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Mass optimization of four bar linkage using genetic algorithms with dual bending and buckling constraints

M.R.A. Hassan[1], I.A. Azid[2]*, M. Ramasamy[2], J. Kadesan[2], K.N. Seetharamu[3], A.S.K. Kwan[4] and P. Arunasalam[5]

[1]*Faculty of Mechanical Engineering, Penang Campus, Universiti Teknologi MARA, 13500 Permatang Pauh, Pulau Pinang, Malaysia*
[2]*School of Mechanical Engineering, Engineering Campus, Universiti Sains Malaysia, 14300 Nibong Tebal, Pulau Pinang, Malaysia*
[3]*M.S. Ramaiah School of Advanced Studies, New B E L Road, Bangalore-560054, India*
[4]*Division of Structural Engineering, Cardiff School of Engineering, PO Box 686 The Parade, Cardiff CF24 3TB, U.K.*
[5]*Department of Mechanical Engineering, T.J. Watson School of Mechanical Engineering, State University of New York at Binghamton, NY 13902, USA*

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Abstract. In this paper, the mass optimization of four bar linkages is carried out using genetic algorithms (GA) with single and dual constraints. The single constraint of bending stress and the dual constraints of bending and buckling stresses are imposed. From the movement response of the bar linkage mechanism, the analysis of the mechanism is developed using the combination of kinematics, kinetics, and finite element analysis (FEA). A penalty-based transformation technique is used to convert the constrained problem into an unconstrained one. Lastly, a detailed comparison on the effect of single constraint and of dual constraints is presented.

Keywords: mass optimization; bar linkage; finite element analysis; genetic algorithms; buckling constraints.

1. Introduction

The four bar linkage is the basis of all mechanism and is made up of links and joints. In most mechanism tasks, the process requires the transfer of a single input to a single output (Erdman and Sandor 1984). Thus, it has a wide range of applications in industries such as in mechanical systems and devices. This four bar linkage mechanism consists of two rotating links as input and output links, a coupler, and a virtual link or ground. Kinematics analysis is carried out to determine the position, velocity, and acceleration for each bar linkage. Several techniques for the kinematics analysis can be used for the four bar linkage mechanism, such as those presented by Hartenberg and Denavit (1964) and Shingley and Uicker (1980). The loop closure method, based on the described vector of close loop obtained from the linkage orientation, is commonly used because it is powerful

*Corresponding author, Ph.D., E-mail: ishak@eng.usm.my

and robust. In addition, path description and path optimization can be directly obtained using this method. As such, the kinetic analysis is carried out to address the problem of determining the forces developed in the bar linkage mechanism in moving condition. Kinetic analysis can be carried out using any of the available methods such as energy method and graphical force analysis or superposition (Norton 1999, Waldron and Kinzel 2004, Yan and Yan 2009, Zhou 2009) and linear simultaneous equation solution (Chong *et al.* 2005, Venanzi *et al.* 2005, Khatait *et al.* 2006). These methods are based on equilibrium equations using the basic principles of the summation of forces and moments that are equal to zero.

The need to include stress analysis in the mechanism of bar linkage has long been proposed, but limited work has been carried out in this field. The developed stress in the bar linkage is induced by the existing dynamics forces in the rotational condition. In most cases, the stress of the part under the dynamics load limits the design, while in some cases, the stress is excessive. Thus, in designing for the minimum mass of the bar linkage mechanism, the dynamic forces analysis is important for stress analysis while maximizing the strength and stiffness needed to withstand dynamics forces. In this paper, the finite element analysis (FEA) is used to determine the stress developed in the bar linkage, as this method is widely used for cases involving structural analysis such as beam analysis (Chen and Yang 2000, Avilés *et al.* 2009) and frame structure (Cameron *et al.* 2000, Robinson 1996).

Many techniques, including genetic algorithms (GA), have been developed and used for the optimization of member in structures such as trusses, bridges, automotive structural optimization, and so on. In this paper, a similar technique of structural optimization using GA is applied in the mass optimization of the bar linkage mechanism. Although in bar linkage mechanism GA is widely applied to the problem of path generation (Laribi *et al.* 2004, Cabrera *et al.* 2002, Smaili and Rick 1996, Nariman-zadeh *et al.* 2009, Shen *et al.* 2009), so far, the authors have not found any work on mass optimization using GA coupled with the FEA. Thus, the present study focuses on mass optimization of bar linkage using GA with FEA to determine the optimal mass constrained by both bending and buckling stress.

This paper is divided into three main sections. The first section gives a literature review of mechanism analysis. The second section describes kinematic analysis and the technique used in FEA to solve the developed stresses in the bar linkages and how this technique can be extended to find the optimum mass through the application of GA. The final section presents the result of the mass optimization of the four bar linkage with both single constraint and dual constraints.

2. Application of optimization procedure to the four bar linkage mechanism

2.1 Analysis of bar linkage mechanism

The analysis of bar linkage mechanism involves kinematics and kinetic analyses (Erdman and Sandor 1984), which analyze a combination of position, velocity, and acceleration to determine the motion properties useful in finding the forces in each bar linkage. The FEA is coupled with such analyses to determine the stresses in each bar linkage. Fig. 1 shows a typical four bar linkage mechanism. The masses of the bearings and the effects of friction are negligible, but the effect of the gravity of the bar is counted. The positions of links 3 and 4, which are represented by angles θ_3 and θ_4, need to be determined, as angle θ_2 is an independent variable. The pivot at the crank or

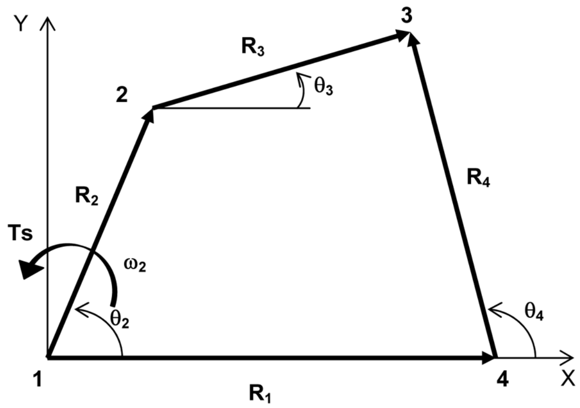

Fig. 1 The four bar linkage mechanism

link1 is taken as the origin with respect to the global XY system. A well-known loop closure method is used for this position analysis. Using the directions of the vectors as shown, the vector loop equation is given by

$$\mathbf{R}_2 + \mathbf{R}_3 - \mathbf{R}_4 - \mathbf{R}_1 = 0 \tag{1}$$

Using the complex notation, each position vector is substituted with a polar form to obtain the following

$$L_2 e^{j\theta_2} + L_3 e^{j\theta_3} - L_4 e^{j\theta_4} - L_1 e^{j\theta_1} = 0 \tag{2}$$

The unknown angle can be solved simultaneously by substituting the Euler identity, as given by

$$L_n \cos\theta_n + jL_n \sin\theta_n \tag{3}$$

into Eq. (2) and by separating the resulting Cartesian form vector equation from its real and imaginary parts, setting each part to zero.

By taking the gradient of position vector Eq. (2) once and twice, the angular velocity and acceleration of each link can be obtained. The angular velocity equation is given by

$$jL_2\omega_2 e^{j\theta_2} + jL_3\omega_3 e^{j\theta_3} = jL_4\omega_4 e^{j\theta_4} \tag{4}$$

while the angular acceleration is given by

$$jL_2\alpha_2 e^{j\theta_2} + j^2 L_2\omega_2^2 e^{j\theta_2} + j^2 L_3\omega_3^2 e^{j\theta_3} + jL_3\alpha_3 e^{j\theta_3} = j^2 L_4\omega_4^2 e^{j\theta_4} + jL_4\alpha_4 e^{j\theta_4} \tag{5}$$

where ω_2 and α_2 are the input or known angular velocity and acceleration, and ω_3 and ω_4, and α_3 and α_4 are the unknown angular velocity and acceleration, respectively. By identifying the input angular velocity and acceleration, the unknown angular velocity and acceleration can be determined by using the same strategy carried out for the position analysis.

After determining the position, velocity, acceleration, and inertia properties, such as mass and mass moment of inertia, for each bar, the force analysis of the bar linkage can be carried out. This force analysis is obtained from the summation of all forces and torques in the system. The general notation for an equilibrium equation from the free body diagram of each link is given by

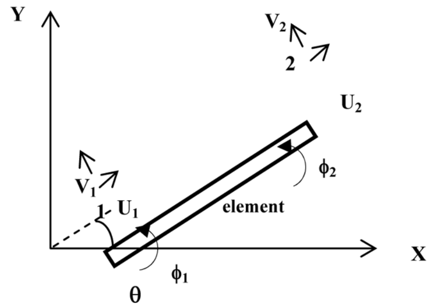

Fig. 2 A beam finite element model and nodal

$$F_{kl} + F_{lm} + \Sigma F_{ext\,l} = m_l a_{Gl} \tag{6}$$

$$(R_{kl} \times F_{kl}) + (R_{lm} \times F_{lm}) + (R_{ext\,l} \times \Sigma F_{ext\,l}) = I_{Gl} \alpha_{Gl} \tag{7}$$

where

$l = 2, 3, \ldots, n$; n = pin connection
$k = l - 1$; i = previous link
$m = l + 1$, $l \neq n$; m = next link
if $l = n$, $m = 1$ $F_{li} = -F_{kl}$; $F_{ml} = -F_{li}$

From Eq. (6), the sum of the forces can be separated into the X and Y component equation. These two equations, along with Eq. (7), can be solved simultaneously. The weight of the link can be treated as an external force acting on the center of gravity of the link at a constant angle. The set of simultaneous equations can be solved by a matrix method using any programming language.

The FEA is carried out to find the stresses in each bar linkage using the stiffness and force matrix. Each link is modeled by one planar beam finite element with three degrees of freedom at each node and is represented by horizontal displacement U, vertical displacement V, and end rotation ϕ, as shown in Fig. 2. The local displacements obtained from the FEA are used to find the axial and bending stresses. The element stiffness matrices are formulated in the local coordinate system. In the 2D beam-element FEA formulation, the local displacements can be related to global displacements by the beam equation

$$\{d\} = [T]\,\{D\} \tag{8}$$

where $\{d\}$ is the local displacement vector with $\{d\}^T = \{U_1\ V_1\ \phi_1\ U_2\ V_2\ \phi_2\}$
and $[T]$ is the transformation matrix.

The global displacement matrix can be derived from the global stiffness matrix as given by

$$\{F\} = [K]\{D\} \tag{9}$$

where $[K]$ is the total global element stiffness matrix, and $\{F\}$ is the total force vector of the whole structure.

In general, the global stiffness matrix $[K]$ for a beam element includes the axial force, shear force, and bending moment effects, as shown in Eq. (10). The element forces $\{F\}$ can be obtained using the set of simultaneous equations from Eqs. (6) and (7).

$$K = \frac{E}{L} \times \begin{bmatrix} AC^2 + \frac{12I}{L^2}S^2 & \left(A - \frac{12I}{L^2}\right)CS & -\frac{6I}{L}S & -\left(AC^2 + \frac{12I}{L^2}S^2\right) & -\left(A - \frac{12I}{L^2}\right)CS & -\frac{6I}{L}S \\ & AS^2 + \frac{12I}{L^2}C^2 & \frac{6I}{L}C & -\left(A - \frac{12I}{L^2}\right)CS & -\left(AS^2 + \frac{12I}{L^2}C^2\right) & \frac{6I}{L}C \\ & & 4I & \frac{6I}{L}S & -\frac{6I}{L}C & 2I \\ & & & AC^2 + \frac{12I}{L^2}S^2 & \left(A - \frac{12I}{L^2}\right)CS & \frac{6I}{L}S \\ & & & & AS^2 + \frac{12I}{L^2}C^2 & -\frac{6I}{L}C \\ symmetry & & & & & 4I \end{bmatrix} \tag{10}$$

where
A = cross sectional area of link
$C = \cos\theta$
$S = \sin\theta$
I = moment of inertia

The local displacements are used to find the axial and bending stresses. The axial stress in each bar is given by Eq. (11)

$$\sigma = \frac{E}{L}[(U_2 - U_1)\cos\theta + (V_2 - V_1)\sin\theta] \tag{11}$$

while the bending moment in each node can be defined by Eq. (12)

$$M = EI\left[\left(-\frac{6}{L^2} + \frac{12x}{L^3}\right)(V_1) + \left(-\frac{4}{L} + \frac{6x}{L^2}\right)(\phi_1) + \left(\frac{6}{L^2} - \frac{12x}{L^3}\right)(V_2) + \left(-\frac{2}{L} + \frac{6x}{L^2}\right)(\phi_2)\right] \tag{12}$$

where at node 1, $x = 0$ and at node 2, $x = L$.

This is particularly important to ensure that the bending stress does not exceed the allowable bending stress. Finally, the bending stress in each nodal is given by

$$\sigma_b = \frac{M}{I_{zz}} \tag{13}$$

where I_{zz} = section modulus of the beam.

In this paper, buckling is taken as another mode of failure, which can be developed with the bending stress. In the calculation of buckling, the design variables, including cross sectional areas, are applied in the moment of inertia because the critical load buckling, P_{cr}, is computed by the following formula

$$P_{cr} = \frac{\pi^2 EI}{(kL)^2} \tag{14}$$

where k = effective length factor. In this case, the bar linkage is assumed as the pin end with $k = 1$ (Hibbeler 2002).
L = length of the link

The critical stress, σ_{cr}, for buckling is then found by

$$\sigma_{cr} = \frac{P_{cr}}{A} \tag{15}$$

2.2 Implementation of genetic algorithms (GA)

Nowadays, GA takes the dominant position in optimization problem. GA is a probabilistic method that applies a search from a population and proposes a set of equally good resolutions to the designer. It uses directed random search to find the optimal solution in the complex area. This randomness offers GA robustness and the capability to improve solutions as well. In GA, the random generation of an initial population is the first step in the optimization process. The initial population is generated randomly between preset lower and upper limits, which comprise binary digits. GA starts by randomly generating an initial population, which is represented by a binary string (chromosomes) designed without any constraint. It works on the initial population for an optimization problem, as measured against the objective function or fitness function. In this paper, binary numbers are formulated to become real numbers and are taken as variables representing the cross sectional of area linkages. The length, ℓ, of the binary number code representing the solution for a number in the range between a and b is given by

$$2^{\ell} - 1 \leq (a - b) \cdot 10^{d} \tag{16}$$

where d is the number of decimal places required.

For example, if the solution can be found between 0 and 31, then a = 31 and b = 0. With 2 decimal places, d = 2, and the length of chromosome, ℓ, is 12. Therefore, a binary string of length 12 can represent the value of real numbers from 0.00 to 40.95. Table 3 shows several examples of randomly generated cross sectional areas of the four bar linkages denoted by A_1, A_2, A_3, B_1, B_2, and B_3.

After the population is created, the string evaluation called fitness is carried out. The fitness value is the performance of the members in the population, in this case, the mass of the bar linkage. A fitness function is derived from the objective function and is used in successive genetic operations.

Table 1 Length of each bar linkage

Bar reference	Length (m)
L_1	0.9144
L_2	0.3048
L_3	0.9144
L_4	0.7620

Table 2 Material properties of the four bar linkage mechanism

Material properties of the four bar linkage mechanism	Magnitude
Modulus of Elasticity, Aluminum (Alloy 6061-T6), E	68.95 GPa
Density of Aluminum (Alloy 6061-T6), ρ	2.757×10^3 kg/m^3
Range of cross sectional area for each solid circular bar, X_s	1 cm^2-20 cm^2

Table 3 Generated cross sectional area for linkages

Area references	Binary	(Cross sectional areas) divide 100
A_1	000100000000	5.12
A_2	001100000000	15.36
A_3	001100000001	15.37
B_1	000000000001	0.01
B_2	000100010000	5.28
B_3	001010000100	12.99

Next, based on individual fitness, a pair of string selected for reproduction, undergoes an exchange of genetic information such as crossovers and mutations, where chromosomes are paired and randomly altered. The selection process is a mimic of the natural phenomenon of survival of the fittest by allocating the fitter individual a higher chance of being chosen for genetic operators. In other words, it is a process of selecting highly fit individuals for the crossover and mutation processes. In GA, a commonly employed "roulette wheel" is used to implement the selection of the string. After the strings have been selected, the population for the next generation is created. The resulting offsprings become part of the next generation, and the process is repeated until a desired convergence limit or number of generation is achieved. This population depends on the selection operator as well as on the crossover and mutation probability.

The crossover is the most important operator in the genetic process. It is applied to produce a new generation by combining ranking and selection. The crossover process is used when two strings are paired up to exchange genetic information. In other words, it is considered the exchange of segments between two parent strings to produce an offspring string accordingly, as shown in Fig. 3. The parent strings are represented by cross sectional areas, as shown in Table 3. The Crossover works with the genetic material provided by the two parents and produces children that possess a genetic resemblance with that of the parent. Applied with high probability, p_c, the crossover will only be invoked if the randomly generated number is within p_c. If not, then the child strings simply adopt the parent genetic code without modification.

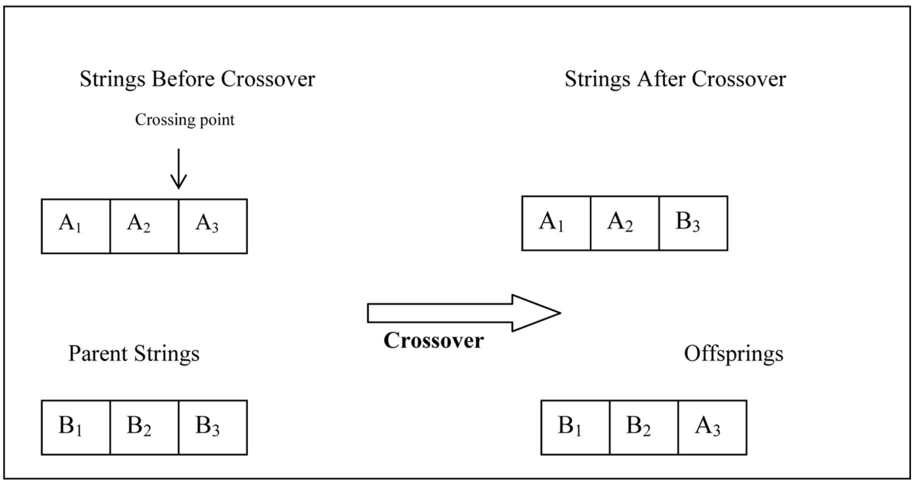

Fig. 3 Representation of the implementation of the crossover

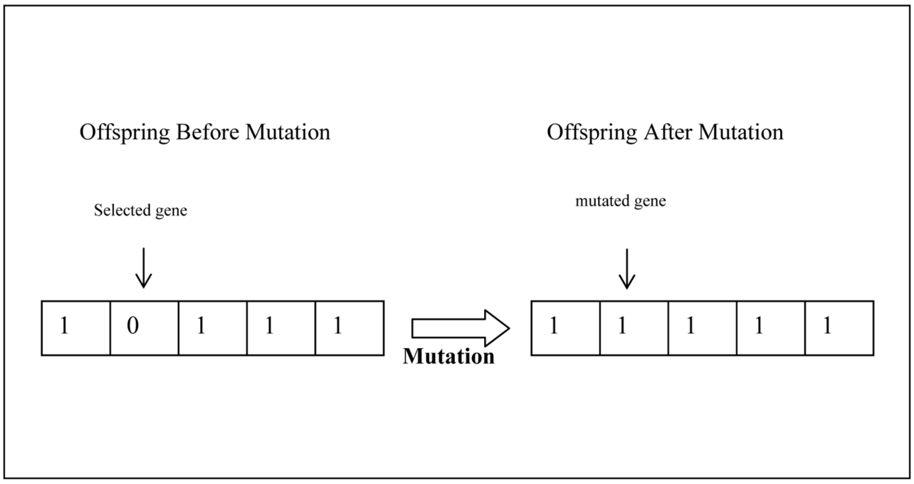

Fig. 4 Representation of the implementation of the mutation

Mutation is the stochastic operator used to protect against the loss of genetic diversity. It is performed after the crossover process. During mutation, a single bit is simply flipped from 1 to 0 or from 0 to 1, as shown in Fig. 4. This is applied at a low probability, p_m. Mutation will take place if a randomly generated number is below such value of probability. The low occurrence of mutation would ensure that wholesome genetic information resulting from the crossover is not largely lost by excessive mutation. In other words, a small rate has the effect of preventing the optimization, narrowing to a small focus without being disruptive to the local search carried out by the crossover.

The algorithm repeats the abovementioned process by generating a new population and by evaluating its fitness and constraint violation. The entire process of evaluation and reproduction continues until either the population converges to an optimal solution for the problem or reaches a certain predetermined number of solutions. In this paper, the individual with the smallest fitness value without violating the constraints in current population represents the optimum mass of the bar linkage.

2.3 Cross sectional area optimization using genetic algorithm (GA) of bar linkage

In this paper, the optimum cross sectional areas of each bar linkage is found for a given stress limit. GA is used in this sizing optimization because it is considered one of the most effective and robust optimization technique (Osyczka and Kundu 1995). As GA is suited for unconstrained optimization problems, a penalty-based transformation method (Rajeev and Krishnamoorthy 1990) is applied in this paper for constrained optimization. The mathematical programming formulation of these problems can be written as follows (Rajeev and Krishnamoorthy 1992)

$$\text{Minimize } f(x) \text{ subject to } g_i(x) \leq 1,\ i = 1, 2, ..., m$$

where m is the number of constraints.

As the objective function is to minimize the mass, $f(x)$ for the problems can be written as

$$f(x) = \sum_{j=1}^{j} \rho A_j L_j \tag{17}$$

where A_j = the cross-sectional area of jth member
L_j = the length of jth member
ρ = the density of material

and $g_i(x)$ can be written as

$$\sigma_{b_j} \le \sigma_a \quad \text{for } j = 1, 2, 3, \ldots \tag{18}$$

where σ_{b_j} = bending stress or buckling stress in member j
σ_a = allowable bending or buckling stress

The formulation based on the violations of normalized constrained is used in the transformation method (Rajeev and Krishnamoorty 1990). The constraints are expressed in normalized form as shown below

$$\frac{\sigma_{b_j}}{\sigma_a} - 1 \le 0 \tag{19}$$

A violation coefficient C is computed in the following manner:
If $g_i(x) > 1$, then $C_i = g_i(x)$; or if $g_i(x) \le 1$, then $C_i = 0$

$$C = \sum_{j=1}^{m} C_j \tag{20}$$

where m = the number of constraints.
The modified objective function $\phi(x)$ is written incorporating the constraint violations as

$$\phi(x) = f(x)(1 + KC) \tag{21}$$

where the parameter K has to be judiciously selected depending on the violated individual in the next generation of GA. A value of 10 is chosen for all the problems as suggested by Rajeev and Krishnamoorthy (1990).

3. Numerical example

The four bar linkage mechanism is considered for the illustration of the current study. The objective is to minimize the mass of the four bar linkages mechanism using the single stress constraint and dual stress constraints. The results of the optimization of the four bar linkage mechanism with single constraint (bending stress constraint) and optimization of the four bar linkage mechanism with dual constraints (bending and buckling stress constraints) will be presented as follows.

3.1 Optimization of four bar linkage mechanism with bending constraint only

Based on Fig. 1, the driving link L_2 is rotated with a constant angular velocity of $\omega = 10\pi$ rad/s. The links are considered to be elastic. Their properties are presented in Tables 1 and 2 (Toporov and Markine 1998).

As common to the optimization using GA, the initial population of 80 chromosomes representing the cross sectional areas of link L_2, L_3 and L_4 is randomly generated. The modified objective

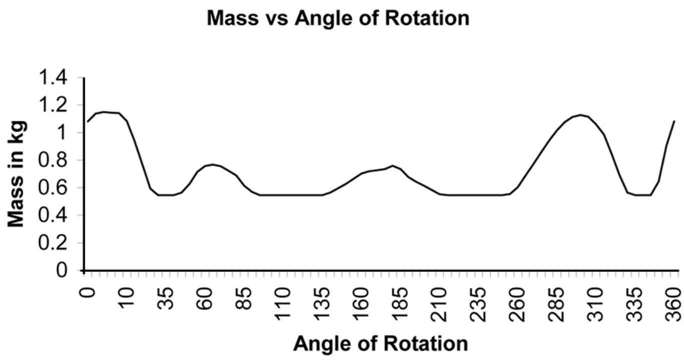

Fig. 5 Optimum mass at each angle of the four bar linkage mechanism (1°-360°)

function, as discussed in Section 2.3, is embedded in the GA as the fitness function. The optimization procedure here seeks the minimum bar linkage cross sectional area with the allowable bending stress, σ_{al}, of 55.5 MPa as constraint in a predetermined number of generations. Meanwhile, the synthesis analysis is carried out first with $\alpha_2 = 0$, as no acceleration is imposed, and angle θ_2 can be determined by

$$\theta_2 = \omega_2 t \quad \text{where} \quad 0 \le t < 0.2s \tag{22}$$

From this equation, the angle rotates from 1° to 360° with angular velocity, ω_2, of 10π rad/s. Physically, the rotation increases if ω_2 is increased. The mass changes at every angle due to the change in angle representing the position of the node of the reference to a fixed axis (node 1 as shown in Fig. 1) and the generated cross sectional area in each bar of linkages. The change in angle will result in different values of global stiffness matrix, as the bending stress depends on the developed force due to the rotation or movement of the bar linkage mechanism.

The optimum mass at every angle at 1° to 360° obtained from the current optimization technique is shown in Fig. 5. From this figure, it is clear that the change in angle gives different values of the optimal mass. The optimal mass varies from 0.548 kg to 1.14 kg, depending on the angle of rotation. The optimal mass of the four bar linkage mechanism is found at 4° of rotation with an optimal mass value of 1.14 kg. Although the mass of 0.548 kg at angle 25° is seen as the lowest optimal mass, as it is the lightest, it will violate the constraint at angles other than 25°. Hence, it can be concluded that the optimal design for the four bar linkage mechanism with a mass of 1.14 kg is obtained at an angle of 4° along with the cross sectional area in each link, which is acceptable at every angle of rotation.

Fig. 6 shows the graph of the modified objective function against the number of generation at an angle of 4° of rotation. From this figure, typical to the optimization with GA, it can be observed that the optimum mass is reduced from 1.93 kg to 1.14 kg as the number of generation increases until the mass becomes constant at generation 40. Therefore, the lightest mass for the four bar linkage is 1.14 kg, with a cross sectional area at links 2, 3, and 4 at 4.99, 2.04, and 1.00 cm^2, respectively.

To validate 1.14 kg as indeed the optimum mass value, a graph of the mass against the number of possible solution is generated, as shown in Fig. 7(a). The value of the mass is generated at every possible value of the cross sectional areas of the bar linkage. The search space for this problem is from 1 cm^2 to 10 cm^2 with an increment of 0.01 cm^2. For example, one set of possible solution, which can be the cross sectional areas for all bar linkages, is taken as 1 cm^2 from which the mass

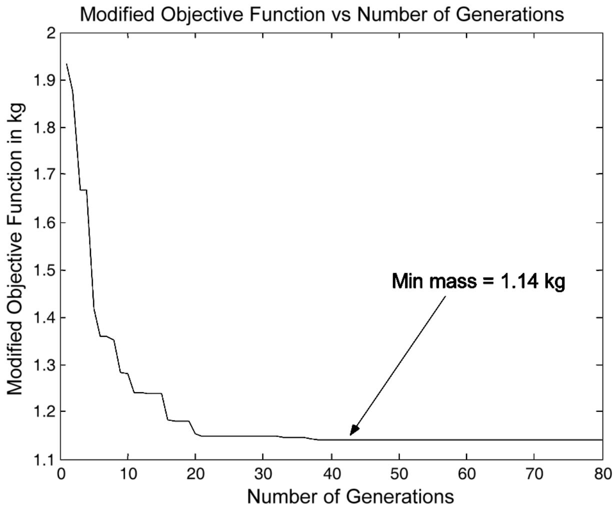

Fig. 6 Minimum mass of the four bar linkage mechanism at an angle of 4° rotation for the number of generations

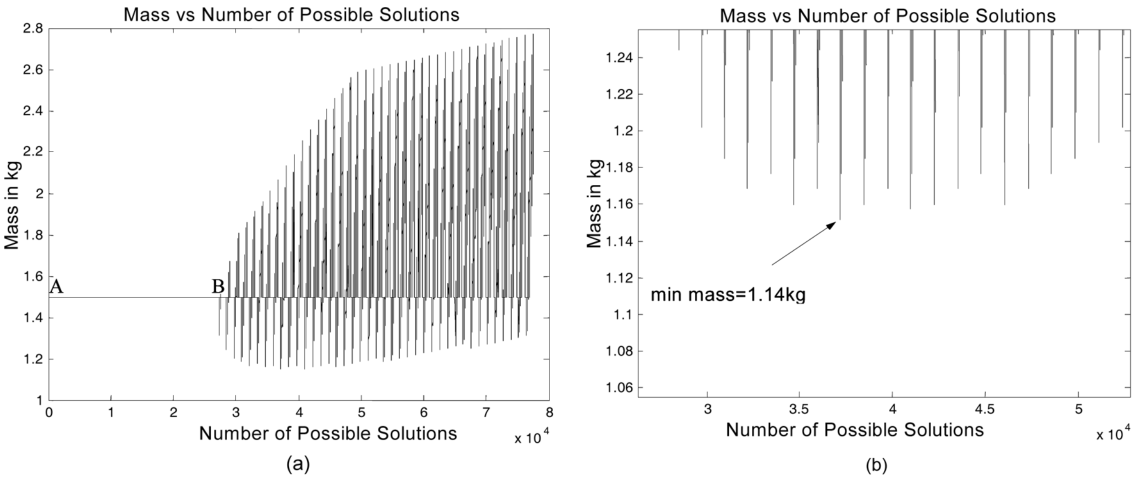

Fig. 7 (a) Number of possible solutions for the mass of the four bar linkage mechanism with a single constraint, (b) Zooming in the lowest section of Fig. 7(a)

0.546 kg is obtained. The second possible solution can then be taken with A_1, A_2, and A_3 equal to 1, 1.01, and 1 cm^2, respectively, in which the mass is 0.549 kg. This process of determination of the possible solution is continuously carried out until all the areas, A, are filled with 10 cm^2 each. From this exercise, it can be concluded that there are very large numbers of possible solutions to this problem, as shown in Fig. 7(a). In this graph, a straight line of 1.5 kg serves as the demarcation line for the feasible solutions, where the possible solutions between A and B violate the constraints. Fig. 7(b) is the enlargement of the lowest portion of Fig. 7(a). This figure shows that from the many feasible solutions available, only one value for the minimum mass 1.14 kg, which is the current method of optimization, can locate this point. Thus, it is proven that the optimal mass of 1.14 kg obtained by GA is indeed the optimal solution.

3.2 Four bar linkage mechanism with bending and buckling stress constraints

In this design optimization, the material is assumed to have a buckling stress limit of 240 MPa, and other parameters, such as length and other properties, are the same as those in Section 3.1. The optimal mass variation of the four bar linkage mechanism is shown in Fig. 8. The optimal mass varies from 0.55 kg to 1.29 kg, depending on the angle from 4^o to 360^o. As observed, the optimum mass of 1.29 kg is recorded at an angle of 4^o. This optimum value can be confirmed from the number of possible solutions from Fig. 9. Further, although there are many possible solutions, only few comply with both the bending and buckling constraints. The straight line of 1.33 kg representing the demarcation of the possible solutions violates the constraints. In the dual constraints, two portions of A to B and C to D of the number of possible solutions violate the

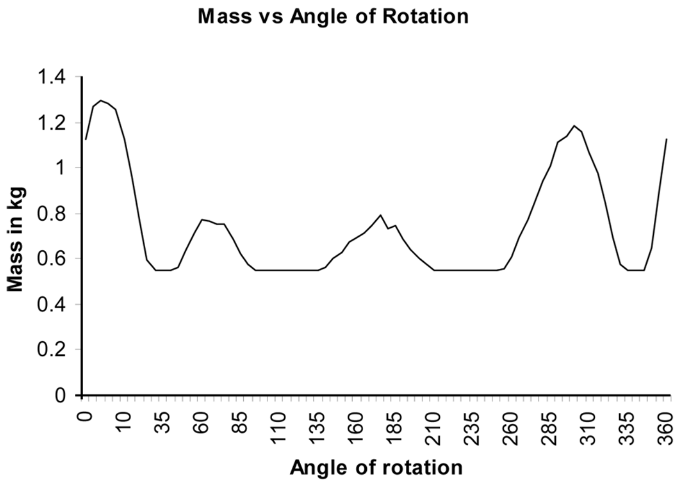

Fig. 8 Optimum mass at each angle of the four bar linkage mechanism (1^o-360^o)

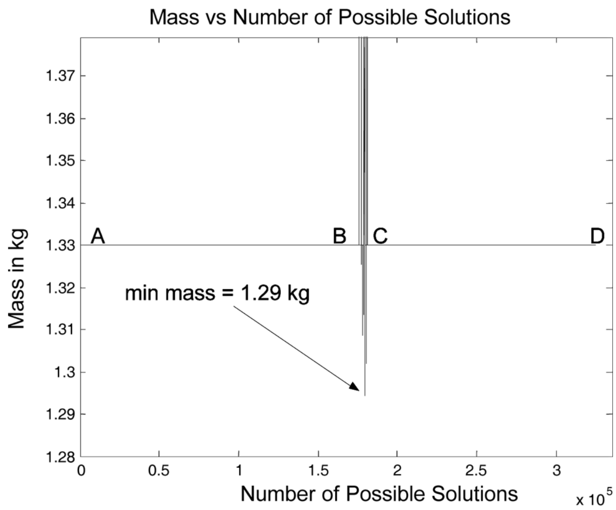

Fig. 9 Number of possible solutions for the mass of the four bar linkage mechanism with dual constraints

Table 4 Computational details of the four bar linkage mechanism at selected generation at an angle of 4^o rotation

Gen	A2	A3	A4	δ1	δ2	δ3	δ4	δB1	δB2	δB3	C1	C2	C3	C4	CB1	CB2	CB3	MOF	Mass
	cm^2	cm^2	cm^2	Pa	Pa	Pa	Pa	Pa	Pa	Pa								kg	kg
1	4.00	3.00	2.00	-953235	-6.3E+07	4.3E+07	1482118	2.3E+08	1.9E+07	1.9E+07	-0.983	0.1264	-0.23	-0.97	-0.028	-0.919	-0.92	5.12	1.51
2	4.00	3.00	2.00	-953235	-6.3E+07	4.3E+07	1482118	2.3E+08	1.9E+07	1.9E+07	-0.983	0.1264	-0.23	-0.97	-0.028	-0.919	-0.92	5.12	1.51
3	4.28	6.00	1.00	-1457680	-5.9E+07	1.9E+07	4379061	2.5E+08	3.9E+07	9326404	-0.974	0.0603	-0.65	-0.92	0.039	-0.838	-0.96	4.91	2.08
4	4.25	1.97	1.35	-595623	-6.2E+07	5.1E+07	1451089	2.5E+08	1.3E+07	1.3E+07	-0.989	0.1239	-0.09	-0.97	0.0316	-0.947	-0.95	4.22	1.14
5	4.25	1.97	1.35	-595623	-6.2E+07	5.1E+07	1451089	2.5E+08	1.3E+07	1.3E+07	-0.989	0.1239	-0.09	-0.97	0.0316	-0.947	-0.95	4.22	1.14
6	4.25	1.97	1.35	-595623	-6.2E+07	5.1E+07	1451089	2.5E+08	1.3E+07	1.3E+07	-0.989	0.1239	-0.09	-0.97	0.0316	-0.947	-0.95	4.22	1.14
7	4.00	3.00	1.00	-842682	-5.6E+07	3.1E+07	2413271	2.3E+08	1.9E+07	9326404	-0.985	0.0139	-0.44	-0.96	-0.028	-0.919	-0.96	1.49	1.30
8	4.00	3.00	1.00	-842682	-5.6E+07	3.1E+07	2413271	2.3E+08	1.9E+07	9326404	-0.985	0.0139	-0.44	-0.96	-0.028	-0.919	-0.96	1.49	1.30
9	4.00	3.00	1.00	-842682	-5.6E+07	3.1E+07	2413271	2.3E+08	1.9E+07	9326404	-0.985	0.0139	-0.44	-0.96	-0.028	-0.919	-0.96	1.49	1.30
10	4.00	3.00	1.00	-842682	-5.6E+07	3.1E+07	2413271	2.3E+08	1.9E+07	9326404	-0.985	0.0139	-0.44	-0.96	-0.028	-0.919	-0.96	1.49	1.30
15	4.15	3.00	1.00	-812358	-5.5E+07	3.1E+07	2412356	2.4E+08	1.9E+07	9326404	-0.985	-0.007	-0.44	-0.96	0.0079	-0.919	-0.96	1.40	1.32
20	4.11	2.99	1.00	-816814	-5.5E+07	3.1E+07	2403267	2.4E+08	1.9E+07	9326404	-0.985	-0.001	-0.44	-0.96	-0.002	-0.919	-0.96	1.31	1.31
25	4.11	2.98	1.00	-815047	-5.5E+07	3.1E+07	2398446	2.4E+08	1.9E+07	9326404	-0.985	-0.001	-0.44	-0.96	-0.002	-0.92	-0.96	1.31	1.31
30	4.11	2.97	1.00	-813191	-5.5E+07	3.1E+07	2393382	2.4E+08	1.9E+07	9326404	-0.985	-0.001	-0.43	-0.96	-0.002	-0.92	-0.96	1.30	1.30
35	4.11	2.94	1.00	-805573	-5.5E+07	3.2E+07	2372617	2.4E+08	1.9E+07	9326404	-0.985	-2E-04	-0.43	-0.96	-0.002	-0.921	-0.96	1.30	1.30
40	4.11	2.93	1.00	-804801	-5.5E+07	3.2E+07	2370514	2.4E+08	1.9E+07	9326404	-0.985	-1E-04	-0.43	-0.96	-0.002	-0.921	-0.96	1.30	1.30
45	4.11	2.93	1.00	-803803	-5.5E+07	3.2E+07	2367794	2.4E+08	1.9E+07	9326404	-0.986	-3E-05	-0.43	-0.96	-0.002	-0.921	-0.96	1.29	1.29
50	4.11	2.93	1.00	-803803	-5.5E+07	3.2E+07	2367794	2.4E+08	1.9E+07	9326404	-0.986	-3E-05	-0.43	-0.96	-0.002	-0.921	-0.96	1.29	1.29
55	4.11	2.93	1.00	-803803	-5.5E+07	3.2E+07	2367794	2.4E+08	1.9E+07	9326404	-0.986	-3E-05	-0.43	-0.96	-0.002	-0.921	-0.96	1.29	1.29
60	4.11	2.92	1.00	-801455	-5.6E+07	3.2E+07	2361401	2.4E+08	1.9E+07	9326404	-0.986	0.0002	-0.43	-0.96	-0.002	-0.921	-0.96	1.29	1.29
65	4.11	2.93	1.00	-803803	-5.5E+07	3.2E+07	2367794	2.4E+08	1.9E+07	9326404	-0.986	-3E-05	-0.43	-0.96	-0.002	-0.921	-0.96	1.29	1.29
70	4.11	2.93	1.00	-803803	-5.5E+07	3.2E+07	2367794	2.4E+08	1.9E+07	9326404	-0.986	-3E-05	-0.43	-0.96	-0.002	-0.921	-0.96	1.29	1.29
80	4.11	2.93	1.00	-803803	-5.5E+07	3.2E+07	2367794	2.4E+08	1.9E+07	9326404	-0.986	-3E-05	-0.43	-0.96	-0.002	-0.921	-0.96	1.29	1.29

constraints. The first portion A to B is violated because of the bending stress constraint, while the second portion C to D is due to the buckling constraint.

Table 4 shows the computational details of the four bar linkage mechanism with bending and buckling stresses at the selected generation at an angle of 4^o of rotation. The cross sectional areas are listed in columns (2), (3), and (4). The bending stresses are listed in columns (5) to (8) as the nodal stresses, whereas columns (9) to (11) display the buckling stresses as the stresses in the links. The coefficient of violations is listed in columns (12) to (15) for both bending and buckling violations. As stated earlier, these columns show whether the mass of the bar is acceptable or violates the constraints imposed. The optimal mass in each generation is shown in the last column. It can be seen that node 2 is violated from the 1^{st} until the 10^{th} generation due to the bending stress. The violation is taken as the solution to show the evolution of the current optimization from having bad fitness (infeasible solutions) at the beginning to having feasible solutions at the latter stage. During the process of searching for the optimum solution, the quality of individuals in the new generation greatly improves compared with the previous one, and the amount of violation is considerably reduced and continues to improve until the convergence is met.

Fig. 10 is the graph of the iteration history of the optimal mass at 4^o angle of rotation. The convergence is quite fast in the first 10 generations where the mass drops from 5.12 kg to 1.49 kg. As the generation proceeds, the mass is continuously reduced up until the optimal solution of 30 generation. The optimal mass of 1.29 kg is obtained, and the cross sectional area of the links for $A2$, $A3$, and $A4$ are 4.11, 2.93, and 1.00 cm^2, respectively.

The result from the optimization technique can be discussed with regard to the effect of the constraints. Different values of the optimal mass are obtained for single and dual constraints. In the optimization of the four bar linkage mechanism, the optimal mass of 1.14 kg is obtained for the single constraint and that for the dual constraints is 1.29 kg. The optimal mass for dual constraints is heavier than that obtained from the single constraint, as the dual constraints find it more complicated to comply due to the incorporated bending and buckling stress constraints. In addition, it should be noted that the dual constraints are more critical than the single constraint when applied

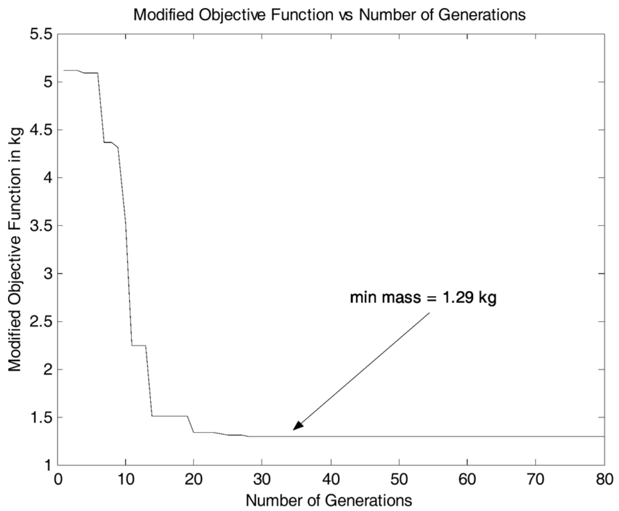

Fig. 10 Minimum mass of the four bar linkage mechanism at an angle of 4^o rotation for the number of generations

to the four bar linkage mechanism. From these comparisons, it can be inferred that the optimal mass decreases when the number of constraints increases.

4. Conclusions

This paper aims to find the minimum mass of the bar linkage mechanism in movement with the imposed dual constraints. The utilization of kinematics, kinetics, and FEA is successfully carried out in the analysis of the bar linkage mechanism. Further, GA is successfully used to optimize the cross sectional area of the four bar linkage mechanism by coupling the mechanism synthesis with FEA. GA with a single constraint successfully optimizes the four bar with an optimal mass of 1.14 kg. As the optimization technique developed in this paper is a robust technique in which any constraint can be incorporated, a single constraint is extended to include the dual constraints, with both bending and buckling constraints imposed. With 55.5 MPa and 240 MPa values for the bending and buckling stress constraints, respectively, the optimal mass of 1.29 kg is obtained. Moreover, it is worthy to note that the search for the optimal mass of the bar linkage mechanism only takes few minutes to be completed.

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