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On the accuracy of estimation of rigid body inertia properties from modal testing results

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Abstract. The rigid body inertia properties of a structure including the mass, the center of gravity location, the mass moments and principal axes of inertia are required for structural dynamic analysis, modeling of mechanical systems, design of mechanisms and optimization. The analytical approaches such as solid or finite element modeling can not be used efficiently for estimating the rigid body inertia properties of complex structures. Several experimental approaches have been developed to determine the rigid body inertia properties of a structure via Frequency Response Functions (FRFs). In the present work two experimental methods are used to estimate the rigid body inertia properties of a frame. The first approach consists of using the amount of mass as input to estimate the other inertia properties of frame. In the second approach, the property of orthogonality of modes is used to derive the inertia properties of a frame. The accuracy of the estimated parameters is evaluated through the comparison of the experimental results with those of the theoretical Solid Work model of frame. Moreover, a thorough discussion about the effect of accuracy of measured FRFs on the estimation of inertia properties is presented.

Keywords: rigid body; inertia properties; frequency response functions; mode shapes; transformation matrix.

1. Introduction

Estimation of the inertia properties of rigid bodies is important in the design of structures which rotate during their motion such as airplanes or satellites. In most cases an accurate analytical model of structure is not available and therefore the computational approaches can not be used to estimate the inertia properties of structure. The alternative approach resides on the use of experimental data available from measurements. Currently there are two different approaches to estimate the inertia properties of a structure from experimental data:

1. Time domain approaches
2. Frequency domain approaches

1.1 Time domain methods

The inertia properties can be obtained by the pendulum methods (Holzweissig and Dresign 1994).

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In this approach the structure is hanged and forced to oscillate within small amplitudes as a pendulum. The moment of inertia of structure can be obtained by measuring the period of oscillation. This is an important approach and is being used widely. However, friction, the effect of surrounding air and the effect of extra masses bias the results.

The experimental modal data can be used in the time domain approaches to calculate the inertia properties. There is no need to transfer the data from time domain to frequency domain and conduct the signal processing procedures. Pandit and Hu (1994) obtained the inertia properties of rigid bodies with damped boundary conditions in the time domain using the rigid body equations of motion. Pandit *et al.* (1992) used the transformation matrices to change the only translational motions to the rotational and translational motions. Hahn (1994) used a 6 Multi-Axis test facility to measure the accelerations and excitation forces and obtain the inertia properties of rigid body. Hou *et al.* (2009) presented an improved approach to identify the inertia parameters of odd-shaped bodies using a trifilar pendulum. The method is efficient and reliable for the assemblies and structures with complex shapes. Using this method, the vibration period of body can be measured simply and accurately.

1.2 Frequency domain methods

There are different approaches to obtain the inertia properties of rigid bodies in frequency domain:
a) Modal parameter methods
b) Direct physical parameter identifications
c) Residual inertia methods

Wei and Reis (1989) developed a method to fit a polynomial to the measured rigid body modes to determine the rigid body properties. Bretl and Conti (1987) proposed two different methods between them, the procedure using masslines as input requires the knowledge of the body mass. Mangus *et al.* (1992) and Nakamura (1995) presented a least square curve fitting method used to obtain the rigid body properties from the measured frequency response functions. The method was applied to two highly damped systems. Link (1985) considered the general methods of the behavior of an elastic system and does not restrict the methods to ideal rigid systems. One pitfall of the method is that all the rigid body modes of system are not excited. Link (1996) also proposed a method in which the inertia properties of the body were identified using the base excitation. The advantage of this method is that the rigid body responses are separated from the elastic modes. Gatzwiller *et al.* (2000) designed the sensors that not only can measure the dynamic forces but also the inertia properties. It seems that Residual inertia methods have more accurate results compared to the other methods. Almeida *et al.* (2008) evaluated and compared three frequency domain methods. This evaluation showed that none of the frequency domain methods is the superior one and depending on the case, one of the frequency methods is more efficient than others. It is advised to use more than one method to obtain the more reliable results. Atchonouglo *et al.* (2008) proposed an algorithm based on least square method and conjugate gradient method to evaluate the inertia properties of a structure.

2. Theory

In the present work the estimation of the inertia parameters of a frame follows closely two methods in the work of Conti and Bretl (1989) and Almeida *et al.* (2007). In both methods the responses are measured in various points and directions due to the impact excitation. The rotational degrees of freedom can not be measured easily. However, the measured translational responses and excitations can be transformed to the equivalent translational and rotational responses and excitations at the center of gravity. The obtained FRFs at the center of gravity are then used to extract the natural frequencies, damping ratios and mass-normalized mode shapes using identification methods (Ewins 1995). The first approach consists of using the amount of mass as input and evaluates three coordinates of center of gravity and six moments of inertia. The second method considers that the coordinates of center of gravity is known and evaluates the mass of structure and six moments of inertia. The inertia properties of frame are evaluated using the theoretical modeling and the results are compared with those of the experimental approaches.

2.1 Method 1

Fig. 1 shows a rigid body with the viscously damped elastic supports. The equation of motion at the center of gravity is

$$M_g\ddot{X}_g + C_g\dot{X}_g + K_gX_g = F_g \tag{1}$$

M_g, K_g and C_g are the mass, the stiffness and the damping matrices about the center of gravity and X_g is the displacement vector at the center of gravity including three translational x_g, y_g, z_g and three rotational $\theta_{gx}, \theta_{gy}, \theta_{gz}$ displacements at the center of gravity, where

$$M_g = \begin{bmatrix} m & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & I_{xx} & -I_{xy} & -I_{xz} \\ 0 & 0 & 0 & -I_{yx} & I_{yy} & -I_{yz} \\ 0 & 0 & 0 & -I_{zx} & -I_{zy} & I_{zz} \end{bmatrix}, \quad X_g = \begin{Bmatrix} x_g \\ y_g \\ z_g \\ \theta_{gx} \\ \theta_{gy} \\ \theta_{gz} \end{Bmatrix}, \quad F_g = \begin{Bmatrix} f_x \\ f_y \\ f_z \\ T_x \\ T_y \\ T_z \end{Bmatrix} \tag{2}$$

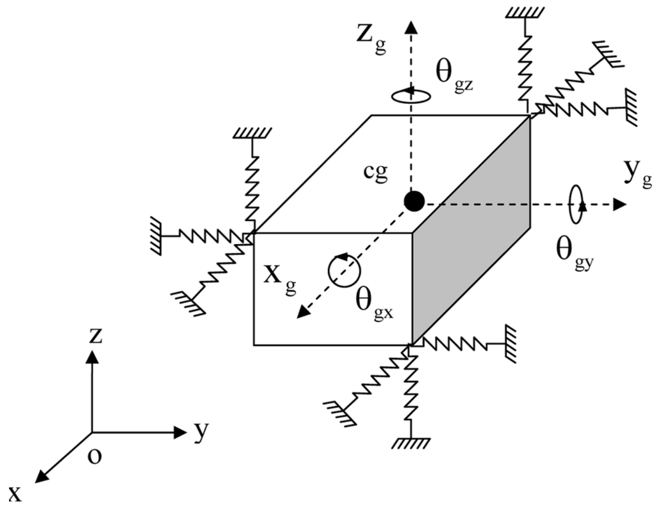

Fig. 1 A rigid body on the elastic supports

Where I_{xx}, I_{yy} and I_{zz} are the moments of inertia with respect to x, y and z coordinates respectively and I_{xy}, I_{xz} and I_{yz} are the cross moments of inertia. Considering the behavior of a rigid body, the relation between the rotational and translational motions of the centre of gravity and translational motions of the measured points can be expressed as

$$X_m = T_{mg} \cdot X_g \tag{3}$$

Where X_m is the displacement vector of measured translational responses at the measurement points and is given by

$$X_m = [X_1\ X_2\ \dots\ X_n]^T \tag{4}$$

n is the number of accelerometer locations. X_i is the translational response measured by an accelerometer in the translational directions at point i. T_{mg} is the transformation matrix between the rigid body motion at the center of gravity and the translational measurement. T_{mg} is formed depending on the direction of measurement of X_i. For instance if X_1 is measured in $+X$ direction, X_2 is measured in $+Y$ direction, X_3 is measured in $+Z$ direction, X_4 is measured in $-X$ direction, X_5 is measured in $-Y$ direction and X_6 is measured in $-Z$ direction, T_{mg} is given by

$$T_{mg} = \begin{bmatrix} 1 & 0 & 0 & 0 & w_1 - w_g & v_g - v_1 \\ 0 & 1 & 0 & w_g - w_2 & 0 & u_2 - u_g \\ 0 & 0 & 1 & v_3 - v_g & u_g - u_3 & 0 \\ -1 & 0 & 0 & 0 & w_g - w_4 & v_4 - v_g \\ 0 & -1 & 0 & w_5 - w_g & 0 & u_g - u_5 \\ 0 & 0 & -1 & v_g - v_6 & u_6 - u_g & 0 \end{bmatrix} \tag{5}$$

Where (u_g, v_g, w_g) are the coordinates of center of gravity and (u_i, v_i, w_i) are the coordinates of i^{th} accelerometer location. The number of measurement points, n is required to be at least 6. For $n \geq 6$, X_g can be calculated using pseudo inverse as

$$X_g = T_{mg}^{+} \cdot X_m \tag{6}$$

Where

$$T_{mg}^{+} = (T_{mg}^{T} \cdot T_{mg})^{-1} \cdot T_{mg}^{T} \tag{7}$$

The equation of motion with respect to the measurement points is

$$M_m \ddot{X}_m + C_m \dot{X}_m + K_m X_m = F_m \tag{8}$$

Where

$$M_m = (T_{mg}^{-1})^T M_g T_{mg}^{-1} \tag{9}$$

$$C_m = (T_{mg}^{-1})^T C_g T_{mg}^{-1} \tag{10}$$

$$K_m = (T_{mg}^{-1})^T K_g T_{mg}^{-1} \tag{11}$$

$$F_m = (T_{mg}^{-1})^T F_g \tag{12}$$

Let the exciting force $F = [F_x, F_y, F_z]^T$ be applied to the structure at point (u_f, v_f, w_f), then the equivalent force vector at the center of gravity can be obtained as

$$F_g = T_f \cdot F \tag{13}$$

Where

$$T_f = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & w_g - w_f & v_f - v_g \\ w_f - w_g & 0 & u_g - u_f \\ v_g - v_f & u_f - u_g & 0 \end{bmatrix} \tag{14}$$

Therefore

$$F_m = (T_{mg}^{-1})^T \cdot T_f \cdot F \tag{15}$$

It can be shown that

$$T_{mg} = T_{mo} \cdot G_{og} \tag{16}$$

Where T_{mo} is the transformation matrix with respect to the global coordinate system and is the same as T_{mg} as shown in Eq. (5) in which the coordinates of center of gravity (u_g, v_g, w_g) are replaced by the coordinates of origin $(0, 0, 0)$.

$$G_{og} = \begin{bmatrix} I & H_{og} \\ 0 & I \end{bmatrix} \tag{17}$$

Where

$$H_{og} = \begin{bmatrix} 0 & -w_g & v_g \\ w_g & 0 & -u_g \\ -v_g & u_g & 0 \end{bmatrix} \tag{18}$$

And I is a 6 × 6 unit matrix. G_{og} has an interesting property that

$$G_{og}^{-1} = \begin{bmatrix} I & -H_{og} \\ 0 & I \end{bmatrix} = \begin{bmatrix} I & H_{go} \\ 0 & I \end{bmatrix} = G_{go} \tag{19}$$

From Eqs. (9), (11) and (16)

$$M_m^{-1} \cdot K_m = T_{mg} \cdot M_g^{-1} \cdot K_g \cdot T_{mg}^{-1} = T_{mo} \cdot G_{og} \cdot M_g^{-1} \cdot K_g \cdot G_{og}^{-1} \cdot T_{mo}^{-1} \tag{20}$$

Or

$$M_g G_{og}^{-1}(T_{mo}^{-1} M_m^{-1} \cdot K_m T_{mo}) = K_g \cdot G_{og}^{-1} \tag{21}$$

Let

$$Q = T_{mo}^{-1} M_m^{-1} \cdot K_m \cdot T_{mo} \tag{22}$$

Then

$$M_g \cdot G_{go} \cdot Q = K_g \cdot G_{go} \tag{23}$$

From the state space model of a vibrating system

$$\begin{bmatrix} -M_m^{-1} \cdot C_m & -M_m^{-1} \cdot k_m \\ I & 0 \end{bmatrix} = \begin{bmatrix} L\mu \\ L \end{bmatrix} \mu \begin{bmatrix} L\mu \\ L \end{bmatrix}^{-1} \tag{24}$$

In which μ is the diagonal matrix of eigenvalues and L is the corresponding mass-normalized mode shapes or eigenvectors about the measurement coordinate. The complex eigenvalues and eigenvectors can be arranged in conjugate pairs as

$$L = [L_1 \ \ L_2] \tag{25}$$

$$\mu = \begin{bmatrix} \mu_1 & 0 \\ 0 & \mu_2 \end{bmatrix} \tag{26}$$

In which L_1, L_2 and μ_1, μ_2 are conjugate pairs. Therefore Eq. (24) can be written as

$$\begin{bmatrix} -M_m^{-1} \cdot C_m & -M_m^{-1} \cdot k_m \\ I & 0 \end{bmatrix} = \begin{bmatrix} L_1\mu_1L_1^{-1} + L_2\mu_2L_2^{-1} & -(L_1\mu_1L_1^{-1})^*(L_2\mu_2L_2^{-1}) \\ I & 0 \end{bmatrix} \tag{27}$$

Thus

$$M_m^{-1} \cdot C_m = -((L_1\mu_1L_1^{-1}) + (L_2\mu_2L_2^{-1})) = -((L_1\mu_1L_1^{-1}) + (L_1\mu_1L_1^{-1})^*) = -2\mathrm{Re}(L_1\mu_1L_1^{-1}) \tag{28}$$

$$M_m^{-1} \cdot K_m = (L_1\mu_1L_1^{-1}) \times (L_2\mu_2L_2^{-1}) = (L_1\mu_1L_1^{-1}) \times (L_1\mu_1L_1^{-1})^* = \mathrm{Re}^2(L_1\mu_1L_1^{-1}) + \mathrm{Im}^2(L_1\mu_1L_1^{-1}) \tag{29}$$

Therefore from Eq. (22) and Eq. (29)

$$Q = T_{mo}^{-1} \cdot [\mathrm{Re}^2(L_1\mu_1.L_1^{-1}) + \mathrm{Im}^2(L_1\mu_1.L_1^{-1})] \cdot T_{mo} \tag{30}$$

Where $(L_1\mu_1.L_1^{-1})^*$ is the conjugate of $(L_1\mu_1.L_1^{-1})$. Eq. (30) shows that Q can be determined if the location of measurement points and the modal parameters are known. If the number of measurement points is greater than 6, Q can be calculated by substituting T_{mo}^{-1} by T_{mo}^{+} in Eq. (30).

From Eq. (23) and Eq. (30)

$$\begin{bmatrix} mI & 0 \\ 0 & M \end{bmatrix} \begin{bmatrix} I & H_{go} \\ 0 & I \end{bmatrix} \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} I & H_{go} \\ 0 & I \end{bmatrix} \tag{31}$$

Where m is the mass of structure and M is the inertia tensor relative to the origin O as shown in Eq. (2). Rearranging Eq. (31)

$$\begin{bmatrix} m(Q_{11} + H_{go}Q_{21}) & m(Q_{12} + H_{go}Q_{22}) \\ MQ_{21} & MQ_{22} \end{bmatrix} = \begin{bmatrix} K_{11} & K_{11}H_{go} + K_{12} \\ K_{21} & K_{21}H_{go} + K_{22} \end{bmatrix} \tag{32}$$

Four equations are obtained

$$m(Q_{11} + H_{go}Q_{21}) = K_{11} \tag{33}$$

$$m(Q_{12} + H_{go}Q_{22}) = K_{11}H_{go} + K_{12} \tag{34}$$

$$MQ_{21} = K_{21} \tag{35}$$

$$MQ_{22} = K_{21}H_{go} + K_{22} \tag{36}$$

As K_{11} is a symmetric matrix, the left hand side of Eq. (33) is also symmetric, it can be concluded that

$$\begin{bmatrix} q_{52}+q_{63} & -q_{42} & -q_{43} \\ -q_{51} & q_{41} & -q_{53} \\ -q_{61} & -q_{62} & q_{41}+q_{52} \end{bmatrix} \begin{bmatrix} u_g \\ v_g \\ w_g \end{bmatrix} = \begin{bmatrix} q_{32}-q_{23} \\ q_{13}-q_{31} \\ q_{12}-q_{21} \end{bmatrix} \tag{37}$$

Where q_{ij} is the i^{th} row and j^{th} column element of the matrix Q defined in Eq. (30). Therefore the coordinates of the center of gravity can be calculated from Eq. (36). If the mass of structure m, is known, K_{11} can be calculated from Eq. (33). Thus K_{12} is calculated from Eq. (34). As $K_{21} = K_{12}$, M is calculated from Eq. (35). Finally K_{22} is calculated from Eq. (36). Therefore knowing the mass of system all other 9 inertia properties can be derived. The stiffness matrix can also be determined. As the location of centre of gravity is known K_g and M_g can be determined. Therefore using this procedure, the inertia properties with respect to the center of gravity can be determined.

2.2 Method 2

The orthogonality of the mass-normalized mode shapes with respect to the mass matrix can be expressed as

$$L_g^T M_g L_g = 1 \tag{38}$$

where M_g is the mass matrix with respect to the center of gravity as shown in Eq. (2) and L_g is the mass-normalized mode shape matrix with respect to the center of gravity. Therefore mass matrix M_g can be obtained as

$$M_g = L_g^{-T} L_g^{-1} \tag{39}$$

In this method it is assumed that the location of the center of gravity is known from theoretical modeling or from method 1. Then the moments of inertia of structure are obtained through calculating M_g from Eq. (39).

3. Experimental test case - steel frame

Fig. 2 shows the experimental case study of a steel frame. The frame is made of 20×20 mm steel bars with the mass of 4.16 kg. The frame was suspended from the elastic supports which control the motion of structure in all directions. The measurement points were selected taking into due account the recommendations proposed by Lee *et al.* (1999) so that all the modes of structure are clearly

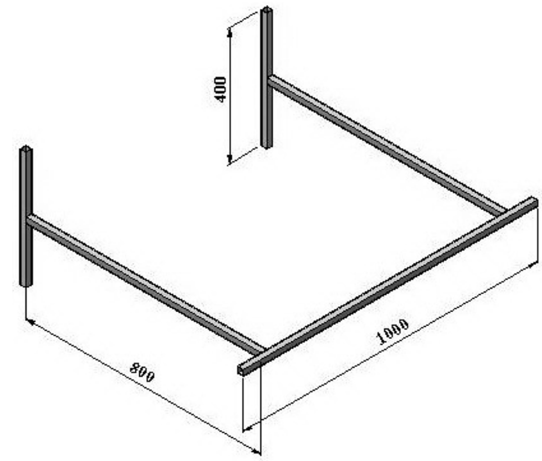

Fig. 2 The steel frame dimensions

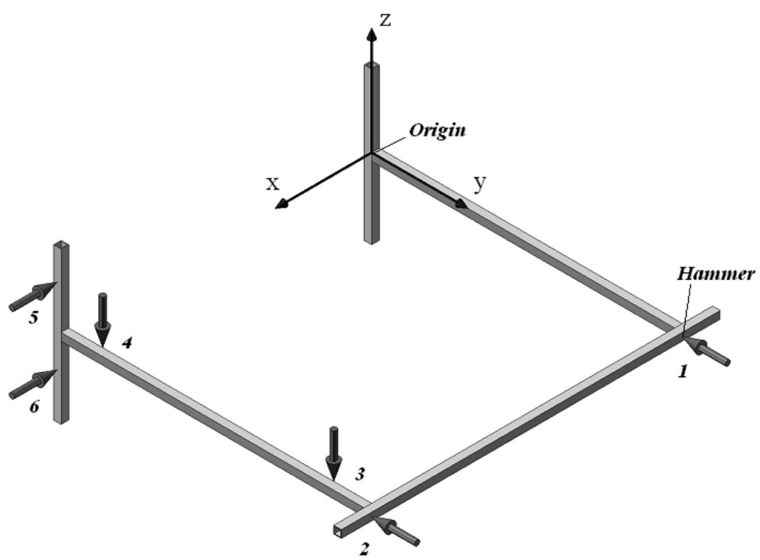

Fig. 3 The location of excitation points and response measurements

excited. Fig. 3 indicates the locations of excitation and response points. The test setup is presented in Fig. 4 showing the support conditions of the frame. Table 1 indicates the measurement points and the direction of measurement at each point with respect to the global coordinates. The force was applied to the structure via an impact hammer BK 8202. Six accelerometers were attached at the point of excitation and 5 other points of structure so that they can show all the rigid body modes. The Frequency Response Functions (FRFs) of frame was measured and processed by a multi channel BK analyzer allowing for simultaneous acquisition of six responses and one force in a conventional modal test. The frequency range was chosen to be 0-1.6 KHz, in order to detect the first elastic mode of frame.

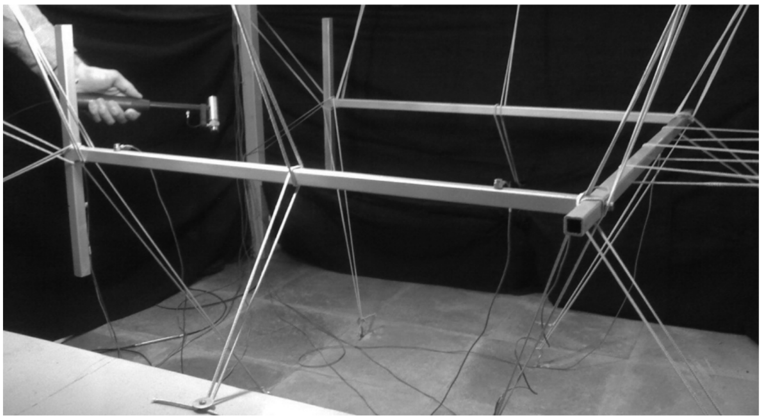

Fig. 4 Test setup

Table 1 The position of measurement points

Point Number	Measurement points			Measurement Direction
	X (mm)	*Y* (mm)	*Z* (mm)	
1	0	820	0	+*Y*
2	800	820	0	+*Y*
3	800	700	0	+*Z*
4	800	90	0	+*Z*
5	820	0	110	+*X*
6	820	0	-90	+*X*

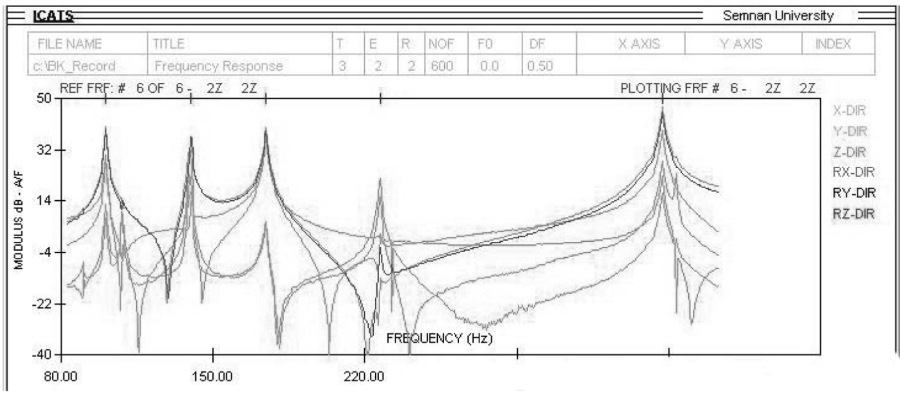

Fig. 5 The FRFs of frame showing its first elastic mode

According to the vibration theory if a structure is excited in the free-free condition, all the six rigid body modes have zero natural frequencies. However, when the structure is hanged from a soft spring, the rigid body modes appear in low frequencies depending on the stiffness of suspension springs. Fig. 5 presents the measured frequency response functions of the frame in the frequency range of 80-430 Hz showing that the first elastic mode of frame is at 100 Hz. Usually the rigid

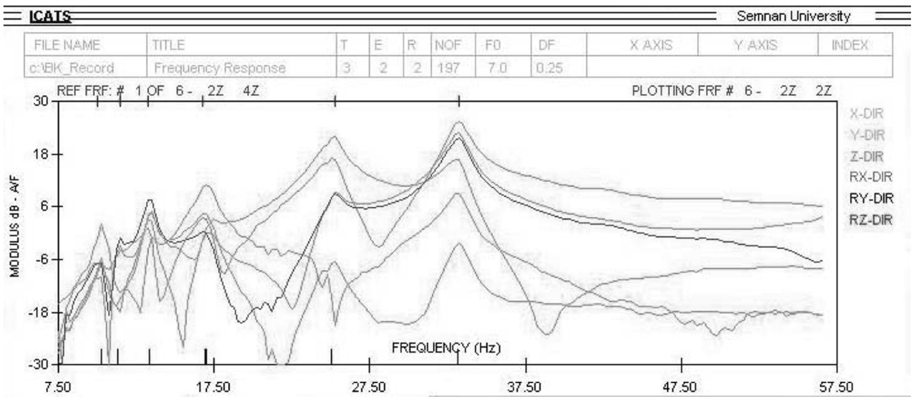

Fig. 6 The six rigid body modes of frame

body modes are in the range of 10-20 percent of the first elastic mode. The FRFs of the frame was measured again in the frequency range of 0-60 Hz in order to obtain a better resolution. Fig. 6 shows the six rigid body modes of structure in the frequency range of 0-60 Hz. The highest rigid body mode is at 33.139 Hz which is not close to the first elastic mode at 100 Hz.

For the set of six measured FRFs, the necessary modal analysis was undertaken in order to extract the modal parameters (natural frequencies, mass-normalized mode shapes and damping ratios). The modal analysis software MODENT module of ICATS (1998-2000) was used for this process. The NLLS (Non Linear Least Square) modal identification method was chosen inside MODENT software. The results of modal analysis were used in the first method introduced in section 2.1 relative to the defined coordinates. The mass of structure was required for the first method which was easily measured by weighting the structure. The coordinates of center of gravity were calculated to be:

$$u_g = 0.4208 \text{ mm},\ v_g = 0.4618 \text{ mm},\ w_g = 0.000217 \text{ mm}$$

The inertia parameters of structure were computed using the Solid Works software for the sake of comparison. The mass of accelerometers (5 gr each) were considered in the Solid Works model showing the negligible effects (maximum 1% error) on the calculated inertia properties. To extract the inertia tensor values from the second method, the coordinate system were transferred to the

Table 2 Comparison of the inertia properties from method 1, method 2 and Solid Work

Characteristics	Test Value (method1)	Test value (method 2)	Solid work value
X_{cg} (m)	0.4208	Known	0.4
Y_{cg} (m)	0.4618	Known	0.424
Z_{cg} (m)	0.000217	Known	0.0007641
I_{xx} (kg.m^2)	0.5323(9.7%)	0.52913(9%)	0.485026
I_{yy} (kg.m^2)	0.5729(5.8%)	0.6327(4%)	0.608268
I_{zz} (kg.m^2)	1.123(5.3%)	0.8423(21%)	1.0665
I_{xy} (kg.m^2)	-0.0008712(8.75%)	-0.000975(21.9%)	0.0008
I_{xz} (kg.m^2)	-0.004725(18%)	-0.00432(8%)	-0.004
I_{yz} (kg.m^2)	0.003251(22.6%)	0.003714(11.6%)	0.0042

center of gravity using the calculated coordinates of center of gravity from Solid Works. The mass-normalized mode shapes were calculated about the new system of coordinates. Then the moment of inertia parameters were extracted using Eq. (40). The percentage of difference of results of methods 1 and 2 compared to Solid Work results are given in Table 2. Comparison of the test results and the theoretical results are also given in Figs. 7 and 8 on the bar graphs.

From the observation of the results in Table 2 and Figs. 7 and 8 it can be concluded that there are small error percentage between the results. The errors in the experimental methods 1 and 2 are due to inherent measurement errors of modal testing. The accuracy of measurement in the modal testing method depends on many parameters such as:

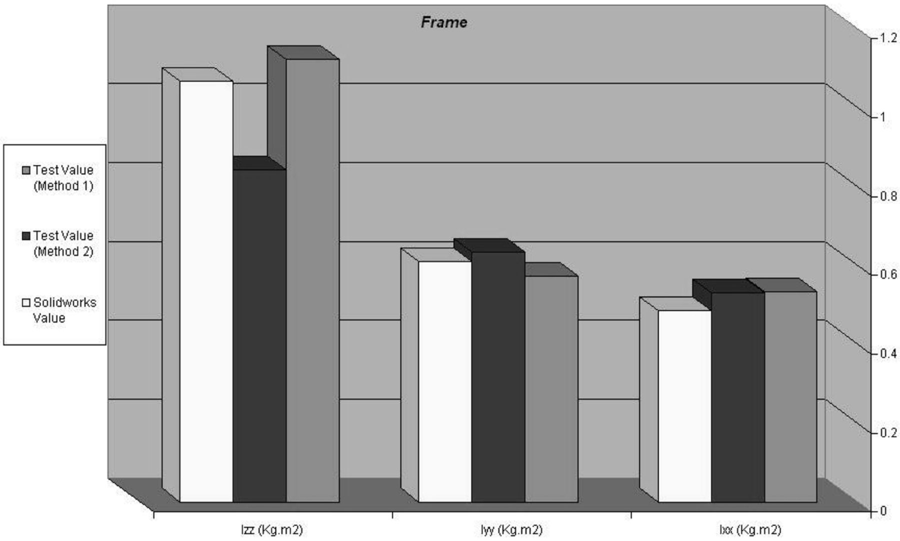

Fig. 7 Comparison of the test results and the theoretical results (I_{xx}, I_{yy}, I_{zz})

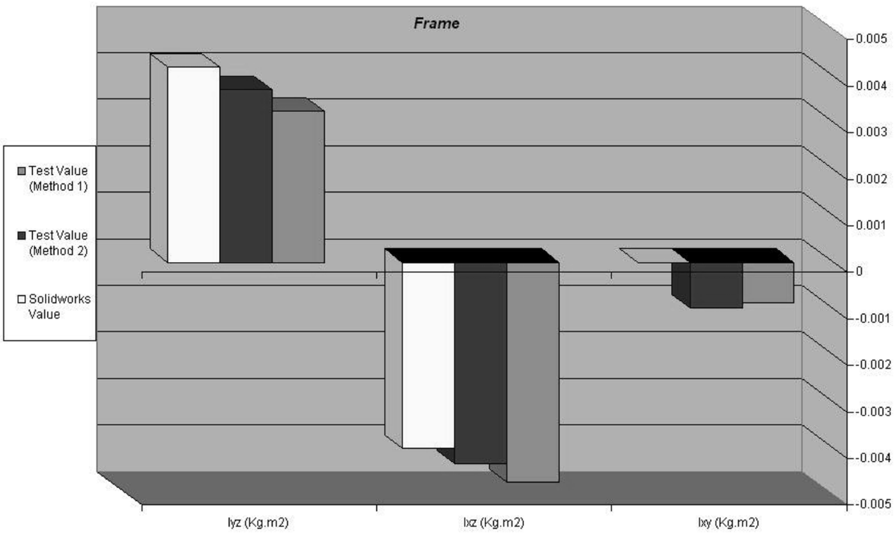

Fig. 8 Comparison of the test results and the theoretical results (I_{yz}, I_{xz}, I_{xy})

1. The accuracy of coordinates
2. The quality of FRFs due to the signal processing errors
3. The resolution of the FRFs
4. The direction of force which should be exactly normal to the surface
5. The location and direction of accelerometers
6. Mass loading effect of accelerometers
7. Modal parameter extraction using modal analysis process

On the other hand the accuracy of the moment of inertia properties from Solid Work depends highly on the geometrical and material properties accuracy of the frame. Also there are some errors at the joints of steel bars which are welded and can not be exactly modeled theoretically. Therefore none of the theoretical and experimental methods can be claimed to be the most accurate. However, Figs. 7 and 8 show that the moment of inertia properties can be obtained using the experimental methods 1 and 2 with reasonable accuracy for engineering design purposes.

4. Conclusions

In this research work, the inertia properties of a frame were estimated using the frequency response functions in the hammer test. Two methods were applied in order to extract the moment of inertia parameter properties of frame. Also Solid Works was used in order to obtain the inertia properties. The results were compared showing that there are minor differences between the results. The accuracy of the obtained parameters using experimental methods depends highly on the accuracy of modal testing method. On the other hand the accuracy of the theoretical method depends on the accuracy of geometrical and material properties of frame. Due to the inherent errors both in the experimental and theoretical methods, the accurate results were not available. However, as there is small error percentage between the results it can be concluded that the experimental methods have reliable outputs for the engineering computations. Also using method 1 the coordinate of center of gravity of frame could be extracted with reasonable accuracy compared to the Solid Work results.

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