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Technical Note

Static configurations of cables in cable stayed bridges

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1. Introduction

Long stay cables in cable-stayed bridges may have large sag under their weight. In this paper, a precise catenary of stay cables is deduced. Furthermore, in order to simplify the calculation, an approximate configuration of parabola is presented. Compared with former studies, the parabola has higher precision.

2. Static solutions to inextensible cable

It is assumed that the bending stiffness of a uniform inextensible cable is small enough to be neglected. Fig. 1 denotes the inclined cable stayed at two fixed points A and B which have the coordinates (0, 0) and (l, h), respectively. Considering a differential segment at (x, z) in Fig. 2, the vertical equilibrium under its self-weight requires (Irvine 1981)

$$H[d^2z/dx^2] = -mg\sqrt{1+(dz/dx)^2} \tag{1}$$

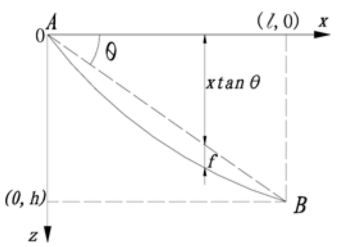

Fig. 1 Coordinate system

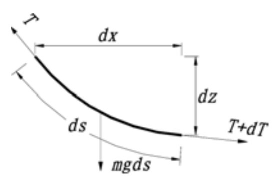

Fig. 2 Forces acting on differential segment

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where H is the horizontal component of cable tensile force T, being a constant everywhere; mg is the self-weight per unit length of cable. The boundary conditions at the cable supports A and B are $z(0) = 0$ and $z(l) = h$, respectively. Substituting $z = x\tan\theta + f$ into Eq. (1), the nondimensional solution to cable sag f can be expressed as

$$\bar{f} = \frac{2}{a} sh\left[arsh\frac{a\tan\theta}{2sh(a/2)} - \frac{a}{2}(\bar{x}-1)\right] sh\frac{a\bar{x}}{2} - \bar{x}\tan\theta \tag{2}$$

In Eq.(2), $\bar{f}$, $\bar{x}$, and a are non-dimensional variables which are defined as f/l, x/l and mgl/H, respectively; θ is the inclination of cable chord with respect to the horizontal direction. Furthermore the tensile force T and nondimensional cable length $\bar{S}$, namely, S/l, can be derived as

$$T = H\sqrt{1 + \left(\frac{d\bar{z}}{d\bar{x}}\right)^2} = Hch\left[a\bar{x} - \frac{a}{2} - arsh\frac{a\tan\theta}{2sh(a/2)}\right] \tag{3}$$

$$\bar{S} = \int_0^1 \sqrt{1 + \left(\frac{d\bar{z}}{d\bar{x}}\right)^2} d\bar{x} = \sqrt{\left(\frac{2sh\frac{a}{2}}{a}\right)^2 + \tan^2\theta} \tag{4}$$

where $\bar{z} = z/l$.

The above expressions can not be used directly due to the unknown parameter H in them. In fact, in cable-stayed bridge design the vertical component of the tensile force at the support B, i.e., V_B, is confirmed firstly and then the cable tension is determined. Therefore Eq. (5) is deduced by the relationship between V_B and the horizontal component H at the support B. From Eq. (5) the tensile force at the support B comes out by iterative calculations. Then the key parameter H and the others in Eq. (2) to Eq. (4) are obtained

$$\left.\frac{d\bar{z}}{d\bar{x}}\right|_{\bar{x}=1} = sh\left\{-\frac{mgl}{2\sqrt{T_B^2 - V_B^2}} + arsh\frac{mgl\tan\theta/\sqrt{T_B^2 - V_B^2}}{2sh[mgl/(2\sqrt{T_B^2 - V_B^2})]}\right\} = \frac{V_B}{\sqrt{T_B^2 - V_B^2}} \tag{5}$$

3. Static solutions to extensible cable

In actual case, stay cables are extensible. In the strained profile the length of the differential cable segment becomes ds_1, and its unit weight changes into $m_1 g$ which equals to $mg \times ds/ds_1$ owing to conservation of mass. Substituting the cable tensile force expressed by Hooke's law into Eq. (1) leads to the differential equation for the strained profile as

$$\left[1 + b\sqrt{1 + \left(\frac{d\bar{z}}{d\bar{x}}\right)^2}\right]\frac{d^2\bar{z}}{d\bar{x}^2} = -a\sqrt{1 + \left(\frac{d\bar{z}}{d\bar{x}}\right)^2} \tag{6}$$

where $b = H/EA$, E is the Young's modulus of cable, A is the uniform cross-sectional area in the unstrained profile; and the definitions of the other parameters are the same as those in the former section. Eq. (6) may be solved by setting $u = d\bar{z}/d\bar{x}$. Considering the boundary conditions, the solutions for nondimensional variables $\bar{x}$ and $\bar{z}$ are

$$\begin{cases} \bar{x} = (arshu_0 + bu_0 - arshu - bu)/a \\ \bar{z} = (\sqrt{u_0^2 + 1} + bu_0^2/2 - \sqrt{u^2 + 1} - bu^2/2)/a \end{cases} \tag{7}$$

where, $u_0 = \left.\frac{d\bar{z}}{d\bar{x}}\right|_{\bar{x}=0} = \sqrt{T_A^2 - (T_B^2 - V_B^2)}/\sqrt{T_B^2 - V_B^2}$, $u_1 = \left.\frac{d\bar{z}}{d\bar{x}}\right|_{\bar{x}=1} = V_B/\sqrt{T_B^2 - V_B^2}$.

As for the support B, Eq. (7) may be rewritten as

$$\begin{cases} l = (arshu_0 + bu_0 - arshu_1 - bu_1)/a \\ h/l = (\sqrt{u_0^2+1} + bu_0^2/2 - \sqrt{u_1^2+1} - bu_1^2/2)/a \end{cases} \tag{8}$$

It is also assumed that V_B is known beforehand. By solving Eq. (8), T_A and T_B can be obtained. The other parameters, may be derived out. Moreover, the iteration is involved in Eqs. (5) and (8), so simplified methods are necessary to be derived.

4. Simplified static solutions

Since the sag at stay cable's mid span is usually small, it is assumed that the length of differential segment, namely ds, is equal to $dx \cdot \sec\theta$. Consequently Eq. (1) may be rewritten as

$$H[d^2z/dx^2] = -mg\sec\theta \tag{9}$$

It may be deduced that the nondimensional solution to Eq. (9) is

$$\bar{z} = -a\sec\theta\bar{x}^2/2 + (a\sec\theta + 2\tan\theta)\bar{x}/2 \tag{10}$$

Eq. (10) points out that the sag curve is a parabola. Now by introducing the given parameter V_B through the formula $d\bar{z}/d\bar{x}|_{X=1} = V_B/H$ the constant parameter H may be obtained as

$$H = mgl\csc\theta/2 + V_B\cot\theta \tag{11}$$

Consequently the tensile force and cable length are respectively

$$T = H\sqrt{1 + [a\sec\theta(\bar{x} - 1/2) - \tan\theta]^2} \tag{12}$$

$$\bar{S} = \frac{1}{a\sec\theta}\left(\frac{u\sqrt{u^2+1}}{2} + \frac{1}{2}arshu\right)\Bigg|_{u_0}^{u_1} \quad \begin{pmatrix} u_0 = V_B/H - 2\tan\theta \\ u_1 = -V_B/H \end{pmatrix} \tag{13}$$

All the parameters can be derived directly once H is known by Eq. (11). Thus the solving efficiency can be greatly improved.

5. An example

An actual cable (J34), which is the longest one in Sutong Yangtze River Bridge with main span of 1088 meters, is adopted to testify the precision of the above formulas. The main parameters of the cable are: V_B = 2478.824(kN); mg = 98.8 (kN/m); l = 532.626 (m); h = 220.564 (m); L(chord length of a cable) = 576.488 (m) ; A = 0.012046 (m^2); E = 1.95e8 (kN/ m^2). The Newton-Raphson method and Broyden method are adopted to solve the nonlinear equation (Eq. (5)) and nonlinear equation set (Eq. (8)), respectively. The computed results are listed in Table 1, in which $f_{l/2}$ is the sag at mid span of the cable; $f_{max.}$ is the maximum sag at the point ($x_{max.}$, $z_{max.}$) which satisfies $d\bar{f}/d\bar{x} = 0$; and θ_A and θ_B represent the tangent inclinations at the ending points A and B, respectively. Method 1,

Table 1 Static solutions with different methods and comparisons

		T_A (kN)	T_B (kN)	S (m)	$f_{l/2}/L$	f_{max}/L	x_{max}/l	$\tan\theta_A$	$\tan\theta_B$
Cable J34	Method 1	7333.834	7115.952	576.616	1/101.439	1/101.439	0.499	0.4570	0.3717
	Method 2	7331.219	7114.005	576.616	1/101.725	1/101.725	0.499	0.4569	0.3717
	Method 3	7336.789	7119.051	576.616	1/101.439		0.5	0.4568	0.3714
	Method 4	7331.231	7114.020	578.388	1/101.694	1/101.685	0.499	0.4569	0.3717
	Method 5	7331.219	7114.005	576.615	1/101.689	1/101.689	0.499	0.4569	0.3717

Method 2 and Method 3 in Table 1 mean those in Sections 2, 3 and 4, respectively. Moreover, some results using the methods by Irvine (1981) and by Wei (1998) are also listed and the two methods are denoted as Method 4 and Method 5, respectively. Cable profile in Method 4 is described by the equations set using curvilinear coordinates which are less convenient than Cartesian coordinates in the paper. As for Method 5, static equilibrium equations are just the same with those in Method 2. But there is some difference of the solving process and as a result the expressions for cable parameters are different. It can be seen from Table 1 that all the results are almost the same. Thereby the validities of the methods in the three sections above are well proved.

From the above discussion, it can be found that the parabolic configurations are very close to the real profiles of stay cables because their sag ratio is small. However with the development of bridge engineering, cables will become longer and longer. Thus discussion of the applicability of parabola and the relevant simplified solutions in more extensive situations are made here.

As for a given cable, the cable nonlinearity induced by cable sag effect is determined by cable tension. Therefore by changing the value of V_B from 500 kN to 5000 kN, cable stress varies from 115 MPa to 1100 MPa accordingly. The solutions to extensile cable by Method 2 are taken as the precise reference to testify the validity of the simplified Method 3. T_{A1}, T_{B1} and f_1 are the results by Method 3 and T_{A2}, T_{B2} and f_2 by Method 2. f_1 and f_2 denote sag value at the cable midpoint. The results indicate that all the differences increase with the decrease of cable tension. But they are no more than 0.5% for the cable tension and 1% for the cable sag. Hence the parabola and the relevant simplified solutions are applicable in more extensive situations.

6. Conclusions

The precise and simplified solutions for sag and cable parameters are both derived in this paper. The solutions show cable tension decreases monotonously along the cable from the top end to the low one. The parabola is very close to the actual sag curve of stay cables by considering chord component of cable weight. Based on the parabola, cable parameters could be obtained conveniently.

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