Structural Engineering and Mechanics, Vol. 34, No. 4 (2010) 525-539 525
DOI: http://dx.doi.org/10.12989/sem.2010.34.4.525

Time-dependent and inelastic behaviors of fiber- and particle hybrid composites

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(Received May 13, 2009, Accepted December 15, 2009)

Abstract. Polymer matrix composites are widely used in many engineering applications as they can be customized to meet a desired performance while not only maintaining low cost but also reducing weight. Polymers can experience viscoelastic-viscoplastic response when subjected to external loadings. Various reinforcements and fillers are added to polymers which bring out more complexity in analyzing the time-dependent response. This study formulates an integrated micromechanical model and finite element (FE) analysis for predicting effective viscoelastic-viscoplastic response of polymer based hybrid composites. The studied hybrid system consists of unidirectional short-fiber reinforcements and a matrix system which is composed of solid spherical particle fillers dispersed in a homogeneous polymer constituent. The goal is to predict effective performance of hybrid systems having different compositions and properties of the fiber, particle, and matrix constituents. A combined Schapery's viscoelastic integral model and Valanis's endochronic viscoplastic model is used for the polymer constituent. The particle and fiber constituents are assumed linear elastic. A previously developed micromechanical model of particle reinforced composite is first used to obtain effective mechanical properties of the matrix systems. The effective properties of the matrix are then integrated to a unit-cell model of short-fiber reinforced composites, which is generated using the FE. The effective properties of the matrix are implemented using a user material subroutine in the FE framework. Limited experimental data and analytical solutions available in the literatures are used for comparisons.

Keywords: viscoelasticity; viscoplasticity; micromechanics; homogenization; hybrid composite.

1. Introduction

Polymer materials have been used in many engineering applications due to their relatively low-cost and lightweight as compared to metal and ceramics. Polymers are known for their time-dependent characteristic, poor impact resistance, and low fracture toughness. Particles are added to increase the toughness and reduce the time-dependent characteristics in the polymer matrix. A combined fiber reinforcements and matrix, having particle fillers and polymer, forms a hybrid composite.

Several experimental studies have been conducted on understanding performance of hybrid composites. Young *et al.* (1986) examined elastic moduli of hybrid composites at different

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temperatures. The studied hybrid composite consisted of glass fiber, rubber particle, and epoxy matrix. The elastic moduli of the composite increased with increasing fiber and particle volume contents and decreasing temperatures. It was also concluded that the surface treatment of glass fibers significantly influenced the effective elastic moduli. Friend *et al.* (1991) examined short-fiber and particulate metal matrix hybrid composites. The strength and hardness of the hybrid composite are controlled by the total volume contents of the fibers and particles, while the fracture toughness depends on the particle contents. It was also found that the ultimate tensile strength and toughness decreased with increasing particle sizes. Yilmazer (1992) studied a hybrid composite consisting of glass fiber (GF), glass bead (GB) and acrylonitrile butadieve styrene (ABS). It was shown that increasing GF or GB contents in the ABS matrix increased the ultimate strength of the composite. Increasing GB contents in the ABS matrix decreased the failure strain of the composite. Moreover, increasing GF in the ABS matrix increased the flexural strength and tensile strength of the composite, but increasing GB in ABS decreased the flexural strength. Oh *et al.* (2007) studied failure behavior of a short-fiber/particle hybrid composite. Increasing particle contents increased the fracture toughness and fatigue thresholds of the composite systems. They concluded that the combined short-fiber and particle hybrid composite could increase damage tolerance in the composites. Arunachaleswaran *et al.* (2007) examined creep behaviors of hybrid composites having alumina short-fiber (saffil) and SiC particle in an alloy AE42 matrix, containing magnesium, aluminum and rare earth elements, at different temperatures. They compared the creep resistant of a composite having 10% volume contents of saffil and 10% volume contents of SiC to the one with 15% saffil and 5% SiC. They concluded that the hybrid composite with 10% saffil and 10% SiC exhibited better creep resistance than the one with 15% Saffil and 5% SiC. Mondal *et al.* (2008) also studied creep and recovery responses of saffil, SiC, and AE42 hybrid composites in the longitudinal fiber direction. They compared creep resistant at different temperatures and stress levels between the composite filled with only 20% volume contents of saffil short-fiber and the hybrid composite having 15% saffil and 5% particle. Replacing a part of the expensive saffil short-fiber by the cheap SiC particle is beneficial in reducing cost while maintaining similar performance. They also found that permanent deformation is associated with the fiber breakage and dislocation of matrix.

Limited micromechanical modeling approaches of hybrid composites have been developed for predicting effective elastic behaviors. Liu (1998) presented multiple-step homogenization method, using the rule of mixture, for determining elastic responses of fiber and particle reinforced hybrid composites. The cylinder model is used for the fiber reinforcement, and the sphere model is used for the particle reinforcement. Kanaun *et al.* (2001) used an effective field approach for a three phase hybrid composite to predict overall elastic response. This method used the Mori-Tanaka micromechanical relation. The composite is divided into sub-regions. The overall elastic responses are predicted by the influence of inclusions and correlation of sub regions. Halpin *et al.* (1971) developed a laminate analogy approach (LAA) to predict effective mechanical properties of a laminated system that consists of layers of short fiber composites. This laminate analogy approach is extended for 2D and 3D composite systems consisting of short fiber reinforcements. Fu *et al.* (2002) applied the LAA for predicting effective properties of a short fiber and particle hybrid composite. The particle filled matrix is considered as an effective matrix and then they applied the LAA to the short fiber and particle reinforced hybrid composite. Fu *et al.* (2002) also used a rule of hybrid mixture (RoHM) to obtain the elastic modulus of hybrid particle/fiber/polymer matrix composite. The hybrid composite systems were divided into two systems which are unidirectional fiber reinforced composite and particle reinforced system.

Available studies on micromechanical formulations of hybrid composite systems focus on predicting effective linear elastic properties. Polymers, which are often used as matrix in the hybrid composites, can exhibit time dependent and inelastic responses. The time-dependent and inelastic behaviors become more pronounced at elevated temperatures. This study introduces an integrated micromechanical and FE unit-cell model for predicting performance of hybrid composites. The studied hybrid systems consist of unidirectional short-fiber reinforcements embedded in a matrix system having solid spherical particle fillers and polymers. The fiber and particle sizes are assumed uniform throughout the hybrid composites. A simplified micromechanical formulation of particle reinforced composite is used to obtain effective properties of the matrix system. Constitutive equations for viscoelastic and viscoplastic deformations are used for the homogeneous polymer in the matrix. This matrix system is integrated to the unit-cell model of short fiber reinforced composites, which is generated using the FE code. Limited experimental data and analytical solutions available in the literatures are used for comparisons.

2. Constitutive model for the viscoelastic-viscoplastic responses

A time integration scheme for isotropic viscoelastic-viscoplastic response having stress-dependent material properties is formulated and integrated to the micromechanical model of particle reinforced composites. Linearized solutions of the nonlinear constitutive equation and iterative schemes, which minimize errors due to the linearization, are derived. For a small deformation gradient problem, the total strains and incremental strains at a material point can be additively decomposed into recoverable viscoelastic and irrecoverable viscoplastic components, which are written as

$$\varepsilon_{ij}^{t} = \varepsilon_{ij}^{ve,(t)} + \varepsilon_{ij}^{vp,(t)} = \varepsilon_{ij}^{ve,(t-\Delta t)} + \varepsilon_{ij}^{ve,(t)} + \varepsilon_{ij}^{vp,(t-\Delta t)} + \varepsilon_{ij}^{vp,(t)} \tag{1}$$

where $\varepsilon_{ij}^{ve,(t)}$ and $\varepsilon_{ij}^{vp,(t)}$ are the viscoelastic and viscoplastic strains at current time t, respectively; $\Delta\varepsilon_{ij}^{ve,(t)}$ and $\Delta\varepsilon_{ij}^{vp,(t)}$ are the current incremental viscoelastic and viscoplastic strains. Detailed time integration algorithm can be found in Kim and Muliana (2009).

The Schapery (1969) nonlinear single integral equation is used for the viscoelastic component. This model is generalized for isotropic viscoelastic response. The viscoelastic strain is decomposed into deviatoric and hydrostatic components. The Schapery integral form is applied separately to the deviatoric and hydrostatic strains, which are written as

$$\varepsilon_{ij}^{ve,t} = e_{ij}^{ve,t} + \frac{1}{3}\delta_{ij}\varepsilon_{kk}^{ve,t} \tag{2}$$

$$e_{ij}^{ve,t} = \frac{1}{2}g_0(\bar{\sigma}^t)J_0S_{ij}^t + \frac{1}{2}g_1(\bar{\sigma}^t)\int_0^t \Delta J^{(\psi^t-\psi^\tau)}\frac{d[g_2(\bar{\sigma}^\tau)S_{ij}^\tau]}{d\tau}d\tau \tag{3}$$

$$\varepsilon_{kk}^{ve,t} = \frac{1}{3}g_0(\bar{\sigma}^t)B_0\sigma_{kk}^t + \frac{1}{3}g_1(\bar{\sigma}^t)\int_0^t \Delta B^{(\psi^t-\psi^\tau)}\frac{d[g_2(\bar{\sigma}^\tau)\sigma_{kk}^\tau]}{d\tau}d\tau \tag{4}$$

Where J_0 and B_0 are the elastic shear and bulk compliances, respectively, ΔJ_0 and ΔB_0 are the transient shear and bulk compliances, respectively, and $\bar{\sigma}^t$ is the effective stress at current time. The shear and bulk compliances are expressed in terms of D^0 and $\Delta D(t)$, which are the uniaxial elastic and transient compliances. The uniaxial transient compliance is expressed in terms of Prony series

$\Delta D(t) = \sum_{n=1}^{N} D_n(1.-e^{t\lambda n})$, where N is a number of Prony series terms, D_n and λ_n are the coefficient and inverse of characteristic of creep time for each Prony term, respectively. The corresponding linear elastic Poisson's ratio for the isotropic viscoelastic material is assumed constant. The variable ψ is the reduced-time (effective time) given by

$$\psi^t \equiv \psi(t) = \int_0^t \frac{ds}{a(\bar{\sigma}^t)} \tag{5}$$

The parameter $a(\bar{\sigma}^t)$ is the time-stress shift factor. The parameter g_0 measures the reduction or increase in elastic compliances due to stresses. The parameter g_1 measures the effect of stress on the transient compliance. The parameter g_2 accounts for the loading rate effect on the time-dependent response. Eqs. (3) and (4) are solved using a recursive-iterative method, which was previously presented by Haj-Ali and Muliana (2004). The incremental deviatoric and volumetric strains are given as

$$\begin{aligned}\Delta e_{ij}^{ve,t} &= \frac{1}{2}\left[g_0^t J_0 + g_1^t g_2^t \sum_{n=1}^{N} J_n - g_1^t g_2^t \sum_{n=1}^{N} J_n \left(\frac{1-\exp[-\lambda_n \Delta\psi^t]}{\lambda_n \Delta\psi^t}\right)\right]\Delta S_{ij}^t - \\ &\frac{1}{2} g_1^t \sum_{n=1}^{N} J_n \left[\exp[-\lambda_n \Delta\psi^t] q_{ij,n}^{t-\Delta t} - g_2^{t-\Delta t} \frac{(1-\exp[-\lambda_n \Delta\psi^t])}{\lambda_n \Delta\psi^t} \Delta S_{ij}^{t-\Delta t}\right]\end{aligned} \tag{6}$$

$$\begin{aligned}\Delta \varepsilon_{kk}^{ve,t} &= \frac{1}{3}\left[g_0^t B_0 + g_1^t g_2^t \sum_{n=1}^{N} B_n - g_1^t g_2^t \sum_{n=1}^{N} B_n \left(\frac{1-\exp[-\lambda_n \Delta\psi^t]}{\lambda_n \Delta\psi^t}\right)\right]\Delta \sigma_{kk}^t - \\ &\frac{1}{3} g_1^t \sum_{n=1}^{N} B_n \left[\exp[-\lambda_n \Delta\psi^t] q_{kk,n}^{t-\Delta t} - g_2^{t-\Delta t} \frac{(1-\exp[-\lambda_n \Delta\psi^t])}{\lambda_n \Delta\psi^t} \Delta \sigma_{kk}^{t-\Delta t}\right]\end{aligned} \tag{7}$$

where $q_{ij,n}^{t-\Delta t}$ and $q_{kk,n}^{t-\Delta t}$ are history variables correspond to the deviatoric and volumetric strains.

Several viscoplasticity theories based on the concept of the yield surface have been derived. The total strain was divided into two components, which are recoverable elastic or viscoelastic strain and an irrecoverable viscoplastic strain. Some polymer used as constituents in composites can exhibit inelastic deformations even at a very low stress level. Therefore, it is quite difficult to accurately define the yield point at a very low stress level. The endochronic theory derived by Valanis (1971) without a yield surface for isotropic materials is used to model viscoplastic responses of polymers. Kerth and Lin (2003) developed an incremental endochronic theory with the effect of temperature that is compatible with FE method. The plastic deformation was incorporated for the deviatoric response. Lee and Chang (2004) extended the endochronic theory to describe collapse of cylindrical tubes under external pressure and axial tension. They used first-order differential equations of the rate-sensitivity function of the intrinsic time measure to investigate the collapse. The rate-sensitivity function is also used to simulate the viscoplastic behavior.

The Valanis viscoplastic deviatoric stress is given as

$$S_{ij}^t = 2G\int_0^t \rho(z^t - z')\frac{de_{ij}^{vp}}{dz'}dz' \tag{8}$$

where S_{ij}^{t} and $e_{ij}^{vp,t}$ are the deviators of the stress and plastic strain tensors, respectively, and G is the shear modulus. The kernel $\rho(z)$ is a material function and z appearing in the equation is the intrinsic time function. The kernel $\rho(z)$ can be written as

$$\rho(z) = \rho_0 \delta(z) + \rho_1(z) \tag{9}$$

$$S_y^0 = 2G\rho_0 \tag{10}$$

where ρ_0 is a material constant, $\delta(z)$ is the Dirac delta function, and ρ_1 is a nonsingular function and is related to a kinematic hardening behavior. The variable S_y^0 is a material constant which can be related to the yield stress. Substituting Eqs. (9) and (10) into (8) yields

$$S_{ij}^{t} = S_y^0 \frac{de_{ij}^{vp,t}}{dz} + 2G\int_0^t \rho_1(z^t - z')\frac{de_{ij}^{vp,t}}{dz'}dz' \tag{11}$$

where

$$dz = \frac{d\zeta}{f(\zeta)} \tag{12}$$

For the isotropic hardening model, the second term of Eq. (11) can be retrieved at all times. Eq. (11) can be rewritten as

$$S_{ij}^{t} = S_y^0 \frac{de_{ij}^{vp,t}}{dz} = S_y^0 \frac{de_{ij}^{vp,t}}{d\zeta} f(\zeta) \tag{13}$$

The intrinsic time scale, ζ, is defined as (Valanis 1971, Khan and Huang 1995)

$$d\zeta = (de_{ij}^{vp,t}, de_{ij}^{vp,t})^{1/2} \tag{14}$$

Substituting Eqs. (14) to (13), the yield criterion is given as

$$S_{ij}^{t} S_{ij}^{t} = (S_y^0)^2 f^2(\zeta) \tag{15}$$

The parameter $f(\zeta)$ is a nonnegative function of the intrinsic time scale, ζ, with $f(0) = 0$. If $f(\zeta)$ equal to 1, then Eq. (15) corresponds to the von Mises yield function. The intrinsic time scale function $f(\zeta)$ can take various forms (Valanis 1971), such as

$$f(\zeta) = 1 + \beta\zeta \quad \text{(Linear form)} \tag{16}$$

$$f(\zeta) = a + (1-a)e^{-b\zeta} \quad \text{(Exponential form)} \tag{17}$$

Where β, a, and b are material constants that have to be characterized from experiments. The total plastic work is defined by

$$W^{vp,t} = \int_0^t S_{ij}^{t} de_{ij}^{vp} \tag{18}$$

The evolution of W^p can be obtained by differentiating Eq. (18)

$$dW^{vp,t} = S_{ij}^{t} de_{ij}^{vp} \tag{19}$$

The material responses are categorized as

$$S_{ij}^{t}S_{ij}^{t} \le (S_y^0)^2 f^2(\zeta) \quad \text{(elastic or viscoelastic response)} \tag{20}$$

$$S_{ij}^{t}S_{ij}^{t} = (S_y^0)^2 f^2(\zeta) \quad \text{and} \quad S_{ij}^{t} de_{ij}^{vp,t} > 0 \quad \text{(viscoplastic response)} \tag{21}$$

The time-integration scheme is formulated based on incremental time step. The time derivative of the deviatoric viscoplastic strain, from Eq. (13), is

$$de_{ij}^{vp,t} = \frac{1}{S_y^0 f(\zeta)} S_{ij}^{t} d\zeta \tag{22}$$

The strain increases monotonically by setting $f(\zeta)$ to be a monotonic decreasing function of ζ. The incremental total strain is defined by additively combining the incremental viscoelastic and viscoplastic components and also their deviatoric and volumetric parts. The expression of $d\zeta$, corresponding to Eq. (4), can be expressed in terms of total incremental stress and strain (Valanis 1971, Khan and Huang 1995), which are given as

$$\begin{gathered} d\zeta = \frac{1}{(S_y^0)^2 f(\zeta) f'(\zeta)} [2G(S_{ij}^{t} de_{ij}^{t}) - 2G S_y^0 f(\zeta) d\zeta] \\ \left[1 + \frac{2G}{S_y^0 f'(\zeta)}\right] d\zeta = \frac{2G}{(S_y^0)^2 f(\zeta) f'(\zeta)} (S_{ij}^{t} de_{ij}^{t}) \end{gathered} \tag{23}$$

Thus

$$d\zeta = \frac{2G}{S_y^0 f(\zeta)[2G + S_y^0 f'(\zeta)]} (S_{ij}^{t} de_{ij}^{t}) \tag{24}$$

Substituting Eqs. (24) to (22), we obtain time-derivative of the viscoplastic strain

$$de_{ij}^{vp,t} = \frac{2G}{(S_y^0)^2 f^2(\zeta)[2G + S_y^0 f'(\zeta)]} (S_{ij}^{t} de_{ij}^{t}) S_{ij}^{t} \tag{25}$$

An incremental formulation of the Valanis viscoplastic constitutive model is formed by assuming $de \approx \Delta e / \Delta t$. Eq. (25) is now written as

$$\Delta e_{ij}^{vp,t} = \frac{2G}{(S_y^0)^2 f^2(\zeta)[2G + S_y^0 f'(\zeta)]} (S_{ij}^{t} \Delta e_{ij}^{t}) S_{ij}^{t} \tag{26}$$

Eq. (26) together with Eqs. (6)-(7) give total incremental strain at current time t.

3. Integrated micromechanical and FE model of hybrid composite systems

The studied hybrid composites consist of unidirectional short fibers and matrix systems, having particle fillers and polymers, as illustrated in Fig. 1. The micromechanical model of particle reinforced composites is used to obtain effective viscoelastic and viscoplastic responses of the matrix systems in the hybrid composite. FE mesh of a unit cell model of the unidirectional short-fiber reinforced composite is generated and the micromodel of particle reinforced composites is

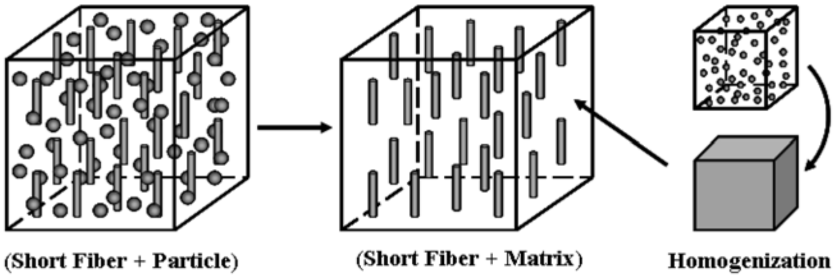

Fig. 1 Hybrid composite systems with short fiber and particles

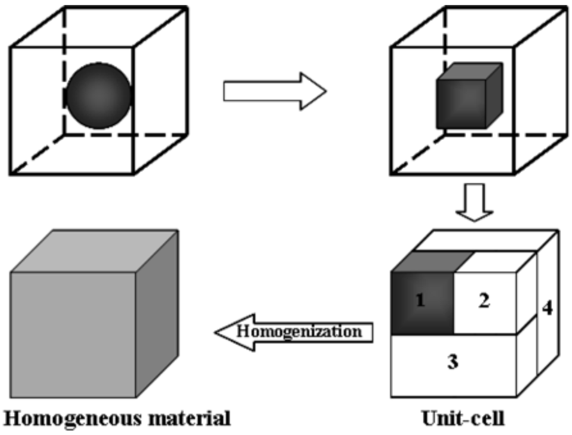

Fig. 2 Representative unit-cell model for the particulate reinforced polymers

implemented for the matrix element via a user material (UMAT) subroutine. In this study, we assume that the microstructures of hybrid composites consist of discontinuous unidirectional short fibers dispersed in homogeneous matrix having particle fillers for the following reason. Although the fibers and particles have approximately the same diameters, the length of the fibers is much higher than the particle and fiber's diameters ($L_f/d_f > 20$ and $L_f/d_p > 6$). When we pick the smallest representative medium of hybrid composites it often contains a single fiber with more than two particles. It should be noted that the order of homogenization could influence the prediction of the effective responses of the hybrid composites. An alternative approach is to homogenize responses of fiber, particle, and polymer constituents all together. The effective properties of a matrix in the hybrid composite are approximated using a volume average of constituents in the particle reinforced composite model. A representative unit-cell model of solid spherical particle is given in Fig. 2.

Different microstructural characteristics can result in large variations in the local field variables. For elastic and plastic deformations in heterogeneous materials the effects of microstructural characteristics on the overall properties and field variables have been studied (examples can be found in Kouznetsova *et al.* 2001, Jiang *et al.* 2002, Bilger *et al.* 2005, Kim and Muliana 2009, Muliana and Kim 2010). A unit-cell model with one inclusion embedded in a matrix medium results in similar overall responses to those of unit-cell models having several inclusions, especially for linear elastic responses. When plastic deformations are considered, the overall responses from the unit-cell model with one inclusion (assuming periodic boundary conditions) and the ones with several arrangements of inclusions show larger variations, but still within acceptable tolerance. The unit-cell model with one inclusion cannot capture shear band and/or localized stress behaviors in the matrix medium. Furthermore, when temperature effects are considered, microstructural characteristics can strongly influence thermal (residual) stresses in the composites. Lee and Sohn (1994) investigated the residual stress distribution in a unidirectional graphite/epoxy laminate composite under different loading conditions and temperatures. The residual stresses depend on the matrix behavior during polymerization and the magnitude of applied load. During a cooling down period, the magnitude of the residual stresses increases, which is due to the mismatches in the thermo-mechanical properties of the constituents in the composites.

A unit-cell model for the particle-reinforced system is composed of four sub-cells as shown in Fig. 2. Following Haj-Ali and Pecknold (1996), in a heterogeneous periodic medium, a basic unit-cell that represents geometrical and material characteristics can be defined. Each unit-cell is divided into a number of subcells and the spatial variation of the displacement field in each subcell is assumed such that the stresses and deformations are spatially uniform. Traction continuity and displacement compatibility at an interface between subcells are satisfied in an average sense. Thus, the average stresses and strains in the unit-cell model are defined by

$$\begin{aligned} \bar{\sigma}_{ij} &= \frac{1}{V}\sum_{\alpha=1}^{N}\int_{V^{(\alpha)}} \sigma_{ij}^{(\alpha)}(x_k^{(\alpha)})dV^{(\alpha)} \approx \frac{1}{V}\sum_{\alpha=1}^{N} V^{(\alpha)}\sigma_{ij}^{(\alpha)} \\ \bar{\varepsilon}_{ij} &= \frac{1}{V}\sum_{\alpha=1}^{N}\int_{V^{(\alpha)}} \varepsilon_{ij}^{(\alpha)}(x_k^{(\alpha)})dV^{(\alpha)} \approx \frac{1}{V}\sum_{\alpha=1}^{N} V^{(\alpha)}\varepsilon_{ij}^{(\alpha)} \end{aligned} \tag{27}$$

The superscript (α) denotes the sub-cell number and N is the number of sub-cells. The stress $\sigma_{ij}^{(\alpha)}$ and strain $\varepsilon_{ij}^{(\alpha)}$ are the average stress and strain within each sub-cell. Following Hill's (1964) micromechanical model formulation, local average stress and strain fields are related to the effective (global) stress and strain fields through concentration tensors. In this study, the sub-cell strain-interaction (concentration) matrix $\mathbf{B}^{(\alpha)}$, which relates the sub-cell average strains, $\boldsymbol{\varepsilon}^{(\alpha)}$, to the effective unit-cell average strain, $\bar{\boldsymbol{\varepsilon}}$ is defined as

$$\varepsilon_{ij}^{(\alpha)} = B_{ijkl}^{(\alpha)}\bar{\varepsilon}_{kl} \tag{28}$$

Using the sub-cell's strain in Eq. (28), the effective strain and stress are given as

$$\begin{aligned} \bar{\varepsilon}_{ij} &= \frac{1}{V}\sum_{\alpha=1}^{N} V^{(\alpha)}B_{ijkl}^{(\alpha)}\bar{\varepsilon}_{kl} \\ \bar{\sigma}_{ij} &= \frac{1}{V}\sum_{\alpha=1}^{N} V^{(\alpha)}C_{ijkl}^{(\alpha)}B_{klrs}^{(\alpha)}\bar{\varepsilon}_{rs} \end{aligned} \tag{29}$$

Finally, the effective stiffness matrix is written as

$$\bar{C}_{ijrs} = \frac{1}{V}\sum_{\alpha=1}^{N} V^{(\alpha)}C_{ijkl}^{(\alpha)}B_{klrs}^{(\alpha)} \tag{30}$$

where $\mathbf{C}^{(\alpha)}$ is the stiffness matrix of the sub-cell, and $\bar{\mathbf{C}}$ is the unit-cell effective stiffness matrix. Up to this point, the strain interaction matrices $\mathbf{B}^{(\alpha)}$ have not been determined. In order to derive the strain-interaction matrices for all subcells, the micromechanical relations together with sub-cells constitutive material models are imposed. The micromechanical model relations can be found in Muliana and Kim (2007). The effective response of the particle reinforced composites are

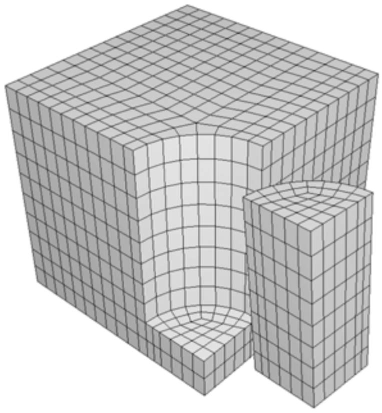

Fig. 3 Representative volume element for short fiber/particle reinforced composite

implemented for matrix materials, using a user material (UMAT) subroutine, in the FE unit-cell model of fiber reinforced matrix composites. It is noted that the above micromechanical formulations give exact solution for a linear elastic response. Due to the nonlinear and time-dependent of the hybrid composite, the micromechanical relations are used for the incremental stress-strain relations, which give linearized (trial) solutions. A correction scheme is added to minimize errors from the linearization.

4. Numerical implementation and verification

The integrated micromechanical model of particle reinforced composites and FE model of a unit-cell model of the short-fiber composites is numerically implemented. Fig. 3 shows a representative unit-cell model of short fiber-particle reinforced composites. The properties of matrix are obtained using the homogenization method of particle reinforced composite. The fiber and particle are assumed to be linear elastic. The polymer constituent follows viscoelastic-viscoplastic responses.

4.1 Elastic responses

The elastic modulus obtained from the proposed approach is compared with the ones obtained using analytical solutions reported by Fu *et al*. (2002). The hybrid composite consists of glass fiber, calcite particle, and ABS matrix. The fiber length and radius are 336 μm and 6.9 μm, respectively, and particle diameter is 13~37 μm. Elastic properties are given in Table 1. The integrated micromechanical and FE unit-cell model is generated for hybrid composites having 10% and 30%

Table 1 Elastic properties of particle/short fiber/ABS composites

Mechanical properties	Material type		
	Calcite particle	ABS matrix	Short glass fiber
Young's modulus E (GPa)	167	2.39	75
Poisson ratio, v	0.25	0.35	0.25

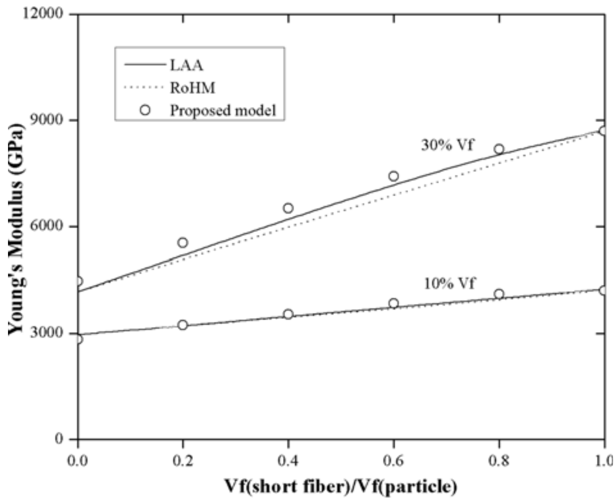

Fig. 4 Effective elastic moduli of hybrid composite

Table 2 Elastic properties of the phases in PP-EPR-SGF hybrid composite

Mechanical properties	Material type		
	PP	EPR	SGF
Young's modulus E (GPa)	1.25	3.0	70
Poisson ratio, v	0.42	0.42	0.23

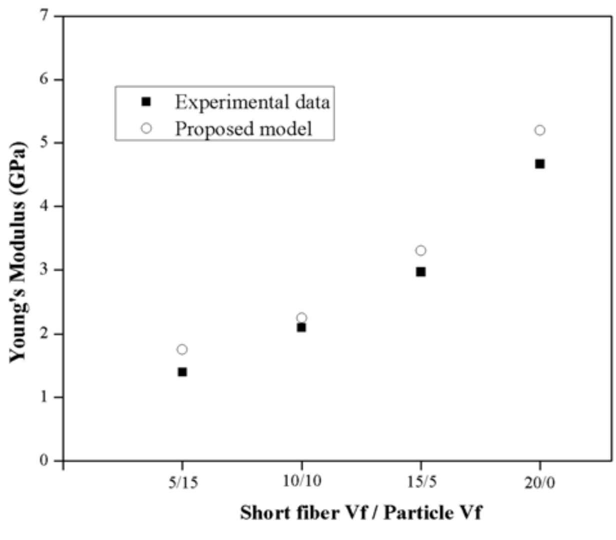

Fig. 5 Comparison for the elastic modulus of PP-EPR-SGF hybrid composite

total volume contents of particles and fibers. The elastic moduli generated using the proposed approach agrees well with the RoHM and LAA results (Fig. 4). In Fig. 4, Vf_fiber indicates a volume content of short fibers and Vf_total denotes total volume contents of short fibers and particles.

The effective elastic moduli generated using the integrated micromechanical and FE framework is verified using experimental data for a combined polypropylene (PP), ethylene propylene rubber (EPR), and short glass fiber (SGF) hybrid composite (Zebarjad *et al*. 2001). The elastic properties for the PP matrix, EPR particles and SGF are given in Table 2. Fig. 5 illustrates effective elastic moduli of hybrid composites at several fiber and particle combinations. The homogenization scheme gives reasonable predictions.

4.2 Time dependent responses

As time-dependent data of polymer hybrid composites with short fibers are not available, creep data on metal based hybrid composites (Arunachaleswaran *et al*. 2007) are used to verify the proposed modeling approach. The studied hybrid composite consists of alumina short fiber (Saffil), particle (SiC), and matrix (alloy AE42). The fibers have an average diameter of 8 μm and an average length of 180 μm. The particles have an average diameter of 30 μm. The viscoelastic parameters for the matrix are characterized by matching the creep response of the matrix in Fig. 6. The elastic properties of the constituents are given Table 3 and the time-dependent parameters of the matrix are given in Table 4. Creep strains of hybrid composite having 10% short fiber + 10% particles and 10% short fiber + 15% particle subject to a constant stress of 100 MPa at 240°C are illustrated in Fig. 6. Overall good predictions of creep responses are observed.

A combined viscoelastic-viscoplastic response of hybrid composites is examined using the proposed modeling approach. In this study, short fiber and particle are assumed to be linear elastic, while polymer matrix exhibit viscoelastic-viscoplastic responses. The time-integration algorithm for

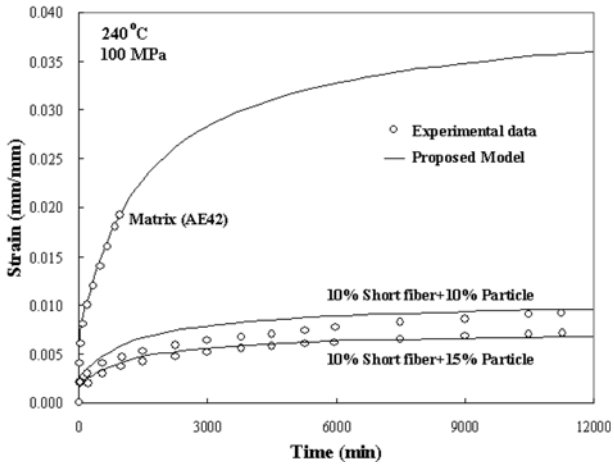

Fig. 6 Comparison with experimental data at 240°C and 100 MPa

Table 3 Elastic properties of the phases in AE42-Saffil-SiC hybrid composite

Mechanical properties	Material type		
	AE42	SiC	Saffil
Young's modulus E (GPa)	45	450	300
Poisson ratio, v	0.3	0.3	0.25

Table 4 Prony series coefficients for the AE42 matrix

n	λ_n (min^{-1})	$D_n \times 10^{-4}$ (MPa^{-1})
1	1	0.111
2	0.07	4.000
3	0.001	1.500
4	0.0002	1.500

Table 5 Elastic properties of the phases in Short fiber-Glass particle-HDPE hybrid composite

Mechanical properties	Material type		
	HDPE	Glass particle	Glass short fiber
Young's modulus E (GPa)	4.535	69	70
Poisson ratio, v	0.3	0.3	0.3

Tabel 6 Prony series coefficients for the HDPE polymer

n	λ_n (min^{-1})	$D_n \times 10^{-4}$ (MPa^{-1})
1	1	2.23
2	10^{-1}	2.27
3	10^{-2}	1.95
4	10^{-3}	3.50
5	10^{-4}	5.50
6	10^{-5}	5.50

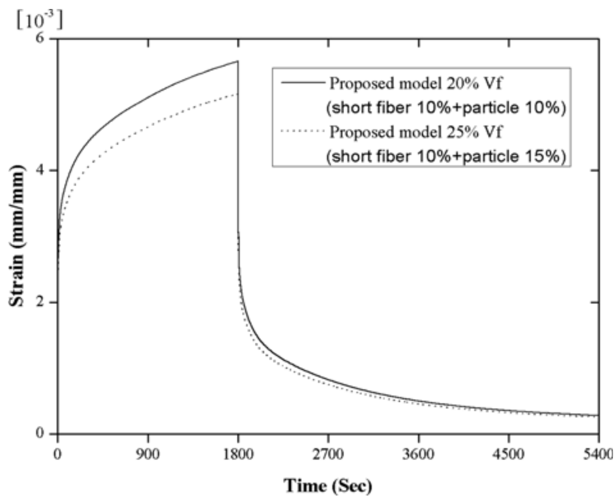

Fig. 7 Creep-recovery responses of Short fiber-particle-HDPE hybrid composite

the combined viscoelastic-viscoplastic model is used for the high density polyethylene (HDPE) matrix. The micromechanical model of particle reinforced composite is implemented in the UMAT subroutine of the ABAQUS FE code. The viscoelastic and viscoplastic parameters of the HDPE are calibrated from experimental data of Lai and Bakker (1995). The elastic properties of short fiber, particle, and HDPE constituents are given in Table 5. The viscoelastic properties for HDPE

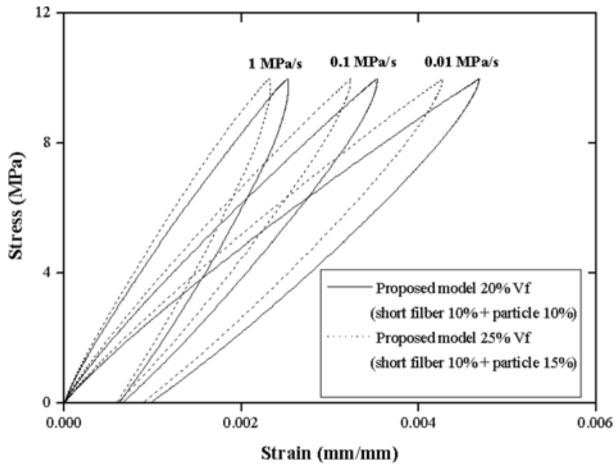

Fig. 8 Stress-strain relation under different loading rate

constituent is given in Table 6. The viscoplastic material constants of the endochronic model S_y^0 and β in Eqs. (15) and (16) are 21 and 10^{-7}, respectively. The variable S_y^0 is related to the magnitude of the plastic strain and β is the time dependent parameter. The integrated micromechanical and FE unit-cell model is used to predict creep-recovery responses at 10 MPa for two hybrid systems. The first system has 10% volume contents of short fiber and 10% volume contents of particle and the second system consists of 10% volume contents of short fiber and 15% volume contents of particles. Fig. 7 illustrates the creep-recovery responses due to loading in the axial fiber direction. Permanent deformations are exhibited after a complete removal of the loads. It is seen that adding elastic particles reduces creep strain during loading, but insignificantly decreases the permanent deformation. It is noted that loadings are applied in the axial fiber direction and majority of the axial loads are carried by the linear elastic fiber and only smart part of the external stress is carried by the matrix. Adding particles to the matrix reduces creep in the matrix, eventually decreasing creep in the overall hybrid systems. The low stress level in the matrix might contribute to small plastic deformations during loadings.

The proposed modeling approach is also used to simulate stress-strain responses during quasi-static loadings at different loading rates: 0.01, 0.1, 1.0 MPa/s. Loadings are applied in the axial fiber direction. Fig. 8 shows predictions of the stress-strain responses under the three different stress rates. The combined viscoelastic-viscoplastic response is more pronounced for a slow loading. It is also seen that increasing total volume contents of the elastic reinforcements reduces the maximum strain, but insignificantly decreases the permanent deformation.

5. Conclusions

An integrated micromechanical model and unit-cell FE analyses has been generated for predicting effective viscoelastic-viscoplastic responses of hybrid composites consisting of short-fiber reinforcements, aligned unidirectionally, and a matrix system. The matrix is composed of solid spherical particle fillers dispersed in a homogeneous polymer constituent. The polymer constituents exhibit a combined viscoelastic-viscoplastic response, while the fiber and particle constituents are linear elastic. A simplified micromechanical model of a solid spherical particle reinforced composite is used to obtained effective response of the matrix systems. The effective properties of the matrix are then integrated to a unit-cell FE model of short-fiber reinforced composites. The proposed approach is shown to be capable of predicting overall behavior of hybrid composites. This approach is also useful for designing a new material system as overall responses of hybrid systems can be simulated by varying properties and compositions of the constituents.

Acknowledgements

This research is sponsored by the National Science Foundation under Grant No. 0546528.

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