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Piecewise exact solution for seismic mitigation analysis of bridges equipped with sliding-type isolators

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Abstract. Recently, earthquake proof technology has been widely applied to both new and existing structures and bridges. The analysis of bridge systems equipped with structural control devices, which possess large degrees of freedom and nonlinear characteristics, is a result in time-consuming task. Therefore, a piecewise exact solution is proposed in this study to simplify the seismic mitigation analysis process for bridge systems equipped with sliding-type isolators. In this study, the simplified system having two degrees of freedom, to reasonably represent the large number of degrees of freedom of a bridge, and is modeled to obtain a piecewise exact solution for system responses during earthquakes. Simultaneously, we used the nonlinear finite element computer program to analyze the bridge responses and verify the accuracy of the proposed piecewise exact solution for bridge systems equipped with sliding-type isolators. The conclusions derived by comparing the results obtained from the piecewise exact solution and nonlinear finite element analysis reveal that the proposed solution not only simplifies the calculation process but also provides highly accurate seismic responses of isolated bridges under earthquakes.

Keywords: multiple friction pendulum system; bridges; base isolator; base isolation system; earthquake engineering; structural control; passive control.

1. Introduction

Bridge engineering facilitates speedy transportation and promotes the development and competitive edge of countries. Previously, the bridges and buildings in the several countries located in earthquake-prone regions suffered from seismic damage, such as during the 1994 Northridge earthquake in USA, 1995 Kobe earthquake in Japan, and 1999 Chi-Chi earthquake in Taiwan. It is difficult to improve the safety and functionality of bridges and buildings under severe seismic loadings by using traditional earthquake resistant design methods. In recent years, the earthquake proof technologies have been acknowledged as powerful and effective tools to update the seismic resistibility of bridges and buildings by installing structure control devices. The friction pendulum system (FPS) proposed by Zayas *et al.* (1987) includes an articulated slider and a concave sliding surface. Tsopelas *et al.* (1996) performed the shaking table tests to investigate the behavior of

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bridges equipped with FPS bearings under earthquakes; in these tests, seismic motions with peak ground accelerations in the range of 0.1-1.1 g were simulated. The experimental results demonstrated a substantial improvement in the ability of isolated bridges to sustain all levels of seismic excitation under elastic conditions (Constantinou *et al.* 1993, Tsopelas *et al.* 1996). Tsai *et al.* (2003b, 2003c, 2004, 2005a, 2006a) proposed the multiple friction pendulum system (MFPS) comprising an articulated slider and the upper and lower spherical concave surfaces, which were used to lengthen the vibration period of a structure and enhance bearing displacement capacities while compared to the FPS isolation bearing; and following up research has also been published by Fenz and Constantinou (2006). In addition, other types of MFPS isolators consisting of numerous sliding interfaces, as shown in Fig. 1, were invented by Tsai in 2003 as well (Tsai 2003a, Tsai *et al.* 2008). It comprises two or more than two spherical concave sliding surfaces and an articulated slider to accommodate the large displacements induced by the earthquakes with long predominant periods. Furthermore, a bridge system may possess quite different natural periods in lateral and longitudinal directions; therefore, the lengthening of the natural periods in these two directions is necessary to satisfy economic constraints and the requirements of bearing displacements in the two different directions. With regard to these requirements, the trench friction pendulum system (TFPS) was proposed by Tsai *et al.* (2006b) to provide different lengthening periods in lateral and longitudinal directions and reduce the maximum lateral bearing displacement. Furthermore, the natural period of the isolator is constant when the radius of curvature of the spherical concave surface has been designed. Tsai *et al.* (2008) proposed the direction optimized-friction pendulum system (DO-FPS), which can continually change the natural periods during earthquakes. Therefore, structures equipped with DO-FPS isolators can avoid resonating with earthquake energy-enriched exciting periods.

Although a bridge equipped with base isolators can reduce a majority of the responses during earthquakes, conducting dynamic analysis for a bridge system with base isolators possessing nonlinear behavior is a complex and time-consuming task due to the numerous degrees of freedom involved. In this study, we derived a piecewise exact solution by simplifying the complex analysis process for bridges. We proposed a simple two-degrees-of-freedom system to reasonably model and describe a bridge system equipped with sliding-type isolators (Tsai *et al.* 2007). A piecewise exact solution has been derived to calculate the behavior of a nonlinear system wherein sliding-type

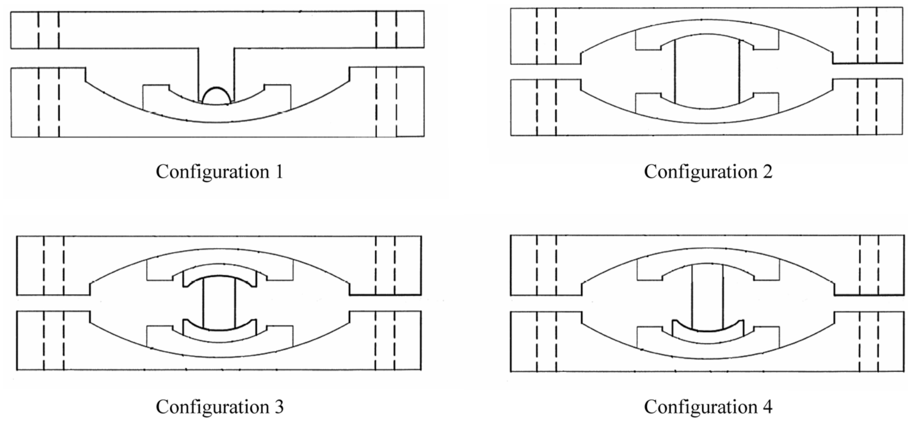


Fig. 1 Four Types of MFPS isolators

isolators were installed between the deck and pier of a bridge during earthquakes. In addition, numerical results obtained using the finite element computer program called Nonlinear Stress Analysis Techniques (NSAT) (Tsai 1996) were compared with the analytical results obtained using the proposed piecewise exact solution. This comparison reveals that the proposed piecewise exact solution not only simplifies the calculation process but also provides reasonably accurate seismic responses of an isolated bridge under various earthquakes.

2. Mechanical characteristics of sliding-type isolators

As shown in Fig. 2, based on the equilibrium of the forces in the horizontal direction of the upper and lower concave surfaces of the MFPS isolator, the horizontal forces in the upper (F_1) and lower (F_2) concave surfaces can be obtained as (Tsai *et al.* 2003c, 2004, 2005a, b)

$$F_1 = \left(\frac{W}{R_1}\right)U_1 + \mu W \mathrm{sgn}(\dot{U}_1) \tag{1}$$

and

$$F_2 = \left(\frac{W}{R_2}\right)U_2 + \mu W \mathrm{sgn}(\dot{U}_2) \tag{2}$$

where W indicates the vertical force acting on the MFPS isolator; R_1 and R_2, radii of curvature of the upper and lower concave surfaces, respectively; μ, the friction coefficient of the concave surface; U_1, the relative displacement between the upper concave surface and articulated slider; and U_2, the relative displacement between the lower concave surface and articulated slider.

The stiffness of the MFPS isolator can be obtained by considering the free body and satisfying force equilibrium. The stiffness of the MFPS isolator can be subsequently expressed as follows (Tsai *et al.* 2003c, 2004, 2005a, b, Fenz and Contantinou 2006)

$$K_b = \frac{W}{R_1 + R_2} \tag{3}$$

According to Eq. (3), the natural period of the MFPS isolator can be given as

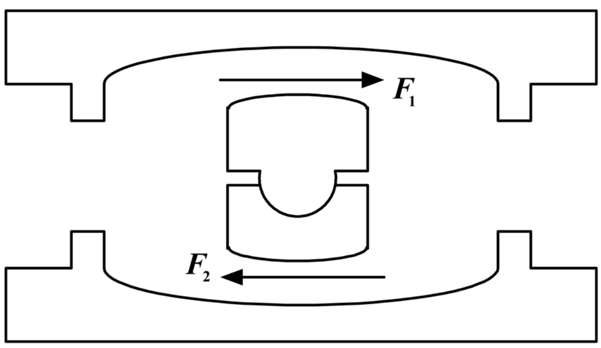


Fig. 2 Horizontal forces of MFPS isolator

$$T = 2\pi\sqrt{\frac{R_1 + R_2}{g}} \tag{4}$$

In addition, the dynamic friction coefficient of the Teflon concave surfaces of the MFPS isolator was presented by Tsai *et al.* (2005b, 2006a) as follows

$$F_t = \frac{WA}{\lambda_1 A + \lambda_2 p} \cdot \{1 + \alpha[1 - \exp(-\beta V)]\} \times Coef \tag{5}$$

where A is the contact area between the slider and the concave surface of the FPS or MFPS isolators; λ_1, λ_2, α, and β, parameters to be determined by experiments; and *Coef*, decay function depicting the phenomenon of the degradation of friction force with the increase in the number of cyclic reversals. The coefficient of *Coef* can be shown as

$$Coef = (1 - \gamma_1) + \gamma_1 \cdot \exp\left(-\gamma_2 \cdot \int_0^t \frac{F_t - F_t^0}{F_t^0} \cdot dU_T\right) \tag{6}$$

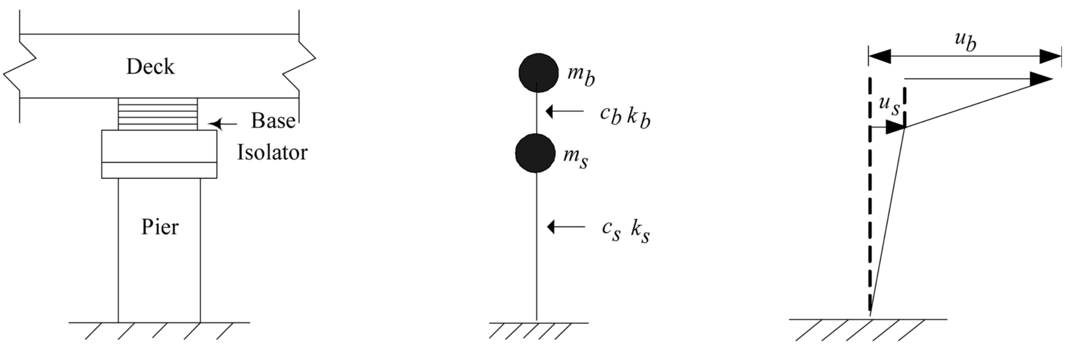
where γ_1 and γ_2 are parameters describing the decay behavior of the friction force at the Teflon interface associated with the energy accumulation in the sliding history; F_t^0, the friction force when the sliding velocity is zero; and dU_T, the displacement increment.

3. Equation of motion of bridge equipped with sliding-type base isolator

In order to simplify the analytical processes and reduce calculation time, the unit-isolated bridge was simply simulated as a system with two degrees of freedom, as shown in Fig. 3. The equation of motion for the bridge deck, in this case, can be expressed as follows (2005a)

$$m_b\ddot{u}_b + c_b(\dot{u}_b - \dot{u}_s) + k_b(u_b - u_s) = -m_b\ddot{u}_g - \mu(\dot{u}_b - \dot{u}_s)m_b g\,\mathrm{sgn}(\dot{u}_b - \dot{u}_s) \tag{7}$$

where m_b is the mass of the deck; c_b and k_b, the damping coefficient and horizontal stiffness of the isolator, respectively; u_b and u_s, the displacements of the deck and pier relative to the ground, respectively; $\mu(|\dot{u}_b - \dot{u}_s|)$, the friction coefficient for the sliding interface, which is a function of the relative velocity between the slider and concave surface; and $\ddot{u}_g$, the ground acceleration.



(a) Simplified bridge model (b) Two-degrees of freedom model (c) Displacement ofpier and deck

Fig. 3 A bridge with sliding type isolators simulated by two degree of freedoms

Further, the function of the friction coefficient for the sliding-type isolator can be expressed as follows (Constantinou *et al.* 1993)

$$\mu(\dot{u}_b - \dot{u}_s) = \mu_{\max} - (\mu_{\max} - \mu_{\min})\exp(-\alpha|\dot{u}_b - \dot{u}_s|) \tag{8}$$

Furthermore, the equation of motion for the pier can be obtained as

$$m_s\ddot{u}_s - c_b\dot{u}_b + (c_s + c_b)\dot{u}_s - k_b u_b + (k_s + k_b)u_s = -m_s\ddot{u}_g + \mu(\dot{u}_b - \dot{u}_s)m_b g\mathrm{sgn}(\dot{u}_b - \dot{u}_s) \tag{9}$$

where m_s, c_s, and k_s indicate the mass, damping coefficient, and horizontal stiffness of the pier, respectively.

On rearranging Eqs. (7) and (9), we get

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = -\mathbf{M}\mathbf{r}\ddot{u}_g - \mathbf{f} \tag{10}$$

where

$$\mathbf{M} = \begin{bmatrix} m_b & 0 \\ 0 & m_s \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} c_b & -c_b \\ -c_b & c_s + c_b \end{bmatrix} \quad \mathbf{K} = \begin{bmatrix} k_b & -k_b \\ -k_b & k_s + k_b \end{bmatrix}$$

$$\mathbf{r} = [1 \;\; 1]^T \quad \mathbf{f} = \begin{bmatrix} \mu(|\dot{u}_b - \dot{u}_s|)m_b g\mathrm{sgn}(\dot{u}_b - \dot{u}_s) \\ -\mu(|\dot{u}_b - \dot{u}_s|)m_b g\mathrm{sgn}(\dot{u}_b - \dot{u}_s) \end{bmatrix} \quad \mathbf{u} = [u_b \;\; u_s]^T$$

The modal frequencies can be solved by using the following equation

$$\det[\mathbf{K} - \omega^2\mathbf{M}] = 0 \tag{11}$$

Subsequently, the characteristic equation can be represented as follows (Naeim and Kelly 1999)

$$(1-\gamma)\omega^4 - (\omega_s^2 + \omega_b^2)\omega^2 + \omega_s^2\omega_b^2 = 0 \tag{12}$$

where

$$M = m_b + m_s \quad \gamma = \frac{m_b}{M} \quad \omega_s^2 = \frac{k_s}{M} \quad \omega_b^2 = \frac{k_b}{m_b}$$

Simultaneously, we can define $\varepsilon = \omega_b^2/\omega_s^2$, and the solution to Eq. (12) can be expressed as

$$\omega_1^2 = \omega_b^2(1 - \gamma\varepsilon) \tag{13}$$

$$\omega_2^2 = \frac{\omega_s^2}{1-\gamma}(1 + \gamma\varepsilon) \tag{14}$$

In addition, the mode shapes of the first and second modes can be expressed as Eqs. (15) and (16), respectively; these are also shown in Fig. 4.

$$\boldsymbol{\varphi}_1 = [1 \;\; \gamma\varepsilon]^T \tag{15}$$

$$\boldsymbol{\varphi}_2 = \left[1 \;\; \frac{\varepsilon - 2\gamma\varepsilon - 1}{\varepsilon(1-\gamma)}\right]^T \tag{16}$$

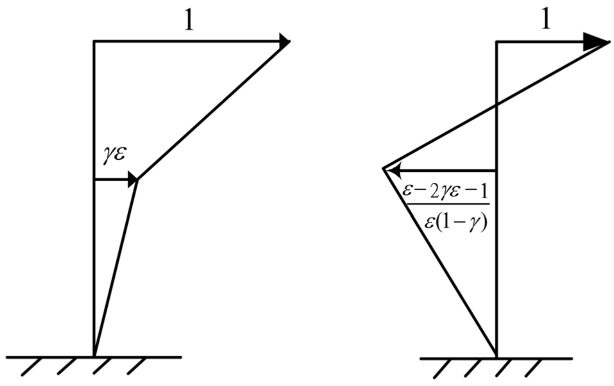


Fig. 4 Mode shapes of a bridge with sliding type of isolators

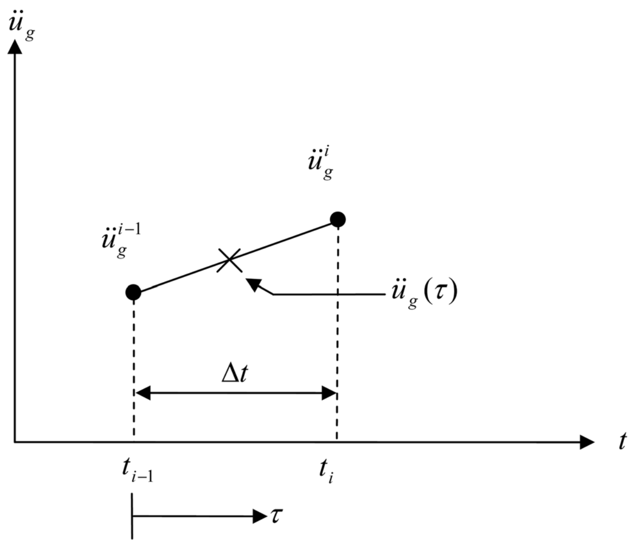


Fig. 5 Linear interpolation of ground acceleration between time steps t_{i-1} and t_i

Subsequently, the total displacement response can be expressed as

$$\mathbf{u} = \begin{bmatrix} u_b \\ u_s \end{bmatrix} = \boldsymbol{\varphi}_1 q_1(t) + \boldsymbol{\varphi}_2 q_2(t) \tag{17}$$

Although the ground motion is a random function of time, it can still be considered a linear function for a small time increment, as shown in Fig. 5. Therefore, the interpolation function is introduced to describe the ground accelerations between time steps t_{i-1} and t_i (Chopra 1995, Tsai *et al.* 2004, 2005a). Furthermore, Eq. (10) can be rewritten by substituting the interpolation functions of the ground motions for the first and second modes. It is obtained as

$$\ddot{q}_1 + 2\xi_1\omega_1\dot{q}_1 + \omega_1^2 q_1 = -L_1\left(\ddot{u}_g + \frac{\ddot{u}_g^i - \ddot{u}_g^{i-1}}{\Delta t}\tau\right) - f_1 \tag{18}$$

and

$$\ddot{q}_2 + 2\xi_2\omega_2\dot{q}_2 + \omega_2^2 q_2 = -L_2\left(\ddot{u}_g + \frac{\ddot{u}_g^i - \ddot{u}_g^{i-1}}{\Delta t}\tau\right) - f_2 \tag{19}$$

where

$$L_1 = \frac{\phi_1^T\mathbf{Mr}}{\phi_1^T\mathbf{M}\phi_1} \quad f_1 = \frac{\phi_1^T\mathbf{f}}{\phi_1^T\mathbf{M}\phi_1} \quad L_2 = \frac{\phi_2^T\mathbf{Mr}}{\phi_2^T\mathbf{M}\phi_2} \quad f_2 = \frac{\phi_2^T\mathbf{f}}{\phi_2^T\mathbf{M}\phi_2}$$

On solving Eqs. (18) and (19), we obtain the modal coordinates for the first and second modes as follows

$$\begin{aligned} q_1(\tau) &= (A_1\cos\omega_{1d}\tau + B_1\sin\omega_{1d}\tau)\exp(-\xi_1\omega_1\tau) \\ &+ \frac{1}{\omega_1^2}\left[-L_1\ddot{u}_g^{i-1} - f_1 + \frac{2L_1\xi_1}{\omega_1\Delta t}(\ddot{u}_g^i - \ddot{u}_g^{i-1})\right] - L_1\frac{\ddot{u}_g^i - \ddot{u}_g^{i-1}}{\omega_1^2\Delta t}\tau \end{aligned} \tag{20}$$

$$\begin{aligned} q_2(\tau) &= (A_2\cos\omega_{2d}\tau + B_2\sin\omega_{2d}\tau)\exp(-\xi_2\omega_2\tau) \\ &+ \frac{1}{\omega_2^2}\left[-L_2\ddot{u}_g^{i-1} - f_2 + \frac{2L_2\xi_2}{\omega_2\Delta t}(\ddot{u}_g^i - \ddot{u}_g^{i-1})\right] - L_2\frac{\ddot{u}_g^i - \ddot{u}_g^{i-1}}{\omega_2^2\Delta t}\tau \end{aligned} \tag{21}$$

where

$$\omega_{1d} = \omega_1\sqrt{1-\xi_1^2} \qquad \omega_{2d} = \omega_2\sqrt{1-\xi_2^2}$$

$$A_1 = q_1^{i-1} - \frac{1}{\omega_1^2}\left[-L_1\ddot{u}_g^{i-1} - f_1 + \frac{2L_1\xi_1}{\omega_1\Delta t}(\ddot{u}_g^i - \ddot{u}_g^{i-1})\right]$$

$$B_1 = \frac{1}{\omega_{1d}}\left[\dot{q}_1^{i-1} + \xi_1\omega_1A_1 + L_1\frac{\ddot{u}_g^i - \ddot{u}_g^{i-1}}{\omega_1^2\Delta t}\right]$$

$$A_2 = q_2^{i-1} - \frac{1}{\omega_2^2}\left[-L_2\ddot{u}_g^{i-1} - f_2 + \frac{2L_2\xi_2}{\omega_2\Delta t}(\ddot{u}_g^i - \ddot{u}_g^{i-1})\right]$$

$$B_2 = \frac{1}{\omega_{2d}}\left[\dot{q}_2^{i-1} + \xi_2\omega_2A_2 + L_2\frac{\ddot{u}_g^i - \ddot{u}_g^{i-1}}{\omega_2^2\Delta t}\right]$$

4. Numerical analyses of bridge equipped with sliding-type isolators

In order to verify the accuracy of the results analyzed by the proposed piecewise exact solution, an RC bridge (Kunde and Jangid 2006) equipped with MFPS isolators is adopted to analyze the responses of isolated bridges under seismic loadings using the NSAT program (Tsai 1996) for the nonlinear finite element analysis. As shown in Fig. 6, the isolated RC bridge possesses two spans of 30 m each; mass density, 2.4×10^3 kg/m^3; Young's modulus of elasticity, 20.64 GPa; height, 8 m; cross area of the deck, 3.57 m^2; cross area of the pier, 4.09 m^2; moment of inertia of the deck, 2.08 m^4; and moment of inertia of the pier, 0.64 m^4. The dead load on the deck is 1.0 t/m^2. In

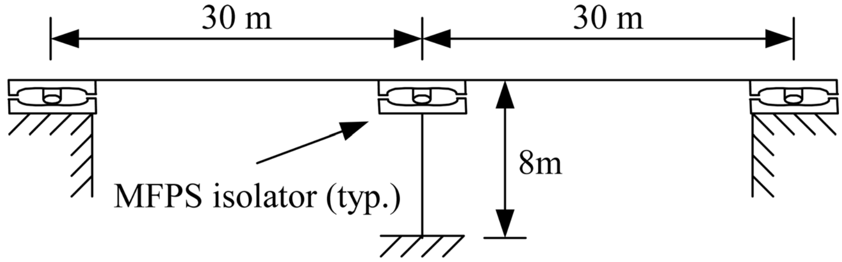


Fig. 6 Sketch of a bridge with sliding type isolators

addition, the MFPS isolators were installed at the two end points of the deck and between the deck and pier, and the radius of curvature of each spherical concave surface is 1.5 m. The parameters for the Telfon material obtained from experimental tests (Tsai *et al.* 2005a, 2006a) for the calculations are given as follows: $\lambda_1 = 21.120$, $\lambda_2 = 1.221\times10^{-7}$ (1/Pa), $\alpha = 1.903$, $\beta = 100.000$ (s/m), $\gamma_1 = 0.1390$ and $\gamma_2 = 7.1537$ (1/m). In the finite element analysis, the beam element was adopted for modeling the deck and pier of the bridge; and the MFPS isolation element (Tsai *et al.* 2004) was adopted for simulating the multiple pendulum systems located beneath the bridge deck. Excitations including the 1940 El Centro earthquake in USA, 1995 Kobe earthquake in Japan, and 1999 Chi-Chi earthquake (TCU084 station) in Taiwan were used as inputs for the analyses. Figs. 7-9 show the comparisons between the bearing displacements of the results obtained from the piecewise exact solution and NSAT analysis under various earthquakes with 0.5 g in PGA. The comparisons in these figures indicate that the piecewise exact solution provides fairly accurate predictions for bearing displacements during earthquakes. Furthermore, the hysteresis loops shown in Figs. 10-12 depict the hysteretic behavior of the sliding-type isolator as obtained from both the piecewise exact solution and NSAT analysis during earthquakes. These figures reveal that the proposed analytical procedure provides not only reasonable predictions for bearing displacements but also accurate hysteretic responses of isolators under seismic loadings.

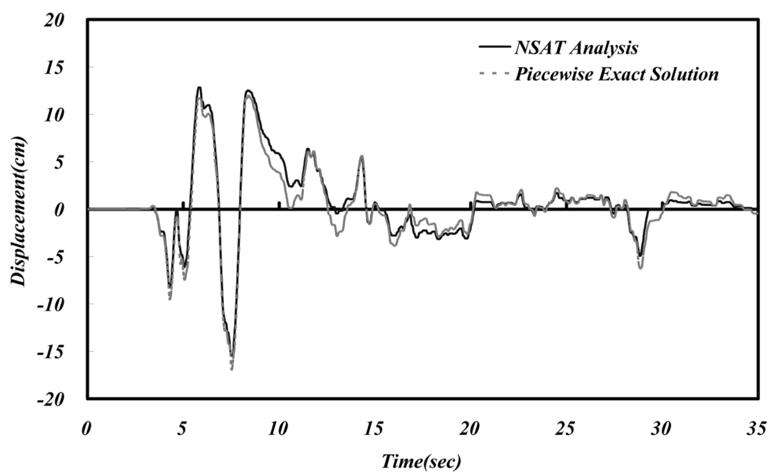


Fig. 7 Comparison of bearing displacements between piecewise exact solution and NSAT analysis under El Centro earthquake of 0.5 g in PGA

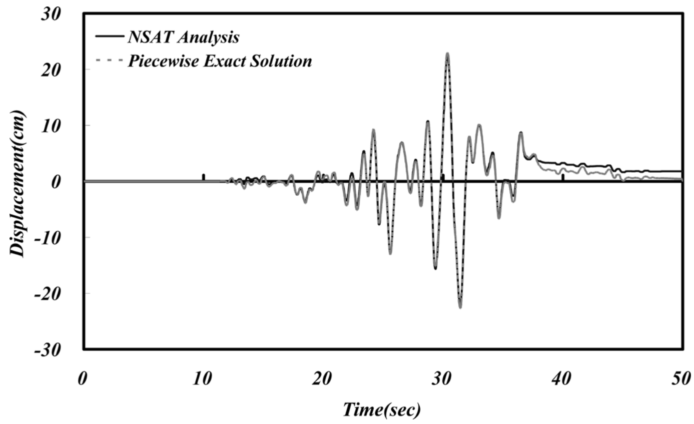


Fig. 8 Comparison of bearing displacements between piecewise exact solution and NSAT analysis under Chi-Chi earthquake of 0.5 g in PGA

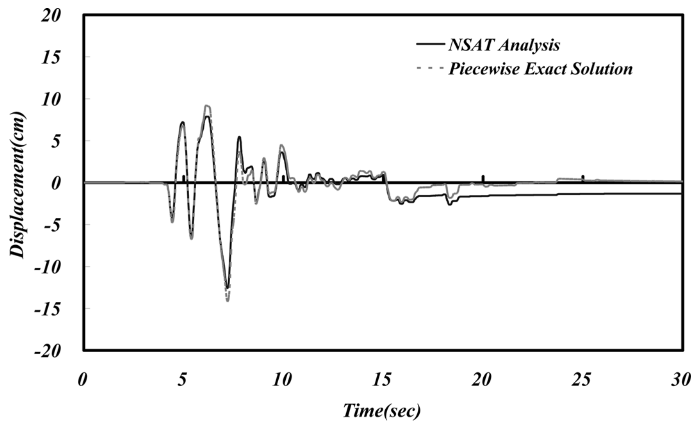


Fig. 9 Comparison of bearing displacements between piecewise exact solution and NSAT analysis under Kobe earthquake of 0.5 g in PGA

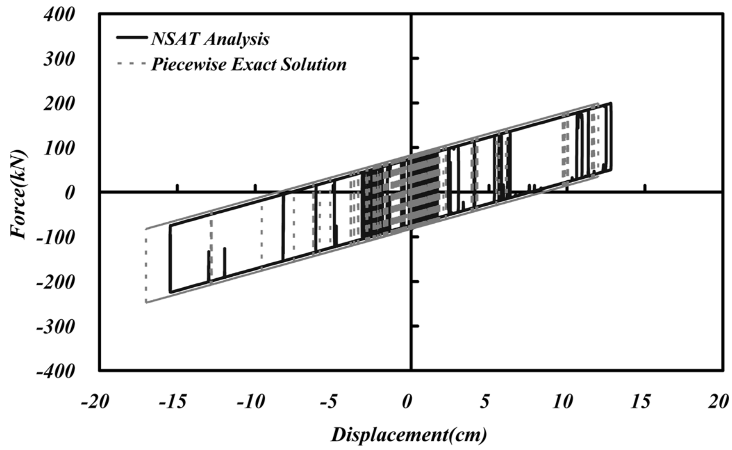


Fig. 10 Comparison of hysteresis loop between piecewise exact solution and NSAT analysis under El Centro earthquake of 0.5 g in PGA

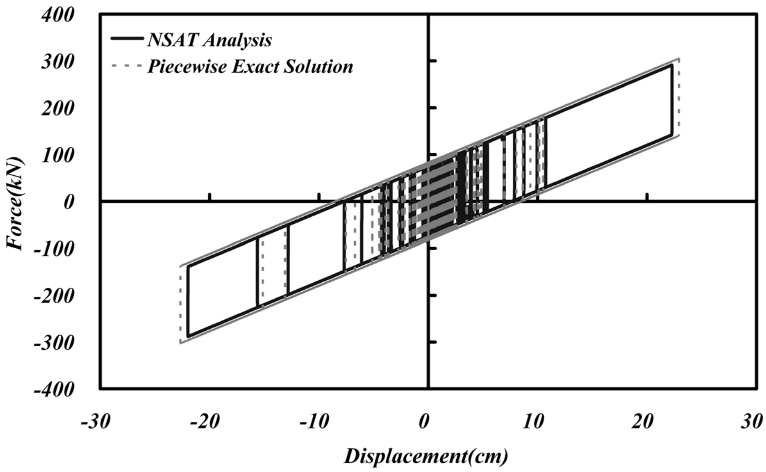


Fig. 11 Comparison of hysteresis loop between piecewise exact solution and NSAT analysis under Chi-Chi earthquake of 0.5 g in PGA

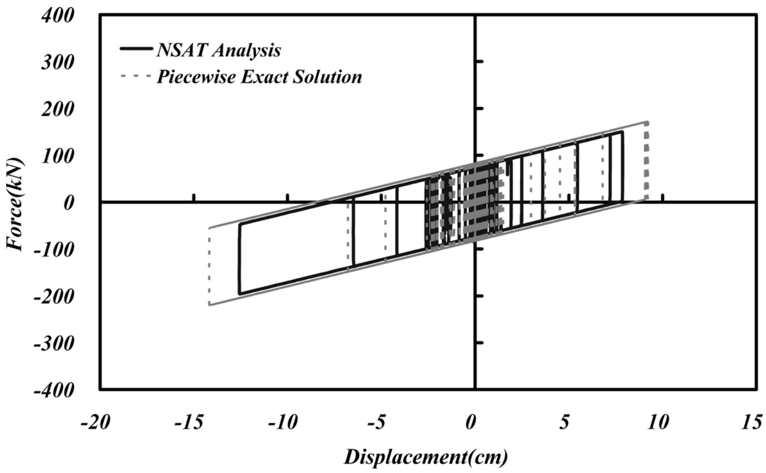


Fig. 12 Comparison of hysteresis loop between piecewise exact solution and NSAT analysis under Kobe earthquake of 0.5 g in PGA

5. Conclusions

In this study, we focus on simplifying the analytical procedure for bridge systems equipped with sliding-type isolators, which possess nonlinear behaviors and massive degrees of freedom. The proposed piecewise exact solution for an isolated bridge provides fairly accurate results for the given example under seismic loadings in the longitudinal direction as well as simplifies the complex analytical processes and reduces calculation time. Therefore, it can be assumed that the proposed algorithm is a good tool for the preliminary design of the bearing displacement, the isolation period, etc.

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