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Optimum design of FRP box-girder bridges

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Abstract. Light weight superstructure is beneficial for bridges in remote areas and in emergency erection. In such weight sensitive applications, combination of fibre reinforced plastics (FRP) as material and box-girders as a structural system have great scope. This combination offers various options to tailor structure and its elements but this flexibility poses greater challenge in optimum design. In this paper a procedure is derived for a generalised optimum design of FRP box-girder bridges, using genetic algorithms (GA). The formulation of the optimum design problem in the form of objective function and constraints is presented. Size, configuration and topology optimization are done simultaneously. A few optimum design studies are carried out to check the performance of the developed procedure and to get trends in the optimum design which will be helpful to the new designers.

Keywords: box-girder; FRP; bridge; optimum design; genetic algorithms.

1. Introduction

Growing problems of maintenance and replacement of bridge infrastructure have led to search for alternate structural materials. In this search, fibre reinforced plastics (FRP) have recently attracted attention of the researchers due to their high specific strength, specific stiffness and durability. The potential for the application of FRP has been particularly high in deployable bridges for the defence, temporary bridges, emergency bridges needed to restore life lines immediately after natural calamities and permanent bridges in remote areas. The reduction in the unit cost of FRP in the post cold-war era and improved understanding of their behaviour through experimental and analytical studies have recently made the material more affordable even in special civil engineering structural systems. The efficiency of FRP as a material in weight sensitive applications, combined with box-girders as efficient structural form, render FRP box-girder bridges attractive in certain bridging applications in infrastructure.

Einde *et al.* (2003) have mentioned that in addition to the potential lower life-cycle cost, FRP decks would be significantly lighter, resulting in large savings in column and foundation cost. FRP deck systems also have a high application potential in areas where longer unsupported deck spans are necessary or where lower weight would translate into lower seismic demand. However, they pointed out that real challenge is to optimise the configuration and use of advanced composite

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materials to match both the performance and cost of traditional decks. Fibre composites, while making it easy to tailor the material according to strength requirements, pose challenging problems in the optimal design of FRP box-girder bridges, due to the flexibility in the design in terms of the size, shape and configuration of lay-ups of fibres in elements and members. Since little prior experience exists in the profession to guide new designs, formal optimum design approaches are essential, in order to efficiently utilise the expensive material in the civil engineering applications. Recent advances in computing hardware and software technologies have made the solutions of such a complex optimum design problem possible. Keller and Gurtler (2005) have described the behaviour of hybrid girders consisting of fiber reinforced polymer bridge decks adhesively connected to steel main girders. They carried out experimental investigations and claimed that the results serve as a basis for the development of a general design method for hybrid FRP/steel bridges. Wight *et al.* (2006) have investigated the behaviour of a short span bridge structural concept with the help of experimental studies. The tests confirmed that the box-beam had sufficient stiffness and strength to function effectively as a single track-way of a small span bridge.

There are a few published and unpublished evidences of use of FRP in bridge engineering. Sotiropoulos *et al.* (1994) have discussed examples wherein FRP is used at component level such as carbon epoxy tensile members in trusses supporting high strength aluminium deck for a military application. They have reported experimental and theoretical investigations on FRP shapes which may be used in bridge systems and illustrated the effectiveness of the analysis and design methods used in conventional structural systems for FRP structures. Also, FRP has been used as tendons for pre- and post-tensioning purposes by Leonard (1990). The Ginzi highway bridge, built in 1982 in Bulgaria, was made of glass fibre reinforced plastic (GFRP) I-beams. Ease of component transportation, comparatively short assembly and erection time, durability and the light weight of the bridge are necessary factors for successful application of FRP components in the bridge members.

Lee *et al.* (1994) have observed that the composite box beams showed no sign of global deterioration and generally behaved as predicted, based on long term experimental study of FRP box-girders. They have recommended highway structures manufactured from composite materials as a viable solution to reduce substantially the high maintenance cost of conventional bridges. Prakash *et al.* (2004) have presented the experimental evaluation of fatigue performance of a FRP bridge deck. The fatigue test served as simple baseline indicator of the long-term durability of the composite deck. The test result showed almost no reduction in stiffness or strength after 2 million cycles of fatigue loading in excess of the design wheel load. Wu *et al.* (2006) have reported durability of FRP composite bridge deck material under freeze thaw and low temperature conditions. They found that freeze thaw cycling between 4.4 and -17.8^{o}C alone and up to 1.25 hour and 625 cycles caused very insignificant change in flexural strength and modulus of the FRP specimens conditioned in dry air, distilled water and salt water. This proves the performance of FRP under environmental effects. Khalifa *et al.* (1996) have described FEM based analysis and design of a FRP cable-stayed pedestrian bridge. Strength, stiffness and buckling were the design criteria. Keller and Gurtler (2006) have evaluated the in-plane compression and shear performance of FRP bridge decks acting as top chord of bridge girders. The influence of cell geometry (trapezoidal or triangular) on the system properties is discussed.

From the brief review it is clear that FRP has great potential for use in bridges, particularly when durability and light weight are important. Box-girders consume less material; possess high flexural and torsional stiffness. FRP on the other hand has high strength to weight and stiffness to weight ratios. Hence, FRP box-girders are ideal for mobile bridges and super structure of bridges in remote areas. Nystrom *et al.* (2003) have studied financial viability of fiber reinforced polymer bridges. They reported that the life-cycle cost of FRP bridge is 39% higher than the life-cycle cost of comparable conventional R. C. bridges. However, there are other significant intangible benefits that have not been considered, such as the increased load carrying capacity due to lower weight of composites, faster replacement and decreased risk of damage due to earthquakes.

The design of FRP box-girder bridges involves large number of design variables compared to isotropic bridges. This combined with the high cost of material and the lack of necessary past experience in their design (Vanderplaats and Weisshaar 1989), make optimum design approach imperative in the development of FRP box-girder bridges. The present work deals with the optimum design of FRP box-girder bridges.

Stiffened panels are extensively utilized in weight sensitive stressed skin structural systems such as light weight bridges, aerospace structures and ship hulls. Many researchers have demonstrated optimum design of FRP panels based on mathematical programming methods (Baburaj and Kalyanaraman 1993, Bushnell 1987, Stroud and Agranoff 1976). In the present work FRP stiffened panels are treated as the basic building block of the FRP box-girder bridge. In the absence of bench mark optimum designs for box-girders the optimization approach for laminated composite structures is already validated by authors by carrying out optimum designs of FRP stiffened panels subjected to in-plane loading (Upadhyay and Kalyanaraman 2000) for which results are available in literature. Box-girders are subjected to transverse loading which lead to longitudinal and transverse flexure, shear, torsion, distortion and shear lag effects. Each of these behavioural aspects is addressed to solve box-girder problem. In this regard, the resulting state of stress in individual panel is different from stiffened panel optimization problem as stress distribution does not remain uniform in transverse as well as in longitudinal direction along with presence of bending.

The design variables in the optimum design of FRP box-girder bridges are of mixed type, involving continuous variables such as overall dimensions, integer variables such as the number of lay-ups in a layer, discrete variables such as the type of stiffener and orientation of fibres in each layer. Consequently, the continuity of the objective function and the constraint surfaces in the design space and the existence of gradients are not guaranteed. Treatment of the fibre orientation as a design variable leads to non-convex design space, frequently trapping optimum design solutions at the number of local minimum points. These factors render the formulation of optimum design of FRP members using the conventional mathematical programming methods inefficient. Enumeration and random search methods may be robust in solving such optimum design problems. However, due to their inefficient convergence towards the optimum solution, large size problems tax heavily, making optimum design in a realistic time frame difficult (Sargent *et al.* 1995).

In this study, genetic algorithms (GA) have been used for optimum design, since they are robust and fairly efficient in solving mixed variable optimisation problem in the design space. Genetic algorithms are computationally simple, but powerful in their search for better solutions (Goldberg 1989). Hajela (1993) has discussed important features of GA. The complexity of the design problem does not pose any great difficulty in seeking optimum solution by this method. Rajeev and Krishnamoorthy (1992) have proved the efficiency of genetic algorithms (GA) in solving discrete design variable optimisation problems, as encountered in trusses and transmission line towers. Adeli and Cheng (1993) have used GA in optimisation of space structures. Sargent *et al.* (1995) have solved the optimum lay-up sequence problem and Punch *et al.* (1995) have solved the problem of optimum design of energy absorbing laminated composite beam, using GA. Upadhyay and Kalyanraman (2000) have done the optimum design of fibre composite stiffened panels. Blade, tee and hat type of stiffeners are considered as design variables along with fibre orientation and various size related variables. Burgueno and Wu (2006) have reported structural optimization of membrane based forms for innovative FRP bridge system. The layered and fibre dominated structure of FRP composites is most efficient when used under in-plane stress demand. They reported shape and laminate stacking sequence optimization of FRP shells independently by using two different approaches of optimization. However, treating all parameters as design variables together may lead to even better and practically feasible designs and the same approach is adopted in present paper.

This paper describes the mathematical formulation of the optimum design of FRP box-girder bridge problem using GA, discusses briefly the details of the program developed for optimum design and presents the results of parametric studies using the software.

2. Optimum design problem formulation

Formulation of an optimum design problem involves transforming verbal description of the problem into a well defined mathematical statement. Such a statement should identify design variables, equations of the constraint surfaces that define the feasible design space and objective function that defines the optimum state of the solution point. These steps are described in the following sections for the problem of the FRP box-girder bridges.

2.1 Design variables

Parameters that define the design space are referred to as design variables. The cross section of a typical single cell box-girder bridge without overhang beyond the web is shown in Fig. 1, along with the serial number of various elements as enumerated in Table 1. The type of each design variable is also listed in Table 1. The design variables are coded and dealt with in binary form in GA. The number of binary digits (Table 1) used to represent each design variable determines the accuracy to which the values can be obtained.

In the box-girder problem, the top flange width in between the webs is assumed to be decided by the functional requirement of the roadway width. Other overall dimensions of the box section are taken as design variables. Transverse stiffeners are provided to improve transverse flexural stiffness of the panels and to improve their buckling strength. The same number of transverse stiffeners in all

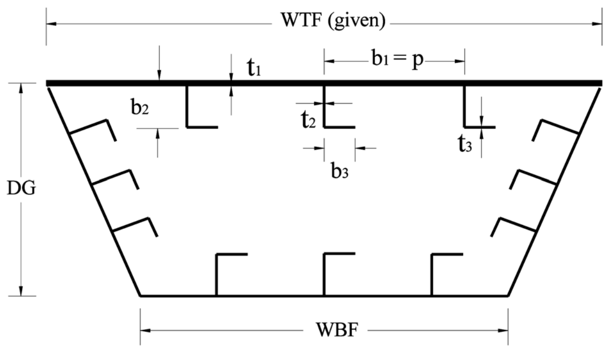

Fig. 1 Design variables of a box-girder section

Table 1 Design variables

Design Variable	Type	Binary digits	Number of variables
Depth of girder	Real	4	1
Width of bottom flange	Real	4	1
No. of transverse stiffeners	Integer	4	1
Dimensions of transverse stiffeners	Real	4	12
Longitudinal stiffener type	Integer	2	3
No. of longitudinal stiffeners	Integer	4	3
Width of the elements	Real	4	3
No. of layers in each element	Integer	2	6
Fibre orientation in each layer	Integer	2	15
No. of laminas in each layer	Integer	4	15

Total number of design variables = 60
Total string length = 192

the panels of the box-girder over the entire length of the bridge was used. Based on a parametric study, only the angle sections, with $\pm 45^{\circ}$ orientation of fibres in the connected leg along with 90% of 0° lay-up and 10% of 90° lay-up in the outstanding leg, were used for the transverse stiffeners. Width and thickness of connected and outstanding legs of the transverse angle stiffeners in each of the three panels contributed 12 design variables to the problem.

The type (blade or angle) and the number of longitudinal stiffeners in the panel width, dictating the stiffener spacing, were also taken as design parameters of the longitudinal stiffeners. All the elements of the cross section of the box-girder were assumed to be made of balanced symmetric lay-ups and hence only half of the elements' thicknesses were included in the design variables. The depth of the connected leg, the width of the out stand (equal to zero in the case of blade stiffener) of the longitudinal stiffeners, the number of layers having different orientation (up to 4 per half thickness), orientation of the fibres in each layer (0°, $\pm 45^{\circ}$, 90°) and the number of lay-ups (lamina) in each layer were the variables that define the design of the longitudinal stiffeners.

2.2 Constraints

Strength, Stability and serviceability considerations contribute to the behavioural constraints in the optimum design of the box-girder bridges. Stability constraints arise from overall panel buckling (buckling of compression flange under combined membrane compression and shear or buckling of webs of the box section under shear and normal stress gradient), referred to as the system level constraint. Local buckling of the skin and other elements of the panels, torsional flexural buckling of stiffeners and strength constraints like material failure of the fibre composite laminate are referred to as the local level constraints. The deflection control was the system level serviceability constraint imposed on the design.

Calculation of the behavioural constraints requires stress and stability analysis of the bridge under different critical load conditions. The longitudinal bending and shear, pure torsion and cross section distortion due to un-symmetric patch loading, shear lag, plate local buckling, stiffener buckling, panel overall buckling are the behavioural aspects that have to be considered in the design of FRP box-girder bridges.

Although finite element method can be used to carry out reasonably accurate analysis for evaluating such constraints satisfactorily, the computational overhead of such a method would be too high in optimum design methods, due to the number of iterative analysis required. The currently available closed form methods of analysis, based on the mechanics of material approaches, are inadequate to deal with all the above behavioural aspects of fibre composite box-girder bridges. In the study (Upadhyay 1997, Upadhyay and Kalyanaraman 2003) that formed the basis for this paper, a computationally efficient procedure for the simplified analysis of FRP box-girder bridges was evolved and validated by comparing the results of the simplified analysis with the finite element analysis results. This simplified analysis procedure was used to evaluate the constraints in the optimum design study.

The in-plane normal and shear stress resultants in each panel of the box-girder due to flexure are obtained by the mechanics of material approach, wherein the panels are treated as equivalent orthotropic plates. Only in-plane shear stresses due to torsion (St. Venant's torsion) of the cross section under eccentric load are considered and warping stresses are disregarded, since they are small compared to flexural and distortional stresses (ASCE-AASHTO Task Committee Report 1971). The local buckling strength of the panels and plate elements under unidirectional compression, in-plane shear and combination of both were evaluated using orthotropic plate buckling equations. The gradient of the longitudinal stress in the panels along the length was considered in the instability analysis.

The cross section of a box section undergoes distortion due to eccentric loading and patch loading. The beam on elastic foundation analogy (Wright *et al.* 1968) along with the orthotropic box frame behaviour was used to account for the distortion effects. The shear lag effect on the panels of the FRP box-girder were considered by modifying the 3 bar method for steel box-girders by Evans and Taherian (1977), to account for the orthotropic nature of FRP panels. Full details of the development of these simplified analysis procedures and their comparison with the finite element analysis results are available in Upadhyay (1997) and Upadhyay and Kalyanaraman (2003).

The stresses in the different layers, along and transverse to the fibre directions, were obtained by the classical laminated plate theory. Different material failure criteria such as the maximum stress theory, maximum strain theory, Tsai-Wu criteria, Tsai-Hill criteria, stress interaction criteria (Upadhyay and Kalyanaraman 2000) are available as alternatives in the design software.

The constraints are expressed in the normalized form as

$$\underline{G}_i(X) = \{1.0 - (g(X)/G(X))\} \geq 0.0 \tag{1}$$

Where $g(X)$ is the actual deflection, stresses, equivalent stresses, stress resultants etc. and $G(X)$ is the limiting deflection, allowable stresses, allowable equivalent stresses, buckling strength, etc.

In addition to the behavioural constraints discussed above, constraints on any of the design variables such as the overall depth restriction and the relationship between design variables, such as the same lay-up being carried from one element to another in order to meet some manufacturing requirements, are the possible side constraints. The side constraints allow practical considerations to be included in the optimum design process.

2.3 Objective function

Due to the high unit cost of FRP, the cost of FRP box-girder is essentially proportional to its mass. Hence, the minimization of mass is relevant from cost reduction point of view. The mass of the bridge per unit span length was taken as the objective function to be minimized in this study.

3. Implementation in genetic algorithms

The genetic algorithms (GA) have philosophical basis on the Darwin's theory of survival of the fittest. In this method, a random search is conducted in those regions of the design space offering the most significant potential for gain (Hajela 1993). A number of design alternatives form the population of a generation. Each design alternative is expressed in the form of a string of binary characters representing the design variables. The set of such strings form the population of a particular generation.

Since the GA can not deal with the constraints of the solution space directly, the violated constraints are augmented to the objective function with a scaling factor, to arrive at the fitness value of a member of the population as given below.

$$F(X) = \{1.0 + \Sigma C_i \underline{G}_i(X)\}\ W(X) \tag{2}$$

In the above expression C_i is a vector of constants and depends on the sensitivity of constraints. $\underline{G}_i(X)$ is considered only in the case of violated constraints and $W(X)$ is the mass index.

It is in the evaluation of the fitness of each member of the population that the domain information comes into play. Thus, the complexity of the mathematical representation of the design domain does not directly affect the GA based optimisation.

Fig. 2 shows the flow chart of the optimum design based on GA. On every generation, genetic operations are carried out to obtain the next improved generation. In the present work initial population was randomly generated within the prescribed limits. Reproduction, cross-over and mutation are the three genetic operators, which were used in the present work. The reproduction operator used a roulette wheel selection process, in which the probability of a member being selected was dictated by the relative fitness of the different members of the population. In this work, two point cross-over is used, wherein the binary strings between two randomly selected cross-over sites in the chosen members of the population are swapped. Mutation operator involves switching (from 0 to 1 or 1 to 0) a randomly chosen binary character in the string and is used to improve the probability of finding the global optimal solution and to avoid early convergence to a local optimum point.

Initially many trial runs were carried out using the software developed, in order to ascertain appropriate values of the various parameters of the genetic algorithms that would facilitate efficient solution of the optimisation problem. Based on this study, the following values were used for the parameters in all the subsequent studies, unless stated otherwise. The number of members in each population = 50, the probability of cross over = 0.80, and the probability of mutation = 0.0018. The roulette wheel selection with areas proportional to the actual fitness of a member of the population was used for the reproduction. Convergence was assumed when 80 % of the design variables in the total population had the same value or the total pre-specified number of generations was completed. More details about the software for the optimum design of FRP box-girders based on GA can be obtained from the thesis of Upadhyay (1997). The complete closed form equations used for stability and strength calculation are given in Upadhyay and Kalyanaraman (2000, 2003).

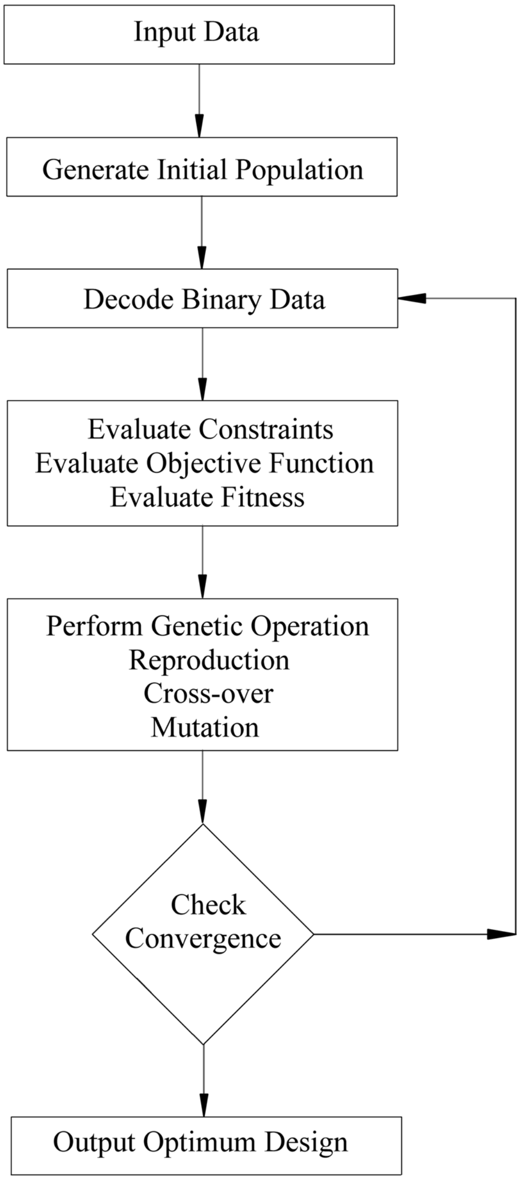

Fig. 2 Genetic algorithm flow chart

4. Parametric design studies

No bench-mark results are available for comparison with the results of FRP bridge optimum designs generated in this study. Hence, the capabilities of the software were evaluated by parametric studies on the design of a single lane, simply supported, single cell box-girder highway bridge subjected to IRC class AA tracked vehicle loading, using carbon fibre reinforced plastics (CFRP). The properties of the material used in the optimum design are given in Table 2. On the basis of

Table 2 Properties of graphite epoxy material

Particulars	Values
Density, ρ (kg/m^3)	1520
Elastic modulus along fibre, E_1 (GPa)	145
Elastic modulus across fibre, E_2 (GPa)	16.5
Shear modulus, G_{12} (GPa)	4.48
Poisson's ratio, ν_{12}	0.314
Poisson's ratio, ν_{21}	0.037
Limiting longitudinal tensile stress, σ_1 (tens.) (MPa)	1240
Limiting longitudinal compressive stress, σ_1 (comp.) (MPa)	−1240
Limiting transverse tensile stress, σ_2 (tens.) (MPa)	55.2
Limiting transverse compressive stress, σ_2 (comp.) (MPa)	−207
Limiting shear stress, τ_{12} (MPa)	82.7

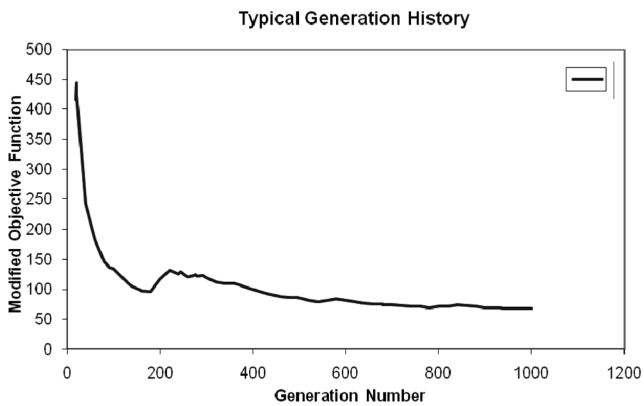

Fig. 3 Typical convergence

guidelines given by Zhang (1993), an overall factor of safety of 3.0 was used for all the modes of failure, in the optimum design studies of the CFRP box-girder.

The analysis was carried out for patch loading corresponding to class AA highway loading of the Indian Road Congress, acting any where on the deck, to obtain the maximum effects. Studies were carried out for the track loads applied concentric as well as eccentric with respect to the centre line of the bridge. The optimum designs were done for obtaining minimum weight at a critical section or the minimum weight of the full bridge. The optimum design studies were carried out without imposing any restriction on the depth as well as by imposing restriction on the depth of the section. Results of these studies are reported in the following sections. Fig. 3 shows a typical convergence observed during optimum designs of the FRP box-girders.

4.1 Optimum design of mid-span section

Optimum design studies were carried out for three simply supported span lengths of 20, 30 and

Table 3 Mass per unit length at mid-span section

Span (m)	Mass (tonne/m)			
	Blade stiffened		Angle stiffened	
	Eccentric load	Concentric load	Eccentric load	Concentric load
20	0.109	0.103	0.103	0.102
30	0.118	0.113	0.113	0.110
40	0.136	0.129	0.125	0.114

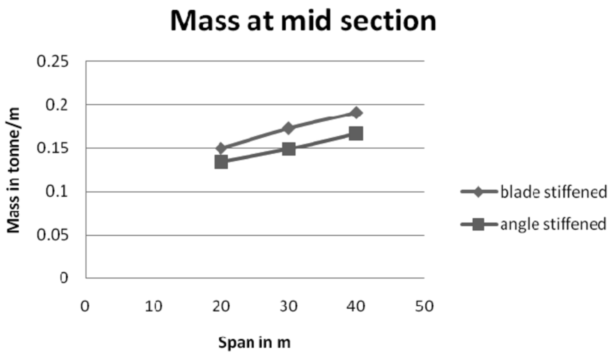

Fig. 4 Mid section mass v/s span

40 m using angles and blades for the longitudinal stiffeners, both for eccentric and concentric loading on the deck. The top flange width was fixed as 3.45 m on the basis of traffic requirements of a deployable single lane bridge and the girder is assumed to be prismatic. Mass per unit length of bridge for different span length, loading eccentricity and type of stiffener, without imposing any depth restriction on the box section, is presented in Table 3. The following observations are made based on these results.

- Unit weight of box section with angle stiffened panels is generally lower than that with blade stiffened panels, particularly as the span length increases.
- Eccentric loading invariably requires heavier section compared to the corresponding concentric loading, again the difference being more in the case of longer spans.
- The unit weight increases at an increasing rate, as the span length increases. The increase is more in the case of sections with blade stiffened panels compared to sections with angle stiffened panels.

The effects of restricting the depth of the girder to be less than 850 mm, which can be fabricated and transported easily, was studied for the case of eccentric loading and the two types of stiffeners. The results presented in Fig. 4 shows that the box sections with the angle stiffened panels are lighter, especially in the case of girders with depth restriction. Further comparison of data presented in Fig. 4 and Table 3 shows that restriction on depth causes nearly 50% increase in the mass of optimally designed girder in the case of girders with blade stiffened panels and about 30% increase in the case of girders with angle stiffened panels.

On the basis of the results of the parametric study on prismatic members, for the case of unrestricted depth and eccentric loading the optimal values of mid-span section dimensional parameters, namely span to mid-span deflection ratio (L/δ), span to mid-span depth ratio (L/D) and ratio of the bottom flange width to the top flange width (W_b/W_t), are presented in Fig. 5. The following are the general observations.

- The span to depth ratio of optimal designs is in the range of 10 to 18. The larger ratios are generally applicable for longer spans and for angle stiffened box sections.
- The ratio of the bottom flange width to the top flange width is in the range of 0.45 to 0.65, the larger values being applicable to longer span lengths.
- The span length to the maximum centre span deflection ratio is in the range of 169 to 383, the larger value being applicable to shorter span lengths. Although such larger deflections may be acceptable in temporary deployable bridges, permanent bridges may require lower deflection than these values, calling for stiffer and hence heavier members.

The top flange of the box-girder sections is subjected to membrane compression in the

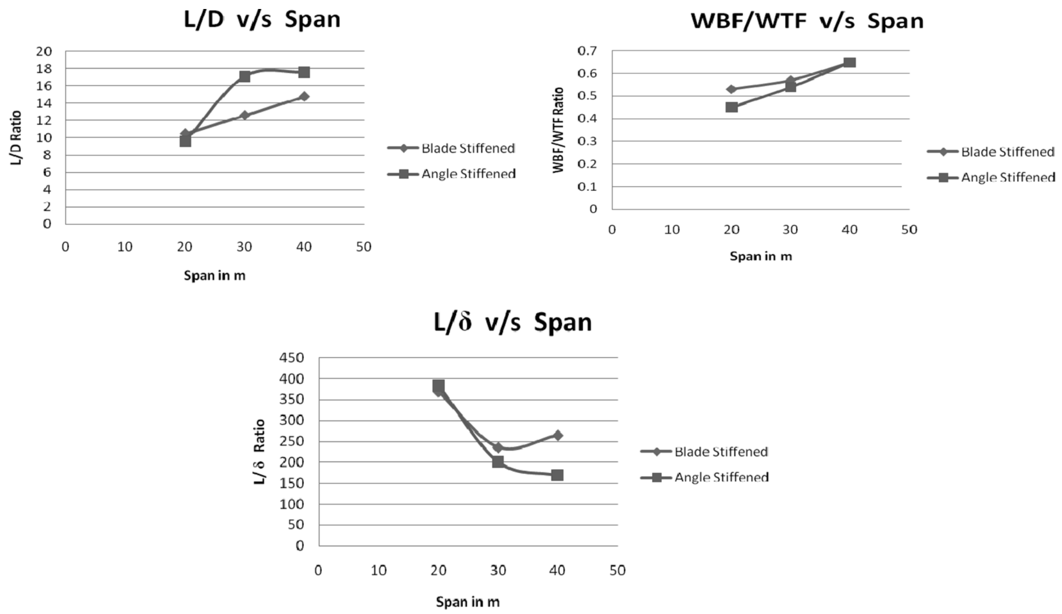

Fig. 5 Optimum value of dimension related parameters

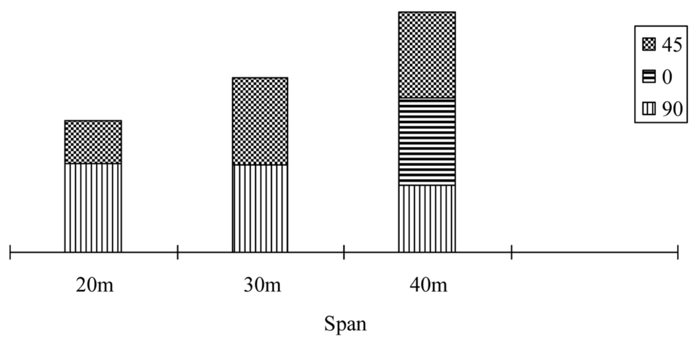

Fig. 6 Percentage, location and orientation of fibres in skin

longitudinal direction due to longitudinal bending moment in the girder, and bending moment in the lateral direction due to patch loading between girder webs. The relative value of stresses due to these two stress resultants increases with increase in the span length. Due to longitudinal compression the skin may undergo local buckling failure or material failure, depending upon the width to thickness ratio. In multi-layered CFRP elements, fibres oriented along the span length (0^o) are more effective in resisting material failure and ±45° layers are more effective in resisting plate buckling. Fig. 6 shows the optimum configuration of the top flange skin obtained during the optimum design studies. The following observations can be made with regard to the fibre orientation in the top flange skin in the optimum design:

- The outermost layers invariably have ±45° lay-up, due to their efficiency in resisting local buckling of the skin. The local buckling strength of skin is usually critical in CFRP box sections.
- The percentage of the 0° fibre increases with increase in the span length, because the higher longitudinal compressive stress encountered in such longer spans is more efficiently resisted by 0^o fibre.
- The percentage of the 90° fibre is higher in shorter span lengths to resist the lateral bending stress efficiently.

4.2 Stiffener type as a design variable

Studies reported so far, required that the type of stiffener in all the panels be the same. Studies were also carried out by allowing the type of stiffener in each panel to be different and to be chosen by the optimization method. The results are presented in Table 4 for mid-span section optimum design, for the case of no depth restriction and eccentric loading. Following observations are based on these results:

- Minimum weight is obtained when the optimisation method is allowed to choose the type of stiffener in each panel independently. The weight saving can be as high as 10% in the case of long span girders.
- The optimum design opts for angle stiffeners in the top (compression) flange which is subjected to membrane compression and is critical with respect to buckling. The blade stiffeners are economical in the webs and the bottom flange and are also chosen by the optimum design procedure.

4.3 Effect of optimising the full length girder

All the optimal designs discussed so far, dealt with optimum design of a section for the maximum design forces. In practice, the design forces would change from section to section and consequently

Table 4 Stiffener type as a design variable

Span (m)	Total mass (tonne/m)			Optimum stiffener type		
	Blade	Angle	Variable	Top	Web	Bottom
20	0.109	0.103	0.099	Angle	Blade	Blade
30	0.118	0.113	0.103	Angle	Blade	Blade
40	0.136	0.125	0.114	Angle	Blade	Blade

Table 5 Mass of box-girder from multiple section design

Span (m)	Total girder mass (tonne)				
	Case I	Case II	Case III	Case IVa	Case IVb
20	2.18	2.62	2.53	2.79	2.56
30	3.54	4.10	3.87	4.01	3.62
40	5.44	5.72	5.53	5.74	5.51

optimum design of different sections of a girder separately as above, may lead to designs which may be difficult to integrate along the span length and hence impractical to manufacture.

It would be desirable to optimally design simultaneously multiple sections of the box-girder for the envelop of design forces acting on the sections due to various loading conditions, simultaneously satisfying some side constraints imposed to obtain practically feasible optimum design. Such side constraints could call for a linear or parabolic variation of the overall dimensions, such as the depth and the bottom flange width between specified sections, or require the slope of the webs to be kept constant as the depth of the girder changes over the span length and the corresponding flange width being used, etc. The resulting optimum design problem would have a larger number of design variables depending upon the number of sections to be considered in the full length girder.

The optimum box-girder section design software was modified to consider up to 3 sections simultaneously. Design studies were carried out to ascertain the performance of the software and the trends in the results. The results for the simply supported girder using blade stiffened panels, with no restriction on the depth, for eccentric loading are presented in Table 5 for three different span lengths. Four different cases are presented. Case I was the prismatic girder designed for forces corresponding to mid-span section only. Case II was the prismatic girder designed for forces corresponding to mid-span section and support section simultaneously.

Case III was the non-prismatic girder designed for forces corresponding to mid-span section and support section, with the depth and the bottom flange width being allowed to vary linearly between these sections and the number of longitudinal stiffeners being maintained the same over the entire span length in all the panels. All the other dimensions were allowed to vary freely. Case IVa was similar to case II except three critical sections were chosen for design, namely support section, quarter span section and mid span section. In case IVa the number of members in each population and the number of generations before stopping the problem were kept the same as in single section design (Population size = 50 and number of generation before stopping = 1000), in spite of the increase in the number of design variables. In case IVb these values in the GA were increased to 100 and 5000 respectively to study their influence.

The following observations are made based on the results presented in Table 5.

- The optimum design can be improved by designing a girder considering forces acting on many sections simultaneously and allowing the girder to be non-prismatic so that the strength of the CFRP girder can be tailored to meet the strength requirements at different sections efficiently.
- When the problem size is increased the population size in the GA and the number of generations to convergence should also be increased in order to obtain the optimum values. In fact the lower values used in case IVa were arrived at as the appropriate values for the size of the problem corresponding to single section design, after many numerical experiments.

Table 6 Effect of variation in seed value

Seed Value	Mass (tonne)	% Difference from mean mass
0.030	2.99	−3.2
0.150	3.17	+2.5
0.280	3.09	0.0
0.570	3.05	−1.3
0.793	3.15	+1.9

In general, common features are required for comparison purpose. As mentioned earlier, benchmark problems having similar configuration of bridge are not available in FRP. However, Naresh and Vijayakumar (2006) have reported similar construction (same span, similar loading and width) by using high strength steel plates. The mass of 40 m span steel bridge comes out to be 26 tonne, while the mass of FRP bridge from the present optimum design comes out to be around 6 tonne. In addition to this, a comparison with an unpublished literature reveals that around 40% saving in mass can be achieved by using FRP box-girder bridge in place of aluminium alloy bridge of 20 m span having the same geometrical features and subjected to the same loading.

4.4 Effect of variation in seed value

The seed values are required for generating randomly the members of the first generation. Initially optimum design of the mid-span section of a 20 m span bridge was carried out for the case of restricted depth, eccentric loading and blade stiffened panels, using a range of seed values, to study the influence of the seed value and the initial population. It is seen in Table 6, that the optimum value of the objective function corresponding to different seed values is nearly the same, the maximum difference from the mean value being only around 3.2%. It proves that the genetic algorithms based optimum design procedure is reliable and that it converges to near global optimum values with out getting trapped in local minima. Further, many near optimum alternatives can be obtained from the procedure, and they offer additional freedom to the designer to choose a suitable near optimum alternate solution, if so desired.

5. Conclusions

The Complexity of the optimum design of FRP box-girder bridge and the need to use genetic algorithms to solve this problem were discussed. The mathematical formulation of the optimum design problem and other details of the solution process using genetic algorithms were presented. Results of several optimum design parametric studies were presented and compared.

The following characteristics are observed:

- The GA is efficient in solving the complex optimum design problem.
- The dependence of the optimum mass on the initial seed value is small.
- The developed optimum design procedure is more general as size, configuration and topology optimization can be carried out simultaneously.
- Results of parametric design studies done on single lane, single cell, simply supported, CFRP

box-girder bridges subjected to class AA tracked vehicle loading, indicate following features:

- Up to 45% penalty is imposed on the mass, if the depth is restricted ($d \leq 0.85$ m), when compared with no depth restriction results.
- Between 6 to 16% higher mass is obtained for blade stiffened FRP box-girders in comparison to angle stiffened box-girders.
- Further economy is possible, if different types of stiffener are allowed in different panels of the same box-girder. Compression flange is likely to require angle stiffener whereas web and tension flange are likely to require blade stiffener in the optimum design.
- In general, below 6% difference in mass is observed when torsion is disregarded in the single lane bridges. This may not be so in multiple lane bridges, wherein torsion could be larger.
- The span length to depth ratio of optimally designed girders is in the range of 9 to 18, the larger values being applicable to longer spans.
- In optimally designed box-girders, the ratio of width of bottom flange to that of the top flange varies in the range of 0.45 to 0.65, as the span length increases from 20 m to 40 m.

This information can be used to accelerate the optimum design process and in learning more about the behaviour of the system. The insight gained can lead to newer concepts and better designs for the FRP box-girder bridges.

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