t_0)}\frac{dz_t(x,t_0)}{ds_t(t_0)} \tag{11}$$

After respective multiplication of Eqs. (11) by $(ds_b(t_0)/dx)^2$ and $(ds_t(t_0)/dx)^2$, one obtains

$$\frac{\Delta H_b(t)\left(\dfrac{ds_b(t_0)}{dx}\right)^3}{E_b(t)A_b} + (\varepsilon_{cb}(t) + \varepsilon_{Tb}(t))\left(\frac{ds_b(t_0)}{dx}\right)^2 = \frac{du_b(x,t)}{dx} + \frac{dw(x,t)}{dx}\frac{dz_b(x,t_0)}{dx}$$

$$\frac{\Delta H_t(t)\left(\dfrac{ds_t(t_0)}{dx}\right)^3}{E_t(t)A_t} + (\varepsilon_{ct}(t) + \varepsilon_{Tt}(t))\left(\frac{ds_t(t_0)}{dx}\right)^2 = \frac{du_t(x,t)}{dx} + \frac{dw(x,t)}{dx}\frac{dz_t(x,t_0)}{dx} \tag{12}$$

It is convenient to use the cable equations in the following integrated form (for in this way we to ensure that $\Delta H_b(t)$ and $\Delta H_t(t)$ are constant, as it must be because no longitudinal loads are acting)

$$\int_{t_0}^{t}\int_0^l \frac{\Delta H_b(t)\left(\dfrac{ds_b(t_0)}{dx}\right)^3}{E_b(t)\,A_b}dx\,dt + \int_{t_0}^{t}\int_0^l \varepsilon_{cb}(t)\left(\frac{ds_b(t_0)}{dx}\right)^2 dx\,dt + \int_{t_0}^{t}\int_0^l \varepsilon_{Tb}(t)\left(\frac{ds_b(t_0)}{dx}\right)^2 dx\,dt =$$
$$= \int_{t_0}^{t}\int_0^l \frac{du_b(x,t)}{dx}dx\,dt + \int_{t_0}^{t}\int_0^l \left(\frac{dw(x,t)}{dx}\frac{dz_b(x,t_0)}{dx}\right)dx\,dt$$

$$\int_{t_0}^{t}\int_0^l \frac{\Delta H_t(t)\left(\dfrac{ds_t(t_0)}{dx}\right)^3}{E_t(t)A_t}dx\,dt + \int_{t_0}^{t}\int_0^l \varepsilon_{ct}(t)\left(\frac{ds_t(t_0)}{dx}\right)^2 dx\,dt + \int_{t_0}^{t}\int_0^l \varepsilon_{Tt}(t)\left(\frac{ds_t(t_0)}{dx}\right)^2 dx\,dt =$$
$$= \int_{t_0}^{t}\int_0^l \frac{du_t(x,t)}{dx}dx\,dt + \int_{t_0}^{t}\int_0^l \left(\frac{dw(x,t)}{dx}\frac{dz_t(x,t_0)}{dx}\right)dx\,dt \tag{13}$$

The following form after integration of Eqs. (13) is obtained

$$\frac{\Delta H_b(t) L_{e,b}(t_0)}{E_b(t) A_b} + \varepsilon_{cb}(t) L_{cb}(t_0) + \alpha \Delta T_b L_{Tb}(t_0) = u_b(l,t) - u_b(0,t) + \int_0^l \left(\frac{dw(x,t)}{dx} \frac{dz_b(x,t_0)}{dx} \right) dx$$

$$\frac{\Delta H_t(t) L_{e,t}(t_0)}{E_t(t) A_t} + \varepsilon_{ct}(t) L_{ct}(t_0) + \alpha \Delta T_t L_{Tt}(t_0) = u_t(l,t) - u_t(0,t) + \int_0^l \left(\frac{dw(x,t)}{dx} \frac{dz_t(x,t_0)}{dx} \right) dx \tag{14}$$

where $u_b(l,t)$ and $u_b(0,t)$ and/or $u_t(l,t)$ and $u_t(l,t)$ are the longitudinal movements of the bottom and top supports, respectively. The left side terms $L_{eb}(t_0), L_{et}(t_0), L_{cb}(t_0), L_{ct}(t_0), L_{Tb}(t_0)$ and $L_{Tt}(t_0)$ of Eq. (14) characterizing the lengths of the unloaded cables at time t_0 are given by

$$L_{e,b,t}(t_0) = \int_0^l \left(\frac{ds_{b,t}(t_0)}{dx} \right)^3 dx \cong l \left[1 + 8 \left(\frac{d_{b,t} - b_{b,t}}{l} \right)^2 \right]$$

$$L_{c,b,t}(t_0) = L_{T,b,t}(t_0) = \int_0^l \left(\frac{ds_{b,t}(t_0)}{dx} \right)^2 dx = l \left[1 + \frac{16}{3} \left(\frac{d_{b,t} - b_{b,t}}{l} \right)^2 \right] \tag{15}$$

where $(ds_{b,t}(t_0))^2 = dx^2 + (dz_{b,t}(x,t_0))^2$, $ds_{b,t}(t_0)/dx = \sqrt{1 + (dz_{b,t}(x,t_0)/dx)^2}$, and the first two terms of the binomial series are considered.

Cable Eq. (14) are sufficient for obtaining closed-form solutions for the dependent variables $\Delta H_b(t)$ and $\Delta H_t(t)$.

2.4 Determination of the additional horizontal components of cable forces $\Delta H_b(t)$ and $\Delta H_t(t)$

Through substitution of the derivatives of expressions (1) and of the deflection function given by Eq. (5) for the corresponding types of load into the linearized system of the cable Eq. (14), after integration and rearrangement, one obtains the following explicit relations

$$\Delta H_b(t) = \frac{K_B(t) K_{T1}(t) - K_T(t) K_{B2}(t)}{K_{B1}(t) K_{T1}(t) - K_{T2}(t) K_{B2}(t)}$$

$$\Delta H_t(t) = \frac{K_{B1}(t) K_T(t) - K_{T2}(t) K_B(t)}{K_{B1}(t) K_{T1}(t) - K_{T2}(t) K_{B2}(t)} \tag{16}$$

for the calculation of the increments or decrements of the horizontal components of the cable tension forces in the bottom $\Delta H_b(t)$ and top $\Delta H_t(t)$ chords, in which, the simplified expressions are

$$K_B(t) = k_b(t) \overline{M} - k_{cb}(t) - k_{Tb}(t)$$

$$K_{B1}(t) = 1 + k_b(t)(d_b - b_b)$$

$$K_{B2}(t) = k_b(t)(d_b - b_b)$$

$$K_T(t) = k_t(t) \overline{M} - k_{ct}(t) - k_{Tt}(t)$$

$$K_{T1}(t) = 1 + k_t(t)(d_t - b_t)$$

$$K_{T2}(t) = k_t(t)(d_t - b_t) \tag{17}$$

The term $\overline{M}$, in Eq. (17), which characterizes a bending moment according to the corresponding load type is equal to:

- $\overline{M} = ql^2/8$ in case of a vertical uniformly distributed load of intensity q, per unit span;
- $\overline{M} = (q/4l)(a^3 - b^3) + (3q/8)(b^2 - a^2)$ in case of a partially uniformly distributed load q applied from $x = a$ to $x = b$;
- $\overline{M} = (l^2/16)(q_1 + q_2)$ in case of a trapezoidal load of intensity q_1 and q_2 at the ends of the span of structure, where $q_1 < q_2$;
- $\overline{M} = q_2 l^2/16$ in case of a triangular load and $\overline{M} = (3P/4l)(al - a^2)$ for a point load P acting at a distance a from the left hand support.

For the simplified terms in Eq. (17) are

$$k_b(t) = \frac{16}{3l} \frac{(d_b - b_b)}{H_b(t_0) + H_t(t_0)} \frac{E_b(t)A_b}{L_{eb}}$$

$$k_{cb}(t) = \frac{E_b(t)A_b}{L_{eb}} \varepsilon_{cb}(t) L_{cb}$$

$$k_{Tb}(t) = \frac{E_b(t)A_b}{L_{eb}} \alpha \Delta T_b(t) L_{Tb}$$

$$k_t(t) = \frac{16}{3l} \frac{(d_t - b_t)}{H_b(t_0) + H_t(t_0)} \frac{E_t(t)A_t}{L_{et}}$$

$$k_{ct}(t) = \frac{E_t(t)A_t}{L_{et}} \varepsilon_{ct}(t) L_{ct}$$

$$k_{Tt}(t) = \frac{E_t(t)A_t}{L_{et}} \alpha \Delta T_t(t) L_{Tt} \tag{18}$$

If the horizontal support flexibility (elastic and viscoelastic flexible supports) at the bottom $f_{xb}(t) = f_{1xb}(t) + f_{2xb}(t)$, and top $f_{xt}(t) = f_{1xt}(t) + f_{2xt}(t)$ suspended chord occur at each end respectively, one may replace the axial tension stiffness $E(t)A$ of the cables by the modified stiffness at time t

$$\overline{E_b(t)A_b} = \frac{E_b(t)A_b}{1 + \frac{E_b(t)A_b f_{xb}(t)}{L_{eb}}}$$

$$\overline{E_t(t)A_t} = \frac{E_t(t)A_t}{1 + \frac{E_t(t)A_t f_{xt}(t)}{L_{et}}} \tag{19}$$

3.6 The application possibilities of the derived equations

All derived equations and results are equally applicable to biconvex and biconcave cable trusses. In case of a biconvex truss $(d_b - b_b)$ and/or $(d_t - b_t)$ are positive and $\Delta H_b(t)$ and $\Delta H_t(t)$, calculated from Eq. (16), are also positive. For the resulting values of the bottom and top horizontal components of cable forces at the investigated time t the following holds true

$$
\begin{aligned}
H_b(t) &= H_b(t_0) + \Delta H_b(t) \\
H_t(t) &= H_t(t_0) - \Delta H_t(t)
\end{aligned} \tag{20}
$$

In case of a biconcave trusses $(d_b - b_b)$ and/or $(d_t - b_t)$ are negative and $\Delta H_b(t)$ and $\Delta H_t(t)$, calculated from Eq. (16) are also negative. For the resulting values of horizontal components of cable forces at the investigated time *t*, after substitution of $-\Delta H_b(t)$ and $-\Delta H_t(t)$ into expressions (20), the following holds true

$$
\begin{aligned}
H_b(t) &= H_b(t_0) - \Delta H_b(t) \\
H_t(t) &= H_t(t_0) + \Delta H_t(t)
\end{aligned} \tag{21}
$$

In the deflection functions given by Eq. (5) it is necessary to use the correct expressions corresponding to biconvex/biconcave truss. The additional deflection is of course still positive.

3. Comparison with finite element results

Numerical verification of accuracy of the closed-form analytical model which leads to Eqs. (16) and (21) was carried out on a biconcave truss with immovable supports shown in Fig. 4, through a comparison with the results obtained by FEM, using the COSMOS/M software (COSMOS/M 2002). The following data were specified: the given geometrical quantities $l = 75$ m, $d_b = d_t = 2.625$ m and $b_d = b_t = 6.375$ m, the cross-sectional areas of the bottom and top cable $A_b = 1.73.10^{-3}\text{m}^2$, $A_t = 2.692.10^{-3}\text{m}^2$, the moduli of elasticity of the cables $E_b = E_t = 1.5.10^8$ kNm^{-2}. The vertical load of $q = 1.9$ kNm^{-1}, uniformly distributed over the horizontal projection was applied. The effect of cable creep is not considered in this example. The initial horizontal components of the pretensions in the bottom and top chord of $H_b(t_0) = H_t(t_0) = 700$ kN were imposed. In case of a closed-form analysis, ties with large axial stiffness are replaced by a continuous diaphragm.

Table 2 gives the calculated values of ΔH_b and ΔH_t, H_b and H_t, and deflection *w* at the middle of span of the truss. Results obtained from Eqs. (16) and (21) and according to Eq. (5) and Table 1

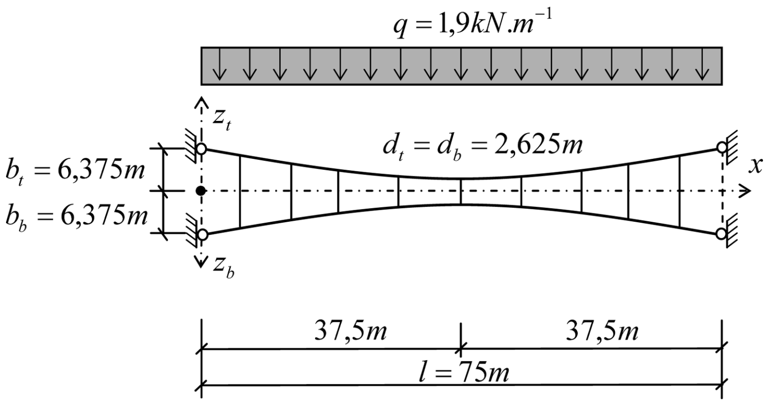


Fig. 4 Geometry and loading of biconcave cable truss

Table 2 Verification of the accuracy of closed-form model for a biconcave cable truss through a comparison with the FEM results

Closed-form model				
ΔH_b (kN) Eq. (16)	ΔH_t (kN) (16)	H_b (kN) (21)	H_t (kN) (21)	w (m) (5)
-119,996	-186,727	580,004	886,727	0,13266
FEM				
-	-	580,950	892,019	0,1313

have been compared with the results obtained by FEM in Table 2 (the corresponding equation numbers are shown). Both results are in a very good agreement.

4. Conclusions

In this paper the time-dependent closed-form static solution of the suspended biconvex and biconcave cable trusses with unmovable and elastic flexible supports subjected to various types of loads has been presented. Irvines forms of the deflection and the cable equations were modified because of the effects of non-linear creep. The concrete forms of the system of two linearized cable equations were derived and presented. From a solution of a vertical equilibrium equation for a loaded cable truss, the additional vertical deflection was determined. The time-dependent closed-form model serves to determine the response, i.e., horizontal components of cable forces and deflection of the cable truss, due to the applied loading, considering effects of elastic deformations, creep strains, temperature changes, and elastic or viscoelastic supports at the investigated time. Verification of the results was performed. Results obtained by the present closed-form solution were compared with those obtained by FEM.

The derived time-dependent closed-form computational model is used for a time-dependent simulation-based reliability assessment of cable trusses as is described in the second part of this paper.

It is believed that the presented time-dependent solution will lead to an improved closed-form analysis of the pre-stressed cable truss with rheological properties and to an improvement of its reliability assessment.

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