# Passive control of seismically excited structures by the liquid column vibration absorber

## Tanmoy Konar<sup>a</sup> and Aparna (Dey) Ghosh\*

Department of Civil Engineering, Bengal Engineering and Science University, Shibpur, Howrah, India

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**Abstract.** The potential of the liquid column vibration absorber (LCVA) as a seismic vibration control device for structures has been explored in this paper. In this work, the structure has been modeled as a linear, viscously damped single-degree-of-freedom (SDOF) system. The governing differential equations of motion for the damper liquid and for the coupled structure-LCVA system have been derived from dynamic equilibrium. The nonlinear orifice damping in the LCVA has been linearized by a stochastic equivalent linearization technique. A transfer function formulation for the structure-LCVA system has been presented. The design parameters of the LCVA have been identified and by applying the transfer function formulation the optimum combination of these parameters has been determined to obtain the most efficient control performance of the LCVA in terms of the reduction in the root-mean-square (r.m.s.) displacement response of the structure. The study has been carried out for an example structure subjected to base input characterized by a white noise power spectral density function (PSDF). The sensitivity of the performance of the LCVA to the coefficient of head loss and to the tuning ratio have also been examined and compared with that of the liquid column damper (LCD). Finally, a simulation study has been carried out with a recorded accelerogram, to demonstrate the effectiveness of the LCVA.

**Keywords:** liquid column vibration absorber; seismic vibration control; power spectral density function; simulation.

#### 1. Introduction

A variation of the conventional liquid column damper (LCD), termed the liquid column vibration absorber (LCVA), is a recent development in the field of passive control devices. Unlike the LCD, the LCVA has different cross-sectional areas in the vertical and horizontal columns of the U-shaped damper container. Due to this, the natural frequency of the oscillating liquid in the LCVA is controlled not only by the length of the liquid column but also by the area ratio between the vertical and horizontal columns. As compared to the LCD, the LCVA thereby affords a wider choice in configuration without compromising on the optimum performance of the damper. Further, the transition effects resulting from the damper liquid moving between the vertical and horizontal portions cause head loss, which is in addition to the energy dissipation due to the passage of the liquid through the orifice(s).

<sup>\*</sup>Corresponding author, Assistant Professor, E-mail: aparna@civil.becs.ac.in

<sup>&</sup>lt;sup>a</sup>Former Post-graduate Student

The LCVA was first proposed by Watkins (1991) who carried out laboratory tests on a number of variations of the basic LCVA system. Watkins and Hitchcock (1992) and Hitchcock et al. (1997a) extended the study to a bi-directional LCVA model. In the latter work, the experimental results were found to be in good agreement with the results obtained by considering an equivalent solid mass vibration absorber model of the LCVA. Hitchcock et al. (1997b) investigated the effect of the geometric configuration of the LCVA, sans orifice, on its natural frequency and damping ratio. There have been several studies on the performance of the LCVA for the mitigation of windinduced vibrations in structures, as by Chang and Hsu (1998), Chang and Ou (1998), Hitchcock et al. (1999) and Samali et al. (2001, 2004), amongst others. Wu et al. (2008) investigated the wind induced interaction between a tuned liquid column damper (TLCD) with non-uniform cross-section, i.e., LCVA, and a bridge deck in pitching motion. Taflinidis et al. (2005) have aimed to study the rotational vibration reduction capacity of the LCD and the LCVA and to find the optimum design parameters of the dampers. Optimal design parameters of LCVAs, have also been studied by Wu et al. (2009) for SDOF systems subjected to harmonic type of wind loading and presented in the form of design tables. Chaiviriyawong et al. (2007) have investigated the effect of the variation in the liquid velocity in the relatively large transition zones between the vertical columns and the horizontal column on the natural frequency of the LCD or LCVA. Taflinidis et al. (2007) have proposed a robust reliability based design of TLCD & LCVA under earthquake excitation for systems that involve model uncertainty. The general conclusion from these works is that the LCVA is a very effective passive control device for flexible structures with several advantages, both performance as well as functional based, over the LCD.

As the LCVA has hitherto been chiefly studied for wind-excited structures, the objective of this paper is to examine its applicability as a seismic vibration control device for structures and compare its performance with the LCD. In this study, the flow of liquid in the LCVA has been taken as unsteady and non-uniform. The governing differential equation for the liquid within the damper container and that for the coupled LCVA-structure system have been derived from dynamic equilibrium. The nonlinear orifice damping has been linearized by a standard stochastic equivalent linearization technique. A transfer function formulation in the frequency domain, relating the structural displacement response to the input ground acceleration, has been carried out on them. The sensitivity of the damper performance to the key design parameters, namely the tuning ratio and the coefficient of head loss, for the LCVA and the LCD, have been examined and compared. The control effectiveness of the LCVA for an example structure has been illustrated both in frequency domain and has also been compared with that of the LCD.

#### 2. Modeling of the structure-LCVA system

A single-degree-of-freedom structure with attached LCVA, subjected to horizontal ground motion, is investigated (see Fig. 1(a)). The effect of vertical ground motion on the LCVA has not been considered in the present study. Fig. 1(b) shows the model of the LCVA alone in detail. The LCVA consists of a U-shaped container having cross-sectional area of vertical columns equal to  $A_{\nu}$ , and that of horizontal column equal to  $A_h$ . The horizontal width, *B*, denotes the centre to centre distance between the vertical limbs of the damper. The vertical height of liquid, *h*, is the distance measured from the central line of the horizontal column to the still water level of the LCVA. The mass of the



Fig. 1(a) Model of structure-LCVA system



damper container is  $m_c$  and the mass of the liquid in the damper is given by  $\{\rho(A_hB+2A_vh)\}$ , where  $\rho$  represents the mass density of the damper liquid. The coefficient of head loss, controlled by the opening ratio of the orifice(s) and the geometry of the damper, is denoted by  $\xi$ . The LCVA is assumed to be connected rigidly to the top of the structure, which is modeled as a linear singledegree-of-freedom (SDOF) system having a natural frequency equal to the most predominant modal frequency of the structure (fundamental or otherwise). The mass, stiffness and damping of the SDOF system, subjected to base acceleration  $\ddot{z}(t)$ , are represented by  $m_o$ ,  $k_o$  and  $c_o$  respectively. The horizontal displacement of the mass  $m_o$ , is denoted by x(t) while the displacement of the liquid in the vertical column of the LCVA is designated by y(t).

### 3. Formulation of transfer function

 $F_3$ 

The different forces acting on the liquid in the horizontal portion of the LCVA are shown in Fig. 2. The inertia force of the liquid in the horizontal portion of the LCVA can be expressed as

$$F_{1} = \rho A_{h} B\{ \ddot{x}(t) + r \ddot{y}(t) + \ddot{z}(t) \}$$
(1)

where, r is the area ratio which is given by  $(A_v/A_h)$  and  $\{\ddot{x}(t) + r\ddot{y}(t) + \ddot{z}(t)\}$ , is the absolute acceleration of the liquid in the horizontal portion of the damper.

The force due to head loss caused by the orifice and sudden change in the cross sectional area between vertical and horizontal portion of the LCVA is given by

Fig. 2 Forces acting on the liquid in the horizontal portion of the LCVA

where,  $\{r\dot{y}(t)\}\$  is the velocity of liquid in the horizontal portion of the LCVA and  $\xi$  is the coefficient of head loss which depends on the geometry of the damper.

The hydrostatic pressure force from the left limb of the damper on the horizontal portion may be written as

$$F_{3} = A_{h}[\{h - y(t)\}\{g - \ddot{y}(t)\}\rho]$$
(3)

Similarly the hydrostatic pressure force from the right limb of the damper on the horizontal portion can be expressed as

$$F_4 = A_h[\{h + y(t)\}\{g + \ddot{y}(t)\}\rho]$$
(4)

On consideration of the horizontal equilibrium of the liquid in the horizontal portion of the LCVA, we get

$$F_1 + F_2 - F_3 + F_4 = 0 \tag{5}$$

which leads to the following expression

$$\rho A_h L_e \ddot{y}(t) + \frac{1}{2} \rho A_h r^2 \xi |\dot{y}(t)| \dot{y}(t) + 2\rho g A_h y(t) = -\rho A_h B\{ \ddot{x}(t) + \ddot{z}(t) \}$$
(6)

where,  $L_e$  denotes the effective length of the LCVA, defined as

$$L_e = \{Br + 2h\}\tag{7}$$

An equivalent linear equation corresponding to the nonlinear equation given by Eq. (6) may be written as

$$\rho A_h L_e \ddot{y}(t) + 2\rho A_h C_p \dot{y}(t) + 2\rho g A_h y(t) = -\rho A_h B\{\ddot{x}(t) + \ddot{z}(t)\}$$
(8)

where,  $C_p$  represents the equivalent linearized damping coefficient which may be obtained by minimizing the mean square value of the error incorporated due to this linearization.  $C_p$  is expressed as

$$C_{p} = \frac{r^{2} \xi < |\dot{y}(t)| \dot{y}(t) >}{4 < \{\dot{y}(t)\}^{2} >}$$
(9)

If  $\dot{y}(t)$  is a zero mean stationary Gaussian process then

$$\langle \dot{y}(t)|\dot{y}(t)\rangle = 2\sqrt{\frac{2}{\pi}\sigma_{\dot{y}}^3}$$
 and (10)

$$\langle \{\dot{y}(t)\}^2 \rangle = \sigma_{\dot{y}}^2 \tag{11}$$

where,  $\sigma_{\dot{y}}^2$  is the standard deviation of  $\ddot{y}$ . The value of  $C_p$  may then be simplified as

$$C_p = \frac{r^2 \xi \sigma_{\dot{y}}}{\sqrt{2\pi}} \tag{12}$$

On normalizing Eq. (8) with respect to  $\rho A_h L_e$  we obtain

$$\ddot{y}(t) + \frac{2C_p}{L_e} \dot{y}(t) + \omega_l^2 y(t) = -\alpha \{ \ddot{x}(t) + \ddot{z}(t) \}$$
(13)

where  $\omega_l [= \sqrt{2g/L_e}]$  is the natural frequency of the LCVA and  $\alpha [= B/L_e]$  is the area ratio, i.e., the

ratio of the length of the horizontal portion of the LCVA to its effective length. The consideration of the horizontal equilibrium of the structure-LCVA system leads to

$$(m_o + m_c + \rho B A_h + 2\rho h A_v) \{ \ddot{x}(t) + \ddot{z}(t) \} + \rho B A_h r \ddot{y}(t) + c_o \dot{x}(t) + k_o x(t) = 0$$
(14)

Normalization of Eq. (14) with respect to  $m_o(1+\mu)$  provides the following equation

$$\ddot{x}(t) + \frac{2\zeta\omega_s}{(1+\mu)}\dot{x}(t) + \frac{\omega_s^2}{(1+\mu)}x(t) = -\ddot{z}(t) - \frac{\rho BA_h r}{m_o(1+\mu)}\ddot{y}(t)$$
(15)

where,  $\mu[=(m_c + \rho B A_h + 2\rho h A_v)/m_o]$  is the mass ratio, defined as the ratio of the mass of the damper angement to that of the structure,  $\omega_s[=\sqrt{k_o/m_o}]$  is the natural frequency of the structure and  $\zeta[=c_o/2m_o\omega_s]$  is the viscous damping ratio of the structure.

The Fourier transformation of Eqs. (13) and (15) lead to the following input-output relations in frequency domain

$$X(\omega) = H_X(\omega)\ddot{Z}(\omega) \tag{16}$$

and

$$Y(\omega) = H_Y(\omega)Z(\omega) \tag{17}$$

where,  $X(\omega)$ ,  $Y(\omega)$  and  $\ddot{Z}(\omega)$  are the Fourier transforms of the corresponding time- dependent variables, x(t), y(t) and  $\ddot{z}(t)$  respectively. Further in Eqs. (16) and (17),  $H_x(\omega)$  is the transfer function relating the displacement of the structure to the input base acceleration and  $H_y(\omega)$  is the transfer function which relates the displacement of the liquid in the vertical column of the LCVA to the input base acceleration. They are expressed as

$$H_{X}(\omega) = \frac{[\rho Br A_{h} \omega^{2} H_{1}(\omega) + m_{0}(1+\mu)] H_{2}(\omega)}{[\rho Br A_{h} \omega^{4} H_{1}(\omega) H_{2}(\omega) - m_{0}(1+\mu)]}$$
(18)

and

where

$$H_{Y}(\omega) = H_{1}(\omega) \{ \omega^{2} H_{X}(\omega) - 1 \}$$
(19)

$$H_1(\omega) = \frac{\alpha}{\omega_l^2 - \omega^2 + i\omega(2C_p/L_e)}$$
(20)

and

$$H_2(\omega) = \frac{(1+\mu)}{\omega_s^2 + 2\zeta \omega_s i\omega - (1+\mu)\omega^2}$$
(21)

Let the earthquake ground acceleration be characterized by a power spectral density function (PSDF),  $S_z(\omega)$ . Hence the PSDF of the displacement response of the structure, denoted by  $S_X(\omega)$ , is expressed by Newland (1993)

$$S_X(\omega) = |H_X(\omega)|^2 S_z(\omega)$$
(22)

Also, the PSDF of the liquid velocity,  $\dot{u}(t)$ , represented by  $S_{\dot{u}}(\omega)$ , is evaluated from the following expression

$$S_{ii}(\omega) = \omega^2 |H_u(\omega)|^2 S_z(\omega)$$
<sup>(23)</sup>

The root-mean-square (r.m.s.) value of the displacement response of the structure,  $\sigma_x$ , and the r.m.s. value of the velocity response of the liquid column, equal to the standard deviation,  $\sigma_u$ , can be numerically evaluated by computing the square root of the area under the corresponding PSDF

curve as given by Eqs. (22) and (23) respectively. With an initial value of  $C_p$  ( $C_p = 0$ , that is no nonlinear damping is a good initial value), the responses are computed and an iterative method is followed to solve the set of nonlinear algebraic equations involved till convergence is achieved.

#### 4. Selection of design parameters of the LCVA

It is clear from the preceding section that the independent parameters that determine the response of a structure with an attached LCVA include the natural frequency of the structure ( $\omega_s$ ), damping ratio of the structure ( $\zeta$ ), the mass of the structure ( $m_a$ ), the mass ratio ( $\mu$ ), the length ratio ( $\alpha$ ), the area ratio (r), the coefficient of head loss ( $\xi$ ), the natural frequency of the LCVA ( $\omega_l$ ), the mass density of the liquid  $(\rho)$  and the input excitation. It should be noted that the additional parameters in case of the LCVA as compared to the case of the LCD are the mass of the structure  $(m_o)$ , the area ratio (r) and the mass density of the liquid ( $\rho$ ). Since  $\omega_s$ ,  $\zeta$ ,  $m_o$  and the input excitation must be stated by the problem itself and  $\mu$  is generally fixed from practical constraints, the remaining five parameters, namely,  $\alpha$ , r,  $\xi$ ,  $\gamma$  and  $\rho$  may be optimized. In the present study, the liquid in the LCVA is assumed to be water and so  $\rho$  is a fixed quantity. The remaining four parameters of the LCVA are examined in this section to obtain the optimal combination of the design parameters of the LCVA that would achieve the maximum reduction in the r.m.s. displacement response of the structure. The study is carried out on an example structure, modeled as a linear, viscously damped SDOF system that is subjected to an earthquake excitation characterized by a white noise PSDF of intensity  $(S_o)$ equal to 100  $\text{cm}^2/\text{s}^3$ . Both the LCD and the LCVA are long period systems and are applicable for the vibration control of flexible structures. Sadek et al. (1996) in their study on single and multiple TLCDs for seismic applications considered an example ten-storey structure with a fundamental natural frequency and damping ratio of 0.5 Hz and 2% respectively. Here too, the natural period  $(T_n)$  and damping ratio ( $\zeta$ ) of the SDOF structural system being analyzed are equal to 2.0 s ( $\omega_s = 3.1416$  rad/s) and 2% respectively.

First, the variation in the percent reduction due to LCVA in the r.m.s. value of the displacement response of the structure, which is chosen as the performance index for the damper, is studied for a



Fig. 3 Variation in the percent reduction in r.m.s. displacement of the structure, due to LCVA, with *r* and  $\alpha$ , for  $\mu = 0.5\%$ 



Fig. 4 Variation in the percent reduction in r.m.s. displacement of the structure, due to LCVA, with r and  $\alpha$ , for  $\mu = 1.5\%$ 



Fig. 5 Variation in the percent reduction in r.m.s. displacement of the structure, due to LCVA, with r and  $\alpha$ , for  $\mu = 2.5\%$ 

Table 1 Maximum area ratio ( $r_{max}$ ) for various length ratio ( $\alpha$ )

α	0.4	0.5	0.6	0.7	0.8
$r_{\rm max}$	2.5	2.0	1.65	1.425	1.25

range of values of r and  $\alpha$ . The results are presented in Figs. 3-5. The three figures correspond to three practically feasible values of mass ratio, namely 0.5%, 1.5%, 2.5% and for each figure, two example values of  $m_o$ , equal to  $5 \times 10^5$  kg and  $7 \times 10^5$  kg, are considered. For these results, the tuning ratio, which is the ratio of the LCVA frequency,  $\omega_l$ , to the structural frequency,  $\omega_s$ , has been considered to be unity and the coefficient of head loss ( $\xi$ ) has been optimized for each set of parameter values. For a particular value of  $\alpha$ , the upper limit of the range of r has been fixed from the constraint that h cannot be negative. The maximum possible values of r ( $r_{max}$ ) for various  $\alpha$  are given in Table 1. However, it should be noted that the practical values of the upper limit of r will be less than these theoretical values (given in Table 1), as the value of h should be at least equal to half the vertical depth of the horizontal column of the LCVA.

From Table 1 it is observed that when  $\alpha$  is low the value of  $r_{\text{max}}$  is higher. This is significant as higher *r* leads to greater response reduction by the LCVA (which also means that the optimum value of *r* ( $r_{opt}$ ) is equal to  $r_{\text{max}}$ ) while lower  $\alpha$  is associated with lesser response reduction (see Figs. 3-5). Since for the LCD the value of *r* is unity, the advantage of the LCVA over the LCD is clear from these figures. Further, in order to achieve a certain response reduction, a wider choice of the damper geometry is available in case of the LCVA as that may be achieved with different combinations of  $\alpha$  and *r*. However, the value of *r* should not normally be less than unity as then the response reduction capacity of the LCVA would be less than that of the LCD. It is also observed from Figs. 3-5 that the performance of the LCVA is independent of the mass of the structure when the other properties of the structure and that of the LCVA are kept unchanged. For  $\alpha = 0.4$ , the maximum increase in the percent response reduction of the LCVA over that of the LCD is about 14%, 18% and 20% for mass ratio equal to 0.5%, 1.5% and 2.5% respectively, whereas, for  $\alpha =$ 0.8, the maximum increase in the percent response reduction of the LCVA over that of the LCD is about 5% for all the mass ratios considered. Hence, the advantage of the LCVA over the LCD is greater for lower length ratio. This is significant as geometric constraints often prohibit the use of a high value of length ratio which is desired because it leads to higher response reduction. Thus, from Figs. 3-5, one can select the optimal combination of r and  $\alpha$  for a given  $\mu$ , for the design of the LCVA. In case of a constraint on  $\alpha$ , as in the case of a very flexible structure, one can fix the values of r and  $\mu$  to obtain a desired response reduction. Again for a relatively stiff system, when from tuning considerations the geometric constraints on the LCD become critical, the LCVA may offer a more feasible geometric configuration by selecting a value of r less than unity. For example, for a system with  $T_n = 1.2$  s,  $\zeta = 2\%$ ,  $m_o = 5 \times 10^5$  kg,  $\mu = 1.5\%$  and  $\alpha = 0.7$ , the percent reduction in the performance index with LCD is 25.1%. For this case, the required values of the length of the liquid column, L and horizontal length, B, are about 71.5 cm and 50.1 cm respectively, which indicate that the clear distance between the two vertical limbs as well as the liquid height in the vertical limbs of the LCD will be very less which may lead to installation and functional difficulties. If the LCVA with r less than unity is used in such a case, these problems can be mitigated to some extent. For the same example system attached with a LCVA having  $\alpha = 0.7$  and r = 0.8, the height of the liquid in the vertical limb will increase by 25% of the liquid height, h. obtained in case of the LCD while the clear distance between the two vertical limbs will increase by 20% of the height of the horizontal column. However, there will be a marginal loss of about 3% in the percent response reduction due to the consideration of r less than unity.

Next, the sensitivity of the performance of the LCVA to the tuning ratio,  $\gamma (= \omega_t/\omega_s)$  has been analyzed and the optimal tuning ratio,  $\gamma_{opt}$ , has been evaluated for varying  $\alpha$  and  $\mu$ , considering  $T_n =$ 2.0 s,  $\zeta = 2\%$ , and  $m_o = 5 \times 10^5$  kg. The maximum possible value of r corresponding to the  $\alpha$ values has been adopted (see Table 1) and  $\xi$  has been optimized for each set of parameters values. The variation in the response reduction with tuning ratio for different values of  $\mu$  and  $\alpha$  is shown in Fig. 6 where it is observed that for a particular value of  $\mu$ , the curves for different values of  $\alpha$ but with the corresponding maximum values of r. This is also indicated in Figs. 3-5. This, however, is not possible in case of the LCD because only one choice of r is possible for a particular  $\alpha$ . The optimum tuning ratio ( $\gamma_{opt}$ ) and the maximum drop in percent response reduction due to  $\pm 10\%$ mistuning for various  $\mu$  is given in Table 2. It is observed that, similar to the other tuned mass and liquid dampers, the optimum tuning ratio ( $\gamma_{opt}$ ) is close to unity for the low mass ratio and the deviation from unity increases with increase in  $\mu$ . Further, the LCVA performance is more robust



Fig. 6 Variation in the percent reduction in r.m.s. displacement of the structure, due to LCVA, with  $\nu$ , for different  $\alpha$  and  $\mu$ 

Mass ratio ( $\mu$ ) (%)	0.5	1.5	2.5			
Optimum tuning ratio ( $\gamma_{opt}$ )	0.99	0.975	0.96			
Maximum drop in performance due to $\pm 10\%$ mistuning	11.87%	10.09%	7.94%			

Table 2 Optimum tuning ratio ( $v_{opt}$ ) and effect of mistuning for various mass ratio



Fig. 7 Comparison of the performance sensitivity to  $\nu$ , between LCD and LCVA



Fig. 9 Variation in the percent reduction in r.m.s. displacement of the structure, due to LCVA, with  $\xi$ , for different  $\alpha$  and  $\mu = 1.5\%$ 



Fig. 8 Variation in the percent reduction in r.m.s. displacement of the structure, due to LCVA, with  $\xi$ , for different  $\alpha$  and  $\mu = 0.5\%$ 



Fig. 10 Variation in the percent reduction in r.m.s. displacement of the structure, due to LCVA, with  $\xi$ , for different  $\alpha$  and  $\mu = 2.5\%$ 

when  $\mu$  is greater. Fig. 7 provides a comparison of the performance sensitivity of the LCD and the LCVA to the tuning ratio. The LCVA exhibits greater robustness than the LCD as for a mistuning of  $\pm 10\%$  from the optimum tuning ratio, the performance of the LCVA deteriorates by only 7.94% whereas that of the LCD decreases by 12.12% for the same amount of mistuning from the optimum tuning ratio.

The sensitivity of the performance of the LCVA to the coefficient of head loss ( $\xi$ ) has also been examined. The same example system as for Fig. 6 with unit value of tuning ratio is considered. The variation in the response reduction with  $\xi$  and  $\alpha$  for various  $\mu$  is presented in Figs. 8-10. The value



Fig. 11 Sensitivity of  $\xi_{opt}$  to r for different  $\alpha$  and  $\mu = 0.5\%$ 



Fig. 12 Comparison of the performance sensitivity to  $\xi$ , between LCD and LCVA, for different mass ratios

of  $\xi$  corresponding to the maximum response reduction is termed  $\xi_{opt}$ , upto which there is a sharp variation in the percent response reduction. Beyond  $\xi = \xi_{opt}$ , the variation is significant for the lower mass ratios ( $\mu = 0.5\%$ , 1.5%) but reduces for the higher mass ratio ( $\mu = 2.5\%$ ). It is also seen that for a particular value of  $\mu$ , the curves for the different values of  $\alpha$  follow the same trend and the values of  $\xi_{opt}$  corresponding to the different values of  $\alpha$  do not vary appreciably. Also, for a given  $\mu$ , the maximum percent reduction is the same for all  $\alpha$  with corresponding  $\xi_{opt}$  because corresponding  $r_{max}$  has been used. The sensitivity of  $\xi_{opt}$  to r for various values of  $\alpha$  has also been examined for the different mass ratios and it has been found that the variation of  $\xi_{opt}$  with r is very nominal. A typical set of results is given in Fig. 11. Thus,  $\xi_{opt}$  is neither significantly dependent on r nor on  $\alpha$  for a particular value of  $\mu$  and the values of  $\xi_{opt}$  increase with  $\mu$ .

A comparison of the percent response reduction for varying  $\xi$  between the LCD and the LCVA for different mass ratios has been presented in Fig. 12 where it is observed that the LCD and the LCVA are almost equally sensitive to  $\xi$  for a given mass ratio.

It may be noted that where there is requirement of multi-mode response suppression, as in cases where higher mode participation is significant, several LCVAs with different tuning conditions may be installed. For each mode, the corresponding modal properties may be used to select the LCVA parameters as described in this section.

#### 5. Response transfer function of structure-LCVA system

The displacement transfer function of an example structure with LCVA has been evaluated as per the formulation presented in this paper. The same example structure and characterization of seismic base input as in Fig. 6 has been considered with  $\mu$  and  $\alpha$  equal to 2.5% and 0.5 respectively. The corresponding optimum tuning ratio,  $\gamma_{opt}$ , is 0.96 (refer Table 2) and the optimum area ratio,  $r_{opt}$ , is 2.0 (refer Table 1). However, as discussed earlier the practical value of r will always be less than  $r_{opt}$ . Here, let us take r equal to 1.55 say. The displacement transfer functions of the structure without damper and with LCVA have been shown in Fig. 13 and have been compared with the transfer function of the structure with LCD having the same values of  $\mu$  and  $\alpha$  as that of the LCVA. The formulation for the structure-LCD transfer function has been given in Ghosh and Basu (2007). The nature of the transfer functions indicates the vibration suppression of the dampers, with the



Fig. 13 Displacement transfer functions of the structure alone, with LCVA and with LCD, for white noise input



Fig. 14 Displacement time history of the structure alone, with LCVA and with LCD, for the Bhuj accelerogram

LCVA performing better than the LCD. The percent response reductions achieved by the LCVA and the LCD are 29.97% and 23.50% respectively.

#### 6. Simulation study of structure-LCVA system

To examine the performance of the LCVA in the time domain and compare the same with that of the LCD, the example systems of Fig. 13 has been subjected to an accelerogram which is the recorded N78E component of the Bhuj (2001) earthquake at the Ahmedabad (23.03°N, 72.63°E) site. The fourth-order Runge-Kutta method has been employed for the time integration of the response of the structure-damper systems. The optimum value of  $\xi$  has been obtained by minimizing the r.m.s. value of displacement of the structure and has been obtained as 0.93 for the LCVA and 1.24 for the LCD. The results of the simulation study have been presented in Fig. 14 and indicate effective vibration control by the dampers with the LCVA performing better than the LCD. Here, the LCVA achieves a reduction of 14.12% in the peak displacement, and 31.07% in the r.m.s. displacement with respect to the response of the structure alone, while for the LCD the corresponding values are 10.21% and 24.41% respectively. The limiting condition of the peak liquid displacement is also satisfied. The maximum liquid displacement is obtained as 0.2044 m, whereas the limiting liquid displacement (which is actually the vertical height of the liquid, h) is 0.2426 m. It should be noted that, for a particular value of  $\alpha$ , if the maximum liquid displacement remains within the limiting value for the LCVA, the maximum liquid displacement for the LCD will also be within the limiting value, since r of the LCVA is greater than unity.

#### 7. Conclusions

In this paper, the performance of the passive damper, the LCVA, has been studied for the seismic vibration control of a structure. In the frequency domain, the transfer function relating the displacement of a structure, modeled as a linear, viscously damped, SDOF system with attached LCVA, to the input ground acceleration, has been formulated. The parameters controlling the

performance of the LCVA have been identified and their effects on the performance of the damper, measured in terms of the reduction in the r.m.s. displacement response of the structure subjected to white noise base input, have been studied.

The increase in response reduction with increase in area ratio, r, corroborates earlier findings that the equivalent damping of the LCVA increases with r. For a particular value of the length ratio,  $\alpha$ , there is a maximum possible value r, which is thus the optimum r, that is independent of the mass ratio. On comparing with the performance of the LCD, for which the value of r is unity, the LCVA, with  $r_{opt}$  greater than unity, proves to be more effective in controlling the seismic response of the structure. Further, the advantage of the LCVA over the LCD in response reduction is greater at lower values of  $\alpha$ . This is significant as geometric constraints often restrict the adoption of a high value of  $\alpha$ , especially in very flexible structures, while higher  $\alpha$  is desired as it leads to better performance of dampers like the LCD and the LCVA. Also, for a given mass ratio, the LCVA offers a wider choice of the damper geometry as several combinations of r and  $\alpha$  may lead to a certain response reduction, which is not possible in case of the LCD. As a corollary to this, for a given mass ratio, the maximum response reduction achieved by the LCVA is the same for different values of  $\alpha$  and corresponding values of  $r_{max}$ . For control of a relatively stiff system, as in case of a higher mode of a structural system, the LCVA offers a more feasible geometric configuration as compared to the LCD by selecting a value of r less than unity.

A study on the sensitivity of the LCVA performance to tuning ratio,  $\gamma$ , indicates that as for the LCD, the optimal tuning ratio of the LCVA is close to unity for lower  $\mu$  and the value of  $\gamma_{opt}$  decreases from unity as  $\mu$  increases. The performance of the LCVA is more robust for higher  $\mu$  and it is more robust than the LCD for a given  $\mu$ . Regarding the variation in LCVA performance with coefficient of head loss,  $\xi$ , it is found that there is a sharp variation in the percent response reduction up to  $\xi = \xi_{opt}$ . Beyond  $\xi = \xi_{opt}$ , the variation is significant for lower mass ratios but reduces for the higher mass ratio. Another important observation is that  $\xi_{opt}$  does not depend significantly on  $\alpha$  or r for a particular value of  $\mu$ .

The displacement transfer functions of an example structure without damper, with LCVA and with LCD have been illustrated, which indicate greater vibration suppression by the LCVA as compared to the LCD. A time-domain study with the Bhuj accelerogram also demonstrates the effective and superior performance of the LCVA as a seismic passive control device.

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