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Vibration analysis of plates with curvilinear quadrilateral domains by discrete singular convolution method

Ömer Civalek*[1] and Baki Ozturk[2]

[1]*Akdeniz University, Faculty of Engineering, Civil Engineering Department, Division of Mechanics, Antalya, Turkiye*
[2]*Niğde University, Faculty of Engineering, Civil Engineering Department, Division of Mechanics, Niğde, Turkiye*

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Abstract. A methodology on application of the discrete singular convolution (DSC) technique to the free vibration analysis of thin plates with curvilinear quadrilateral platforms is developed. In the proposed approach, irregular physical domain is transformed into a rectangular domain by using geometric coordinate transformation. The DSC procedures are then applied to discretization of the transformed set of governing equations and boundary conditions. For demonstration of the accuracy and convergence of the method, some numerical examples are provided on plates with different geometry such as elliptic, trapezoidal having straight and parabolic sides, sectorial, annular sectorial, and plates with four curved edges. The results obtained by the DSC method are compared with those obtained by other numerical and analytical methods. The method is suitable for the problem considered due to its generality, simplicity, and potential for further development.

Keywords: discrete singular convolution; natural frequency; plates; numerical models; curvilinear domain.

1. Introduction

The analysis of non-rectangular plates or generally called curvilinear or straight-sided quadrilateral plates has been the subject of research of structural and mechanical engineering (Lam *et al*. 1990). The difficulties of mapping boundaries of curvilinear quadrilateral domains could be eliminated firstly by Gordon and Hall (1973) using blending functions which permit exact transformation. Li *et al*. (1986) presented a spline finite strip analysis of arbitrary shaped general plates. Cheung *et al*. (1988) also developed a finite strip analysis for static and vibration analysis of general plates. Cubic serendipity shape functions were first employed for arbitrary shaped general plates by finite strip method (Li *et al*. 1986, Cheung *et al*. 1988). Subsequently, Wang and Cheng-Tzu (1994) and Geannakakes (1990) used a similar approach to analyze irregular plates using the finite strip method in conjunction with orthogonal polynomials and linear serendipity shape functions, respectively. Orthogonal polynomials based on Rayleigh-Ritz method for flexural vibration of annular sector and trapezoidal plates of arbitrary shape have been presented by Liew and Lam (1991, 1993a, b). Liew

*Corresponding author, Professor, E-mail: civalek@yahoo.com

and Han (1997), and Han and Liew (1997) introduced a mapping technique to apply the differential quadrature (DQ) method for analysis of plates in conjunction with the Reissner-Mindlin thick plate theory. Blending functions were employed by Shu *et al*. (2000) for vibration analysis of curvilinear quadrilateral plates using the DQ method. Bert and Malik (1996) improved the numerical accuracy by using the DQ method for plate vibration with irregular domain. Benchmark vibration solutions for polygonal plates are given by Liew *et al*. (1995). Vibration analysis of sectorial plates having corner stress singularities has been investigated by Leissa *et al*. (1993). Exact analytical solutions for free vibrations of sectorial plates have been investigated by Huang *et al*. (1993). A comparative study of the vibration analysis of skew plates has been presented by Wang *et al*. (1994). Two-dimensional orthogonal polynomials for vibration analysis of circular and elliptical plates are investigated by Lam *et al*. (1992). For completely free plates, a meshfree least squares-based finite difference approach was presented by Wu *et al*. (2006). Long list of references on vibration of plates having different geometries are given, for example, in References (Cheung and Cheung 1971, Gorman 1988, Civalek 2004, Singh and Chakraverty 1992, Liew 1992, Wang *et al*. 2004). To the authors' knowledge, it is the first time the DSC method has been successfully applied to plate problems having curvilinear domain for the analysis of free vibration.

2. Discrete singular convolution (DSC)

Discrete singular convolutions (DSC) algorithm was introduced by Wei (1999). As stated by Wei *et al*. (1998) singular convolutions (SC) are a special class of mathematical transformations. They appear in many science and engineering problems, such as the Hilbert, Abel and Radon transforms. In fact, the theory of wavelets and frames, a new mathematical branch developed in recent years, can also find its root in the theory of distributions (Wei *et al*. 1998, Wei and Yun 2002).

The DSC algorithm to solve solid and fluid mechanics problems was first applied by Wei and his co-workers (Wei and Gu 2002, Wei 2001a, b, c). Zhao *et al*. (2002) analyzed the high frequency vibration of plates using the DSC algorithm. Zhao and Wei (2002) adopted the DSC in vibration analysis of rectangular plates with non-uniform boundary conditions. More recently, a good comparative accuracy of DSC and generalized differential quadrature methods for vibration analysis of rectangular plates is presented by Ng *et al*. (2004). Lim *et al*. (2005a, b) presented the DSC-Ritz method for the free vibration analysis of plates and thick shallow shells. Numerical solutions of free vibration problem of rotating and laminated conical shells and plates on elastic foundation have been proposed by author lately (Civalek 2005, 2006, 2007a, b, c, d). These studies indicate that the DSC algorithm work very well for the vibration analysis of plates, especially for high-frequency analysis of rectangular plates. Furthermore, it is also concluded that the DSC algorithm has global methods' accuracy and local methods' flexibility for solving differential equations in applied mechanics. Mathematical foundation of the DSC algorithm is the theory of distributions and wavelet analysis (Wei 1999). Consider a distribution, T and $\eta(t)$ as an element of the space of test function. A singular convolution can be defined by Wei *et al*. (1998)

$$F(t) = (T*\eta)(t) = \int_{-\infty}^{\infty} T(t-x)\,\eta(x)dx \tag{1}$$

where $T(t-x)$ is a singular kernel. For example, singular kernels of delta type

$$T(x) = \delta^{(n)}(x) \quad (n = 0, 1, 2, \ldots) \tag{2}$$

Kernel $T(x) = \delta(x)$ is important for interpolation of surfaces and curves, and $T(x) = \delta^{(n)}(x)$ for $n > 1$ are essential for numerically solving differential equations (Wei 2001). With a sufficiently smooth approximation, it is more effective to consider a discrete singular convolution (Wei *et al.* 2002a)

$$F_\alpha(t) = \sum_k T_\alpha(t - x_k) f(x_k) \tag{3}$$

where $F_\alpha(t)$ is an approximation to $F(t)$ and $\{x_k\}$ is an appropriate set of discrete points on which the DSC is well defined. The mathematical property or requirement of $f(x)$ is determined by the approximate kernel T_α. Recently, the use of some new kernels and regularizer such as delta regularizer (Wei *et al.* 2002b) was proposed to solve applied mechanics problem. The Shannon's kernel is regularized as (Zhao *et al.* 2002)

$$\delta_{\Delta,\sigma}(x - x_k) = \frac{\sin[(\pi/\Delta)(x - x_k)]}{(\pi/\Delta)(x - x_k)} \exp\left[-\frac{(x - x_k)^2}{2\sigma^2}\right]; \quad \sigma > 0 \tag{4}$$

where D is the grid spacing. It is also known that the truncation error is very small due to the use of the Gaussian regularizer, the above formulation given by Eq. (4) is practical, and has an essentially compact support for numerical interpolation (Wei 2001b). Eq. (4) can also be used to provide discrete approximations to the singular convolution kernels of the delta type (Wei *et al.* 2002b)

$$f^{(n)}(x) \approx \sum_{k=-M}^{M} \delta_\Delta(x - x_k) f(x_k) \tag{5}$$

where $\delta_\Delta(x - x_k) = \Delta\delta_\alpha(x - x_k)$ and superscript (n) denotes the n*th*-order derivative. The $2M$+1 is the computational bandwidth which is centered around x, and is usually smaller than the whole computational domain. In the DSC method, the function $f(x)$ and its derivatives with respect to the x coordinate at a grid point x_i are approximated by a linear sum of discrete values $f(x_k)$ in a narrow bandwidth $[x - x_M, x + x_M]$. This can be expressed as (Wei 2001c)

$$\left.\frac{d^n f(x)}{dx^n}\right|_{x = x_i} = f^{(n)}(x) \approx \sum_{k=-M}^{M} \delta^{(n)}_{\Delta,\sigma}(x_i - x_k) f(x_k) \quad (n = 0, 1, 2, \ldots) \tag{6}$$

where superscript n denotes the nth-order derivative with respect to x. The x_k is a set of discrete sampling points centere around the point x, σ is a regularization parameter, Δ is the grid spacing, and $2M + 1$ is the computational bandwidth which is usually smaller than the size of the computational domain. For example, the second order derivative at $x = x_i$ of the DSC kernels for directly given (Wei 2001a)

$$\delta^{(2)}_{\Delta,\sigma}(x - x_j) = \left.\frac{d^2}{dx^2}[\delta_{\Delta,\sigma}(x - x_j)]\right|_{x = x_i} \tag{7}$$

The discretized forms of Eq. (7) can then be expressed as (Wei 2001c)

$$f^{(2)}(x) = \left.\frac{d^2 f}{dx^2}\right|_{x = x_i} \approx \sum_{k=-M}^{M} \delta^{(2)}_{\Delta,\sigma}(k\Delta x_N) f_{i+k,j} \tag{8}$$

3. The DSC method for curvilinear domains

There have been several studies on the application of numerical and analytical methods for free vibration of plates having different geometries (Gorman 1988, Liew *et al*. 1994, Liew and Sumi 1998, Bert and Malik 1996, Wang *et al*. 2001, Karami and Malekzadeh 2003). Two-dimensional orthogonal plate function was applied for frequency analysis of rectangular (Liew *et al*. 1990) and skew plates by Liew and Lam (1990). Xiang *et al*. (1993) proposed a Rayleigh-Ritz procedure for annular sector plates. Consider an eight-node curvilinear quadrilateral domain as shown in Fig. 1(a). Thus, the following equations are used for the coordinate transformation (Han and Liew 1997)

$$x = \sum_{i=1}^{8} \Psi_i(\xi, \eta) x_i \tag{9}$$

$$y = \sum_{i=1}^{8} \Psi_i(\xi, \eta) y_i \tag{10}$$

Using the chain rule, first-order, and second order derivatives of a function are given

$$\begin{Bmatrix} f_x \\ f_y \end{Bmatrix} = [J_{11}]^{-1} \begin{Bmatrix} f_\xi \\ f_\eta \end{Bmatrix} \tag{11}$$

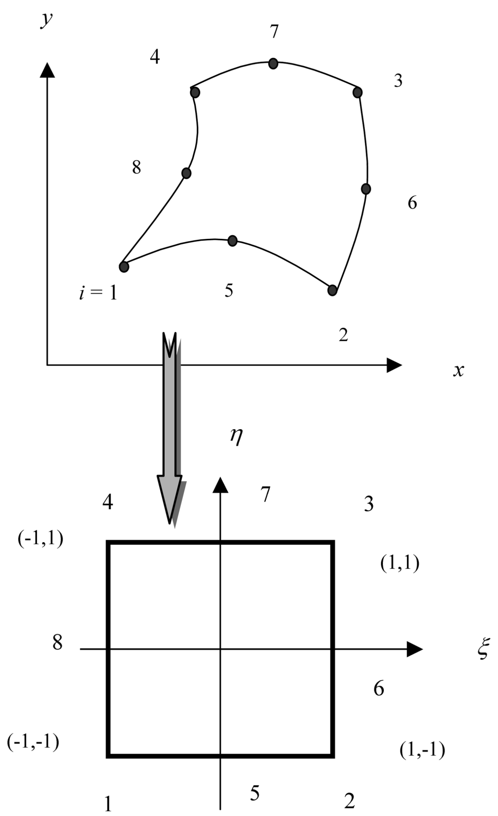

Fig. 1 Mapping of arbitrary curvilinear plates into natural coordinates

$$\begin{Bmatrix} f_{xx} \\ f_{yy} \\ 2f_{xy} \end{Bmatrix} = [J_{22}]^{-1}\begin{Bmatrix} f_{\xi\xi} \\ f_{\eta\eta} \\ 2f_{\xi\eta} \end{Bmatrix} - [J_{22}]^{-1}[J_{21}][J_{11}]^{-1}\begin{Bmatrix} f_{\xi} \\ f_{\eta} \end{Bmatrix} \tag{12}$$

where ξ_i and η_i are the coordinates of Node i in the $\xi - \eta$ plane, and J_{ij} are the elements of the Jacobian matrix. These are expressed as follows

$$[J_{11}] = \begin{bmatrix} x_{\xi} & y_{\xi} \\ x_{\eta} & y_{\eta} \end{bmatrix} \tag{13}$$

$$[J_{21}] = \begin{bmatrix} x_{\xi\xi} & y_{\xi\xi} \\ x_{\eta\eta} & y_{\eta\eta} \\ x_{\xi\eta} & y_{\xi\eta} \end{bmatrix} \tag{14}$$

$$[J_{22}] = \begin{bmatrix} x_{\xi}^2 & y_{\xi}^2 & x_{\xi}y_{\xi} \\ x_{\eta}^2 & y_{\eta}^2 & x_{\eta}y_{\eta} \\ x_{\xi}x_{\eta} & y_{\xi}y_{\eta} & \frac{1}{2}(x_{\xi}x_{\eta} + x_{\eta}x_{\xi}) \end{bmatrix} \tag{15}$$

The above transformations will be used later to transform the governing differential equations and related boundary conditions from the physical domain $x - y$ into the computational domain $\xi - \eta$. Thus, an arbitrary-shaped quadrilateral plate may be represented by the mapping of a square plate defined in terms of its natural coordinates. From Eq. (6), the equations of any-order derivatives can be easily written at any point.

$$\Psi_i(\xi, \eta) = \frac{1}{4}(1 + \xi\xi_i)(1 + \eta\eta_i)(\xi\xi_i + \eta\eta_i - 1) \quad \text{for} \quad i = 1, 3, 5, 7 \tag{16}$$

$$\Psi_i(\xi, \eta) = \frac{1}{4}(1 - \xi^2)(1 + \eta\eta_i) \quad \text{for} \quad i = 2, 6 \tag{17}$$

$$\Psi_i(\xi, \eta) = \frac{1}{4}(1 + \xi\xi_i)(1 - \eta^2) \quad \text{for} \quad i = 4, 8 \tag{18}$$

4. Fundamental equations

For free vibration analysis of thin rectangular plate, the governing equation can be given by (Leissa 1969)

$$D\left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right) - \rho h\frac{\partial^2 w}{\partial t^2} = 0 \tag{19}$$

where D is the bending rigidity for plate, h is the plate thickness, w is the deflection, ρ is the density, x and y are the midplane Cartesian coordinate. The transverse displacement w for free vibration is taken as

$$w(x,y,t) = W(x,y)e^{iwt} \tag{20}$$

Substituting Eq. (20) into Eq. (19), one obtains the normalized equation

$$\frac{\partial^4 W}{\partial X^4} + 2\lambda^2 \frac{\partial^4 W}{\partial X^2 \partial Y^2} + \lambda^4 \frac{\partial^4 W}{\partial Y^4} = \Omega^2 W \tag{21}$$

Where $X = x/a$, $Y = y/b$, $\lambda = a/b$, $\Omega^2 = \rho h a^4 \omega^2 / D$. Now introducing

$$\nabla^2(\bullet) = \frac{\partial^2(\bullet)}{\partial X^2} + \lambda^2 \frac{\partial^2(\bullet)}{\partial Y^2} \tag{22}$$

where ∇^2 is the Laplace operator. Thus, Eq. (21) takes the following simple form

$$\nabla^2 \nabla^2 (W_{XY}) = \Omega^2 W \tag{23}$$

Consider the following differential operators before discretizing the governing differential equations

$$\Re = \frac{\partial^2 W}{\partial X^2} \quad \text{and} \quad S = \frac{\partial^2 W}{\partial Y^2} \tag{24}$$

Thus, the fourth-order derivatives can be given in terms of the second order derivatives, that is

$$\frac{\partial^4 W}{\partial X^4} = \frac{\partial^2}{\partial X^2} \Re \tag{25}$$

$$\frac{\partial^4 W}{\partial Y^4} = \frac{\partial^2}{\partial Y^2} S \tag{26}$$

$$\frac{\partial^4 W}{\partial X^2 \partial Y^2} = \frac{\partial^2}{\partial X^2}\left[\frac{\partial^4 w}{\partial Y^2}\right] = \frac{\partial^2}{\partial X^2} S \tag{27}$$

After the transformation process, the following form can be given for the first-, second, and the fourth-order derivatives, respectively

$$\frac{\partial W}{\partial X} = [J_{11}]^{-1} \frac{\partial W}{\partial \xi} \tag{28}$$

$$\frac{\partial W}{\partial Y} = [J_{11}]^{-1} \frac{\partial W}{\partial \eta} \tag{29}$$

$$\frac{\partial^2 W}{\partial X^2} = [J_{22}]^{-1} \frac{\partial^2 W}{\partial \xi^2} - [J_{22}]^{-1} [J_{21}] [J_{11}]^{-1} \frac{\partial W}{\partial \xi} \tag{30}$$

$$\frac{\partial^2 W}{\partial Y^2} = [J_{22}]^{-1} \frac{\partial^2 W}{\partial \eta^2} - [J_{22}]^{-1} [J_{21}] [J_{11}]^{-1} \frac{\partial W}{\partial \eta} \tag{31}$$

$$\frac{\partial^4 W}{\partial X^4} = \frac{\partial^2 \Re}{\partial \xi^2} = [J_{22}]^{-1}\frac{\partial^2 \Re}{\partial \xi^2} - [J_{22}]^{-1}[J_{21}][J_{11}]^{-1}\frac{\partial \Re}{\partial \xi} \tag{32}$$

$$\frac{\partial^4 W}{\partial Y^4} = \frac{\partial^2 S}{\partial \eta^2} = [J_{22}]^{-1}\frac{\partial^2 S}{\partial \eta^2} - [J_{22}]^{-1}[J_{21}][J_{11}]^{-1}\frac{\partial S}{\partial \eta} \tag{33}$$

$$\frac{\partial^4 W}{\partial X^2 \partial Y^2} = \frac{\partial^2 S}{\partial X^2} = [J_{22}]^{-1}\frac{\partial^2 S}{\partial \xi^2} - [J_{22}]^{-1}[J_{21}][J_{11}]^{-1}\frac{\partial S}{\partial \xi} \tag{34}$$

Using the differential operators in Eqs. (25)-(27) the normalized governing equation, i.e., Eq. (21), takes the following form

$$\frac{\partial^2 \Re}{\partial X^2} + 2\lambda^2\frac{\partial^2 S}{\partial X^2} + \lambda^4\frac{\partial^2 S}{\partial Y^2} = \Omega^2 W \tag{35}$$

or

$$\nabla^2(W_{\xi\eta}) = \Omega^2 W \tag{36}$$

Employing the transformation rule, the governing Eq. (35) becomes

$$[J_{22}]^{-1}\frac{\partial^2 \Re}{\partial \xi^2} - [J_{22}]^{-1}[J_{21}][J_{11}]^{-1}\frac{\partial \Re}{\partial \xi} + 2\lambda^2\left([J_{22}]^{-1}\frac{\partial^2 \Re}{\partial \eta^2} - [J_{22}]^{-1}[J_{21}][J_{11}]^{-1}\frac{\partial \Re}{\partial \eta}\right)$$
$$+\lambda^4\left([J_{22}]^{-1}\frac{\partial^2 S}{\partial \eta^2} - [J_{22}]^{-1}[J_{21}][J_{11}]^{-1}\frac{\partial S}{\partial \eta}\right) = \Omega^2 W \tag{37}$$

DSC rules from Eq. (6) in Eq. (37), one obtains the DSC analog of the governing equations as

$$[J_{22}]^{-1}\left[\sum_{k=-M}^{M}\delta^{(2)}_{\Delta,\sigma}(k\Delta\xi)\Re_{kj} + 2\lambda^2\sum_{k=-M}^{M}\delta^{(2)}_{\Delta,\sigma}(k\Delta\eta)\Re_{ik} + \lambda^4\sum_{k=-M}^{M}\delta^{(2)}_{\Delta,\sigma}(k\Delta\eta)S_{ik}\right]$$
$$-[J_{22}]^{-1}[J_{21}][J_{11}]^{-1}\left(\sum_{k=-M}^{M}\delta^{(1)}_{\Delta,\sigma}(k\Delta\xi)\Re_{kj} + 2\lambda^2\sum_{k=-M}^{M}\delta^{(1)}_{\Delta,\sigma}(k\Delta\eta)\Re_{ik} + \lambda^4\sum_{k=-M}^{M}\delta^{(2)}_{\Delta,\sigma}(k\Delta\eta)S_{ik}\right) = \Omega^2 W_{ij} \tag{38}$$

For convenience and simplicity, the following new variables are introduced

$$\Im = (k\Delta\xi)\Re_{kj} + 2\lambda^2(k\Delta\xi)\Re_{ik} + \lambda^4(k\Delta\eta)S_{ik} \tag{39}$$

$$\Xi = (k\Delta\xi)\Re_{kj} + 2\lambda^2(k\Delta\xi)\Re_{ik} + \lambda^4(k\Delta\eta)S_{ik} \tag{40}$$

Such that the governing equations of plate for free vibration can be expressed

$$[J_{22}]^{-1}\left[\sum_{k=-M}^{M}\delta^{(2)}_{\Delta,\sigma}\Im\right] - [J_{22}]^{-1}[J_{21}][J_{11}]^{-1}\left[\sum_{k=-M}^{M}\delta^{(1)}_{\Delta,\sigma}\Xi\right] = \Omega^2 W_{ij} \tag{41}$$

To obtain the discretized form of Eqs. (35)-(36) in its natural coordinate, we apply Eq. (41) to equation below

$$\nabla^4(W_{\xi\eta}) = \nabla^2\nabla^2(W_{\xi\eta}) = \Omega^2 W \tag{42}$$

Substituting Eq. (41) into Eq. (42) the governing equation can now be given by

$$\left([J_{22}]^{-1}\left[\sum_{k=-M}^{M}\delta^{(2)}_{\Delta,\sigma}\Im\right]-[J_{22}]^{-1}[J_{21}][J_{11}]^{-1}\left[\sum_{k=-M}^{M}\delta^{(1)}_{\Delta,\sigma}\Xi\right]\right.$$

$$\left.\times[J_{22}]^{-1}\left[\sum_{k=-M}^{M}\delta^{(2)}_{\Delta,\sigma}\Im\right]-[J_{22}]^{-1}[J_{21}][J_{11}]^{-1}\left[\sum_{k=-M}^{M}\delta^{(1)}_{\Delta,\sigma}\Xi\right]\right)=\Omega^2 W_{ij} \tag{43}$$

Therefore, the governing equation is given by the matrix notation as

$$(\mathbf{D}_\xi^4\otimes\mathbf{I}_\eta+2\lambda^2\mathbf{D}_\xi^2\otimes\mathbf{D}_\eta^2+\lambda^4\mathbf{I}_\xi\otimes\mathbf{D}_\eta^4)\mathbf{W} = \Omega^2\mathbf{W} \tag{44}$$

where $\mathbf{I}_\xi$ and $\mathbf{I}_\eta$ are the $(N_r+1)^2$; $(r = \xi, \eta)$ unit matrix and $\otimes$ denotes the tensorial product.

4.1 Boundary conditions

Two types of boundary conditions, i.e., simply supported (S) and clamped (C) are taken into consideration. The related formulations and their DSC form are given below in detail.

For simply supported edge (S)

$$W = 0, \text{ and } -D\left(\frac{\partial^2 W}{\partial n^2}+v\frac{\partial^2 W}{\partial s^2}\right)=0 \tag{45}$$

For clamped edge (C)

$$W = 0, \quad \frac{\partial W}{\partial n} = 0 \tag{46}$$

where n and s denote the normal and tangential directions of the plate, respectively. It is known that to obtain a unique solution for a differential equation, appropriate boundary conditions must be satisfied. In application of the DSC method, Wei *et al*. (2002a, b) and Zhao *et al*. (2002) proposed a practical method both for the simply supported and the clamped boundary conditions. Recently, a new approach called the iteratively matched boundary method for applications in boundary conditions in the DSC method was also proposed by Zhao *et al*. (2005), and was applied to the free boundary condition of beams. We used the same procedure proposed by Wei *et al*. (2002a, b), Zhao *et al*. (2002) and Zhao and Wei (2002), in this study. For these purposes, consider a uniform grid having the following form (Wei *et al*. 2002c)

$$0 = X_0 < X_1 < \ldots < X_{N_x} = 1 \tag{47}$$

$$0 = Y_0 < Y_1 < \ldots < Y_{N_y} = 1 \tag{48}$$

Consider a column vector $\mathbf{W}$ given as

$$\mathbf{W} = (W_{0,0}, \ldots, W_{0,N}, \ldots, W_{1,0}, \ldots, W_{N,N})^T \tag{49}$$

with $(N_x+1)(N_y+1)$ entries $W_{i,j} = W(X_i, Y_j)$; $(i = 0, 1, \ldots, N_x; j = 0, 1, \ldots, N_y)$. Let us define the $(N_x+1)(N_y+1)$ differentiation matrices $\mathbf{D}_r^n (r = X, Y;\ n = 1, 2, \ldots)$ with their elements given by

$$[\mathbf{D}_x^{(n)}]_{i,j} = \delta_{\sigma,\Delta}^{(n)}(x_i - x_j) \tag{50}$$

$$[\mathbf{D}_y^{(n)}]_{i,j} = \delta_{\sigma,\Delta}^{(n)}(y_i - y_j) \tag{51}$$

where $\delta_{\sigma,\Delta}^{(n)}(r_i - r_j), (r = x, y)$ is a DSC kernel of delta type. For regularized Shannon's delta kernel, the differentiation in Eqs. (50)-(51) can be given by Zhao *et al*. (2002b)

$$[\mathbf{D}_x^{(n)}]_{i,j} = \delta_{\sigma,\Delta}^{(n)}(x_i - x_j) = \left[\left(\frac{d}{dx}\right)^n \delta_{\sigma,\Delta}(x - x_j)\right]_{x = x_i} \tag{52}$$

$$[\mathbf{D}_y^{(n)}]_{i,j} = \delta_{\sigma,\Delta}^{(n)}(y_i - y_j) = \left[\left(\frac{d}{dy}\right)^n \delta_{\sigma,\Delta}(y - y_j)\right]_{y = y_i} \tag{53}$$

At this stage, we consider the following relationship between the inner nodes and the outer nodes on the left boundary

$$W(X_{-i}) - W(X_0) = a_i[W(X_i) - W(X_0)] \tag{54}$$

or

$$W(X_{-i}) - W(X_0) = W(X_0)\left(\sum_{j=0}^{J} a_i X_{-1}\right)[W(X_i) - W(X_0)] \tag{55}$$

After rearrangement, one obtains

$$W(X_{-i}) = a_i W(X_i) + (1 - a_i) W(X_0) \tag{56}$$

where parameter a_i, $(i = 1, 2, \ldots, M)$ are to be determined by the boundary conditions. Thus, the first and second order derivatives of W on the left boundary are approximated by

$$W'(X_0) = \left(\delta_{\sigma,\Delta}^{(1)}(X_i - X_0) - \sum_{j=0}^{J}(1 - a_i)\delta_{\sigma,\Delta}^{(1)}(X_i - X_j)\right) W(X_0) + \sum_{j=0}^{J}(1 - a_i)\delta_{\sigma,\Delta}^{(1)}(X_i - X_j) W(X_i) \tag{57}$$

$$W''(X_0) = \left(\delta_{\sigma,\Delta}^{(2)}(X_i - X_0) - \sum_{j=0}^{J}(1 - a_i)\delta_{\sigma,\Delta}^{(2)}(X_i - X_j)\right) W(X_0) + \sum_{j=0}^{J}(1 + a_i)\delta_{\sigma,\Delta}^{(2)}(X_i - X_j) W(X_i) \tag{58}$$

Similarly, the first and second order derivatives of f on the right boundary (at X_{N-1}) are approximated by

$$W(X_{N-1+i}) - W(X_{N-1}) = a_i[W(X_{N-1-i}) - W(X_{N-1})] \tag{59}$$

or

$$W(X_{N-1+i}) - W(X_{N-1}) = W(X_{N-1-i})\left(\sum_{j=0}^{J} a_i X_{-1}\right)[W(X_i) - W(X_N)] \tag{60}$$

Consequently, we obtain the following relation

$$W(X_{N-1+i}) = a_i W(X_{N-1-i}) + W(X_{N-1})[1-a_i] \tag{61}$$

Hence, the first and second order derivatives of f on the right boundary are given by

$$W'(X_{N-1}) = \left(\delta^{(1)}_{\sigma,\Delta}(X_i - X_{N-1}) - \sum_{j=0}^{J}(1-a_i)\delta^{(1)}_{\sigma,\Delta}(X_i - X_j)\right)W(X_{N-1}) + \sum_{j=0}^{J}(1-a_i)\delta^{(1)}_{\sigma,\Delta}(X_i - X_j)W(X_i) \tag{62}$$

$$W''(X_{N-1}) = \left(\delta^{(2)}_{\sigma,\Delta}(X_i - X_{N-1}) + \sum_{j=0}^{J}(1-a_i)\delta^{(2)}_{\sigma,\Delta}(X_i - X_j)\right)W(X_{N-1}) + \sum_{j=0}^{J}(1+a_i)\delta^{(2)}_{\sigma,\Delta}(X_i - X_j)W(X_i) \tag{63}$$

For simply supported boundary conditions, the related equations are given by

$$W(X_0) = 0, \quad W''(X_0) = 0 \tag{64}$$

As stated by Wei *et al*. (2002b) and Zhao *et al*. (2002) Eqs. (62)-(63) are satisfied by choosing $a_i = -1$ for i = 1, 2,…, M. This is generally called anti-symmetric extension. For clamped edge, similar statements can be given as

$$W(X_0) = 0, \quad W'(X_0) = 0 \tag{65}$$

Also, these equations given by (65) are satisfied by choosing $a_i = 1$ for i = 1, 2,…, M. This is called symmetric extension (Wei *et al*. 2002c). Thus, DSC form of the related boundary conditions can be given as below.

For simply supported edge (S)

$$W_{ij} = 0 \tag{66}$$

$$\begin{aligned} &-\left(\delta^{(2)}_{\sigma,\Delta}(X_i - X_0) + \sum_{j=0}^{J}(1-a_i)\delta^{(2)}_{\sigma,\Delta}(X_i - X_j)\right)W(X_0) + \sum_{j=0}^{J}(1+a_i)\delta^{(2)}_{\sigma,\Delta}(X_i - X_j)W(X_i) \\ &+ \nu\left\{\left(\delta^{2}_{\sigma,\Delta}(Y_i - Y_0) + \sum_{j=0}^{J}(1-a_i)\delta^{2}_{\sigma,\Delta}(Y_i - Y_j)\right)W(Y_0) + \sum_{j=0}^{J}(1+a_i)\delta^{(2)}_{\sigma,\Delta}(Y_i - Y_j)W(Y_i)\right\} = 0 \end{aligned} \tag{67}$$

For clamped edge (C)

$$W_{ij} = 0 \tag{68}$$

$$\left(\delta^{(1)}_{\sigma,\Delta}(X_i - X_{N-1}) - \sum_{j=0}^{J}(1-a_i)\delta^{(1)}_{\sigma,\Delta}(X_i - X_j)\right)W(X_{N-1}) + \sum_{j=0}^{J}(1-a_i)\delta^{(1)}_{\sigma,\Delta}(X_i - X_j)W(X_i) \tag{69}$$

Finally, after the boundary conditions are implemented, the differentiation matrix, in Eqs. (52)-(53) is given as $\mathbf{D}_r^{*n}(r = X, Y;\ n = 1, 2, \ldots)$. Here $\mathbf{D}_r^{*n}$ is a $(N-2)\times(N-2)$ differential matrix and superscript * is introduced to avoid confusion in differential matrix with $\mathbf{D}_r^n$ in Eq. (44). Thus, Eq. (44) is rewritten as

$$(\mathbf{D}_\xi^{*4} \otimes \mathbf{I}\eta + 2\lambda^2 \mathbf{D}_\xi^{*2} \otimes \mathbf{D}_\eta^{*2} + \lambda^4 \mathbf{I}_\xi \otimes \mathbf{D}_\eta^{*4})\mathbf{W} = \Omega^2 \mathbf{W} \tag{70}$$

in which $\mathbf{W}$ is the column vector, that is

$$\mathbf{W} = (W_{1,1}, \ldots, W_{1,N-2}, \ldots, W_{2,1}, \ldots, W_{N-2,N-2})^T \tag{71}$$

5. Numerical results

In order to establish the accuracy and applicability of the described approach, numerical results were computed for a number of plate problems having different geometries for which comparison values were available in the literature (Liew 1993a, b, Liew and Lim 1993, Malik and Bert 2000, Wang 1994). Detailed reviews have been made by Leissa (1977, 1981, 1987). The performance of the presented DSC method is demonstrated through the vibration solution of plates shown in Figs. 2-6 (Bert and Malik 1996). The presented DSC formulation was used for the free vibration of rectangular plates. The results obtained for the rectangular plates were found to be in excellent agreement with those available in the literature (Leissa 1969). However, these results and comparisons are not given here.

The numerical results for free vibration of plates with irregular domains is tabulated (Tables 1-8) and comparison of the present results with exact or other numerical values available in the literature, when possible, is made. In order to simplify the presentation S and C represent simply supported and clamped, respectively. To show computational efficiency, first four frequencies of SSSS trapezoidal plate (Fig. 4) are analyzed. Results obtained by the differential quadrature (DQ)

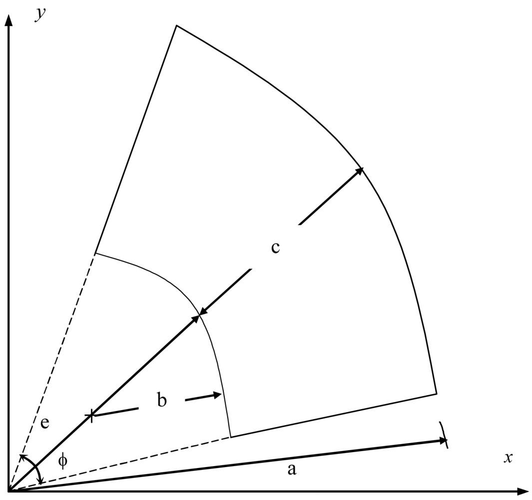

Fig. 2 An eccentric annular sectorial plate

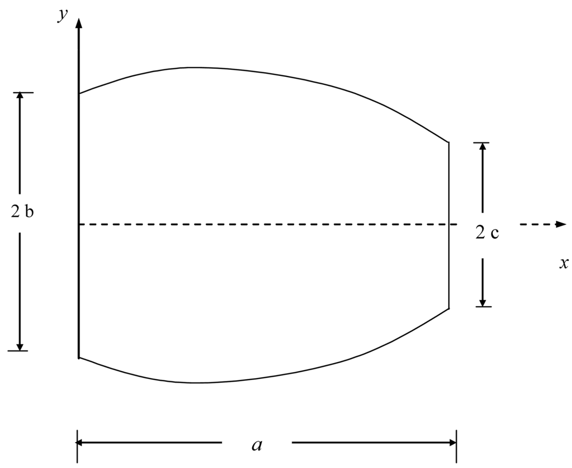

Fig. 3 A symmetric and parabolic trapezoidal plate

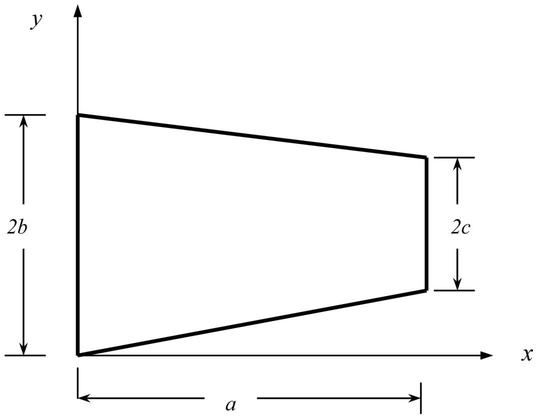

Fig. 4 A symmetric trapezoidal plate

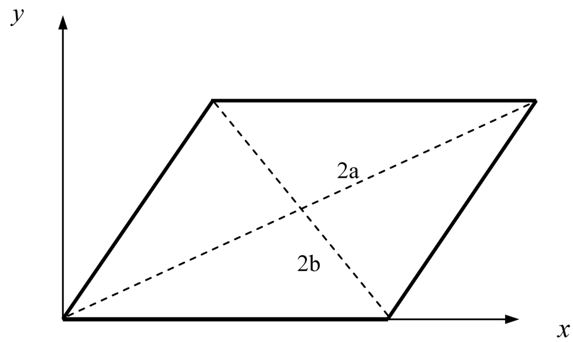

Fig. 5 A typical rhombic plate

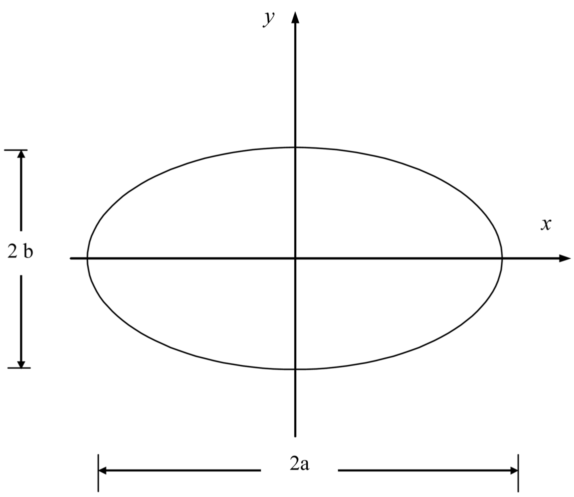

Fig. 6 An elliptic plate

method (Bert and Malik 1996) with N = 25 are given in this table. The analytical results given by Wang and Cheng-Tzu (1994) are also listed in Table 1. It is seen that the present method yields accurate results. In comparison with the results of Bert and Malik (1996), the DSC results using 18 grid points are very accurate. From the results presented in this table, it is clear that the present DSC results are in excellent agreement with those obtained using a variety of numerical methods. Fig. 7 shows the normalized frequency parameters $\Omega_{DSC}/\Omega_{analytical}$ for the first three mode for CCCC rhombic plates (b/a = 1.5). In this figure, $\Omega_{analytical}$ are the analytical solutions taken from Gorman (1988). It is shown in this figure that the DSC results converge to the corresponding analytical solutions when increasing the number of grid points. Reasonable accurate results have been obtained for $N = 18$.

Table 2 presents the numerical results obtained by the DSC method using different grid points for trapezoidal plate having different boundary conditions. It is shown that the solutions converge as the grid number is increased. A reasonably converged solution may be achieved for 18 grids by the

Table 1 Comparisons of first four frequencies ($\Omega = (\omega a^2/\pi^2)\sqrt{\rho h/D}$) of a SSSS trapezoidal plate ($a/b = 3.0$; $b/c = 2.5$)

Grid numbers	Mode			
	1	2	3	4
8×8	5.4524	9.4819	14.9385	16.1008
11×11	5.4213	9.4594	14.7511	16.0043
13×13	5.4067	9.4342	14.7408	15.9505
15×15	5.3904	9.4335	14.7352	15.9401
18×18	5.3897	9.4302	14.7339	15.9283
21×21	5.3897	9.4302	14.7339	15.9282
Wang *et al*. (1994)	5.3906	9.4311	14.727	15.936
Bert and Malik (1996)	5.3890	9.4219	14.680	15.908

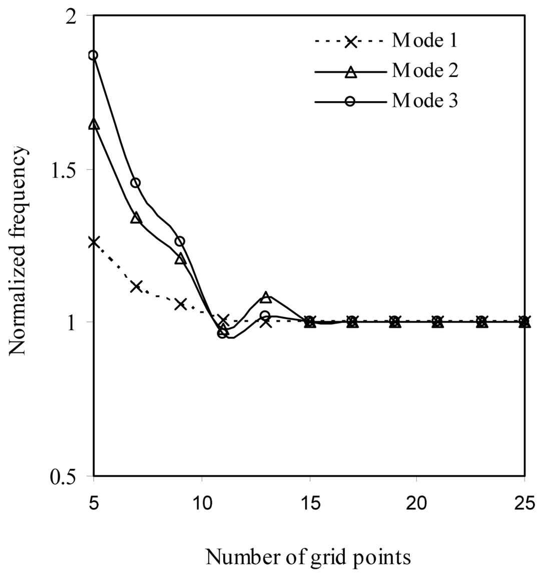

Fig. 7 Variation of normalized frequencies and grid numbers for the first three mode sequence for CCCC rhombic plates (b/a = 1.5)

DSC. Table 3 provides the first four frequencies of CCCC sectorial plate (Fig. 2) for the CCCC boundary condition. The following geometric properties are used for the sectorial plate: $e/b = 0$; $\phi = 30^{\circ}$. Four different a/b ratios for trapezoidal plate are considered. It is shown that as the a/b ratio is increased, the frequencies are found to increase. First two frequencies of an eccentric sectorial plate are listed in Table 4 for three different boundary conditions for the values of e/b = 1.0; a/b = 8/3; ϕ = 45°. The results given by Bert and Malik (1996) are also given in this Table for comparisons. Bert and Malik (1996) presented an eight node transformation based on differential quadrature (DQ) method. The present results agreed well with those values computed by the DQ method (Bert and Malik 1996). First four frequencies of symmetric parabolic (Fig. 3) trapezoidal plate are listed in Table 5. Four different boundary conditions are considered. Results given in this

Table 2 First four frequencies ($\Omega = (\omega a^2/\pi^2)\sqrt{\rho h/D}$) of trapezoidal plate for different boundary conditions ($a/b = 3.0$; $b/c = 2.5$)

CCCC				
$N_\xi = N_\eta$	Mode			
	1	2	3	4
15×15	10.4385	15.6881	21.5473	24.078
18×18	10.4382	15.6739	21.5206	23.910
21×21	10.4382	15.6739	21.5206	23.910
SSSS				
$N_\xi = N_\eta$	Mode			
	1	2	3	4
15×15	5.4129	9.4345	14.7352	15.9401
18×18	5.4018	9.4328	14.7339	15.9386
21×21	5.4017	9.4328	14.7339	15.9382
SCSC				
$N_\xi = N_\eta$	Mode			
	1	2	3	4
15×15	9.4503	14.409	20.005	22.481
18×18	9.4458	14.404	19.906	22.476
21×21	9.4458	14.402	19.905	22.476
CSCS				
$N_\xi = N_\eta$	Mode			
	1	2	3	4
15×15	5.5801	10.002	16.032	17.813
18×18	5.5789	9.9838	15.736	17.307
21×21	5.5784	9.9836	15.730	17.305

Table 3 First four frequencies ($\Omega = (\omega c^2)\sqrt{\rho h/D}$) of a CCCC sectorial plate ($e/b = 0$; $\phi = 30^o$)

a/b	Modes			
	1	2	3	4
1.25	23.848	28.772	37.471	50.329
1.5	29.074	49.411	67.963	81.762
2.0	49.016	85.532	104.409	142.817
4.0	105.923	168.925	234.854	241.682

Table are obtained by setting $b/c = 2.5$; $a/b = 3$. The CCCC trapezoidal plate has the highest frequency parameter Ω followed by the SCSC and CSCS plates. First four frequency parameters of a clamped elliptic plate are given in Table 6. It is observed that a good agreement between the

Table 4 First two frequencies ($\Omega = (\omega a^2/\pi^2)\sqrt{\rho h/D}$) of a eccentric sectorial plate ($e/b = 1.0$; $a/b = 8/3$; $\phi = 45^o$)

Boundary Conditions	Present study		Bert and Malik (1996)	
	1	2	1	2
SSSS	17.596	23.134	17.592	23.130
SCSC	35.371	37.788	35.352	37.794
CCCC	36.368	40.459	36.360	40.452

Table 5 First four frequencies ($\Omega = (\omega a^2/\pi^2)\sqrt{\rho h/D}$) of a symmetric parabolic trapezoidal plates ($b/c = 2.5$; $a/b = 3$) with different boundary conditions

Boundary Conditions	Modes			
	1	2	3	4
SSSS	4.8457	8.6115	13.873	14.546
SCSC	8.5703	12.894	18.158	20.752
CCCC	9.3675	13.982	19.784	21.847
CSCS	5.4843	9.9541	15.438	16.183

Table 6 Comparisons of first four frequencies ($\Omega = (\omega a^2)\sqrt{\rho h/D}$) of a clamped elliptic plate ($a/b = 2.0$)

Method	Mode			
	1	2	3	4
Present DSC results	27.281	39.488	56.042	69.688
Lam *et al*. (1992)	27.477	39.498	55.977	69.856
Bert and Malik (1996)	27.273	39.482	56.029	69.687
Rayleigh-Ritz (Singh and Chakraverty 1992)	27.377	39.497	55.985	69.858

Table 7 First four frequencies ($\Omega = \omega c^2\sqrt{\rho h/D}$) of a sectorial plate ($a/b$ = 2.0; e/b = 0; $\phi = 45^o$)

Boundary Conditions	Mode			
	1	2	3	4
SSSS	68.382	151.056	189.627	278.482
SCSC	107.573	179.104	270.019	306.103

present calculated results and the results of literature (Bert and Malik 1996, Lam *et al*. 1992, Singh and Chakraverty 1992) has been obtained. The frequencies obtained by Lam *et al*. (1992), Singh and Chakraverty (1992) and Bert and Malik (1996) are also provided in the Table. Bert and Malik (1996) have used 25 grid points in their analysis via differential quadrature (DQ) method. From the results presented in this table, it is clear that the present DSC results are in excellent agreement with those obtained using a variety of numerical methods. First four frequencies of a sectorial plate (Fig. 2) are listed in Table 7 for the values of a/b = 2.0; e/b = 0; $\phi = 45^o$. Numerical results are obtained for two different boundary conditions. Table 8 shows the comparison of the first four

Table 8 Comparison of first four frequencies $\Omega = (\omega a^2)\sqrt{\rho h/D}$ of CCCC rhombic plates ($b/a = 1.5$)

Method	Mode			
	1	2	3	4
Present results	12.7035	23.3692	28.2773	34.7418
Analytic results (Gorman 1988)	12.70	23.37	28.25	34.74
Bert and Malik (1996)	12.703	23.369	-	34.738
Shu *et al*. (2000)	12.703	23.369	28.254	34.738

Table 9 First two frequencies ($\Omega = (\omega a^2)\sqrt{\rho h/D}$) of a rhombic plates

Boundary conditions	Mode	b/a		
		3/2	4/2	5/2
SSSS	1	6.9082	5.6839	5.0357
	2	15.2793	11.4638	9.4306
CCCC	1	12.7035	10.5834	9.4701
	2	23.3692	18.0350	15.2142

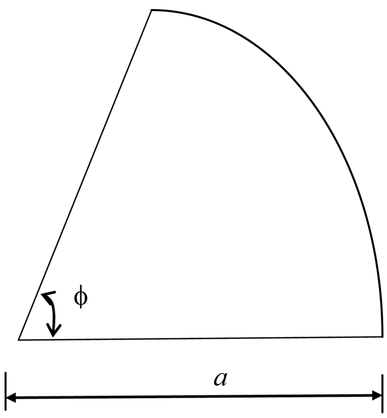

Fig. 8 A typical sectorial plate

frequencies of CCCC rhombic plate. The analytical results given by Gorman (1988) and results given by some numerical approaches (Shu *et al*. 2000, Bert and Malik 1996) are also listed in this table for comparison. It is observed that the results obtained by the DSC method are in good agreement with the analytical and numerical solutions in the literature. First two frequencies of rhombic plates (Fig. 5) are given in Table 9 for different *a/b* ratios and two different boundary conditions. It is shown that the increasing value o*f a/b* ratios always increases the frequency parameter. Another example is the free vibration analysis of sector plate (Fig. 8). Variation of frequency parameter ($\Omega = \omega a^2\sqrt{\rho h/D}$) with mode numbers with different sector angles has been shown in Fig. 9. Table 10 lists the first four frequencies of SSS sector plates. The obtained results are compared with the analytical results given by Huang *et al*. (1993), and with the DQ results given by Liu and Liew (1999). It can be seen that the values calculated by the present method show

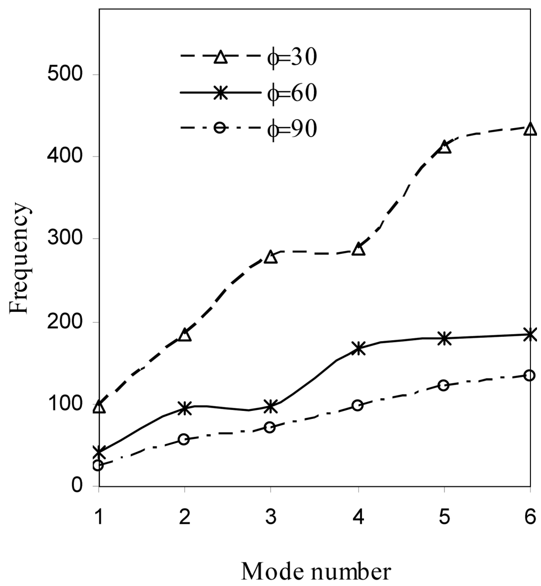

Fig. 9 Variation of frequency parameter of SSS plate with mode number for different sector angle

Table 10 First four frequencies ($\Omega = (\omega a^2)\sqrt{\rho h/D}$) of a SSS sector plates (ϕ = 60)

Method	Mode			
	1	2	3	4
Present results	40.048	94.461	97.782	168.359
Liu and Liew (1999)	39.928	94.383	97.774	168.15
Huang *et al*. (1993)	40.068	94.672	-	168.740

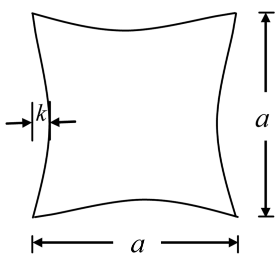

Fig. 10 Geometry of plates with four curved edges

Table 11 Fundamental frequency of a SSSS plate with four curved edges

k/a	Grid Numbers			
	9×9	11×11	15×15	18×18
0.1	21.984	21.738	21.729	21.729
0.2	24.690	23.635	23.441	23.440
0.25	26.721	24.881	24.875	24.873

very good agreement. Finally, the present method is applied to plates with four curved edges shown in Fig. 10. The results for fundamental frequency of SSSS plate with four curved edges are listed in Table 11 for different values of *k/a*. It is shown in this table that it is possible to obtain sufficiently convergent values by use of 15 grid points.

6. Conclusions

In the present study, application of the DSC method, a numerical approach for the free vibration analysis of plates with curvilinear quadrilateral planforms, is presented. By using the geometric transformation, the governing equations and boundary conditions of the plate are transformed from the physical domain into a square computational domain. Solutions of several examples are presented to demonstrate the convergence of the method. Excellent convergence behavior and accuracy in comparison with exact results and/or results obtained by other numerical methods are obtained. Numerical results indicate that the convergence can be achieved by an increase in number of grid points and accurate results can be obtained with 18 grids. It is also important to note that the present method provides a controllable numerical accuracy by using the suitable bandwidth. The implementation of boundary conditions, programming and formulation procedures are found to be straightforward and simple. The required computing time is very small. Although not provided here, the method is also useful in providing vibration solutions of thick rectangular plates. The present study is being further developed to overcome the convergence problems encountered in the curvilinear domains for linear and nonlinear vibration analysis of thick plates. Furthermore, the approach outlined here marks a significant advance in the use of the DSC method for vibration, buckling and static analyses of straight-sided or curvilinear quadrilateral plates.

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