

## Neural networks for inelastic mid-span deflections in continuous composite beams

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**Abstract.** Maximum deflection in a beam is a design criteria and occurs generally at or close to the mid-span. Neural networks have been developed for the continuous composite beams to predict the inelastic mid-span deflections (typically for 20 years, considering cracking, and time effects, i.e., creep and shrinkage, in concrete) from the elastic moments and elastic mid-span deflections (neglecting instantaneous cracking and time effects). The training and testing data for the neural networks is generated using a hybrid analytical-numerical procedure of analysis. The neural networks have been validated for four example beams and the errors are shown to be small. This methodology, of using networks enables a rapid estimation of inelastic mid-span deflections and requires a computational effort almost equal to that required for the simple elastic analysis. The neural networks can be extended for the composite building frames that would result in huge saving in computational time.

**Keywords:** neural network; creep; shrinkage; composite beam; concrete cracking.

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### 1. Introduction

Change in the mid-span elastic deflection in a span of a continuous composite beam (Fig. 1) occurs due to instantaneous cracking in concrete in hogging moment regions, as well as time effects (creep and shrinkage) in concrete. Methods are available in the literature for analysis of the beams, which take into account this change. These methods are based either on incremental or iterative approach. Both the approaches require a computational effort, which is many times more than that required for the elastic analysis (neglecting instantaneous cracking and time effects). The use of neural networks may be made in such cases to rapidly estimate the quantities of design interest for use in everyday design.

Principles and applications of neural networks in civil engineering have been summarized in the works by Flood and Kartam (1994a, b) and Adeli (2001). Hajela and Berke (1991, 1992) have examined the role of computing strategies in structural analysis and design. Some of the typical applications of neural networks in the field of structural engineering include prediction of behavior of

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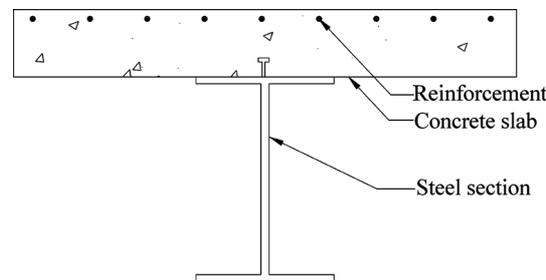


Fig. 1 Composite cross-section

framed shear wall (Mo and Lin 1994) and prestressed concrete frame (Mo and Han 1995), prediction of effect of welding on mechanical behavior of rebars (Mo and Koan 1998), prediction of behavior of concrete confined in hollow bridge columns (Mo *et al.* 2002), prediction of seismic response of prestressed concrete bridges (Jeng and Mo 2004), prediction of time effects in reinforced concrete frames (Maru and Nagpal 2004), modeling of infilled steel frames (Subramnian *et al.* 2005), estimation of hysteretic energy demand in steel moment resisting frames (Akbas 2006), prediction of force reduction factor of prefabricated industrial buildings (Arslan *et al.* 2007), response prediction of geometrically nonlinear truss (Cheng *et al.* 2007), structural damage diagnosis and detections (Tsai and Hsu 2002, Qu *et al.* 2003, Lee *et al.* 2005, Cho *et al.* 2004, Yeung and Smith 2005, Jiang *et al.* 2006, Bakhary *et al.* 2007), prediction of inelastic moments in continuous composite beams considering the concrete cracking (Chaudhary *et al.* 2007a) and prediction of shear lag in composite box beams (Chandak *et al.* 2008). These studies reveal the strength of the neural network in predicting the solutions of different structural engineering problems.

In this paper, neural networks have been developed for estimating the inelastic mid-span deflection,  $D^i$  (considering the instantaneous cracking and time effects in concrete) from the elastic mid-span deflection,  $D^e$  (neglecting the instantaneous cracking and time effects in concrete).  $D^e$ , in turn, can be obtained from any of the readily available software. This methodology of using networks enables rapid estimation of  $D^i$  and requires a computational effort almost equal to that required for the simple elastic analysis. The networks have been validated for four example beams. The errors are shown to be small for practical purposes. The networks can easily be extended for large composite building frames, where a huge saving in computational effort would result.

## 2. Analysis of continuous composite beams

For generalized and efficient neural networks, a huge number of training data sets are required; for the generation of which, a highly efficient analysis procedure is desirable. Recently such a procedure, a hybrid analytical-numerical procedure, for beams and frames, has been developed (Chaudhary *et al.* 2007b, c). The procedure is highly computationally efficient and takes into account the non-linear effects of concrete cracking and time-dependent effects of creep and shrinkage. A cracked span length beam element consisting of an uncracked zone in the middle and cracked zones at the ends (Fig. 2(a)) has been used in the procedure.

The analysis in the hybrid procedure is carried out in two parts. In the first part, an instantaneous analysis is carried out using an iterative method. In the second part, a time-dependent analysis is

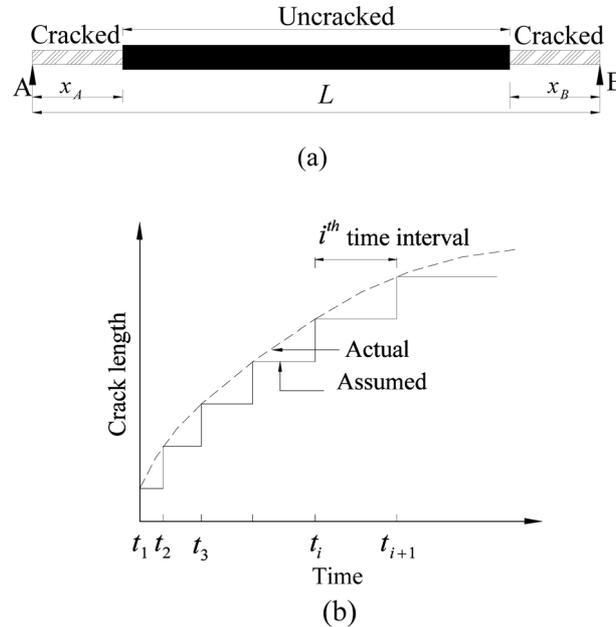


Fig. 2 (a) Cracked span length beam element, (b) progressive nature of cracking

carried out by dividing the time into a number of time intervals to take into account the progressive nature of cracking of concrete (Fig. 2(b)). As shown in the figure, crack lengths are assumed to be constant in a time-interval and revised at the end of each time interval. The age-adjusted effective modulus method, AAEMM (Bazant 1972) is used for predicting the creep and shrinkage effects. CEB-FIP MC 90 (1993) is used for predicting the short term as well as time-dependent properties of the concrete.

### 3. Significant extent of propagation of the effect of cracking

Cracking in continuous composite beams occurs in the end portions (hogging moment regions) of spans at internal supports when subjected to sufficiently high loading. This instantaneous cracking may further progress due to time effects. The mid-span elastic deflections  $D^e$  at the instantaneous state gets changed to  $D^i$  owing to cracking and gets further changed owing to time effects, at the final state (typically 20 years).

The change in the mid-span deflection in spans along the length of a beam due to the instantaneous cracking at a support reduces along the distance from the support. A preliminary numerical study is therefore carried out to estimate the significant extent of the propagation of the effect of the instantaneous cracking at a support, at the instantaneous and the final states. For the study, a typical multi-span (number of spans =  $n$ ) continuous composite beam, shown in Fig. 3(a), is considered. The cross-sectional properties throughout the beam are kept constant unless otherwise stated. The nature of the elastic moment diagram for the beam with equal spans ( $l_1 = l_2 = l_3 = \dots = l_n$ ) and the same load intensities ( $w_1 = w_2 = \dots = w_j = w_{n-1}$ ) is shown in Fig. 3(b) with the maximum moment occurring at the penultimate supports.

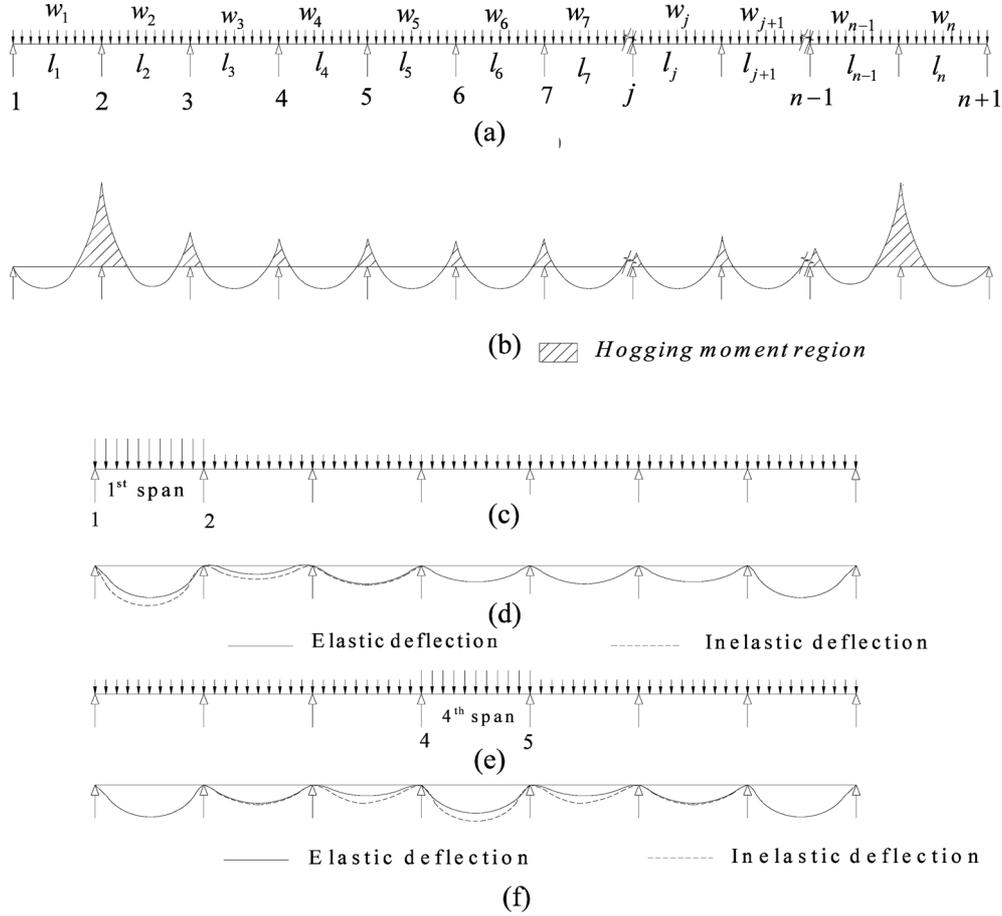


Fig. 3 A typical multi-span continuous composite beam (a) geometry and loading, (b) elastic bending moment diagram, (c) first span with increased loading, (d) elastic and inelastic deflection increased loading, (e) fourth span with increased loading, (f) elastic and inelastic deflection with increased loading

Let the loading be such that the instantaneous cracking occurs at a joint  $j$ . Let the ratio of the cracking moment  $M^c$  to the elastic moment at joint  $j$ , (neglecting cracking)  $M_j^e$  be  $R_j$  (cracking moment ratio); thus a smaller value of  $R_j$ , indicates greater cracking. The change  $(d_k^{ie})_{R_j}$  in mid-span deflection  $D_k^e$  of any span  $k$  at instantaneous or final state depends upon  $R_j$ , and is expressed as

$$\{d_k^{ie}\}_{R_j} = \{D_k^i - D_k^e\}_{R_j} \quad (1)$$

Further the change in  $(d_k^{ie})_{R_j}$ ,  $\{\Delta d_k^{ie}\}_{R_j}$ , from its value at  $R_j = 1$ , is the effect of instantaneous cracking occurring at joint  $j$ . Its normalized value,  $\zeta_{k,j}$  (the value of  $\zeta$ , at the centre of span  $k$  due to cracking at the support  $j$ ) may be taken as a measure of the extent of propagation of the effect of cracking. The span length,  $l_k$  may be taken as the normalizing factor. The expression for  $\zeta_{k,j}$  may therefore be written as

$$\zeta_{k,j} = \{\Delta d_k^{ie}\}_{R_j} / l_k \quad (2)$$

$$\{\Delta d_k^{ie}\}_{R_j} = \{D_k^i - D_k^e\}_{R_j} - \{D_k^i - D_k^e\}_{R_j=1} \quad (3)$$

It may be noted that the second term in Eq. (3) is equal to zero at the instantaneous state.

For the preliminary numerical study, a beam with  $n=7$ ,  $l_k$  ( $k=1$  to  $n$ ) = 8.0 m and  $M^e = 25.5$  kN·m is considered. Two cases are considered to identify the significant extent of the propagation of the effect of the instantaneous cracking on the instantaneous state and the final state. In the first case, the beam is made to crack at the instantaneous state over a penultimate support (support 2,  $j=2$ ) by increasing the load on the first span, whereas in the second case cracking at the instantaneous state is made to occur simultaneously over internal supports (supports 4 and 5,  $j=4, 5$ ) of 4th span, by increasing the load on 4th span. For both the cases, initially the loading on the spans is kept equal to the cracking load,  $w^{cr}$  (the load at which the moment,  $M^e$  at any section of a beam just becomes equal to  $M^e$ ).

Consider the first case. As stated above, the load on the first span (Fig. 3(c)) is increased keeping the loads on other spans constant such that the instantaneous cracking takes place at support 2. The natures of the elastic and the inelastic deflections are shown in Fig. 3(d).  $\zeta_{k,j}$  varies with the cracking moment ratio  $R_2$  (cracking moment ratio at support 2). Seven values of  $R_2$  (1.0, 0.90, 0.75, 0.60, 0.50, 0.40, and 0.25), in the practical range, resulting from the increase in  $w_1$  are considered. Values of  $R_2$  equal to 1.0 and 0.25 indicate initiation of the cracking and the maximum cracking respectively. The variations of  $\zeta_{k,2}$  ( $k=1, 2, 3$ ), at the instantaneous state and at the final state, are shown in Fig. 4(a). It is seen that the nature of variations at the two states are nearly the same. Further, only the variations of  $\zeta_{1,2}$  and  $\zeta_{2,2}$  for the first adjacent spans of the cracked support 2 may be considered to be significant at both the states.

Now, consider the second case in which the instantaneous cracking is made to occur simultaneously at internal supports 4 and 5 of 4th span. For this purpose,  $w_4$  is increased (Fig. 3(e)). The natures of elastic deflection,  $D^e$  and inelastic deflection,  $D^i$  diagrams for this case are shown in Fig. 3(f). Again, seven values of  $R_4$  ( $R_5$  being equal to  $R_4$  for the case considered) are considered. The variations of  $\zeta_{3,4-5}$ ,  $\zeta_{4,4-5}$  and  $\zeta_{5,4-5}$  (for first adjacent spans of the cracked support 4 and 5 due to simultaneous cracking of the two supports) and  $\zeta_{2,4-5}$ ,  $\zeta_{6,4-5}$  (for the second adjacent spans of cracked supports 4 and 5 respectively) are shown in Fig. 4(b). It is again seen that the natures of variations at the two states are nearly the same. Also, only the variations  $\zeta_{3,4-5}$ ,  $\zeta_{4,4-5}$ ,  $\zeta_{5,4-5}$  (for the first adjacent spans of the cracked supports 4 and 5) may be considered to be significant.

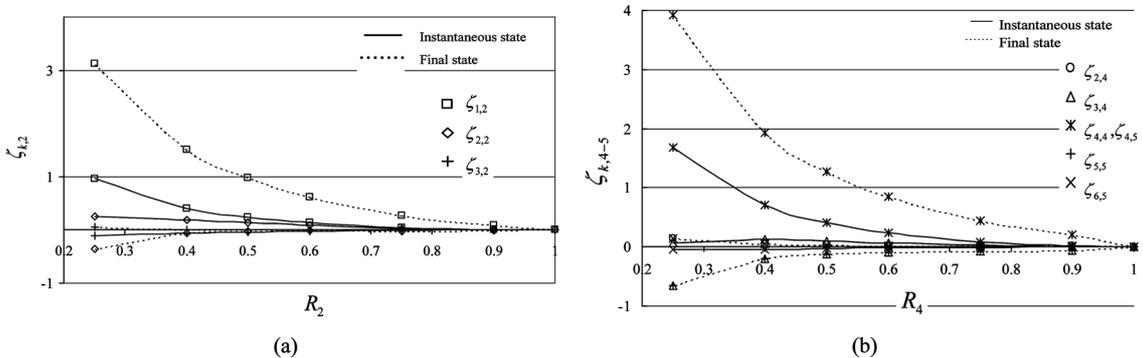


Fig. 4 (a) Variation of  $\zeta_{k,2}$  with  $R_2$  for different spans for cracking at penultimate support (support 2), (b) variation of  $\zeta_{k,4-5}$  with  $R_4$  for different spans for cracking at internal supports (supports 4, 5)

Therefore, it can be stated that, for both the cases, the significant extent of propagation of the effect of instantaneous cracking at a support is limited to the first adjacent spans.

Similar numerical studies are carried out for beams with different  $l_k$ ,  $n$  and  $M^{cr}$ . From these studies also (not reported), it is again observed that the extent of propagation of the effect of instantaneous cracking and time effects is significant for the first adjacent spans of a cracked support.

It therefore follows that in order to establish the change in the mid-span deflection of a span  $j$ , the effect of cracking at the supports  $j$  and  $j+1$  only needs to be considered.

#### 4. Structural parameters

The change in the mid-span deflection of a span  $j$  may be expressed in terms of a ratio designated as inelastic deflection ratio,  $\delta_j \{ \delta_j = (D_j^i - D_j^e) / D_j^f \}$  (where  $D_j^f (= M^{cr} l_j^2 / 32EI)$  is the mid-span deflection of span  $j$  with both ends assumed to be fixed and subjected to uniformly distributed cracking load,  $w^{cr}$ ). This ratio would be the output parameter for the neural networks.

It may be noted that the minimum number of spans in a continuous composite beam in which cracking occurs is two. Further, the seven span beams may represent all the beams in which number of spans is greater than three. For spans greater than three, non linear effect of cracking, creep and shrinkage in internal spans as well as in end spans are similar, for the same structural parameters, irrespective of the number of spans. It may, therefore, be assumed that a beam with seven spans would represent all the beams with more than three spans. Any beam with number of spans greater than three, say four or twelve, can also be chosen; e.g., Chaudhary *et al.* (2007a) used nine span beam to represent all the beams with number of spans greater than three. Therefore, three sets of beams with number of spans equal to two, three and seven may be considered to represent continuous composite beams with any number of spans.

As has been observed in the previous section, cracking at a support  $j$  significantly affects the mid-span deflection of the first adjacent spans (span  $j-1$  and  $j$ ). Accordingly, the probable structural parameters, which may influence cracking at a support  $j$  and thus the inelastic deflection ratios  $\delta_{j-1}$  (left span) and  $\delta_j$  (right span) are listed below:

1. Age of loading,  $t_0$ .
2. Stiffness ratio of adjacent spans,  $S_{j-1}/S_j$  ( $S_j = EI^{tr}/l_j$ , where  $E$  = modulus of elasticity of concrete, and  $I^{tr}$  = transformed moment inertia of composite section about top fiber, the reference axis).
3. Cracking moment ratio at the support,  $R_j$ .
4. Load ratio of the adjacent spans,  $w_{j-1}/w_j$ .
5. Composite inertia ratio,  $I^{cr}/I^{tr}$  ( $I^{cr}$  = transformed moment of inertia of steel section and reinforcement about top fiber, the reference axis).
6. Cracking moment ratio at left adjacent support,  $R_{j-1}$ .
7. Cracking moment ratio at right adjacent support,  $R_{j+1}$ .
8. Grade of concrete,  $Gr$ .

The practical ranges for the different structural parameters are considered as (Chaudhary *et al.* 2007a, Pendharkar 2007):  $t_0 = 7$  days–21 days;  $S_{j-1}/S_j = 0.25$ –4.0;  $R_j = 0.25$ –4.0;  $w_{j-1}/w_j = 0.25$ –4.0;  $I^{cr}/I^{tr} = 0.38$ –0.54;  $R_{j-1} = 0.25$ –4.0;  $R_{j+1} = 0.25$ –4.0;  $Gr = 20$  N/mm<sup>2</sup>–40 N/mm<sup>2</sup>.

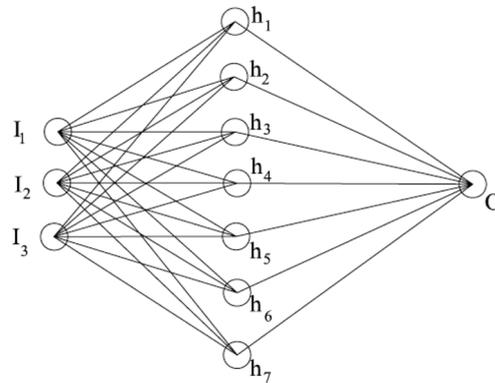


Fig. 5 A typical Neural network model

## 5. Configuration of neural networks

The neural network model chosen in the present study is a multilayered feed-forward network with neurons in all the layers fully connected in feed-forward manner (Fig. 5). Sigmoid function is used as an activation function and the back propagation-learning algorithm is used for training. The back propagation algorithm has been used successfully for many civil engineering applications (Mo and Koan 1998, Mo *et al.* 2002, Maru and Nagpal 2004, Jeng and Mo 2004, Chaudhary *et al.* 2007a, Cheng *et al.* 2007, Chandak *et al.* 2008) and is considered as one of the most efficient algorithm for the engineering applications (Tsai and Hsu 2002).

It has been shown earlier that the cracking at a support affects the change in mid-span deflections of the first adjacent spans. Therefore, the structural parameters which influence the change in the mid-span deflection of a span  $j$  are those which influence the cracking at the supports  $j$  and  $j + 1$ .

For an end span of the two span beam, the input consists of seven parameters,  $t_0$ ,  $S_1/S_2$ ,  $R_2$ ,  $w_1/w_2$ ,  $I^r/I^{un}$ ,  $Gr$  and  $R_j$  (for left end span  $j = 1$ , and for right end span  $j = 3$ , and the output is  $\delta_j$ ). The parameter  $R_{j(=1,3)}$  has been assigned a constant value equal to 10, to model the end supports of the two span beam.

The inelastic deflection ratio,  $\delta_j$ , for an end span and an internal span may be different for both the three span and the seven span beams, therefore independent neural networks are proposed for the end spans and the internal spans of these beams.

The parameters that govern  $\delta_j$  are:  $t_0$ ,  $S_{j-1}/S_j$ ,  $w_{j-1}/w_j$ ,  $R_{j-1}$ ,  $R_j$ ,  $R_{j+1}$ ,  $I^r/I^{un}$  and  $Gr$  owing to cracking at the left support (support  $j$ ) and  $t_0$ ,  $S_j/S_{j+1}$ ,  $w_j/w_{j+1}$ ,  $R_j$ ,  $R_{j+1}$ ,  $R_{j+2}$ ,  $I^r/I^{un}$  and  $Gr$  owing to cracking at the right support (support  $j + 1$ ). Out of these sixteen parameters, five parameters are common i.e.,  $t_0$ ,  $I^r/I^{un}$ ,  $R_j$ ,  $R_{j+1}$  and  $Gr$ . Therefore the eleven parameters for the inelastic deflection ratio of span  $j$  are:  $t_0$ ,  $S_{j-1}/S_j$ ,  $S_j/S_{j+1}$ ,  $w_{j-1}/w_j$ ,  $w_j/w_{j+1}$ ,  $R_{j-1}$ ,  $R_j$ ,  $R_{j+1}$ ,  $R_{j+2}$ ,  $I^r/I^{un}$  and  $Gr$ .

For generality, for both the end spans and the internal spans eleven input parameters are considered. For inelastic deflection ratio of left end span ( $j = 1$ ), the parameters  $S_{j-1}/S_j$ ,  $w_{j-1}/w_j$ ,  $R_{j-1}$  and  $R_j$ , involving left span ( $j - 1$ ) and supports  $j - 1$  and  $j$  have been assigned a constant value equal to 10, since these parameters are fictitious. Similarly for inelastic deflection ratio of right end span ( $j = n$ ), the parameters  $S_j/S_{j+1}$ ,  $w_j/w_{j+1}$ ,  $R_{j+1}$  and  $R_{j+2}$  have been assigned a constant value equal to 10.

## 6. Training of neural network

Since the training of the neural network is an essential step in its performance, a sufficiently large database should be generated for the training and testing. Input data sets have been chosen to cover the entire practical range of parameters and sufficiently large number of values of each of the parameters. The number of data sets chosen is 6750 for the end spans of the two span beam, 33750 for the two end spans of the three span and the seven span beam each, 33750 for the internal span of the three span beam and 1,68,750 for the internal spans of the seven span beam. The variation in number of data sets for each of these cases is due to different number of input parameters and some of the parameters being kept constant e.g.,  $R_3$  for left end span of two span beam (see section 5).

Five neural networks, one each for the end span of the two span beam, the three span beam and the seven span beam and designated as Net-2span-end, Net-3span-end and Net-7span-end respectively and two networks for internal span(s) of the three span and the seven span beams, designated as Net-3span-internal and Net-7span-internal are trained.

In order to bring all the input parameters and output parameters in the range 0.0 to 1.0, the input as well the output data are divided by normalization factors given in Table 1. Since the output parameter can be negative in some cases (e.g., in a span having much larger adjacent spans) the biases 9.7, 10.9, 41.2, 11.7 and 193 are first added to the output parameters, for the networks Net-2span-end, Net-3span-end, Net-3span-internal, Net-7span-end and Net-7span-internal respectively, before applying the normalization factors given in Table 2.

The training is carried out using the Stuttgart Neural Network Simulator (SNNS 1998). For training, several trials with different numbers of neurons in the hidden layer are carried out. Two third of data sets are used for training as training patterns whereas one third of data sets are used for testing. For this partitioning, hold out method (Reich and Barai 1999), in which partitioning is done randomly, has been adopted. The configurations of five optimum networks (number of input parameters-number of neurons in hidden layer-number of output parameters) along with mean square error MSE, square of coefficient of correlation  $R_c^2$ , and number of epochs are given in Table 2. The value of  $R_c^2$  for all the networks is greater than 0.9 for both training and testing data sets which indicates the good generalization capability of network for any new input. Typically, for network NET-2span-end, variation of the mean square error with the epochs is shown in Fig. 6.

Table 1 Normalization factors for parameters

Network	Parameters											
	Input										Output	
	$t_0$	$S_{j-1}/S_j$	$S_j/S_{j+1}$	$w_{j-1}/w_j$	$w_j/w_{j+1}$	$R_{j-1}$	$R_j$	$R_{j+1}$	$R_{j+2}$	$I^r/I^n$	$Gr$	$\delta$
Net-2span-end	22	-	4.05	-	4.05	-	10.5	10.5	-	1	41	19.85
	22	4.05	-	4.05	-	-	10.5	10.5	-	1	41	
Net-3span-end	22	10.5	10.5	10.5	10.5	10.5	10.5	10.5	10.5	1	41	21.00
	22	10.5	10.5	10.5	10.5	10.5	10.5	10.5	10.5	1	41	
Net-3span-internal	22	4.05	4.05	4.05	4.05	10.5	4.05	4.05	10.5	1	41	48.25
Net-7span-end	22	10.1	10.1	10.1	10.1	10.1	10.1	10.1	10.1	1	41	148.5
	22	10.1	10.1	10.1	10.1	10.1	10.1	10.1	10.1	1	41	
Net-7span-internal	22	4.1	4.1	4.1	4.1	10.1	4.1	4.1	10.1	1	41	247.0

Table 2 Configuration of networks, mean square errors, square of coefficient of correlation and number of epochs

Network	Configuration	MSE		$R_c^2$		Epochs
		Training	Testing	Training	Testing	
Net-2span-end	7-12-1	0.00019	0.00297	0.948	0.907	45000
Net-3span-end	11-15-1	0.00043	0.00105	0.955	0.928	45000
Net-3span-internal	11-15-1	0.00042	0.00119	0.941	0.919	45000
Net-7span-end	11-15-1	0.00091	0.00173	0.926	0.911	45000
Net-7span-internal	11-16-1	0.00008	0.00098	0.987	0.961	45000

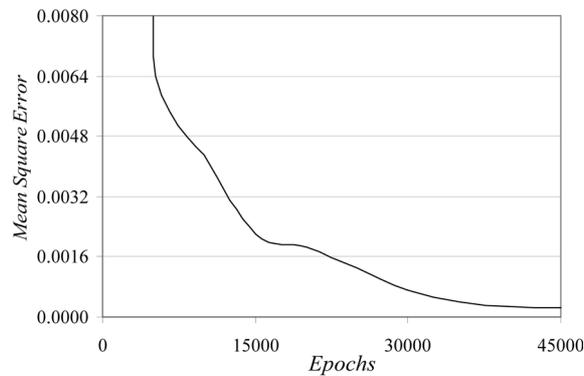


Fig. 6 Variation of mean square error with epochs

### 7. Validation of neural networks

A two span continuous composite validation beam (each span 5.8 m long and subjected to  $w = 6.67$  kN/m), the experimental results for which are available (Gilbert and Bradford 1995), is considered. The network predicts inelastic deflection at centre of spans as 8.17 mm, against 8.00 mm reported by Gilbert and Bradford (1995). The two values are quite close for practical purposes.

Trained neural networks are validated with a number of beams with a wide variation of input parameters. Four example beams, EB1-EB4, are considered (Fig. 7) with  $t_0 = 10$  days,  $M^{cr} = 47.25$  kN·m,  $I^{cr}/I^{un} = 0.4332$  and  $Gr = 32$  N/mm<sup>2</sup>. Inelastic deflections (20 years) for end spans

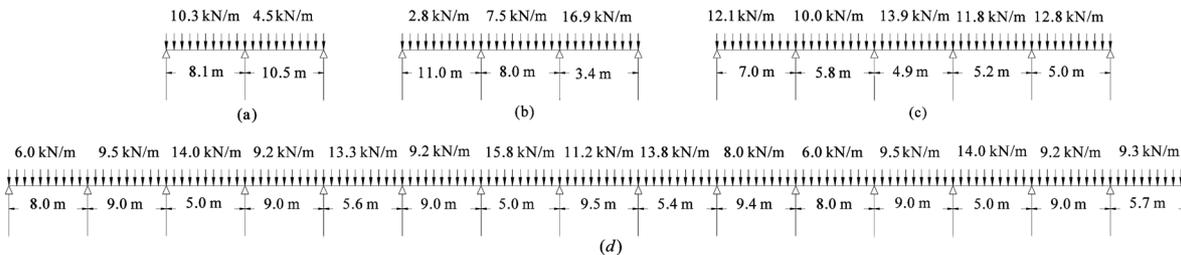


Fig. 7 Example beams (a) EB1, (b) EB2, (c) EB3, and (d) EB4

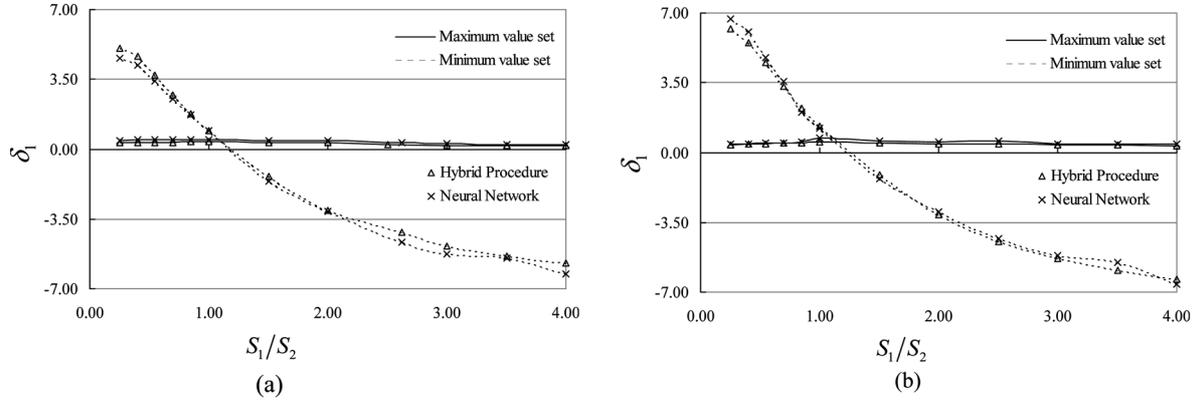


Fig. 8 Variation of  $\delta_1$  with  $S_1/S_2$  for (a) two span beam, and (b) seven span beam

Table 3 Comparison of inelastic deflections obtained from the hybrid procedure and the neural networks

Beam	Span No.	Elastic Deflection (mm)	Longterm inelastic deflections (mm)		Span/Deflection
			Actual	Network	
EB1	1	6.33	10.30	10.96	1263
	2	-0.64	0.37	0.42	-10197
EB2	1	4.14	7.20	7.61	1859
	2	0.13	0.10	0.29	68997
	3	3.31	5.07	5.37	1934
EB3	1	2.08	4.06	4.39	2744
	2	3.77	5.81	6.09	2360
	3	1.09	1.47	1.59	5892
	4	-0.23	-0.64	-0.47	-23542
	5	6.33	11.03	11.47	1248
EB4	1	22.3	34.96	34.41	390
	2	-0.04	2.91	3.97	-246942
	3	11.53	19.12	19.54	815
	4	7.55	17.51	18.59	1246
	5	17.37	30.15	30.96	627
	6	9.32	18.55	19.17	1223
	7	10.44	18.84	19.97	948
	8	8.52	15.63	16.43	985
	9	5.34	5.70	6.02	1574
	10	4.65	5.05	4.92	1701
	11	10.29	14.43	13.59	816
	12	0.21	6.80	6.07	31031
	13	9.87	15.64	14.69	851
	14	5.82	12.30	11.39	1702
	15	22.09	36.07	38.08	470

have been obtained using Net-2span-end for EB1, Net-3span-end for EB2 and Net-7span-end for EB3 and EB4. Inelastic deflections for internal spans have been obtained using Net-2span-internal for EB1, Net-3span-internal for EB2 and Net-7span-internal for EB3 and EB4. Thus as indicated earlier for beams with spans greater than three (EB3 and EB4), the networks Net-7span-end and Net-7span-internal developed for seven span beams have been used.

Inelastic deflections obtained from the hybrid analytical-numerical procedure and the neural networks, along with elastic deflections, are reported in Table 3. The root mean square percentage errors in prediction of the inelastic deflections, on neglecting very high span to deflection ratios (greater than 3000), are 6.41%, 5.69%, 5.93%, and 4.97% for EB1-EB4 respectively. The root mean square percentage error for all the beams, on neglecting very high span to deflection ratios (greater than 3000), is 5.03% which is acceptable for practical design.

In practice, span/deflection ratio is limited to 250 to 350. For spans having a value of span to deflection ratio close to this range, the error is smaller. For example, consider EB4. For span 1 with span to deflection ratio equal to 390, the error is 1.57%, whereas for span 14 with span to deflection ratio 1702, the error is 7.40%.

## 8. Sensitivity analysis

Sensitivity studies are carried out using the developed neural networks and the hybrid analytical numerical procedure and results compared. These studies show the influence of variation of input parameters at a support on the output parameters of the adjacent spans. Also the values of the output parameters obtained from the hybrid procedure and neural networks are compared. The studies are carried out for the adjacent spans of a penultimate end support (support 2) of two span, three span and seven span beams and for the adjacent spans of a typical internal support (support 4) of a seven span beam.

In these studies, only one parameter is varied at a time keeping the other parameters constant, either equal to the minimum or the maximum values of the parameters. These sets are designated as the minimum value set and the maximum value set respectively. It may be further noted that since the cross section of beams is the same throughout the length, the required stiffness ratios  $S_{j-1}/S_j$  for the studies are achieved by varying the length of spans.

### 8.1 Penultimate support

Sensitivity analysis in detail is reported here for the left span (span 1) of a penultimate end support (support 2) of two span ( $n=2$ ) and seven span beams ( $n=7$ ) for two of the input parameters namely,  $S_1/S_2$  and  $R_2$ .

The variation of  $\delta_1$  with the probable structural parameters of two span and seven span beams is studied. The lengths,  $l_1(j=2$  to  $n)$ , are taken as 8.0 m. The span length,  $l_1$  is varied to achieve the specific stiffness ratios. The cracking moment ratio,  $R_1$  is not considered for both two span and seven span beams as the moment at support 1 is zero. Additionally, for two span beam,  $R_3$  is not considered as again the moment is zero at support 3.

#### 8.1.1 Effect of stiffness ratio, $S_1/S_2$

For the two span beam, the values of the other structural parameters, for the two sets considered,

are: the minimum value set:  $t_0 = 7$  days,  $w_1/w_2 = 0.25$ ,  $R_2 = 0.25$ ,  $I^{cr}/I^{un} = 0.38$ ,  $Gr = 20$  N/mm<sup>2</sup>; and the maximum value set:  $t_0 = 21$  days,  $w_1/w_2 = 4.00$ ,  $R_2 = 4.0$ ,  $I^{cr}/I^{un} = 0.54$ ,  $Gr = 40$  N/mm<sup>2</sup>. The two sets for the seven span beam are: the minimum value set:  $t_0 = 7$  days,  $w_1/w_2 = 0.25$ ,  $R_2 = 0.25$ ,  $I^{cr}/I^{un} = 0.38$ ,  $R_3 = 0.25$ ,  $Gr = 20$  N/mm<sup>2</sup>; and the maximum value set:  $t_0 = 21$  days,  $w_1/w_2 = 4.00$ ,  $R_2 = 4.0$ ,  $I^{cr}/I^{un} = 0.54$ ,  $R_3 = 4.0$ ,  $Gr = 40$  N/mm<sup>2</sup>. It may be noted that,  $R_2 = 0.25$  in the minimum value set, represents maximum cracking; and  $R_2 = 4.0$  in the maximum value set represents absence of cracking. The variations of output parameter  $\delta_1$  for the beams are shown in Figs. 8(a)-(b). It is seen that the values of  $\delta_1$  obtained from the hybrid procedure and neural networks are quite close. Further, for the maximum value set, the variation of  $\delta_1$  with  $S_1/S_2$  is significant.

**8.1.2 Effect of cracking moment ratio,  $R_2$**

For the two span beam, the minimum value set and the maximum value set chosen are:  $t_0 = 7$  days,  $S_1/S_2 = 0.25$ ,  $w_1/w_2 = 0.25$ ,  $I^{cr}/I^{un} = 0.38$ ,  $Gr = 20$  N/mm<sup>2</sup>; and  $t_0 = 21$  days,  $S_1/S_2 = 4.0$ ,  $w_1/w_2 = 4.0$ ,  $I^{cr}/I^{un} = 0.54$ ,  $Gr = 40$  N/mm<sup>2</sup>; respectively. For the seven span beam, the minimum value set and the maximum value set are:  $t_0 = 7$  days,  $S_1/S_2 = 0.25$ ,  $w_1/w_2 = 0.25$ ,  $I^{cr}/I^{un} = 0.38$ ,  $R_3 = 0.25$ ,  $Gr = 20$  N/mm<sup>2</sup>; and  $t_0 = 21$  days,  $S_1/S_2 = 4.0$ ,  $w_1/w_2 = 4.0$ ,  $I^{cr}/I^{un} = 0.54$ ,  $R_3 = 4.0$ ,  $Gr = 40$  N/mm<sup>2</sup> respectively. The variations are shown in Figs. 9(a)-(b). It is again seen that the values of output parameter  $\delta_1$  obtained from the hybrid procedure and neural networks are in close agreement. Again, for the maximum value set, the variation of  $\delta_1$  with  $R_2$  is significant.

Sensitivity studies are also carried out for  $t_0$ ,  $w_1/w_2$ ,  $I^{cr}/I^{un}$ ,  $R_3$  and  $Gr$ . As observed earlier, the values of output parameter  $\delta_1$  obtained from the hybrid procedure and neural networks are in close agreement. Also, again the variation of  $\delta_1$  with these parameters is found to be significant for the maximum value sets. Further, the variation of  $\delta_1$  with each of the three parameters  $t_0$ ,  $I^{cr}/I^{un}$  and  $Gr$  is found to be almost linear.

**8.2 Internal support**

Similar studies, as performed for the sensitivity analysis of parameters at penultimate support (support 2), are carried out for the sensitivity analysis of parameters at a typical internal support (support 4) of a seven span beam, again considering the two extreme value sets of the parameters,  $t_0$ ,  $S_3/S_4$ ,  $w_3/w_4$ ,  $R_3$ ,  $R_4$ ,  $R_5$ ,  $I^{cr}/I^{un}$  and  $Gr$ . It is observed that, for internal support also, the values of output parameters  $\delta_3$  and  $\delta_4$  obtained from the hybrid procedure and neural networks are in close

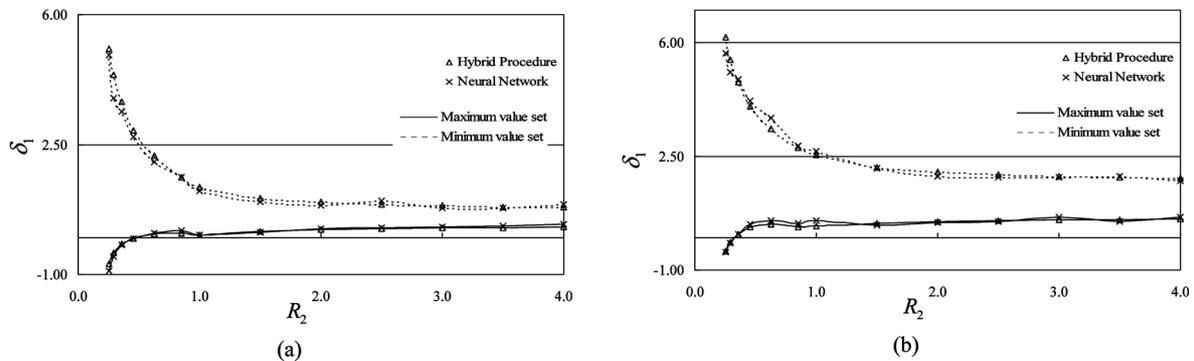


Fig. 9 Variation of  $\delta_1$  with  $R_2$  for (a) two span beam, and (b) seven span beam

agreement. It is also observed that the natures of variations of the output parameter of adjacent spans with the parameters at internal support are similar to the corresponding variations at penultimate support, though the numerical values are different.

## 9. Conclusions

1. Neural networks have been developed for predicting the inelastic mid-span deflections of a continuous composite beam. In development of the networks, use is made of the finding that the cracking of a support influences the mid-span inelastic deflections of first adjacent spans, therefore, the mid-span inelastic deflection of a span due to cracking of concrete and time effects can be obtained with sufficient accuracy if the effect of cracking at the supports of the span, is considered.

2. The neural networks developed with the eleven structural parameters:  $t_0$ ,  $S_{j-1}/S_j$ ,  $S_j/S_{j+1}$ ,  $w_{j-1}/w_j$ ,  $w_j/w_{j+1}$ ,  $R_{j-1}$ ,  $R_j$ ,  $R_{j+1}$ ,  $R_{j+2}$ ,  $I^c/I^m$  and  $Gr$  are shown to yield sufficiently accurate inelastic deflections from the elastic deflections and require a computational effort almost equal to that required for the simple elastic analysis.

3. The networks for the seven span beams, Net-7span-end and Net-7span-internal can be used to predict the mid-span inelastic deflections for beams having any number of spans greater than three.

The development of the neural networks for predicting the long-term inelastic deflections of composite beams is a step towards rapid estimation of long-term inelastic deflections in large composite building frames where a huge computational effort is required.

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**Notations**

$E$	: modulus of elasticity of concrete
$I^r$	: transformed moment of inertia of steel section and reinforcement about top fiber
$I^{un}$	: transformed moment of inertia of composite section about top fiber
$D^e$	: mid-span elastic deflection
$D^f$	: mid-span elastic deflection of fixed beam subjected to cracking load
$D^i$	: mid-span inelastic deflection
$M^{cr}$	: cracking moment
$S_{j-1}/S_j$	: stiffness ratio
$l$	: span length
$n$	: number of spans
$w$	: uniformly distributed load
$w^{cr}$	: cracking load
$w_{j-1}/w_j$	: load ratio
$\delta$	: inelastic deflection ratio
$\zeta$	: normalized variation in mid-span deflection

**Subscript**

$j, k$	: support or span number
$n$	: number of spans