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Nonlinear free vibration of heated corrugated annular plates with a centric rigid mass

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Abstract. A computational analysis of the nonlinear free vibration of corrugated annular plates with shallow sinusoidal corrugations under uniformly static ambient temperature is examined. The governing equations based on Hamilton's principle and nonlinear bending theory of thin shallow shell are established for a corrugated plate with a concentric rigid mass at the center and rotational springs at the outer edges. A simple harmonic function in time is assumed and the time variable is eliminated from partial differential governing equations using the Kantorovich averaging procedure. The resulting ordinary equations, which form a nonlinear two-point boundary value problem in spatial variable, are then solved numerically by shooting method, and the temperature-dependent characteristic relations of frequency vs. amplitude for nonlinear vibration of heated corrugated annular plates are obtained. Several numerical results are presented in both tabular and graphical forms, which demonstrate the accuracy of present method and illustrate the amplitude frequency dependence for the plate under such parameters as ambient temperature, plate geometry, rigid mass and elastic constrain.

Keywords: corrugated annular plate; nonlinear vibration; temperature change; elastic constraint; central rigid mass; shooting method.

1. Introduction

The corrugated thin plates are extensively used in engineering structures. The corrugations reinforce the plates and improve their strength to weight ratio, these distinct mechanical features make the structures more cost-efficient, and lead the corrugated plates more popular in decking, roofing and sandwich plate core structures in order to economize on the plate materials or lighten the plates, especially when used in the precision instruments and sensors as very sensitive elastic elements. Due to their high strength and large flexibility, the corrugated plates are usually required to withstand large amplitude of vibration. In such situations, it is necessary to include the geometrically-induced non-linearity when investigating the structural dynamic behavior.

In most of the studies carried out on the geometrically nonlinear flexural vibrations of plates and shells with various geometries and boundary constraints, the common approach has been to use an assumed space or time function (Huang and Huang 1989, Huang and Aurora 1979), it being supposed that the space and time functions can be separated (Haterbouch and Benamar 2003, 2004).

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In the assumed space mode method, a spatial function which satisfies the related boundary conditions is assumed to eliminate the space co-ordinate, the nonlinear partial differential governing equations are then reduced to a set of nonlinear ordinary differential equations, with time as an independent variable (Dumir *et al.* 1984, 1985). For example, the finite element method (Huang 1998, Rao *et al.* 1993, Stoykov and Ribeiro 2008) and the Galerkin method (Dumir 1986) have been to use an assumed space mode. In the assumed time mode method, the dependence on time is assumed to be harmonic. Then, by utilizing the Kantorovich time-averaging procedure, the nonlinear partial differential governing equations are converted into corresponding ordinary ones, which form a nonlinear boundary value problem in spatial variable (Huang and Walker 1988). Several numerical methods have been proposed for solving this problem. The shooting method is more popular among them. In this case, the boundary value problem is converted to an initial value one, by integrating it using the Runge-Kutta method and at the same time, by performing the successive corrections according to the Newton-Raphson method, an assumed-time-mode solution can be achieved (Li and Zhou 2001, Li and Zhou 2002, 2003, Allahverdizadeh *et al.* 2008). It should be mentioned that the shooting method is applicable not only in the spatial domain like what follows here, it has also been successfully applied in the time domain to investigate the nonlinear vibration of plates and shells. Further information about this time domain procedure can be found in Ribeiro's works (Ribeiro 2004, 2005, 2006, 2008) and references therein. From the view of past literature available on the topic of large amplitude vibration modeling and analysis for plates and shells, it is found that a variety of numerical and analytical methods have been proposed and adopted. Extensive literature reviews on the nonlinear vibration of plates and shells have been reported in (Chia 1980, Sathyamoorthy 1987, Liew *et al.* 1997). Among the commonly used computational approaches, the shooting method has been proved to be an efficient method. This method is extended here to the nonlinear vibration problem of a heated sinusoidally corrugated annular plate with a rigid mass at the center and rotational springs at the outer edges.

Although the geometrically nonlinear problem of a corrugated plate has received a great attention, problems involved are quite difficult to solve satisfactorily and adequately because of the complicated geometry of the plates as well as the nonlinear mathematics. Review of the literature indicates that different investigators studied the nonlinear problems in a corrugated plate, analytically and numerically, based on two nonlinear bending theories: isotropic shallow shell theory and anisotropic plane plate theory. The consideration in nonlinear bending theory of thin shallow shells was first forwarded by Panov (1941) who studied the large deflection problem of shallow corrugated membrane. Axelrad (1964) solved the membrane with deep sinusoidal corrugations using Galerkin's method. Hamada *et al.* (1968) discussed the bending problem of a diaphragm with a boundary corrugation using the finite difference method. Bihari and Elbert (1978) obtained the deflection and radial displacement of a corrugated plate by directly solving 6 coupled first-order differential equations. Chen (1980) solved the large deflection equations of shells for a corrugated circular plate with shallow sinusoidal corrugation using the modified iteration method. Liu and Yuan (1997, 2003) investigated the bending problem of a corrugated plate with a large boundary corrugation under actions of various loads based on the simplified Reissner's equation of axisymmetric shells of revolution by means of the integral equation method. The second theory assumes a corrugated plate as an anisotropic plate and the large deflection theory of thin anisotropic plane plates had been adopted. The pioneer treatment in this theory was suggested by Haringx (1956) who developed a new means to transform a corrugated circular plate into an equivalent orthotropic circular plate. Employing this theory, Liu and Li (1989, 1990) studied the nonlinear bending and free vibration for corrugated circular plate with or without plane central regions via Galerkin's method and modified iteration method. Wang *et al.* (1987) dealt with the nonlinear free vibration of corrugated circular plates and gained the analytical solutions for the amplitude-frequency relationship through perturbation-variation method. Generally, the nonlinear bending theory of plates based approach is applicable to corrugated circular plate with dense corrugations of various types, e.g., toothed corrugations and sinusoidal corrugations, while the shells based way proves to be independent of the density of corrugations (Chen 1980, Zheng 1994).

The studies on geometric nonlinearity in a corrugated plate referred to in the previous paragraph and a few more are devoted mainly to the problem of large deflection bending, where the plate undergoes various laterally static loadings (Liew *et al.* 2007, Peng *et al.* 2007). However, only limited attention has been given to the large amplitude vibration of these plates due to the more complex nature of resulting governing equations when time variable is involved. Some, but much fewer, attempts have been made to predict the nonlinear vibration behavior of corrugated plates in an environment of changing temperatures. More recently, Wang *et al.* investigated the nonlinear vibrations for heated clamped circular plates with full sinusoidal corrugations based on isotropic shallow shell theory using the perturbation-variatiom method (Wang *et al.* 2008) and shooting method (Wang *et al.* 2009).

The objective of present paper is to investigate the axisymmetrically nonlinear vibration of a uniformly heated sine-shaped corrugated annular plate with a concentric mass attached rigidly at the inner edge and rotational springs restrained elastically at the outer edge. The corrugated plate is assumed to be a thin plate with small initial deflection under a distributed time-independent temperature load, and the partial differential equations governing the nonlinear free vibration of corrugated annular plates are formulated from Hamilton's principle in terms of Von Kármán's theory. Assuming the existence of harmonic vibration, the time variable is eliminated by means of the Kantorovich time-averaging method (Huang and Walker 1988, Li and Zhou 2001, 2002, 2003, Allahverdizadeh *et al.* 2008). The governing equations thus reduce to a pair of nonlinear ordinary differential equations with two-point boundary conditions. A numerical analysis is then accomplished by shooting method. The comparison with available published results shows that the proposed approach is of good reliability. A detailed parametric study is conducted involving the dependency of nonlinear frequency on the depth and density of corrugations, temperature change, central mass and radius, along with the edge constraints. Effects of these variables on the trend of nonlinearity are plotted and discussed.

2. Dynamic governing equations

Consider a corrugated annular plate with shallow sinusoidal corrugations having an outer radius a, inner radius b, constant thickness h, corrugation semi-wave length l, wave height f, and an attached concentric rigid mass M_c. The outer edge of the plate is immovably simply supported, restrained elastically against rotation by rotational springs of stiffness k_φ. Let (r, θ, z) denote a set of cylindrical co-ordinates, as shown in Fig. 1, then the wave of plate can be expressed as

$$\overline{w}(r) = f\cos\left(\frac{\pi}{l}r + \varphi\right) \tag{1}$$

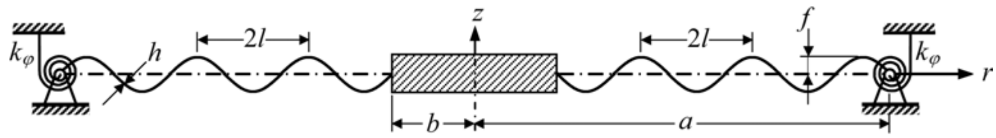

Fig. 1 Corrugated annular plate with shallow sinusoidal corrugations

The corrugated plate is taken as a thin plate with small axisymmetric initial deflection $\overline{w}(r)$, and its material is presumed to be elastic, homogeneous and isotropic. On the basis of Von Kármán's theory, the partial differential equations governing the axisymmetrically large amplitude vibration for a corrugated annular plate under a uniform temperature change T can be derived from Hamilton's principle, when neglecting the longitudinal and rotary inertias (Wang and Dai 2004, Haterbouch and Benamar 2005, Gupta and Ansari 1998, 2002), as follows

$$\int_{t_1}^{t_2}\int_b^a\left\{D\tilde{\nabla}^4 w-\frac{1}{r}\frac{\partial}{\partial r}\left[rN_r\left(\frac{\partial w}{\partial r}+\frac{d\overline{w}}{dr}\right)\right]+\rho h\frac{\partial^2 w}{\partial t^2}\right\}\delta w dr dt = 0 \tag{2}$$

$$\tilde{J}(rN_r) = -\frac{1}{2}Eh\frac{\partial w}{\partial r}\left(\frac{\partial w}{\partial r}+2\frac{d\overline{w}}{dr}\right) \tag{3}$$

with the associated boundary conditions (Huang 1998)

$$\frac{\partial w}{\partial r} = 0,\ Dr\left(\frac{\partial^3 w}{\partial r^3}+\frac{1}{r}\frac{\partial^2 w}{\partial r^2}\right)+\frac{M_c}{2\pi}\frac{\partial^2 w}{\partial t^2} = 0,\ \frac{\partial}{\partial r}(rN_r)-\nu N_r+(1-\nu)\alpha T = 0 \quad \text{at} \quad r = b \tag{4}$$

$$w = 0,\ -D\left(\frac{\partial^2 w}{\partial r^2}+\frac{\nu}{r}\frac{\partial w}{\partial r}\right) = k_\varphi\frac{\partial w}{\partial r},\ \frac{\partial}{\partial r}(rN_r)-\nu N_r+(1-\nu)\alpha T = 0 \quad \text{at} \quad r = a \tag{5}$$

In the foregoing expressions, t is the time variable, $w(r,t)$ and $N_r(r,t)$ signify the transverse deflection and the radial membrane force respectively. $D = Eh^3/[12(1-\nu^2)]$ is the flexural rigidity, ρ designates the mass density of the plate material, E, ν and α indicate the elastic modulus, Poisson's ratio and thermal expansion coefficient of the plate respectively. $\tilde{J}$ and $\tilde{\nabla}^4$ are two partial differential operators with respect to r in the following forms

$$\tilde{J} = r\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}-\frac{1}{r^2}\right),\quad \tilde{\nabla}^4 = \frac{\partial^4}{\partial r^4}+\frac{2}{r}\frac{\partial^3}{\partial r^3}-\frac{1}{r^2}\frac{\partial^2}{\partial r^2}+\frac{1}{r^3}\frac{\partial}{\partial r}$$

It is convenient to introduce the dimensionless variables as follows

$$(R,c) = \frac{1}{a}(r,b),\quad \eta = \frac{\pi a}{l},\quad (W,K) = \frac{\sqrt{12(1-\nu^2)}}{h}\left(w,\frac{\pi a f}{l}\right),\quad S = \frac{arN_r}{D}$$

$$\lambda = \frac{(1-\nu)\alpha a^2}{D}T,\quad \tau = t\sqrt{\frac{D}{\rho h a^4}},\quad \gamma = \frac{M_c}{\rho h\pi b^2},\quad K_\varphi = \frac{a}{D}k_\varphi$$

these quantities, in conjunction with Eq. (1), result the governing equations and the boundary conditions in the following non-dimensional forms

$$\int_{\tau_1}^{\tau_2}\int_c^1\left\{\nabla^4 W-\frac{1}{R}\frac{\partial}{\partial R}\left[S\left(\frac{\partial W}{\partial R}-K\sin(\eta R+\varphi)\right)\right]+\frac{\partial^2 W}{\partial\tau^2}\right\}\delta W R dR d\tau = 0 \tag{6}$$

$$J(S) = -\frac{1}{2}\frac{\partial W}{\partial R}\left[\frac{\partial W}{\partial R} - 2K\sin(\eta R + \varphi)\right] \tag{7}$$

$$\frac{\partial W}{\partial R} = 0,\ \frac{\partial^3 W}{\partial R^3} + \frac{1}{R}\frac{\partial^2 W}{\partial R^2} + \frac{\gamma c}{2}\frac{\partial^2 W}{\partial \tau^2} = 0,\ \frac{\partial S}{\partial R} - \frac{\nu}{R}S + \lambda = 0 \quad \text{at} \quad R = c \tag{8}$$

$$W = 0,\ \frac{\partial^2 W}{\partial R^2} + \left(\frac{\nu}{R} + K_\varphi\right)\frac{\partial W}{\partial R} = 0,\ \frac{\partial S}{\partial R} - \frac{\nu}{R}S + \lambda = 0 \quad \text{at} \quad R = 1 \tag{9}$$

here, J and ∇^4, that are similar to the former defined operators $\tilde{J}$ and $\tilde{\nabla}^4$, are partial differential operators with respect to R

$$J = R\left(\frac{\partial^2}{\partial R^2} + \frac{1}{R}\frac{\partial}{\partial R} - \frac{1}{R^2}\right), \quad \nabla^4 = \frac{\partial^4}{\partial R^4} + \frac{2}{R}\frac{\partial^3}{\partial R^3} - \frac{1}{R^2}\frac{\partial^2}{\partial R^2} + \frac{1}{R^3}\frac{\partial}{\partial R}$$

The initial conditions are taken to be

$$W = y(R),\ \frac{\partial W}{\partial \tau} = 0 \quad \text{when} \quad \tau = 0 \tag{10}$$

3. Method of solution

An exact solution to the problem defined by governing Eqs. (6)-(10) is not yet available because of the coupled membrane force and transverse deflection as well as the nonlinear terms. Herein, the space and time functions are supposed to be separable and the motion is assumed to be harmonic, the approximate solutions to large amplitude vibration of corrugated annular plates are obtained by the aforementioned assumed-time-mode method along with shooting method.

3.1 Kantorovich averaging method

Assuming that the vibration is prior to the buckling of the plate, and that the harmonic temporal function for $W(R, \tau)$ exists, the approximate solutions are taken in the following form (Huang and Walker 1988, Wang *et al.* 2002)

$$W(R, \tau) = y(R)\cos\omega\tau \tag{11}$$

$$S(R, \tau) = S_0(R) + S_1(R)\cos\omega\tau + S_2(R)\cos^2\omega\tau \tag{12}$$

in which $W(R, \tau)$ has already satisfied the initial conditions (10). Here ω is the dimensionless radian frequency, $y(R)$ denotes the shape function of vibration, S_1 and S_2 designate the membrane forces due to the geometric corrugations and the geometric nonlinearity of the plate, respectively, while S_0 denotes the membrane force related to the following static thermal stress problem of the heated plate

$$\frac{d^2 S_0}{dR^2} + \frac{1}{R}\frac{dS_0}{dR} - \frac{1}{R^2}S_0 = 0 \tag{13}$$

$$\frac{dS_0}{dR} - \frac{\nu}{R} S_0 + \lambda = 0 \quad \text{at} \quad R = c, 1 \tag{14}$$

In case of uniform temperature change, one obtains analytically the solution of S_0 without any difficulty

$$S_0 = -\frac{\lambda}{1-\nu} R \tag{15}$$

In addition to the boundary conditions, a normal relationship is imposed on the system, i.e.

$$y(c) = \varepsilon \tag{16}$$

where ε is the dimensionless amplitude of the inner edge of the plate.

Substituting Eqs. (11) and (12) into Eqs. (6)-(10), and applying Kantorovich time-averaging method to Eq. (6), taking Eqs. (15)-(16) into account and noting δy can be arbitrary, one summarizes the following nine coupled set of first-order, nonlinear ordinary differential equations relating to the field equations together with the boundary condition and normalization condition

$$\frac{d\mathbf{Y}}{dR} = \mathbf{H}(R, \mathbf{Y}; \lambda, K, \eta, \gamma, c, K_{\varphi}), \quad R \in (c, 1) \tag{17}$$

$$\mathbf{B}_1 \mathbf{Y}(c) = \{\varepsilon, 0, 0, 0, 0\}^T, \quad \mathbf{B}_2 \mathbf{Y}(1) = \{0, 0, 0, 0\}^T \tag{18}$$

with the denotations of the forms

$$\mathbf{Y}\{Y_1, Y_2, Y_3, Y_4, Y_5, Y_6, Y_7, Y_8, Y_9\}^T = \left\{y, \frac{dy}{dR}, \frac{d^2y}{dR^2}, \frac{d^3y}{dR^3}, S_1, \frac{dS_1}{dR}, S_2, \frac{dS_2}{dR}, \omega^2\right\}^T$$

$$\mathbf{H} = \{Y_2, Y_3, Y_4, \varphi_1, Y_6, \varphi_2, Y_8, \varphi_3, 0\}^T$$

$$\varphi_1 = -\frac{2}{R}Y_4 + \frac{1}{R^2}Y_3 - \frac{1}{R^3}Y_2 + \frac{1}{R}\left(\frac{dS_0}{dR} + \frac{3}{4}Y_8\right)Y_2 + \frac{1}{R}\left(S_0 + \frac{3}{4}Y_7\right)Y_3$$
$$-\frac{1}{R}[Y_5 \eta\cos(\eta R + \varphi) + Y_6 \sin(\eta R + \varphi)]K + Y_1 Y_9$$

$$\varphi_2 = \frac{1}{R}KY_2\sin(\eta R + \varphi) + \frac{1}{R^2}Y_5 - \frac{1}{R}Y_6, \quad \varphi_3 = -\frac{1}{2R}Y_2^2 + \frac{1}{R^2}Y_7 - \frac{1}{R}Y_8$$

and

$$\mathbf{B}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{c} & 1 & 0 & 0 & 0 & 0 & -\frac{\gamma c}{2}\varepsilon \\ 0 & 0 & 0 & 0 & -\frac{\nu}{c} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\nu}{c} & 1 & 0 \end{bmatrix}, \quad \mathbf{B}_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & \frac{1}{\nu + K_{\varphi}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\nu & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\nu & 1 & 0 \end{bmatrix}$$

The problem concerned comes down to solve the nonlinear ordinary differential equations with two-point boundary conditions. For a full corrugated circular plate without a central rigid mass, to avoid the singularity in numerical computation when R tends to zero, a very small positive quantity ΔR is introduced approximately to take the place of $R = 0$ (Li and Zhou 2001, Allahverdizadeh *et al.* 2008). Moreover, because of the continuity conditions for deflections and membrane forces at the center of the plate, there exists $y(\Delta R) \approx y(0) = \varepsilon$ and $S_i(\Delta R) \approx S_i(0) = 0 \ (i = 1, 2)$ when ΔR is sufficiently small.

3.2 Shooting method

The resulting nonlinear spatial boundary value problem (17)-(18) is solved numerically by the shooting method. The related initial value problem may be expressed as William *et al.* (1986)

$$\frac{d\mathbf{Z}}{dR} = \mathbf{H}(R, \mathbf{Z}; \lambda, K, \eta, \gamma, c, K_\varphi), \quad R > c \tag{19}$$

$$\mathbf{Z}(c) = \mathbf{I}(\varepsilon, \mathbf{V}) = \{\varepsilon, 0, V_1, (-V_1/c + \gamma c \varepsilon V_4/2), V_2, \nu V_2/c, V_3, \nu V_3/c, V_4\}^T \tag{20}$$

with

$$\mathbf{Z} = \{Z_1, Z_2, Z_3, Z_4, Z_5, Z_6, Z_7, Z_8, Z_9\}^T, \quad \mathbf{V} = \{V_1, V_2, V_3, V_4\}^T$$

where $\mathbf{V}$ is an unknown vector related to the missing initial values of $\mathbf{Y}$ at $R = c$. Thus a solution of initial value problem (19)-(20) can be symbolically indicated by

$$\mathbf{Z} = \mathbf{Z}(R; \varepsilon, \mathbf{V}, \lambda, K, \eta, \gamma, c, K_\varphi) = \mathbf{I}(\varepsilon, \mathbf{V}) + \int_c^R \mathbf{H}(\zeta, \mathbf{Z}; \lambda, K, \eta, \gamma, c, K_\varphi) d\zeta \tag{21}$$

Now, for prescribed parameters of ε, λ, K, η, γ, c, and K_φ, one seeks values of the components of $\mathbf{V}$, such that solution (21) also satisfies the four finial conditions at $R = 1$, namely

$$\mathbf{B}_2\mathbf{Z}(1; \varepsilon, \mathbf{V}, \lambda, K, \eta, \gamma, c, K_\varphi) = \{0, 0, 0, 0\}^T \tag{22}$$

Clearly, if $\mathbf{V} = \mathbf{V}^*$ is a root of Eq. (22), the solution for two-point boundary value problem is then obtained as

$$\mathbf{Y}(R; \varepsilon, \lambda, K, \eta, \gamma, c, K_\varphi) = \mathbf{Z}(R; \varepsilon, \mathbf{V}^*, \lambda, K, \eta, \gamma, c, K_\varphi) \tag{23}$$

and consequently, for known parameters, λ, K, η, γ, c, and K_φ, the characteristic relation of frequency versus amplitude is obtained as

$$\omega^2 = V_4^*(\varepsilon, \lambda, K, \eta, \gamma, c, K_\varphi) \tag{24}$$

For the linear vibration problem of heated corrugated plates, the natural frequency $\omega = \omega_0$ can be achieved by setting the amplitude parameter ε to be a very small value. Especially, one has $\omega_0 = 0$ when $\lambda = \lambda_{cr}$. It is obvious that λ_{cr} is a critical dimensionless temperature parameter at which buckling occurs in the plate.

The above process of numerical computation is carried out by applying the fourth order Runge-Kutta integration method with variable steps to integrate Eq. (21) and, at the same time, by using the Newton-Raphson iteration method to find the root $\mathbf{V}^*$ of algebraic Eq. (22). The ε-dependent family of solution for Eqs. (17)-(18) is obtained by the analytical continuation method (Huang and Aurora 1979, Huang and Walker 1988), if ε is repeatedly increased by a small step with given values of λ, K, η, γ, c, and K_φ. These measures permit us to get the solutions of two-point boundary value problem (17)-(18), and further the nonlinear characteristic relations of the frequency versus amplitude or nonlinear mode shape with no difficulties.

4. Numerical results and discussions

Numerical investigations for a corrugated annular plate with shallow sinusoidal corrugations are now performed when Poisson's ratio $\nu = 0.3$ and $\varphi = \pi$ throughout the following computation. The numerical integration of Eq. (21) and the successive correction of Eq. (22) are carried out until the error norm become less than 10^{-6}, and ΔR takes 10^{-5} to remove the singularity at the center of the plate for a full corrugated plate without a central rigid mass, ε takes 10^{-5} to deal with the corresponding linear vibration problem. The classical boundary conditions for simply supported and clamped edge are obtained by taking the flexibility parameter $K_\varphi = 0$ and $K_\varphi = 10^{20} \cong \infty$, respectively.

4.1 Comparison with published results

To validate the present formulation and ensure the accuracy and convergence of the proposed solution method, the following special cases have been examined by the comparison of present results with those that are available in the open literature.

First, the present numerical method is verified by the consideration of the linear free vibration for a uniformly heated shallow sinusoidally corrugated annular plate. A clamped immovable corrugated plate with its geometrical parameters $a = 25$ mm, $h = 0.1$ mm, $l = 5$ mm, and $f/h = 1.0$ is examined. For this case, one has the non-dimensional parameters $\eta = 5\pi$, $K = 51.91$. The characteristic

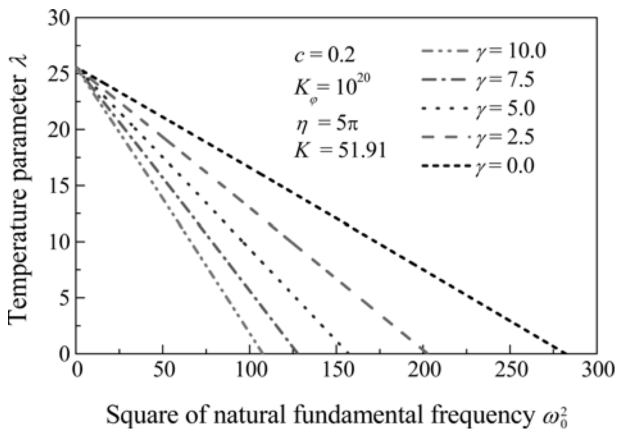

Fig. 2 Characteristic curves of the square of linear fundamental frequency ω_0^2 vs. the temperature parameter λ for a corrugated annular plate with shallow sinusoidal corrugations

relations of the square of fundamental frequency ω_0^2 versus the temperature parameter λ for some assigned values of rigid mass parameters γ when $c = 0.2$ are computed and plotted in Fig. 2, in which, $\lambda = 0$ indicates the square of natural frequency of an unheated plate, $\omega_0^2 = 0$ defines the critical temperature parameter λ_{cr} of a heated plate. It is noted from this figure that the square of natural frequency decreases monotonically and almost linearly with the increment of the temperature parameter, this qualitative result is the same as that obtained by the perturbation-variation method (Wang and Dai 2003) and shooting method (Li and Zhou 2001) for a heated plane plate. It is also indicated that, at the same λ, the natural frequency with a lower γ is higher than that with a higher γ. As expected, the critical temperature parameter λ_{cr} is independent of the rigid mass parameter γ.

The aforementioned natural frequencies are now compared quantitatively for $K = 0$. Under this circumstance, the corrugated plate considered degenerates into a plane one. The first three natural frequencies for an unheated circular plate without a central rigid mass are compared with those obtained by Ritz method (Gupta and Ansari 2002) for various values of rotational flexibility parameters K_φ and presented in Table 1. Good agreements can be seen between the present values and existing results.

Comparisons concerning the relation between the central non-dimensional amplitude and the nonlinear vibration frequency for an unheated circular plane plate without rigid mass are also made.

Table 1 Comparison of the first three natural frequencies ω_{0i} $(i = 1, 2, 3)$ of an unheated circular plate for various values of flexibility parameters K_φ

ω_{0i} / K_φ	0		10		10^2		10^{20}	
ω_{01}	4.9351	4.9351†	8.7519	8.7519†	10.0193	10.0192†	10.2158	10.2158†
ω_{02}	29.7200	29.7200†	35.2190	35.2190†	39.0288	39.0288†	39.7711	39.7711†
ω_{03}	74.1561	74.1560†	80.6870	80.6869†	87.4901	87.4900†	89.1041	89.1041†

†Values taken from Gupta and Ansari (2002)

Table 2 Comparison of the ratios between the nonlinear and linear fundamental frequencies ω / ω_0 for various dimensionless vibration amplitudes of an unheated circular plate

$w(0,0)/h$	Simply supported plate ($K_\varphi = 0$)				Clamped plate ($K_\varphi = 10^{20}$)			
	Present	Haterbouch and Benamar (2005)		Yamaki (1961)	Present	Haterbouch and Benamar (2003)	Haterbouch and Benamar (2004)	Huang and Al-Khattat (1977)
0.2	1.0268	1.0268†	1.0267‡	1.0273	1.0075	1.0072	1.0075	1.0075
0.4	1.1034	1.1034†	1.1025‡	1.1047	1.0296	1.0284	1.0296	1.0296
0.5	1.1578	1.1577†	1.1557‡	/	1.0459	1.0439	/	1.0459
0.6	1.2209	1.2209†	1.2172‡	1.2217	1.0654	1.0623	1.0654	1.0654
0.8	1.3694	1.3693†	1.3606‡	1.3677	1.1135	1.1073	1.1135	1.1135
1.0	1.5402	1.5401†	1.5244‡	1.5342	1.1724	1.1615	1.1724	1.1724
1.5	2.0288	2.0288†	1.9887‡	/	1.3567	1.3255	1.3568	1.3568
2.0	2.5664	2.5664†	2.4962‡	/	1.5787	1.5147	1.5790	1.5790

†Multidimensional model solutions; ‡Single-mode solutions

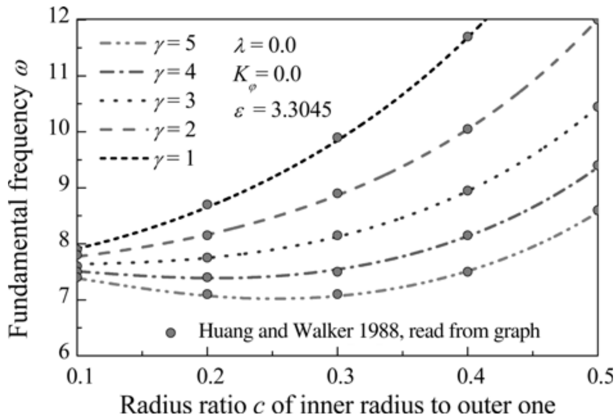

Fig. 3 Characteristic curves of the fundamental frequency ω vs. radius ratio c for an unheated simply supported annular plate with some prescribed values of central rigid mass γ

A set of central dimensionless amplitude defined here by $w(0,0)/h$, and the ratio of the dimensionless nonlinear fundamental frequency ω to the corresponding linear one ω_0 is summarized in Table 2. The available published results that were based on various analytical assumptions and numerical solution techniques are found to have very close agreement to present result.

When a rigid mass occupying a finite area is concentrically added to a thin annular plate, it produces a change in kinetic energy which must match with a change in potential energy due to the additional stiffening effect at the inner boundary. Besides, the stiffness of an annular plate depends on the radius ratio. Thus, the behavior of the plate-mass system depends upon not only the specified rigid mass but also the radius ratios of the system under consideration (Huang and Huang 1989). An examination of the nonlinear vibration characteristics is carried out for various mass parameters γ and radius ratios c. Taken the relative amplitude $w(c, 0)/h = 1.0$, or $\varepsilon = 3.3045$, the fundamental frequencies of an unheated simply supported annular plate with some specified central rigid masses are presented as functions of radius ratio in Fig. 3. The results agree identically with previous findings (Huang and Walker 1988).

Thus, one can confirm from these comprehensive numerical tests that the present approach can yield accurate and reliable solutions for the heated corrugated annular plate with a centric rigid mass.

4.2 Amplitude frequency dependence

Numerical examples are now demonstrated. A sine-shaped corrugated annular plate, with its geometries being the same as what has already been analyzed above, is investigated.

The characteristic curves of ω vs. ε for the heated clamped as well as simply supported corrugated annular plates under some specific values of temperature parameter λ are displayed in Fig. 4, in which, $\varepsilon = 0$ indicates the natural frequencies of a heated plate. Pronounced influences of the ambient temperature changes on the nonlinear vibration behavior of plates are observed from this figure. It is found that the vibration of a heated corrugated annular plate with shallow sinusoidal corrugations possesses a hardening spring behavior on the whole; moreover, owing to the reduction in the flexural rigidity of the plate due to the presence of a compressive membrane force induced by

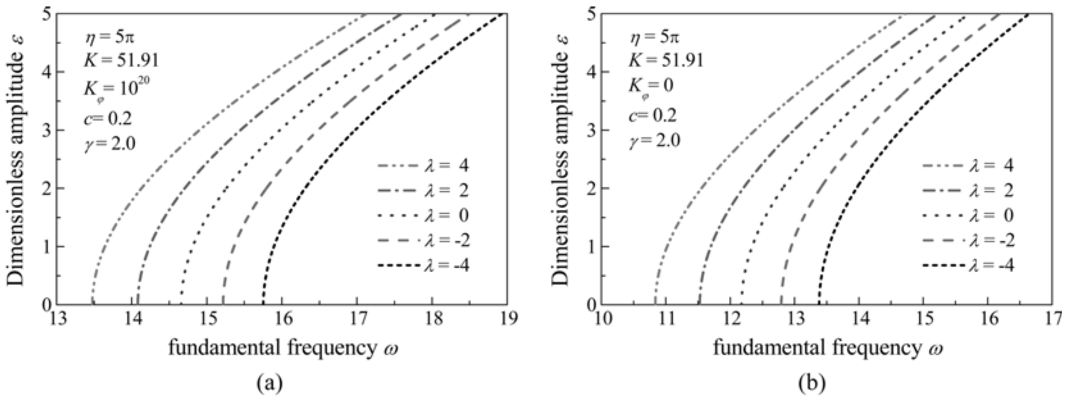

Fig. 4 Characteristic curves of the fundamental frequency ω vs. amplitude ε under various temperature parameters λ for (a) clamped and (b) simply supported corrugated annular plates with shallow sinusoidal corrugations

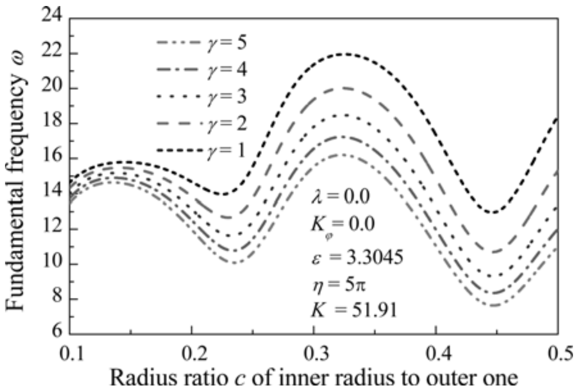

Fig. 5 Influences of radius ratio c on the fundamental frequency ω for unheated simply supported corrugated annular plates with shallow sinusoidal corrugations under some given values of central rigid mass γ

the temperature rise, the nonlinear fundamental frequency decreases with the increase in temperature parameter, and vice versa. The effect of the temperature parameter λ on the nonlinear frequency ω when amplitude ε is smaller is more significant as compared to that when ε is larger for both boundary constraints. In addition, the fundamental frequency for a clamped plate is always greater than that for a simply supported one, other plate parameters being fixed.

Fig. 5 illustrates how the effects of the rigid mass γ and radius ratios c influence the fundamental frequency parameter ω of the corrugated plate carrying a concentric rigid mass. The unheated simply supported corrugated annular plate with its central mass parameter specifies as $\gamma = 1, 2, 3, 4$ and 5 are computed when varying the radius ratio c from 0.1 to 0.5. The dimensionless amplitude of the inner edge of the plate takes $\varepsilon = 3.3045$ (i.e., relative amplitude $w(c,0)/h = 1.0$) to involve the nonlinear consideration. Unlike the change curve of fundamental frequency versus radius ratio in Fig. 3 for a annular plane plate with a rigid core, the ω~c curve here changes in a wavelike form, this may be attributed to the simultaneously increase in stiffness and mass of the plate-mass system when a rigid core is inserted. In addition to this, at different radial position, the plate possesses different height of corrugation, which leads the stiffness vary with the radius ratio. So the

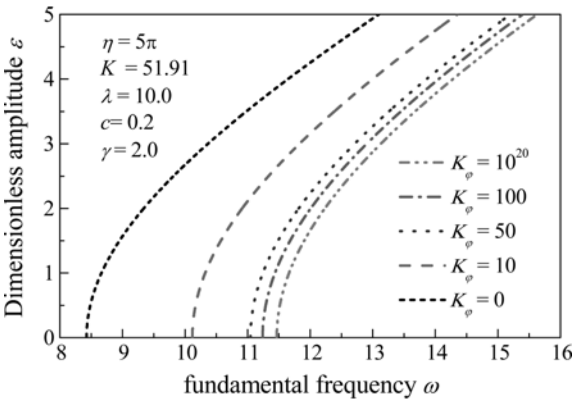

Fig. 6 Characteristic curves of the fundamental frequency ω vs. amplitude ε under various flexibility parameters of outer edge K_φ for a corrugated annular plate with shallow sinusoidal corrugations

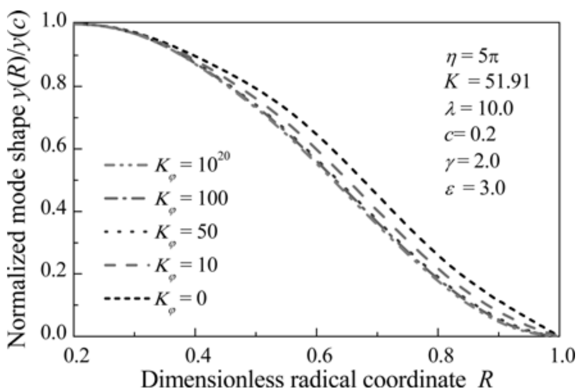

Fig. 7 Normalized nonlinear fundamental mode shape $y(R)/y(c)$ of a corrugated annular plate with shallow sinusoidal corrugations under various flexibility parameters of outer edge K_φ

fundamental frequency increases if the influence of the stiffening effect produced by insertion of a rigid core becomes dominant over any addition of mass, in opposition to this, it decreases if the effect of additional mass has more influence than the increase in stiffness on the frequency. The alternant increase or decrease in the effect of stiffness and additional mass makes the frequency changing sinuously.

Fig. 6 is plotted for frequency parameter ω versus amplitude parameter ε with different values of flexibility parameter K_φ. The frequency ω is found to increase with the increasing values of flexibility parameter K_φ as well as dimensionless inner edge amplitude ε, keeping all other plate parameters fixed. It is seen that the influence of flexibility parameter on frequency parameter for large K_φ is less pronounced than that for small one.

Profile of nonlinear normalized fundamental mode shape $y(R)/y(c)$ for a corrugated plate with the same parameter conditions as those used in Fig. 6 at $\varepsilon = 3.0$ is displayed in Fig. 7. It is evident that the shapes vary with the flexibility parameter K_φ, which is equivalent to stating that they vary with the frequencies of vibration. The nonlinear fundamental frequencies in this case are ω = 10.367, 11.830, 12.713, 12.904, 13.130 corresponding to five boundary flexibility parameters K_φ = 0, 10, 50, 100, 10^{20}. It can also conclude that the flexibility parameters have a minor effect on mode shape when they become large.

The influences of the depth and density of corrugations on the vibration behaviors of the corrugated annular plate, which are not exhibited here, are also investigated by examining the characteristic relationships of K-dependent and η-dependent ω/ω_0 vs. ε. As a general rule, the nonlinear effect for the vibration of a corrugated annular plate weakens when the corrugations become deeper, and strengthens when the corrugations become shallower; the increase of the density parameter of corrugations has an effect of increasing the hardening spring behavior for the vibration of a corrugated annular plate. Such an observation has been mentioned in the previous literatures (Liu and Li 1989, Wang *et al.* 2008, 2009) for a corrugated circular plate with full corrugations.

5. Conclusions

This paper has introduced the assumed-time-mode method in conjunction with shooting method for the nonlinear free vibration analysis of a uniformly heated thin corrugated annular plate with rigidly attached concentric mass and elastically restrained outer edge rotational springs. Numerical results obtained by present approach are compared with those from the available published literatures and a good agreement is found.

Considering the nonlinear characteristic relation of frequency versus amplitude, a hardening spring effect has been observed. Increase in dimensionless amplitude results in an increase in nonlinear frequency, which decreases with an increase in depth of corrugation while increases with an increase in density of corrugation. Decrease in central rigid mass and increase in outer edge stiffness of rotational springs induce an increased fundamental frequency. In general, the nonlinear fundamental frequency is found to decrease with the increasing temperature parameter and vice versa. Parametric study reveals that the radius of central rigid mass influences the vibration frequency of a corrugated annular plate in a wavelike form if all other plate parameters being fixed.

When $K = 0$, the problem involved degenerates into the large amplitude vibration analysis for a heated plane plate with a rigid mass at the center and elastic constraints at the outer edge.

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