Structural Engineering and Mechanics, Vol. 35, No. 5 (2010) 631-643
DOI: http://dx.doi.org/10.12989/sem.2010.35.5.631

Numerical analysis of second-order effects of externally prestressed concrete beams

*Tiejiong Lou[1] and Yiqiang Xiang[2]

[1]*Department of Structural and Geotechnical Engineering, Politecnico di Torino, Torino 10129, Italy*
[2]*Department of Civil Engineering, Zhejiang University, Hangzhou 310058, P.R. China*

(Received September 29, 2009, Accepted March 3, 2010)

Abstract. A numerical procedure for the geometrical and material nonlinear analysis of concrete beams prestressed with external tendons is described, where the effects of external prestressing are treated as the equivalent loads applied on the concrete beams. The geometrical nonlinearity is considered not only the eccentricity variations of external tendons (second-order effects) but also the large displacement effects of the structure. The numerical method can predict the nonlinear response of externally prestressed concrete beams throughout the entire loading history with considerable accuracy. An evaluation of second-order effects of externally prestressed concrete beams is carried out using the proposed analysis. The analysis shows that the second-order effects have significant influence on the response characteristics of externally prestressed concrete beams. They lead to inferior ultimate load and strength capacities and a lower ultimate stress increase in tendons. Based on the current analysis, it is recommended that, for simply-supported externally prestressed beams with straight horizontal tendons, one deviator at midspan instead of two deviators at one-third span be furnished to minimize these effects.

Keywords: external tendons; prestressed concrete beams; second-order effects; response; numerical analysis.

1. Introduction

In an externally prestressed concrete beam, the prestressing tendons are integrated into the concrete beam through end anchorages and deviator points; thus the tendons between adjacent anchorages and/or deviators remain rectilinear throughout the whole loading history. When the beam deforms under external loads, the eccentricities or effective depths of external tendons except at the points of anchorages and deviators would change, causing what is referred to as second-order effects (Harajli *et al.* 1999). The second-order effects are one of the most important features of externally prestressed concrete beams, since they make the beams have different response characteristics and strength capacities from the ones with internal tendons, hence bringing new issues for the analysis and design of these structures. It is necessary to carry out an in-depth investigation of the second-order effects to well understand this important feature inherent in external tendon systems.

*Corresponding author, Ph. D., E-mail: tiejiong.lou@polito.it

Many experimental researches on the behavior of beams with external prestressing have been carried out, including the recent works by Chen *et al.* (2009) and Ahn *et al*. (2010), and great achievements have already been obtained. However, laboratory experiments have their limitations such as high cost and limited number and size of the specimens. These limitations could be well overcome by the numerical analysis.

Analysis of externally prestresed concrete beams is considerably complicated due to the eccentricity variations of external tendons (second-order effects) and the strain incompatibility between the external tendons and the adjacent concrete. Over the last two decades, a number of models (Alkhairi and Naaman 1993, Ariyawardena and Ghali 2002, Aziz *et al*. 2005, El-Ariss 2004, Ng and Tan 2006, Pisani 2005, Ramos and Aparicio 1996) for nonlinear analysis of externally prestressed concrete beams have been developed, where the geometrical nonlinearity was taken into account by the variations in the tendon eccentricity. For slender beams such the ones used for externally prestressed concrete beams, the nonlinear geometrical effects (large displacement effects) of the structure play also an important role in the member behavior. Dall'Asta *et al*. (1996, 2007) and Zona *et al.* (2008) presented a set of works that include the nonlinear geometrical behavior of the structure in the analysis of beams prestressed with external tendons. So far, however, few of the available models have been used to investigate the behavior of beams prestressed with external tendons.

In this study, a numerical procedure for the geometrical and material nonlinear analysis of concrete beams prestressed with external tendon is described, where the geometrical nonlinearity is considered not only the variations in the eccentricity of external tendons but also the large displacement effects of the structure. A parametric evaluation is then conducted by the proposed analysis to increase the depth of understanding of second-order effects of externally prestressed concrete beams.

2. Numerical procedure

2.1 Finite element formulation

Consider a beam element which are defined in the local coordinate system (x, y), as shown in Fig. 1. Assuming that a plane section remains plane after bending and that the shear deformation is negligible, the axial strain ε at any fibre of a concrete section is defined by

$$\varepsilon = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial v}{\partial x}\right)^2 - y\frac{\partial^2 v}{\partial x^2} \tag{1}$$

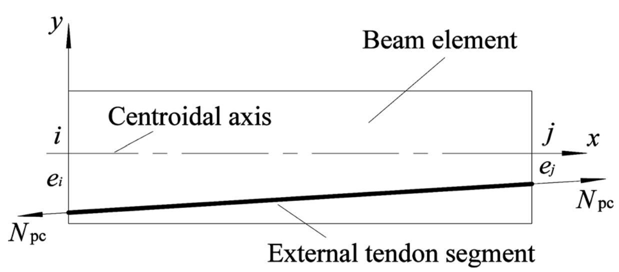


Fig. 1 Contribution of the external tendon segment to the beam element

where u and v are axial and transverse displacements, respectively; and the second term of the right side represents the large displacement effects.

Assuming u is a linear function and v is a cubic polynomial function of x, and applying the principle of virtual work, the following element equilibrium equation can be determined (Lou and Xiang 2006)

$$d\boldsymbol{P}^e = \boldsymbol{K}_T^e d\boldsymbol{u}^e \tag{2}$$

where $\boldsymbol{P}^e$ is the element equivalent nodal loads; $\boldsymbol{u}^e$ is the element nodal displacements; and $\boldsymbol{K}_T^e$ is the element tangent stiffness matrix which, based on the Total Lagrangian description, consists of three components, namely, the material stiffness matrix, the geometrical stiffness matrix and the large displacement stiffness matrix. If the Updated Lagrangian description is used, only two components of the stiffness matrix, the material stiffness matrix and the geometrical stiffness matrix, would be considered.

2.2 Contribution of external prestressing

The external tendon can be considered as an assemblage of a series of tendon segments each of which spans a beam element, as shown in Fig. 1, in which e_i and e_j are the tendon eccentricities at element nodes i and j, respectively. For externally prestressed concrete beams, the eccentricities e_i and e_j change with increasing member deformation, except at anchorages and deviators. The location of each tendon segment determined in terms of the current nodal displacements at anchorages and deviators, together with the nodal displacements of the beam element are used to update the eccentricities e_i and e_j, thus allowing the second-order effects of externally prestressed concrete beams to be considered in the numerical procedure.

For each step, the current tendon strain increment calculated by the elongation ratio of the entire tendon is added to the previous total to obtain the current tendon strain. The current tendon stress is then obtained by using the stress-strain relationship for the prestressing steel. Once the force in external tendons is determined, its contribution to the beam element can be conveniently obtained by converting the tensile force N_{pc} into equivalent nodal loads (Lou and Xiang 2006), as shown in Fig. 1.

2.3 Implementation

The member tangent equilibrium equations are assembled in the global coordinate system, which is fixed in space, from the contributions of all the elements. After imposing appropriate boundary conditions, the entire nonlinear equilibrium path for the structure is traced by an updated normal plane arc-length solution technique (Lam and Morley 1992). This solution algorithm deals well with the changes of response such as cracking, yielding and crushing, and it can easily proceed past the possible limit points during the loading history.

In a numerical analysis, the concrete beam is divided into a number of beam elements interconnected by nodes. The spacing between adjacent nodes relies on several factors such as load geometry and deviator configurations. In general, a spacing of one to two times the beam depth can be used. The cross section of a beam element is subdivided into discrete layers to include varied material properties. The layer number is dependent on the section shape and analysis precision required. Generally, 10, 12 and 16 layers can be respectively used in a typical analysis for rectangular, T and double-T sections.

The nonlinear stress-strain relationships for concrete in compression (Hognestad 1951) and in tension (Kwak and Kim 2002), prestressing steel (Menegotto and Pinto 1973) and ordinary reinforcement (elasto-perfectly plastic behavior) as selected by Lou and Xiang (2006) are adopted here in this study.

The proposed numerical method has been validated by different experimental tests available in the literature. The comparisons between numerical predictions and experimental results of the entire load-deflection response and moment versus stress increase in external tendons for some beam specimens can be seen in Lou and Xiang (2006), and they showed good agreement.

3. Evaluation of second-order effects

Simply-supported prestressed concrete beams with straight horizontal tendons having an initial eccentricity of 200 mm all along the span, as shown in Fig. 2, are used for the analysis to evaluate the second-order effects of externally prestressed beams. Three different deviator configurations are chosen: no deviators, one deviator at midspan and two deviators at one-third span. Besides, a corresponding unbonded internally prestressed concrete beam is also used for comparison. The beams are of a rectangular section with 300 mm wide and 600 mm depth and with tendon area A_p, ordinary tension reinforcement area A_s and ordinary compression reinforcement area A_s', respectively, of 500, 720 and 360 mm^2. The effective depths of ordinary tension and compression reinforcement are equal to 565 and 35 mm, respectively. The material properties are as follows: the concrete cylinder compressive strength and elastic modulus are of 40.0 MPa and 36.3 GPa, respectively; the yield strength and elastic modulus of ordinary reinforcement are of 450 MPa and 200 GPa, respectively; and the ultimate tensile strength and elastic modulus of prestressing steel

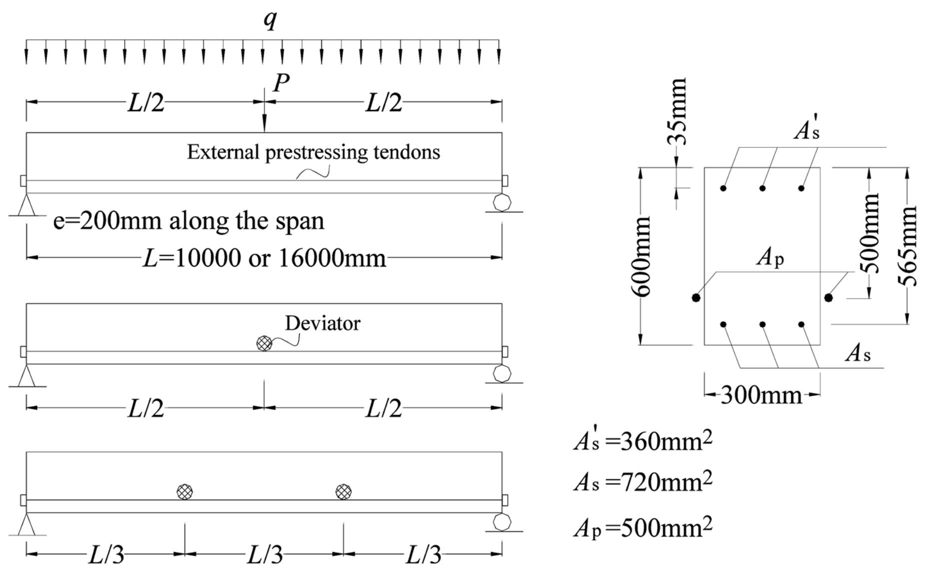


Fig. 2 Details of externally prestressed concrete beams used for second-order effect analysis

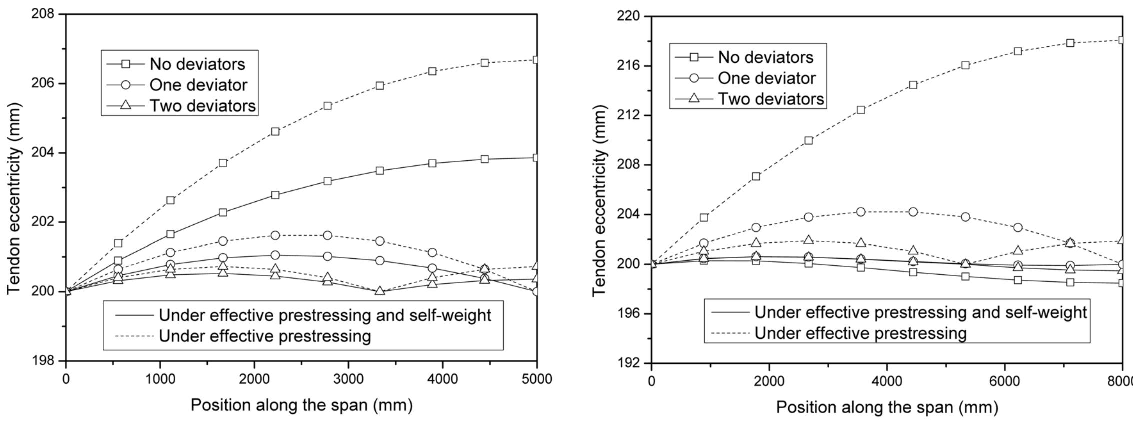
tendons are of 1860 MPa and 195 GPa, respectively. The effective prestress in external tendons is taken as 1120 MPa.

Two levels of span-depth ratio L/d_p (ratio of the span length to the initial tendon depth) are selected. In addition to a usual span-depth ratio of 20, a span-depth ratio of as large as 32 is also selected to illustrate more clearly the significant influence of second-order effects on the flexural behavior of very slender externally prestressed concrete beams and to further demonstrate a reasonable measure for minimizing the second-order effects. Two types of load application, single concentrated load and uniform load, are used in the analysis.

3.1 Eccentricities of external tendons

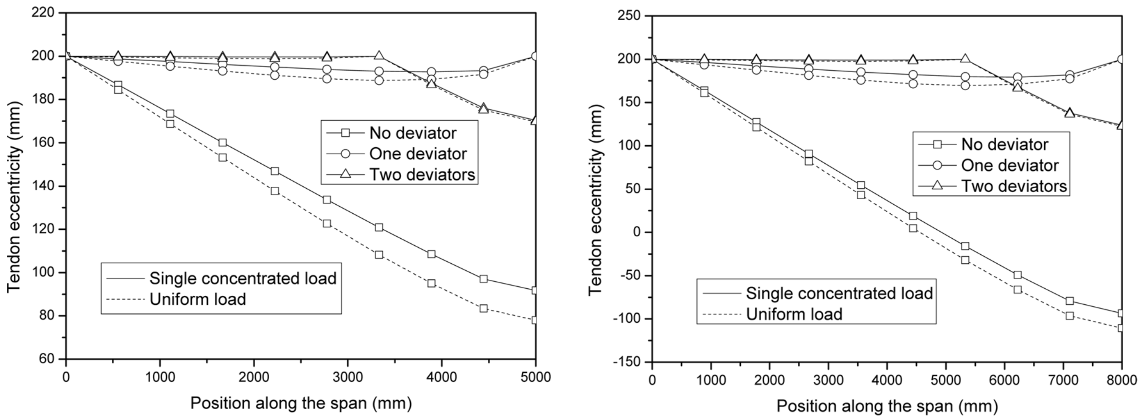
Prior to the analysis, it is necessary to designate a reference state corresponding to the effective prestress f_{pe} in tendons. There are two different reference states frequently used: one is the equilibrium state of the beam under combined effective prestressing and self-weight, and the other is the state of self-equilibrium of the beam under the effects of effective prestressing, not considering the self-weight effects. Since the self-weight is always existent, the first one is more practical and is selected in this study.

The eccentricities of external tendons, at the reference state under effective prestressing and self-weight, along the beam span for span-depth ratios of 20 and 32 are plotted in Figs. 3(a) and (b), respectively (only half span plotted due to symmetry). It is seen from Fig. 3(a) that at the reference state, for a span-depth ratio of 20, eccentricities of the tendons, particularly of the tendons with no deviators, have a certain increase from the initial eccentricity of 200 mm, leading to more pre-compressive stresses in the concrete. Therefore, at this state, the second-order effects would play a positive role on the behavior of externally prestressed concrete beams with a normal span-depth ratio. However, for a span-depth ratio of 32, the effects of self-weight become more important, and may be larger than those of effective prestressing, resulting in a reduction in tendon eccentricities near the critical regions, as shown in Fig. 3(b). For comparison, the variations in tendon eccentricities along the span at another reference state (under effective prestressing) are also plotted



(a) (b)

Fig. 3 Tendon eccentricities along the span at the reference state (a) span-depth ratio of 20, (b) span-depth ratio of 32



(a) (b)

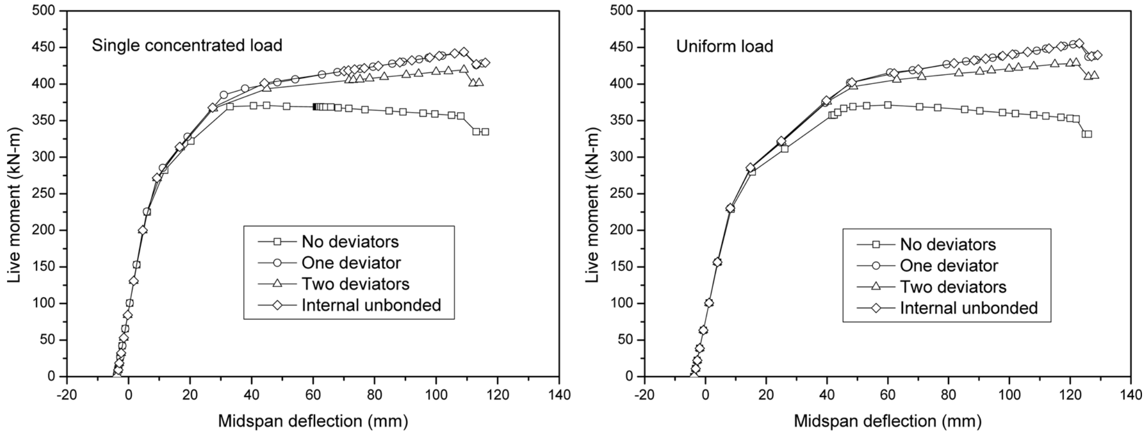
Fig. 4 Tendon eccentricities along the span at the ultimate limit state (a) span-depth ratio of 20, (b) span-depth ratio of 32

in Fig. 3. Due to not considering the effects of self-weight, much larger tendon eccentricities are rendered as compared to the reference state selected in this study.

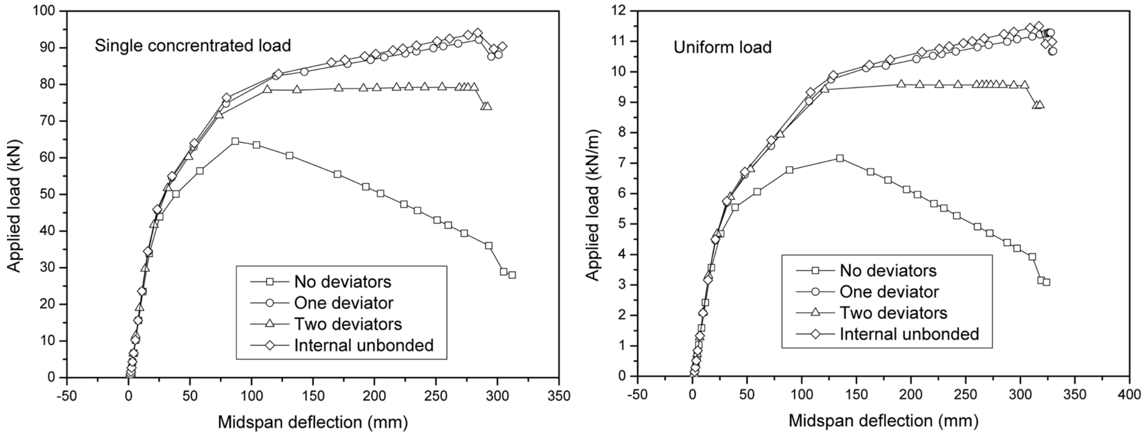
Figs. 4(a) and (b) present the tendon eccentricities at the ultimate limit state for span-depth ratios of 20 and 32, respectively. From Fig. 4, it is seen that, due to a larger ultimate deflection developed, the second-order effects for a uniform load is more serious than that for a single concentrated load. In addition, the figure shows how significant the second-order effects exist in the beams without deviators: for a span-depth ratio of 20, the tendon eccentricities at the midspan section reduce remarkably from an initial eccentricity of 200 mm to 91.7 mm for a single concentrated load and to 78.0 mm for a uniform load, while for a span-depth ratio of 32, the tendon eccentricities are sharply reduced even to negative values of −93.5 mm and −110.6 mm for single concentrated load and uniform load, respectively. It can also be seen in Fig. 4 that providing one or two deviators can effectively diminish the second-order effects. However, the eccentricity variation mode of tendons with one deviator is quite different from that of tendons with two deviators, resulting in different influences of second-order effects for these two deviator configurations. On the one hand, although tendons with one deviator register a larger reduction in eccentricities along most of the span, their eccentricity variations are not serious at the critical regions around the midspan. On the other hand, for tendons with two deviators, although the eccentricity variations can be negligible at the regions between each end support and 1/3 span, the reduction in eccentricities is much more significant around the midspan section. Since the beams are crushed at the midspan section, it may be concluded that, for minimizing the second-order effects of a prestressed concrete beam with external straight horizontal tendons, providing one deviator at midspan is more effective than providing two deviators at one-third span. This conclusion will be further verified by the nonlinear response characteristics of the beams as illustrated below.

3.2 Load (moment)-deflection response

Figs. 5(a) and (b) show the load (moment)-deflection response of externally prestressed concrete beams with span-depth ratios of 20 and 32, respectively, where the live moment refers to the



(a)



(b)

Fig. 5 Load (moment)-deflection response (a) span-depth ratio of 20, (b) span-depth ratio of 32

midspan moment caused by the live load. It can be seen from this figure that prior to the beam collapse followed by a sudden drop of the applied load due to the crushing of the concrete at the critical section, the beams experience almost three straight-stage behaviors with transitions at cracking and yielding points, respectively. Once the beams are loaded to be cracked, the beam stiffness is reduced considerably. Thereafter, the deflection increases rapidly with increasing load up to the yielding of ordinary tension reinforcement. After that, the beam stiffness becomes very weak, accompanying with a sharp increase in deflection until failure.

For a given span-depth ratio and load application, comparing the responses of beams with different deviators, it is seen that the load (moment)-deflection responses for all three deviator configurations are largely identical up to the cracking load. Beyond that, tendons without deviators register larger beam deflection at any load level than do the other two deviator configurations, tendons with one deviator at midspan and with two deviators at one-third span, which exhibit almost identical response characteristics up to the yielding load. After the yielding of ordinary tension reinforcement, negative stiffness occurs in the beam without any deviators, that is, the load (moment) decreases with continuously increasing deflection. This response characteristic is particularly significant for the beam with a span-depth ratio of 32, as can be seen in Fig. 5(b). Also, because two deviators are not placed at the section of the maximum deflection, the beam with two deviators at one-third span mobilizes more significant second-order effects than the beam with one deviator at midspan does.

The corresponding entire load (moment)-deflection responses of unbonded internally prestressed concrete beams are also plotted in the figure for comparisons. They are almost identical to the responses of the externally prestressed concrete beam with one deviator. This indicates that for externally prestressed concrete beams with straight horizontal tendon profile, as long as one deviator is provided at the section of the maximum deflection, the second-order effects of the beams can be negligible even for a span-depth ratio of as large as 32.

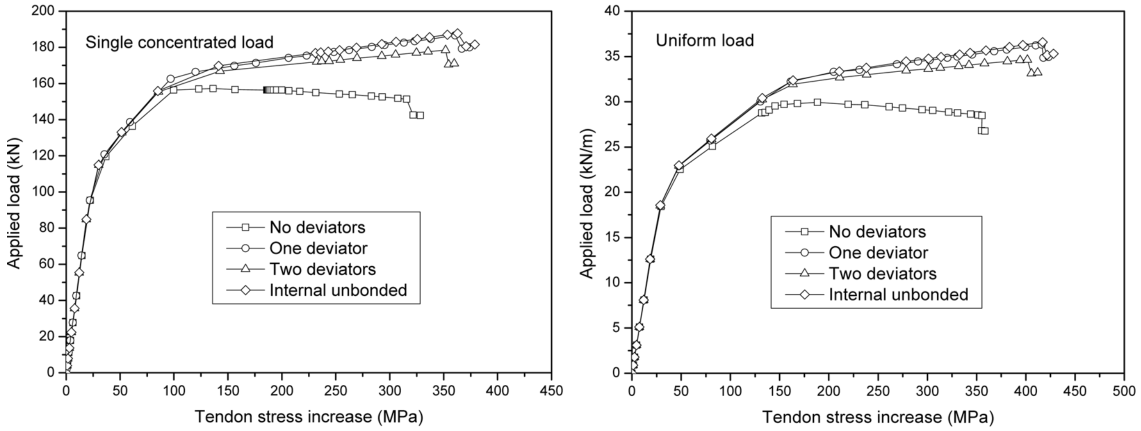
At the ultimate limit state, it can be seen that the second-order effects, associated with the reduction in the tendon eccentricities, seems to have little influence on the ultimate beam deflection, but it leads to much lower ultimate load-carrying capacity of the beams, particularly of the very slender beam. It is suggested that one deviator at midspan instead of two deviators at one-third span, as discussed above, be used to minimize the second-order effects. In this analysis, with provision of a deviator at the critical section, the ultimate single concentrated load and uniform load increase by 23.2% and 27.5%, respectively, for a span-depth ratio of 20, and by 156.1% and 187.8%, respectively, for a span-depth ratio of 32, as compared to the cases of a beam without deviators.

From Fig. 5, it can also be seen that, besides the span-depth parameter, the load type parameter also has an important influence on the deflection development, and hence on the degree of second-order effects. Compared to a single concentrated load, a uniform load registers a larger beam deflection due to a larger length of plastic zone developed at the ultimate limit state, leading to relatively more serious second-order effects if they are not minimized by furnishing one deviator at midspan.

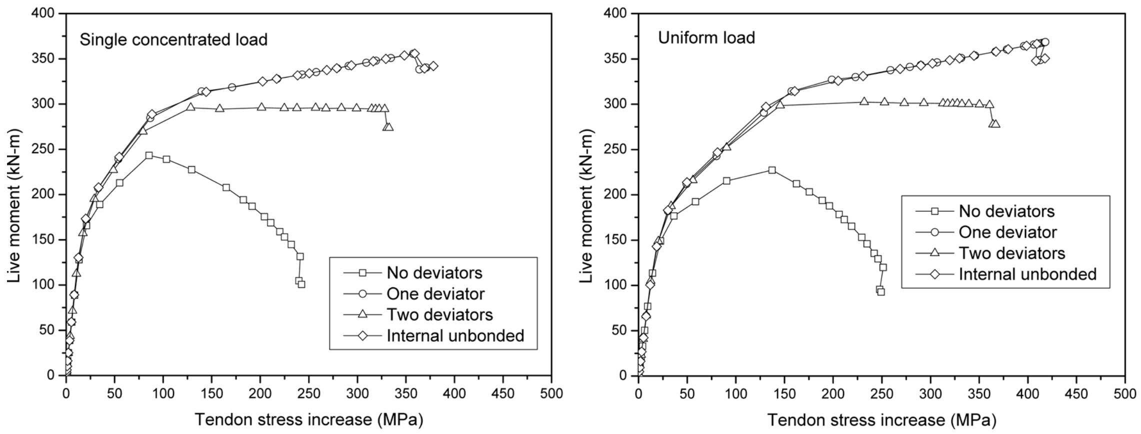
3.3 Response of load (moment) versus stress increase in tendons

Figs. 6(a) and (b) present the response characteristics of stress increase in tendons above the effective stress with increasing load (moment) for externally prestressed concrete beams having span-depth ratios of 20 and 32, respectively.

It is seen in this figure that, similar to the load (moment)-deflection response, the response characteristics of stress increase in tendons also consist of three different stages, attributed to that either concrete cracking or ordinary reinforcement yielding leads to much more stress increase in tendons to resist the applied load (moment). For a given span-depth ratio and load application, prior to cracking, all beams exhibit the same response, indicating at this stage the second-order effects are negligible. After cracking, the second-order effects of the beam without deviators become more and more important, leading to a larger increase in tendons at a given load (moment) as compared to the beams with deviators, of which the second-order effects are still insignificant. When the ordinary tension reinforcement reaches its yield point, the stress in tendons without any deviators rapidly increases with decreasing load (moment) due to the severe second-order effects, which are extremely significant for the very slender beam. Meanwhile, the second-order effects of the beam with two deviators are no longer negligible, resulting in a larger stress increase in tendons in comparison with the beam with one deviator, whose response characteristics of stress increase in



(a)



(b)

Fig. 6 Response of load (moment) versus tendon stress increase (a) span-depth ratio of 20, (b) span-depth ratio of 32

tendons are basically identical to the corresponding beam with internal unbonded tendons.

At the ultimate limit state, it can be seen in the figure that the second-order effects result in a much less ultimate stress increase in external tendons, which, together with a smaller eccentricity in tendons, would produce a much smaller resisting moment. Therefore, the second-order effects cause an inferior ultimate moment capacity of the beams, particularly of the very slender beam as shown in Fig. 6(b). It is also seen that the provision of one deviator at midspan can effectively minimize the second-order effects, even for a span-depth ratio of as large as 32. In this analysis, providing one deviator at the critical section of a beam without deviators, the ultimate live moment and stress increase in tendons induced by the single concentrated load increase by 24.5% and 14.3%, respectively, for a span-depth ratio of 20, and by 170.3% and 48.2%, respectively, for a span-depth ratio of 32; and those induced by the uniform load increase by 29.3% and 17.1%, respectively, for a span-depth ratio of 20, and by 207.3% and 66.4%, respectively, for a span-depth ratio of 32.

From Fig. 6, it can be seen that, because of a larger plastic zone length developed at ultimate, a

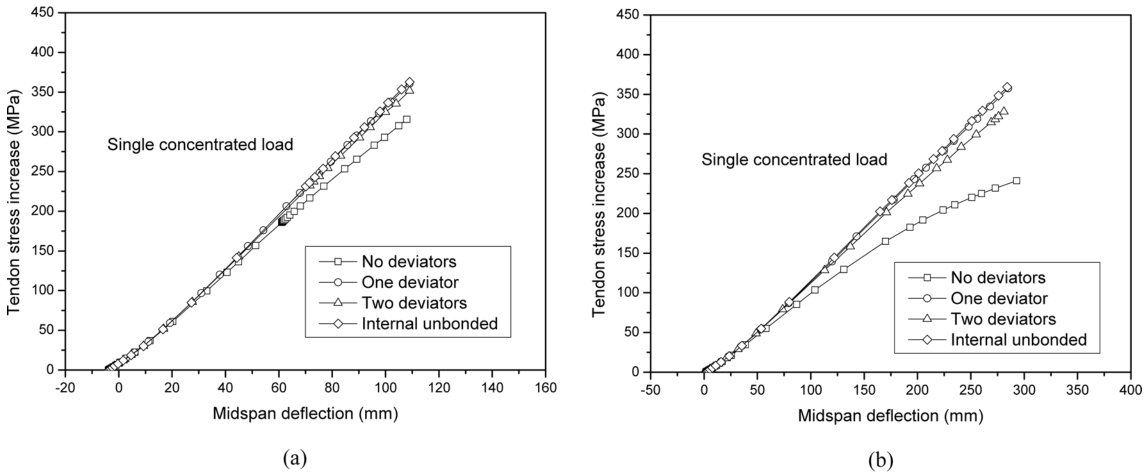


Fig. 7 Relationship curves between tendon stress increase and deflection (a) span-depth ratio of 20, (b) span-depth ratio of 32

uniform load registers larger ultimate tendon stress increase than a single concentrated load does. However, this difference is less obvious for tendons without deviators. This is attributed to that for tendons without any deviators a uniform load results in more significant second-order effects which would reduce more tendon stress. Combining Figs. 6(a) and (b), it is seen that the span-depth ratio parameter hardly influences the ultimate tendon stress increase, provided that the second-order effects are negligible by furnishing one deviator at the section of the maximum deflection. However, if the second-order effects are not effectively minimized, a larger span-depth ratio would mobilize a less ultimate tendon stress increase because of the influence of second-order effects.

3.4 Relationship between deflection and tendon stress increase

The relationship curves between the tendon stress increase and the midspan deflection for span-depth ratios of 20 and 32 are shown in Figs. 7(a) and (b), respectively. It is observed in Fig. 7 that, since the external prestresssing tendons are still at the elastic range at the ultimate limit state, the stress increase in tendons is almost linear with the increase of midspan deflection throughout the whole loading history. It can be also seen from Figs. 7(a) or (b) that, at a given post-cracking deflection, the beam without deviators mobilizes a smaller tendon stress increase in comparison with the beams with deviators due to the influence of second-order effects. Furthermore, after yielding of ordinary reinforcement, the second-order effects of the beam with two deviators at one-third span are no longer negligible as mentioned previously, causing a smaller tendon stress increase as compared to the beam with one deviator at midspan, whose complete response is identical to that of the corresponding unbonded internally prestressed concrete beam.

3.5 Stress in ordinary reinforcement

Figs. 8(a) and (b) show the load-ordinary reinforcement stress response for span-depth ratios of 20 and 32, respectively. It is seen that before the application of the live load, a certain level of precompressive stress exists in the midspan ordinary tension reinforcement, with the magnitude

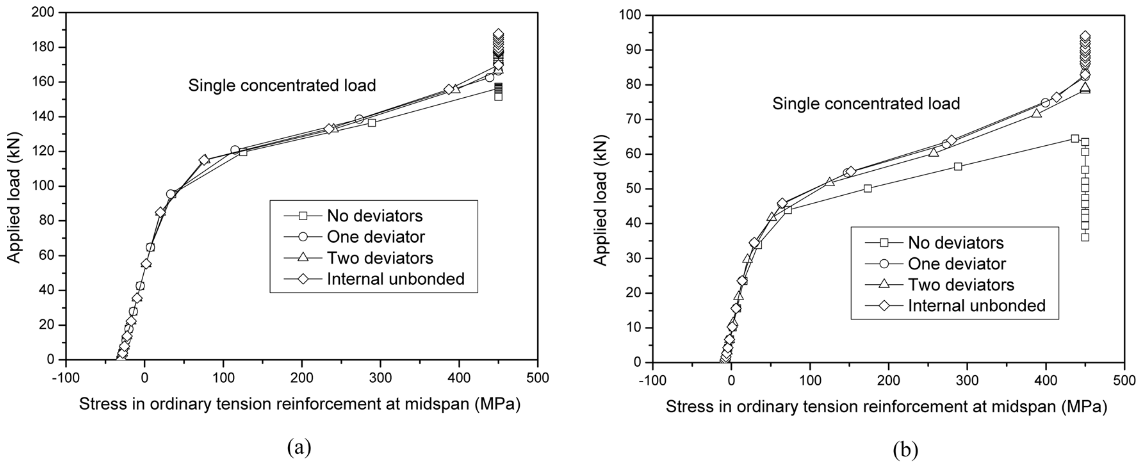


Fig. 8 Response of load versus ordinary reinforcement stress (a) span-depth ratio of 20, (b) span-depth ratio of 32

depending on combined effects of the effective prestressing (producing precompressive stress) and self-weight (producing tensile stress), as well as the variations in tendon eccentricities (second-order effects). The stress in the reinforcement increases linearly with increasing load up to cracking of the concrete and after that, the reinforcement stress increases more quickly up to its yield strength of 450 MPa, and from then on this stress would remain constant since the elasto-perfectly plastic behavior is assumed for the ordinary reinforcement. It is also seen that before cracking, the curves for the beams with different deviators are almost identical, which verify that in the elastic range, the second-order effects are negligible. However, the second-order effects of the beam without deviators become more and more significant after cracking, leading to larger stress in the ordinary reinforcement at a particular level of loading as compared to the other beams.

4. Conclusions

A numerical procedure based on the nonlinear beam flexural theory is described, which is capable of conduct the geometrical and material nonlinear analysis of prestressed concrete beams with external tendons throughout the entire loading history. The geometrical nonlinearity considered in the numerical procedure includes not only the eccentricity variations of external tendons (second-order effects) but also the large displacement effects of the structure. A numerical investigation of second-order effects is conducted using the proposed analysis to increase the understanding of this important feature inherent in the external tendon system.

The second-order effects are the most important characteristic that distinguishes an external tendon system from an internal unbonded one. If the second-order effects are negligible by proper measures, the response of externally prestressed concrete beams is basically identical to that of unbonded internally prestressed concrete beams. The analysis shows that the second-order effects have significant influence on the response characteristics of externally prestressed concrete beams. Under a particular level of loading, these effects lead to a larger beam deflection as well as larger stresses in external tendons and in ordinary reinforcement. Due to these effects, the stiffness of beams without deviators may become negative after the yielding of ordinary tension reinforcement. At failure, the second-order effects lead to much lower ultimate load and moment capacities of the beams and a less ultimate stress increase in tendons, but they seem to have little influence on the ultimate beam deflection. For simply-supported prestressed concrete beams with external straight horizontal tendons as used in this study, response of a beam with one deviator at midspan is nearly identical to that of a corresponding unbonded internally prestressed beam, indicating that the provision of one deviator at the section of the maximum beam deflection can effectively minimize the second-order effects even for a very slender beam.

Acknowledgements

The research has been supported by a grant from the Politecnico di Torino. The first author would like to thank Prof. G. Mancini, Dr. G. Bertagnoli and many other colleagues at the Department of Structural and Geotechnical Engineering of the Politecnico di Torino for their assistance in the research.

References

Ahn, J.H., Jung, C.Y. and Kim, S.H. (2010), “Evaluation on structural behaviors of prestressed composite beams using external prestressing member”, *Struct. Eng. Mech.*, **34**(2), 247-275.

Alkhairi, F.M. and Naaman, A.E. (1993), “Analysis of beams prestressed with unbonded internal or external tendons”, *J. Struct. Eng.-ASCE*, **119**(9), 2680-2700.

Ariyawardena, N. and Ghali, A. (2002), “Prestressing with unbonded internal or external tendons: Analysis and computer model”, *J. Struct. Eng.-ASCE*, **128**(12), 1493-1501.

Aziz, M.A., Abdel-Sayed, G., Ghrib, F., Grace, N.F. and Madugula, M.K.S. (2005), “Analysis of concrete beams prestressed and post-tensioned with externally unbonded carbon fiber reinforced polymer tendons”, *Can. J. Civil Eng.*, **31**, 1138-1151.

Chen, S., Jia, Y. and Wang, X. (2009), “Experimental study of moment redistribution and load carrying capacity of externally prestressed continuous composite beams”, *Struct. Eng. Mech.*, **31**(5), 605-619.

Dall'Asta, A. (1996), “On the coupling between three-dimensional bodies and slipping cables”, *Int. J. Solids Struct.,* **33**(24), 3587-3600.

Dall’Asta, A., Ragni, L. and Zona, A. (2007), “Analytical model for geometric and material nonlinear analysis of externally prestressed beams”, *J. Eng. Mech.-ASCE*, **133**(1), 117-121.

El-Ariss, B. (2004), “Stiffness of reinforced concrete beams with external tendons”, *Eng. Struct.*, **26**, 2047-2051.

Harajli, M., Khairallah, N. and Nassif, H. (1999), “Externally prestressed members: evaluation of second-order effects”, *J. Struct. Eng.-ASCE*, **125**(10), 1151-1161.

Hognestad, E. (1951), “A study of combined bending and axial load in reinforced concrete members”, Bulletin No. 399, University of Illinois Engineering Experiment Station, Urbana, IL, USA.

Kwak, H.G. and Kim, S.P. (2002), “Nonlinear analysis of RC beams based on moment-curvature relation”, *Comput. Struct.*, **80**, 615-628.

Lam, W.F. and Morley, C.T. (1992), “Arc-length method for passing limit points in structural calculation”, *J. Struct. Eng.-ASCE*, **118**(1), 169-185.

Lou, T.J. and Xiang, Y.Q. (2006), “Finite element modeling of concrete beams prestressed with external tendons”, *Eng. Struct.*, **28**(14), 1919-1926.

Menegotto, M. and Pinto, P.E. (1973), “Method of analysis for cyclically loaded reinforced concrete plane frames”, IABSE Preliminary Report for Symposium on Resistance and Ultimate Deformability of Structures Acted on Well-defined Repeated Loads, Lisbon.

Ng, C.K. and Tan, K.H. (2006), "Flexural behaviour of externally prestressed beams. Part I: Analytical model", *Eng. Struct.*, **28**, 609-621.

Pisani, M.A. (2005), "Geometrical nonlinearity and length of external tendons", *J. Bridge Eng.-ASCE*, **10**(3), 302-311.

Ramos, G. and Aparicio, A.C. (1996), "Ultimate analysis of monolithic and segmental externally prestressed concrete bridges", *J. Bridge Eng.-ASCE*, **1**(1), 10-17.

Zona, A., Ragni, L. and Dall'Asta, A. (2008), "Finite element formulation for geometric and material nonlinear analysis of beams prestressed with external slipping tendons", *Finite Elem. Anal. Des.,* **44**, 910-919.