Structural Engineering and Mechanics, Vol. 35, No. 2 (2010) 127-140
DOI: http://dx.doi.org/10.12989/sem.2010.35.2.127

Accuracy of structural computation on simplified shape

P. Marin*

*3SR Laboratory, National Polytechnic Institute of Grenoble, B.P. 95,
38402 St., Martin d'Hères, France*

(Received July 25, 2007, Accepted October 12, 2009)

Abstract. This paper focuses on a number of criteria that enable controlling the influence of geometric simplification on the quality of finite element (FE) computations. To perform the mechanical simulation of a component, the corresponding geometric model typically needs to be simplified in accordance with hypotheses adopted regarding the component's mechanical behaviour. The method presented herein serves to compute an *a posteriori* indicator for the purpose of estimating the significance of each feature removal. This method can be used as part of an adaptive process of geometric simplification. If a shape detail removed during the shape simplification process proves to be influential on mechanical behaviour, the particular detail can then be reinserted into the simplified model, thus making it possible to readapt the initial simulation model. The fields of application for such a method are: static problems involving linear elastic behaviour, and linear thermal problems with stationary conduction.

Keywords: adaptive modelling; geometric simplification; a posteriori mechanical indicator; structural simulation; finite element; CAD; feature removal.

1. Introduction

The use of CAD in design applications makes it possible to more and more precisely represent mechanical components containing a large number of details. For use in finite element (FE) computations, these models are often overly refined; moreover, their direct use could cause several disadvantages. If the user were to prescribe a smaller element size, then the mesh would contain too many finite elements and the time required to run the computation would be excessive. On the contrary, if a larger element size were prescribed, this would generate poorly-shaped elements and convergence issues with respect to the computation. A preliminary model simplification step is thus necessary. Various software applications serve to automate this step to a partial extent. Several categories of approaches have been proposed in order to solve the problems involved in preparing FE models from CAD data.

A first category addresses configurations for which small features must be removed in order to derive a geometric model more compatible with the required FE size (Dabke *et al.* 1994, Mobley *et al.* 1997, François *et al.* 2000, Zhu *et al.* 2002, Joshi *et al.* 2003). In these references, the approaches are highly dependent on the modelling history of the specific part and focus on both the building tree of the object and removal of selected features.

*Ph.D., E-mail: philippe.marin2@grenoble-inp.fr

A second category of approaches begins with a polyhedral model of the part (Schroeder *et al.* 1992, Cohen *et al.* 1995, Véron *et al.* 1997, Fine *et al.* 2000, Fine *et al.* 2005, Hamri *et al.* 2006). In order to adapt the model, various adaptation functions are applied to the initial polyhedral model; these functions combine the decimation process with removal of topological details.

Another category is characterised by idealisation treatments. Such operations are often required to transform a volume into an open surface for the purpose of modelling plate behaviour. Similar operations apply to transforming a volume feature into a line when modelling beam behaviour of a structure. Our paper will focus solely on the errors produced by the first two categories of geometric simplification approaches, the idealisation process will not be addressed herein.

For the engineer, finite element computation accuracy is a vital concern. Sources of error are multi-fold: discretisation error, uncertainty on boundary conditions and the constitutive relation, shape simplification (Szabo 1996), etc. Over the past twenty years, considerable work has been devoted to discretisation errors and adaptive meshing (Babuska *et al.* 1978, Zhienkiewicz *et al.* 1987, Ladeveze *et al.* 1992). Geometric simplification can also strongly influence the quality of finite element results. The choice and control of these simplifications are thus of primary importance. When model preparation is manual, the ultimate model quality will depend on the engineer's expertise. Many industries implement simplified models for their own specific applications; these models are built from experience or simulations using the complete model (Livne 1994, Yoshimura 1998, Kim *et al.* 2001, Lee *et al.* 2003, Machnik *et al.* 1998). In an automatic simplification process, the monitoring step uses input such as geometric criteria, curve and size. According to an *a priori* approach, geometric criteria relative to the mechanical properties of the problem may be added, these would include variations in: mass, volume, cross-section, and centre of inertia (Foucault *et al.* 2004, Léon *et al.* 2004).

Yet these *a priori* criteria cannot quantify the real influence of a geometric simplification on finite element simulations. For example, the errors generated by a hole removal will depend not only on the hole dimension, but also on its location relative to the component. This shape feature could lie in an area containing low or high stresses. To take into account the feature position, a mechanical criterion requires information on the highly-stressed areas and hence a sketch of the set of analytical results.

A real mechanical criterion relies upon an *a posteriori* process. After a simulation run on the simplified part, the mechanical criterion is computed to evaluate the influence of each feature removal. The engineer obtains information in order to quantify the influence of each simplification on the accuracy of finite element results. Such a simplification process was introduced in Véron *et al.* (1998). Following an initial analysis of the simplified problem, the authors undertook an adaptive meshing process (the h-method), during which a size map was computed so as to define the optimal mesh. The authors then used this map to monitor the geometric simplification process, the program then removed the detail if its size was smaller than that given in this area by the size map. This criterion yields incorrect information in the area where stresses are high yet remain smooth. The prescribed optimal mesh sizes are too large in this specific area. For the thermal simulation, Gopalakrishnan *et al.* (2007) also presented an *a posteriori* criterion for estimating defeaturing-induced engineering analysis errors, this criterion requires computing two dual problems on the simplified geometry.

Our criterion is also *a fortiori* and it approximates the energy norm of the difference between the solution on the initial part and the solution on the simplified part. This error indicator necessitates a local finite element computation in the vicinity of each feature. With this information, the user (or program, in an automatic simplification process) is able to redefine the simplified model and choose to accept or not each simplification. The proposed criterion relates to either static analysis problems with linear behaviour or thermal problems for stationary linear conduction. This paper has been organised as follows. Section 2 will describe our proposed error estimator, and Section 3 will highlight its efficiency for various types of problems. In Section 4, we will discuss an adaptive modelling process that integrates CAD software tools.

Another adaptive modelling process has been developed through use of a software environment based on a mixed representation that takes polyhedral models as a reference, yet still enables retaining the semantic of the input CAD models. By combining basic operators acting on the polyhedral model, a number of efficient tools and operators for handling shape sub-domains have been derived (Ferrandes *et al.* 2007, Ferrandes *et al.* 2009) in order to automate the adaptive modelling process. This development has allowed for an automatic simplification process, yet the kinds of features potentially suppressed are much more limited.

2. Influence of geometric simplification

2.1 Error definition

We shall consider herein the framework of a static computation applied to a linear elastic structure. As shown in Fig. 1, two categories of shape changes can be distinguished, i.e., additive or subtractive, according to the geometric domain variations, thus resulting in either addition of the Ω_3 domain or removal of the Ω_4 domain.

Let's assume that the solution on the initial domain Ω_1 (see Fig. 1) yields the displacement field $\vec{U}_1$, stress field $\bar{\bar{\sigma}}_1$ and strain field $\bar{\bar{\varepsilon}}_1$. We denote $\partial\Omega_1$ the boundary of Ω_1, similarly, $\vec{U}_2$, $\bar{\bar{\sigma}}_2$ and $\bar{\bar{\varepsilon}}_2$ are the solution fields of the problem on simplified domain Ω_2 with boundary $\partial\Omega_2$.

Let's now assume that this simplified problem exactly matches the first one, i.e., that the error equals zero if:

- at the intersection of the two domains, i.e., $(\Omega_1 \cap \Omega_2)$, the initial and simplified problem solutions are equal,
- over domain Ω_3, the stress and strain fields, i.e., $\bar{\bar{\sigma}}_2$ and $\bar{\bar{\varepsilon}}_2$ respectively, equal zero,
- over domain Ω_4, the stress and strain fields, i.e., $\bar{\bar{\sigma}}_1$ and $\bar{\bar{\varepsilon}}_1$ respectively, equal zero.

To estimate the influence of these shape modifications, it is necessary to measure:

- the difference $(\vec{U}_1 - \vec{U}_2)$ on the common domain $(\Omega_1 \cap \Omega_2)$,
- stresses $\bar{\bar{\sigma}}_2$ over Ω_3,
- stresses $\bar{\bar{\sigma}}_1$ over Ω_4.

We used the energy norm (Ladeveze *et al.* 1983, Ladeveze *et al.* 1991) to measure these quantities. The corresponding error, denoted *e*, is given by

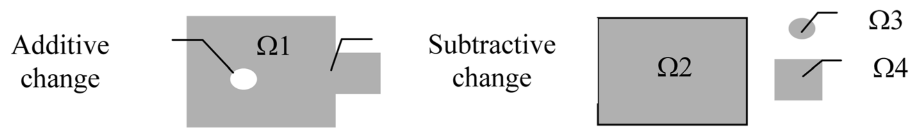

Fig. 1 Simplification example: initial domain Ω_1 and simplified domain Ω_2. Ω_3 is the domain added to Ω_1 to produce Ω_2. Ω_4 is the domain removed from Ω_1 to produce Ω_2

$$e^2 = \frac{1}{2}\int_{\Omega 1 \cap \Omega 2} (\bar{\bar{\sigma}}_1 - \bar{\bar{\sigma}}_2):(\bar{\bar{\varepsilon}}_1 - \bar{\bar{\varepsilon}}_2)d\Omega + \frac{1}{2}\int_{\Omega 3} \bar{\bar{\sigma}}_2:\bar{\bar{\varepsilon}}_2 d\Omega + \frac{1}{2}\int_{\Omega 4} \bar{\bar{\sigma}}:_1\bar{\bar{\varepsilon}}_1 d\Omega \tag{1}$$

This expression can then be transformed to obtain Eq. (2). To simplify this expression, we assumed that the boundary of the simplified domain (i.e., Ω_3 or Ω_4) feature is free, i.e., no boundary conditions applied on it. In the case of a constrained boundary, a similar demonstration could be easily performed, though the expression and demonstration would be more elaborate.

$\vec{n}_\Omega$ designates the unit normal pointing outward from domain Ω, while $\vec{fd}$ designates the volumetric fields of forces acting on Ω.

$$e^2 = \frac{1}{2}\int_{\Omega 3} \vec{fd}\cdot\vec{U}_2 d\Omega + \frac{1}{2}\int_{\Omega_4} \vec{fd}\cdot\vec{U}_1 d\Omega + \frac{1}{2}\int_{\partial\Omega 3 \cap \partial\Omega 1} [\bar{\bar{\sigma}}_2 \cdot \vec{n}_{\Omega 3}]\cdot\vec{U}_1 d\partial\Omega + \frac{1}{2}\int_{\partial\Omega 4 \cap \partial\Omega 2} [\bar{\bar{\sigma}}_1 \cdot \vec{n}_{\Omega 4}]\cdot\vec{U}_2 d\partial\Omega \tag{2}$$

Proof:

To simplify this demonstration, we shall only consider a simplification of the additive type (i.e., feature Ω3). In this case, Eq. (1) is reduced to Eq. (3)

$$e^2 = \frac{1}{2}\int_{\Omega 1} (\bar{\bar{\sigma}}_1 - \bar{\bar{\sigma}}_2):(\bar{\bar{\varepsilon}}_1 - \bar{\bar{\varepsilon}}_2)d\Omega + \frac{1}{2}\int_{\Omega 3} \bar{\bar{\sigma}}_2:\bar{\bar{\varepsilon}}_2 d\Omega \tag{3}$$

We have assumed that the boundary of the simplified domain (i.e., Ω_3 or Ω_4) feature is free, i.e., no boundary conditions applied on it ($\bar{\bar{\sigma}}_1 \cdot \vec{n}_{\Omega 3} = \vec{0}$).

Each integral can thus be transformed by using Green's theorem and integration by parts, leading to this expression (4)

$$e^2 = \frac{1}{2}\int_{\Omega 1} div[\bar{\bar{\sigma}}_1 - \bar{\bar{\sigma}}_2]:(\vec{U}_2 - \vec{U}_1)d\Omega - \frac{1}{2}\int_{\Omega 3} div[\bar{\bar{\sigma}}_2]\cdot\vec{U}_2 d\Omega +$$
$$\frac{1}{2}\int_{\partial\Omega 1} [(\bar{\bar{\sigma}}_1 - \bar{\bar{\sigma}}_2)\cdot\vec{n}_{\Omega 1}]\cdot(\vec{U}_1 - \vec{U}_2)d\partial\Omega + \frac{1}{2}\int_{\partial\Omega 3} [\bar{\bar{\sigma}}_2 \cdot \vec{n}_{\Omega 3}]\cdot\vec{U}_2 d\partial\Omega \tag{4}$$

We can now divide the domain boundary into two parts $\partial^*\Omega$ or $\partial^{**}\Omega$, where respectively the displacement $\vec{Ud}$ or surface tension $\vec{Fd}$ is given. On each domain, local Eq. (5) relate the stresses, body forces $\vec{fd}$ and boundary load $\vec{Fd}$

$$\begin{aligned} &div[\bar{\bar{\sigma}}_1] = -\vec{fd} && \text{on} \quad \Omega \\ &\bar{\bar{\sigma}}\cdot\vec{n}_\Omega = \vec{Fd} && \text{on} \quad \partial^*\Omega \\ &\vec{U} = \vec{Ud} && \text{on} \quad \partial^{**}\Omega \end{aligned} \tag{5}$$

Eq. (5) yield $div[\bar{\bar{\sigma}}_1] = div[\bar{\bar{\sigma}}_2] = -\vec{fd}$ on Ω_1. The first integral of (4) therefore equals zero. The boundary $\partial\Omega 1$ can be divided into $\partial\Omega 2 \cap \partial\Omega 1$ and $\partial\Omega 3 \cap \partial\Omega 1$. On $\partial\Omega 3 \cap \partial\Omega 1$, $\vec{n}_{\Omega 1}$ stands for the opposite of $\vec{n}_{\Omega 3}$. It also becomes possible to transform (4) into (6)

$$e^2 = \frac{1}{2}\int_{\Omega 3} \vec{fd}\cdot\vec{U}_2 d\Omega + \frac{1}{2}\int_{\partial\Omega 2 \cap \partial\Omega 1} [(\bar{\bar{\sigma}}_1 - \bar{\bar{\sigma}}_2)\cdot\vec{n}_{\Omega 2}]\cdot(\vec{U}_1 - \vec{U}_2)d\partial\Omega$$
$$+\frac{1}{2}\int_{\partial\Omega 3} [\bar{\bar{\sigma}}_2 \cdot \vec{n}_{\Omega 3}]\cdot\vec{U}_2 d\partial\Omega - \frac{1}{2}\int_{\partial\Omega 3 \cap \partial\Omega 1} [(\bar{\bar{\sigma}}_1 - \bar{\bar{\sigma}}_2)\cdot\vec{n}_{\Omega 3}]\cdot(\vec{U}_1 - \vec{U}_2)d\partial\Omega \tag{6}$$

On $\partial\Omega 1 \cap \partial\Omega 2$, the boundary conditions involve $\vec{U}_1 = \vec{U}_2$ or $\bar{\bar{\sigma}}_1 \cdot \vec{n}_{\Omega 2} = \bar{\bar{\sigma}}_2 \cdot \vec{n}_{\Omega 2}$, and the second integral of (5) equals zero. Moreover, in order to simplify this expression, we assumed that the boundary of the removed feature is free, i.e., $\bar{\bar{\sigma}}_1 \cdot \vec{n}_{\Omega 3} = \vec{0}$, which yields Eq. (6)

$$e^2 = \frac{1}{2}\int_{\Omega 3} \vec{f}d \cdot \vec{U}_2 d\Omega + \frac{1}{2}\int_{\partial\Omega 3 \cap \partial\Omega 1} [\bar{\bar{\sigma}}_2 \cdot \vec{n}_{\Omega 3}] \cdot \vec{U}_1 d\partial\Omega \qquad (7)$$

In the general case and with simplifications of both the fist and second types, a similar demonstration results in expression (2).

The error e is an absolute error. It proves more meaningful and simpler to introduce a relative error, denoted ε and expressed in percent of strain energy in the problem, as follows

$$\varepsilon = \frac{2e^2}{\int_{\Omega 1} Tr[\bar{\bar{\sigma}}_1 \bar{\bar{\varepsilon}}_1] d\Omega} \approx \frac{2e^2}{\int_{\Omega 2} Tr[\bar{\bar{\sigma}}_2 \bar{\bar{\varepsilon}}_2] d\Omega} \qquad (8)$$

2.2 Error estimator

The computation of error e with Eq. (2) would require knowing the exact solution (quantities with subscript 1) on the boundary of the removed shape feature. This solution is obviously unknown. The following subsections will present two possibilities for evaluating this particular solution.

2.2.1 Direct estimator from finite element results

For a feature simplification of the additive type, $\vec{U}_1$ can be replaced by the solution obtained on the modified part $\vec{U}_2$. The test constitutes a two-dimensional problem with stress plane elements. From the initial domain, the final domain is obtained by removing three features. In order to evaluate error estimation efficiency, the solution is computed on the initial part and then on the simplified part, with these solutions then being used to compute the relative error. Fig. 2 displays the initial domain, the simplified domain and the corresponding boundary conditions. Table 1 demonstrates that this error estimator effectively indicates the importance of each simplification.

Two drawbacks with this direct error estimation procedure can easily be identified:

- For problems such as those depicted in Fig. 3, our criterion cannot differentiate between the two simplifications. For the two tension problems, the simplified problem and error estimation are identical, yet the relative error is ten times higher for the second problem.
- For feature simplifications of the subtractive type, this criterion would not be suitable.

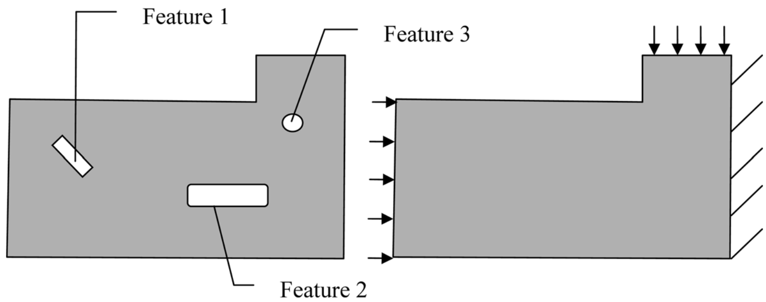

Fig. 2 Two dimensional problem, initial problem with related form features (1, 2, 3) (at left) and simplified problem with prescribed boundary conditions (at right)

Table 1 Comparison between the relative error (ε) and its estimation (ε_{est}) for each removed feature (see in Fig. 2 the location and shape of each feature)

	a = relative error ε	b = error indicator ε_{est}	Robustness index a/b
Feature 1	1, 4%	0, 8%	1.75
Feature 2	4, 6%	3%	1.5
Feature 3	0.65%	0.5%	1.3

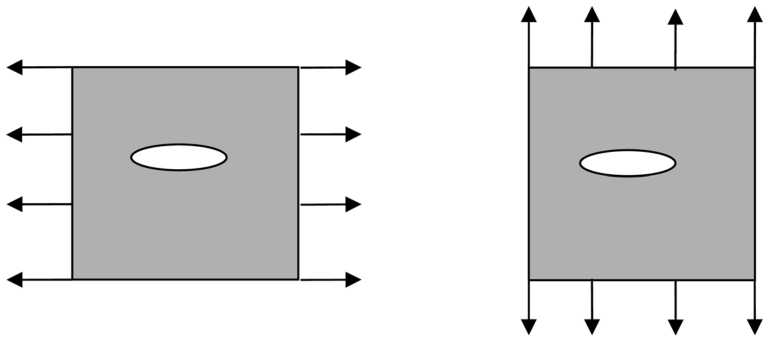

Fig. 3 Two traction problems that generate the same simplified problem after the hole removal

2.2.2 Proposed error estimator

This solution was estimated on the initial domain by using a local computation over a domain surrounding each suppressed detail. Fig. 2 shows an example of such domains (Ω_5 and Ω_6), which correspond to sub-domains Ω_3 and Ω_4 in Fig. 1. The boundary conditions of local problems consist of the displacements $\vec{U}_2$ obtained from the FE computation on the simplified problem Ω_2. The bold lines in Fig. 2 define the boundaries where prescribed displacements from $\vec{U}_2$ serve as boundary conditions for local FE computations in the vicinity of Ω_3 and Ω_4.

By introducing the local FE computations described above, the relative error ε becomes ε_{est}, i.e., the value of the proposed error estimator.

From this approximation, it is now possible to differentiate the two problems in Fig. 3 and obtain a precise error estimation for the two load cases (see Table 2).

2.2.3 Proposed error estimator and discretisation error

The demonstration set forth in Eq. (2) uses properties of the exact solution. The proposed error estimator replaces the exact solution on the simplified problem with the FE results; hence, discretisation errors influence both FE results and our proposed error estimator. A range of tests with various mesh sizes show that these discretisation errors exert only a minor influence on the

Table 2 Comparison between the relative error (ε) and its estimation (ε_{est}) for the removed hole (see in Fig. 3) following the load direction

	a = relative error ε	b = error estimation ε_{est}	Robustness index a/b
Horizontal forces	1, 8%	1, 8%	1
Vertical forces	17%	18, 6%	0, 91

proposed error estimator. In general, the influence of each simplification is quite different and our proposed estimator is able to maintain the right order of magnitude.

3. Effectiveness of error estimator

3.1 Two-dimensional problem

To demonstrate the effectiveness of the adopted error estimator, let's consider the two-dimensional problem with stress plane elements, as illustrated in Fig. 3. The proposed model features a simple geometry, making it possible to easily compute the solution to the FE problem on both the initial and simplified domains, following removal of each detail, i.e., shape feature (see Fig. 3). The FE problem boundary conditions are the same as those given in the previous problem (Fig. 2). Using the corresponding FE mesh, the real relative error ε given by each simplification can be computed and compared with the error estimation obtained from the proposed indicator ε_{est}. Table 3 provides the results for each removed detail. It can be noted that the error and its estimation produced very similar results, i.e., of the same order of magnitude. Due to the discretisation error, $\vec{U}_1$ is not the exact solution to the problem, and the computed error includes not only the error introduced by geometric simplification but also a number of discretisation error effects. In order to test discretisation influence, this same information was related using a thinner mesh (Table 4); using both meshes, the error estimators yield a good assessment of error after each simplification. The coarser mesh offers greater precision, while the user can compare and evaluate the influence of each feature removal thanks to the two tables.

Table 3 Comparison between the relative error (ε) and its estimation (ε_{est}) for each removed feature (see in Fig. 5 the location and shape of each feature)

	a = relative error ε	b = error estimator ε_{est}	Robustness index a/b
Feature 1	0, 28%	0, 22%	1, 27
Feature 2	0, 61%	0, 6%	1, 01
Feature 3	1, 4%	1, 64%	0, 85
Feature 4	0, 65%	0, 78%	0, 83
Feature 5	4, 6%	4%	1, 15

Table 4 Comparison between the relative error (ε) and its estimation (ε_{est}) for each removed feature (see in Fig. 5 the location and shape of each feature), F.E. computation on a finer mesh

	a = relative error ε	b = error estimation ε_{est}	Robustness index a/b
Feature 1	0, 13%	0, 16 %	0, 81
Feature 2	0, 67%	0, 8 %	0, 8
Feature 3	1, 4%	1, 64 %	0, 85
Feature 4	0, 95%	0, 78 %	1, 3
Feature 5	5, 2 %	3, 3 %	1, 57

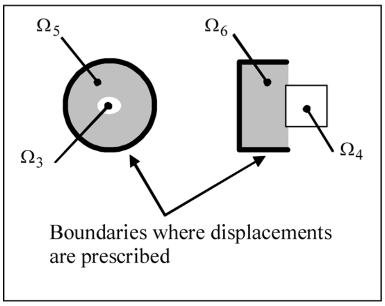

Fig. 4 Neighbouring sub domains, Ω_5 and Ω_6, for the FE local computations around Ω_3 and Ω_4 respectively, used for the simplified problem Ω_2 of Fig. 1

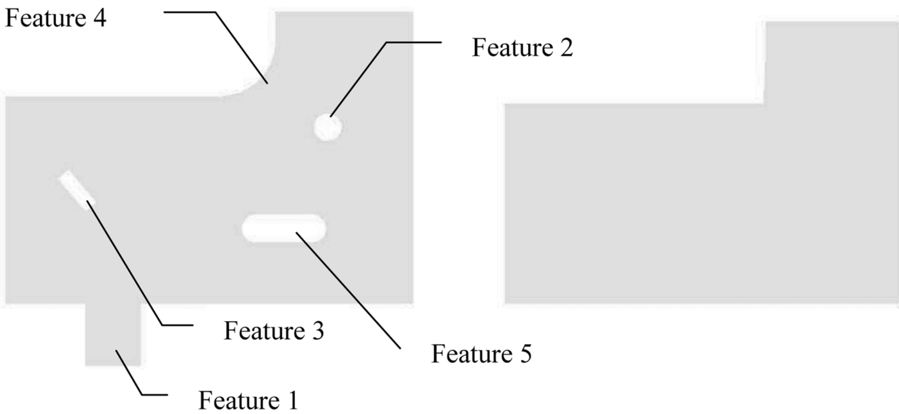

Fig. 5 Two dimensional static problem, initial problem with related form features (1, 2, 3, 4, 5) (at left) and conditions and simplified problem (at right initial part)

3.2 Thermal shell problem

For the second test of adopted error estimator effectiveness, let's consider a linear thermal problem with stationary conduction. This simulation makes use of a shell element (see Fig. 6). The temperature is given on edge 1 and a conduction flux is imposed on edge 2. Table 5 lists the various feature-specific influences and robustness indices: good correlation between the error and its estimation can be observed.

3.3 Three-dimensional problem

The last test is a three-dimensional problem of the linear static type (Fig. 7), in the form of a machine part. The FE problem boundary conditions are a clamped zone (bottom surface) and a uniform pressure zone (left-hand side surface), as depicted in the figures. Table 3 presents the results for each removed detail. It can be remarked that the error and its estimation produced very close results (of the same order of magnitude), and the user can note the limited influence of certain feature removals. It can therefore be assumed that the proposed indicator ε_{est} enables estimating the influence of each shape simplification with good accuracy.

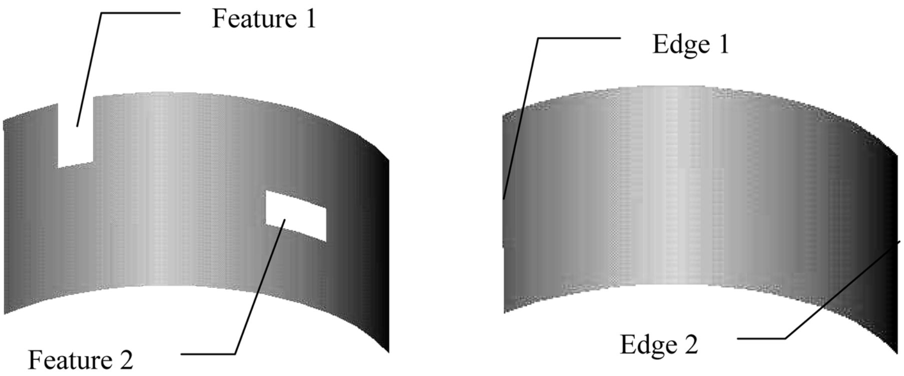

Fig. 6 Shell thermal problem, initial problem with related form features (1, 2) (at left) and and simplified problem (at right initial part)

Table 5 Comparison between the relative error (ε) and its estimation (ε_{est}) for each removed feature (see in Fig. 6 the location and shape of each feature)

	a = relative error ε	b = error estimation ε_{est}	Robustness index a/b
Feature 1	0.64%	0.69%	0.93
Feature 2	2.1%	2.2%	0.96

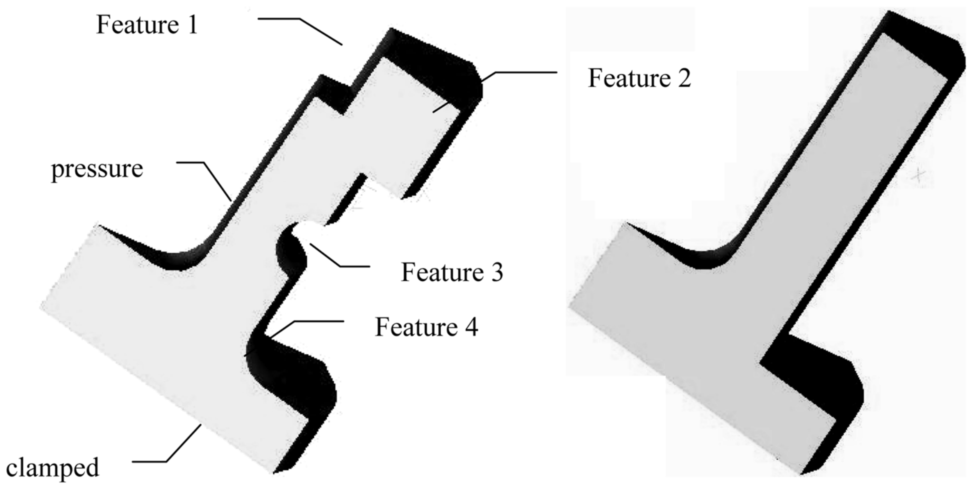

Fig. 7 Three dimensional static problem, initial problem with related form features (1, 2, 3, 4) (at left) and with prescribed boundary conditions and simplified problem (at right)

4. Adaptive simplification process

The *a posteriori* FE error estimator may be incorporated into an adaptive process of shape simplification. The shape of the simplified part can then be refined after an initial FE simulation, depending on the influence of its removed details on FE analysis results. Fig. 8 summarises the complete adaptive modelling process. A simulation is first conducted on a very coarse modelling of the structure, with the user responsible for inputting the accuracy bounds. According to the influence indicator of each simplification and the given accuracy, a selected feature is subsequently added to the first coarse domain.

Such an adaptive process has been illustrated in Fig. 9, which displays the initial CAD model along with the related boundary conditions. This problem is still three-dimensional and of the linear static type. The boundary conditions are as follows: all four feet of the table are clamped, and a force is applied on cylinder 1. Fig. 10 shows the initial simplified model. During the first stage of simplification, 26 shape sub-domains, of both the subtractive and additive types, are suppressed. The initial model cannot be meshed directly because some of the details are too small. After performing a simulation on the simplified domain, the influence indicator for each removed shape feature could be computed. For this example, if the user requires an accuracy of 20%, the simplified model would not need to be readapted. Otherwise, if a 5% accuracy were required, three details

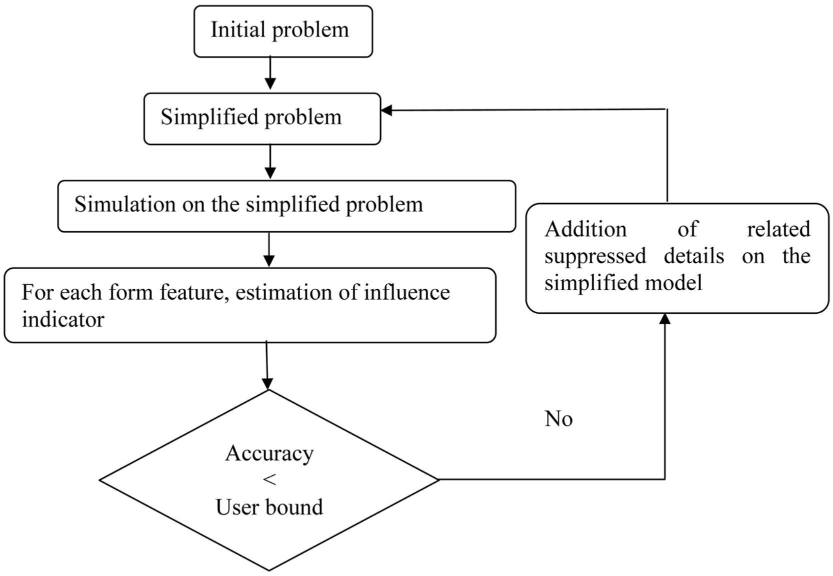

Fig. 8 Preparation of models for the FE analysis, adaptive modelling process

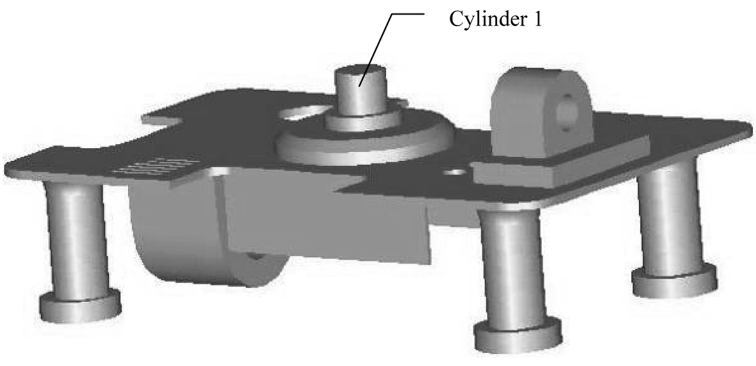

Fig. 9 Adaptive modelling process, initial CAD model

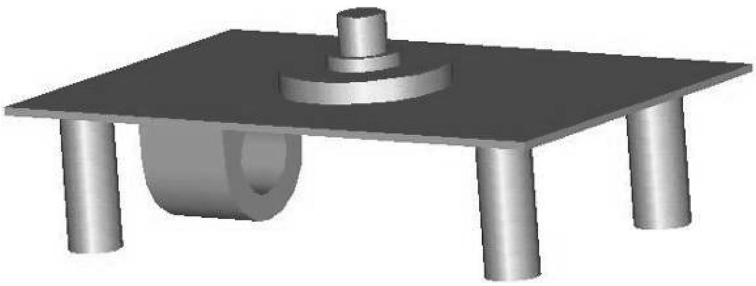

Fig. 10 Adaptive modelling process, simplified model

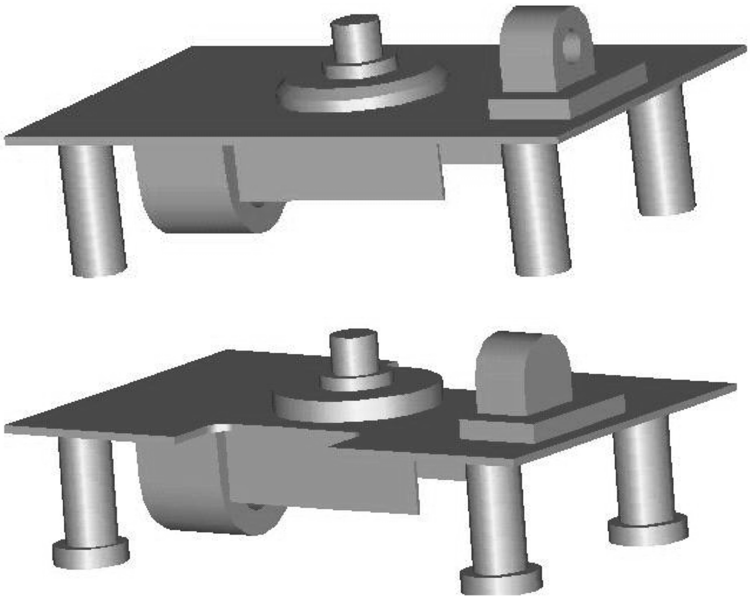

Fig. 11 Adaptive simplified domain following the accuracy given by the user, 5% on the top domain and 1% on the bottom domain

would need to be reinserted into the simplified domain in order to obtain the new adapted domain, as indicated on the top of Fig. 11; moreover, if an accuracy of 1% were required, nine shape features would have to be reintroduced to produce the new adapted domain (bottom of Fig. 11).

To be useful, such an adaptive process needs to be automatic. The adaptive modelling platform requires a different set of tools for each step of the process (Fig. 8).

For the simplification step or to define the adaptive domain, automatic tools are relied upon to suppress or add features. In the example presented, the history tree has been used manually.

For the indicator computation, several tools are introduced for purposes of:

- locating each simplified feature,
- identifying a domain around each simplified feature,
- meshing the feature and its surrounding domain,
- transporting the FE result onto the domain boundary to obtain boundary conditions for the local computation.

Table 6 Comparison between the relative error (ε) and its estimation (ε_{est}) for each removed feature (see in Fig. 7 the location and shape of each feature)

	a = relative error ε	b = error estimation ε_{est}	Robustness index a/b
Feature 1	0, 014%	0, 009%	1, 56
Feature 2	0, 27%	0, 26%	0, 96
Feature 3	8, 4%	8, 3%	1, 01
Feature 4	16%	18%	0, 89

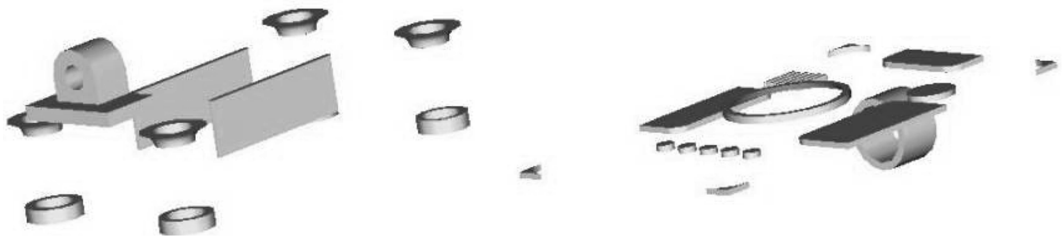

Fig. 12 Results of the Boolean operations, Ω1-Ω2 on the left and Ω2-Ω1 on the right

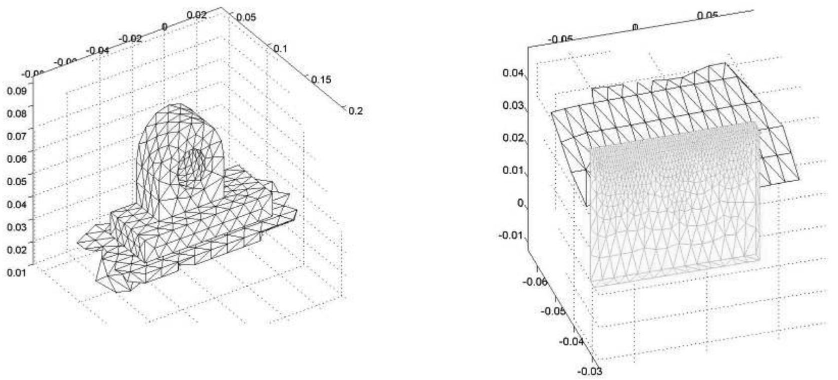

Fig 13 Examples of mesh around simplified feature for the local computation

For this example, the tools on hand use commercial CAD software and the process involves:
- conducting Boolean operations between the simplified and initial CAD models (Fig. 12),
- meshing the Boolean operation result,
- partitioning this mesh in the various feature meshes,
- using both the simplified mesh and the feature mesh to define a new mesh around the feature (Fig. 13). For this local problem, we used an FE mesh of the removal detail along with a subset of the FE mesh of the simplified problem; this subset was formed by the subset of FE elements from this mesh lying closest to the boundary of the suppressed feature and could be defined using a finite number of FE element layers in the vicinity of this feature. However, since FE size in the vicinity of a removed sub-domain is typically large, a small number of layers should always enable achieving a correct transmission of the mechanical fields, in accordance with

Saint-Venant's principle. According to tests performed in the past, the use of 2 or 3 FE layers appears to offer an acceptable solution.

- The two meshes introduced for the local computation are non-conforming (note: an FE mesh is considered conforming if the intersection result of two distinct elements is either empty or a vertex or an edge or a face - should the FE elements be volumes - common to the considered elements). A kinematic linear relation can then be defined to link the FE meshes. Tests have shown that the accuracy of error estimation is not significantly influenced by this non-conformity.

Let's also point out that the goal here is to simply evaluate the order of magnitude of the influence of each shape detail. The various examples presented have indicated that these orders of magnitude differ greatly, implying that the error estimation does not need to be extremely accurate.

5. Conclusions

Result accuracy monitoring represents one of the main problems associated with numerical simulations. This accuracy depends on a number of factors, including discretisation, boundary condition modelling, the constitutive law and geometric simplifications. We have proposed an *a posteriori* error estimator to quantify the influence of geometric simplification. Several tests have shown that this error and its estimation yield very similar results, i.e., of the same order of magnitude. It can therefore be assumed that the proposed indicator enables estimating the influence of each shape simplification with good accuracy. Use of this indicator in an adaptive modelling process helps define the appropriate adaptive domain for the level of accuracy prescribed by the user.

References

Babuska, I. and Rheinbold, W.C. (1978), "A posteriori error estimates for the finite element method", *Int. J. Numer. Meth. Eng.*, **12**, 1597-1615.

Cohen, J., Manocha, D., Varshney, A. and Turk, G. (1995), "Efficient model simplification with global error bounds", *Proceedings of the 5th MSI Worshop on Computational Geometry*, Stony Brook, October.

Dabke, P., Prabhakar, V. and Sheppard, S. (1994), "Using features to support finite element idealisation", *Proceedings of the International Conference ASME*, Minneapolis.

Ferrandes, R., Léon, J.C., Marin, P. and Giannini, F. (2006), "Semantic operators for handling shape sub-domains in the FE model preparation", *Proceedings of ASME DETC International Conference,* Las Vegas.

Ferrandes, R., Marin, P., Léon, J.C. and Giannini, F. (2009), "A posteriori evaluation of simplification details for finite element model", *Comput. Struct.*, **87**, 73-80.

Fine, L. and Leon, J.C. (2005), "A new approach to the Preparation of models for F.E. analyses", *Int. J. Comput. Appl.*, **23**(2/3/4), 166-184.

Fine, L., Rémondini, L. and Léon, J.C. (2000), "Automated generation of FEA models through idealization operators", *Int. Numer. Meth. Eng.*, **49**(1-2), 83-108.

Foucault, G., Marin, P. and Léon, J.C. (2004), "Mechanical criteria for the preparation of finite element model", *Proceeding of the 13th International Meshing Roundtable*, Williamsburg, USA.

François, V. and Cuillière, J.C. (2000), "Automatic remeshing applied to model modification", *Comput. Aided Design,* **32**(7), 377-388.

Gopalakrishnan, S.H. and Suresh, K. (2007), "A formal theory for estimating defeaturing-induced engineering analysis errors", *Comput. Aided Design*, **39**(1), 60-68.

Hamri, O., Léon, J.C., Giannini, F., Falcidieno, B., Poulat, A. and Fine, L. (2006), "Interfacing product views through a mixed shape representation. Part 1: Data structures and operators", *Proceedings of Virtual Concept 2006*, Playa Del Carmen, Mexico.

Joshi, N. and Dutta, D. (2003), "Feature simplification techniques for Freeform surface models", *J. Comput. Inf. Sci. Eng.,* **3**(3), 177-186.

Kim, C., Mijar, A. and Arosa, J. (2001), "Development of simplified models for design and optimization of automotive structures for crashworthiness", *Struct. Multidiscip. O*., **22**(4), 307-321.

Ladeveze, P. and Leguillon, D. (1983), "Error estimate procedure in finite element method and application", *SIAM J. Numer. Anal.*, **20**(3), 485-509.

Ladeveze, P., Marin, P., Pelle, J.P. and Gastine, J.L. (1992), "Accuracy and optimal meshes in finite-element computation for nearly incompressible materials", *Comput. Method. Appl. M*., **94**(3), 303-315.

Ladeveze, P., Pelle, J.P. and Rougeot, P. (1991), "Error estimation and mesh optimization for classical finite elements", *Eng. Computation,* **8**, 69-80.

Lee, K., Chong, T. and Park, G.J. (2003), "Development of a methodology for a simplified finite element model and optimum design", *J. Comput. Struct.*, **81**, 1449-1460.

Livne, E. (1994), "Equivalent plate structural modelling for wing shape optimization including transverse shear", *AIAA J.*, **32**(6), 1278-1288.

Léon, J.C., Marin, P. and Foucault, G. (2004), "Operators and criteria for integrating FEA in the design workflow: Toward a multi-resolution mechanical model Mechanical criteria for the preparation of finite element model", *Proceeding of the 13th European Conference on Mathematics for Industry (ECMI2004),* Eindhoven, The Netherlands.

Machnik, A., Marin, P. and Souza, E. (1998), "Simplification of FEM models within Integrated Environment. Application to automobile crashworthness analysis", *Proceeding of the 2nd International Conference on Integrated Design and Manufacturing (IDMME'98)*, Compiègne, France.

Mobley, A.V., Caroll, M.P. and Canann, S.A. (1998), "An object oriented approach to geometry defeaturing for Finite Element Meshing", *Proceeding of the 7th International Meshing Roundtable*, Dearborn, USA.

Schroeder, W.J., Zarge, J.A. and Lorensen, W.E. (1992), "Decimation of triangle meshes", *Proceedings of SIGGRAPH92 Conference*, *Computer Graphics*, July.

Szabo, B.A. (1996), "The problem of model selection in numerical simulation", *Advances in Computational Methods For Simulation* (Ed. Topping, B.H.V.), Civil-Comp Press, Edinburgh.

Veron, P. and Léon, J.C. (1997), "Static polyhedron simplification using error measurements", *Comput. Aided Design*, **29**(4), 287-298.

Véron, P. and Léon, J.C. (1998), "Shape preserving polyhedral simplification with bounded error", *Comput. Graphic.*, **22**(5), 565-585.

Yoshimura, M. (1998), "Detailed machine structure shapes generated from simplified models", *Proceeding of the IUTAM Symposium on Structural Optimization*, 387-394.

Zhienkiewicz, O.C. and Zhu, J.Z. (1987), "A simple error indicator and adaptive procedure for practical engineering analysis", *Int. J. Numer. Meth. Eng.*, **24**, 337-357.

Zhu, H. and Menq, C.H. (2002), "B-Rep model simplification by automatic fillet/round suppressing for efficient automatic feature recognition", *Comput. Aided Design*, **34**(2), 109-123.