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Postbuckling response and failure of symmetric laminated plates with rectangular cutouts under in-plane shear

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Abstract. This paper deals with the buckling and postbuckling responses, and the progressive failure of square laminates of symmetric lay-up with a central rectangular cutout under in-plane shear load. A detailed investigation is made to show the effects of cutout size and cutout aspect ratio on the buckling and postbuckling responses, failure loads and failure characteristics of $(+45/-45/0/90)_{2s}$, $(+45/-45)_{4s}$ and $(0/90)_{4s}$ laminates. The 3-D Tsai-Hill criterion is used to predict the failure of a lamina while the onset of delamination is predicted by the interlaminar failure criterion. In addition, the effects of boundary conditions on buckling loads, failure loads, failure modes, and maximum transverse deflection for a $(+45/-45/0/90)_{2s}$ laminate with and without a square cutout have been presented. It is concluded that because of early onset of delamination at the net section of cutouts before first-ply failure, total strength of the laminate with very small cutouts can not be utilized.

Keywords: buckling; postbuckling; in-plane shear; composite laminates; cutouts; strength; failure.

1. Introduction

The ultimate failure of a laminated composite panel does not always occur at the load corresponding to the first-ply failure because the failure characteristics of heterogeneous and anisotropic composite laminates differ from that of the isotropic ones. Composite laminates can sustain a much higher load after the occurrence of localized damage such as matrix cracking, fiber breaks or delamination (Aggarwal and Broutman 1990). Due to practical requirements, cutouts are often required in composite structural panels, such as wing spars and cover panels of aircraft structures to provide access for hydraulic lines, electrical lines, fuel lines, damage inspection, and to reduce the overall weight of the aircraft. The presence of these cutouts forms free edges in the composite laminates, which in turn cause high interlaminar stresses (Jones 1975) leading to loss of

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stiffness and premature failure of laminates due to onset of delamination Therefore, stability, overall strength, and failure characteristics of composite panels with cutouts are some of the important parameters for an improved design of structures fabricated with laminated panels and subjected to in-plane loads such as axial compression and in-plane shear loads. Previous investigations related to shear buckling and postbuckling responses of laminated plates without cutouts are the works of Kaminski and Ashton (1971), Prabhakar and Kennedy (1979), Kobayashi *et al.* (1981), Aggrawal (1981), Zhang and Matthews (1984), Stein (1985), Kosteletos (1992). Singh *et al.* (1998a, b) investigated the progressive failure of square symmetric laminates under in-plane shear (positive and negative, respectively) loads. In another investigation, Singh and Kumar (1998) studied the postbuckling and progressive failure of thin, rectangular symmetric laminates with various lay-ups and plate aspect ratios, under in-plane shear loads. Jha and Kumar (2002) studied the response and failure of square laminates under transverse load combined with axial compression and in-plane shear and obtained failure envelopes in the form of load interaction curves using Tsai-Hill failure criteria. Iyengar and Chakraborthy (2004) also studied the interaction curves for composite laminates under uniaxial and shear loadings using simple higher order shear deformation theory.

Herman (1982) presented the first work on buckling and postbuckling behaviour of shear webs with a central circular cutout using finite element technique. Turvey and Sadeghipour (1988) presented a study of shear buckling of square graphite-epoxy and glass-epoxy plates with a central circular cutout. Likewise, Owen and Klang (1990), Vellaichamy *et al.* (1990) and Britt (1993) investigated the buckling behaviour of square plates with cutout under in-plane shear load whereas Rouse (1990) presented an experimental investigation of the buckling and postbuckling behaviour of square graphite-epoxy and graphite-thermoplastic plates loaded in shear. An exhaustive overview of the past research on the buckling and postbuckling behaviour of composite panels with cutouts by Nemeth (1996) showed that the most of early investigations deal with the study of buckling and postbuckling responses of composite laminates with circular cutouts. Further, Romeo (2001) conducted an experimental and analytical study on laminated panels with rectangular opening under bi-axial tension, compression and shear loads. Ghannadpour *et al.* (2006) studied the effects of a cutout on the buckling behaviour of polymer matrix composite laminates. Anil *et al.* (2007) carried out the stability analysis of composite laminates with and without rectangular cutout under biaxial loading and studied load interaction curves. Guo (2007) conducted numerical and experimental studies to investigate the effect of reinforcements around cutouts on the stress concentration and buckling behaviour of a carbon/epoxy composite panel under in-plane shear load. Very lately, Guo *et al.* (2009) studied the performance of cutout and various edge reinforcements in a composite C-section beam under static shear load and demonstrated that the cutout induced stress concentration can be reduced significantly by appropriate cutout shape and edge reinforcements.

Thus, it is observed that there is lack of research investigations relating to the postbuckling response and failure characteristics of laminated plates with non-circular cutouts under in-plane shear loads (positive and negative). The objective of this paper is to investigate the effects of rectangular cutout size and cutout aspect ratio on buckling and postbuckling responses, failure loads, and failure characteristics of $(+45/-45/0/90)_{2s}$, $(+45/-45)_{4s}$ and $(0/90)_{4s}$ laminates with square/ rectangular cutouts under in-plane shear (positive and negative). In addition, the effects of edge boundary conditions on buckling and failure loads, and the maximum transverse deflections associated with failure loads for a $(+45/-45/0/90)_{2s}$ quasi-isotropic laminate with and without cutout have also been presented.

2. Present study

A special purpose computer program was developed to carry out the study based on the finite element formulation (Singh 1996, Ochoa and Reddy 1992) using the first order shear deformation theory with nine noded Lagrangian element having five degrees of freedom per node. Geometric nonlinearities based on von Karman's assumptions (Reddy 2004) and in-plane shear stress and strain material nonlinearities (Hu 1995) have been incorporated. The non-linear algebraic finite element equations are solved by Newton-Raphson technique. The calculation of stresses is done on the nodal points as well as on the Gauss points. All the six components of stress are calculated at each nodal and Gauss point. Nodal point stresses refer to the average value of stresses obtained from various elements associated with a particular node. However, to predict the failure of a lamina, only five stress components (three in-plane stresses and two transverse stresses) at mid plane of each layer are used in the tensor polynomial form of the 3-D Tsai-Hill criterion (Tsai 1984). To predict the ultimate failure of laminate, a progressive failure procedure as used by Singh and Kumar (1998) has been implemented. In this progressive failure procedure, at each load step, gauss point stresses are used in tensor polynomial form of the Tsai-Hill failure criterion. If failure occurs at a gauss point in a layer of an element, a reduction in the appropriate lamina stiffness is introduced in accordance with the mode of failure which is based on the failure indices having largest value. The laminate stiffness is recomputed and failure is checked again at the same load step. If no failure occurs, the process is repeated at next load step. The onset of delamination is predicted by the interlaminar failure criterion (Singh 1996). Ultimate failure is assumed to occur when the onset of delamination occurs or when the plate is no longer able to carry any further increase in load due to very large transverse deflection. The details of tensor polynomial form of 3-D Tsai-Hill criterion and interlaminar failure criterion are presented in appendix.

The properties of the material (Reddy and Reddy 1992) of the laminate are presented in Table 1. In this study, a full square plate of size 279 mm × 279 mm × 2.16 mm with ply thickness 0.135 mm is used with a central cutout of size $c \times d$; where, c and d refers to the dimensions of the cutout along x- and y-direction, respectively (Fig. 1 and Table 2). Convergence study was conducted to decide number of elements for the present investigation. It was found that both, buckling and first-ply failure load show convergence for 78 elements (i.e., 364 nodes). Schematics of finite element mesh and element node numbers are shown in Fig. 1. Three types of flexural edge boundary conditions, namely BC1, BC2 and BC3 are considered; BC1 refers to a plate with all edges simply supported, BC2 refers to a plate with two longitudinal edges (y = 0 and y = b) simply supported and the other two edges clamped, and BC3 refers to a plate with all edges clamped. In all three

Table 1 Material properties of T300/5208 (pre-peg) graphite-epoxy

Mechanical properties	Values	Strength properties	Values
E_1	132.58 GPa	X_t	1.52 GPa
E_2	10.80 GPa	X_c	1.70 GPa
E_3	10.80 GPa	$Y_t = Z_t$	43.80 MPa
$G_{12} = G_{13}$	5.70 GPa	$Y_c = Z_c$	43.80 MPa
$\nu_{12} = \nu_{13}$	0.24	R	67.60 MPa
ν_{23}	0.49	$S = T$	86.90 MPa

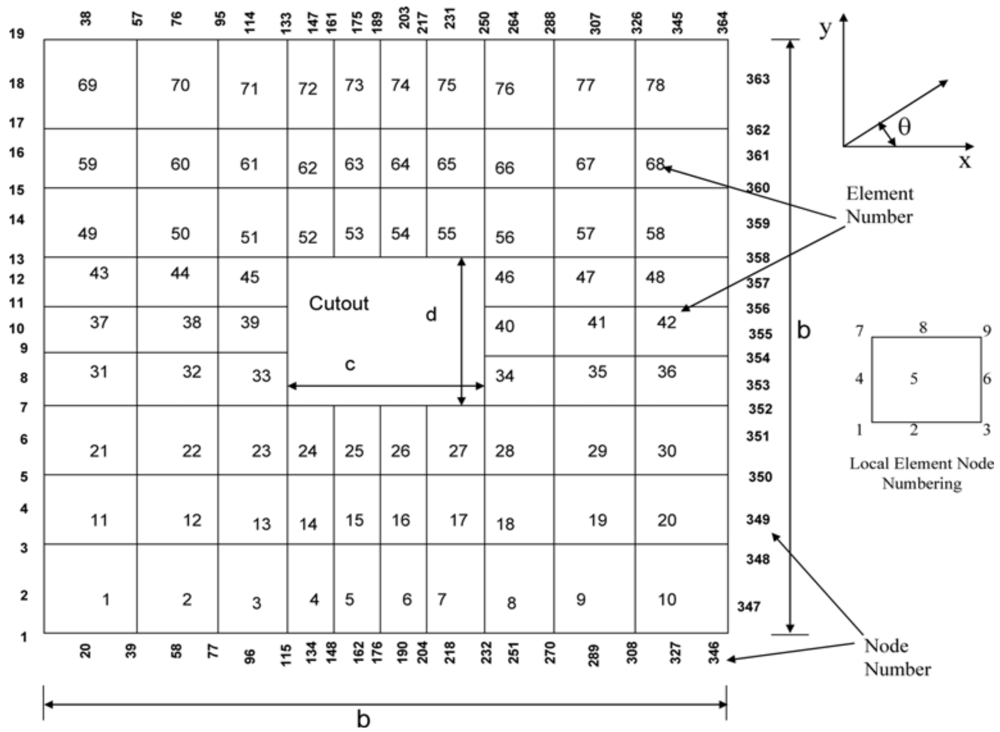

Fig. 1 Typical finite element mesh pattern for the laminate

Table 2 Details of cutout dimensions and aspect ratios

Width of square laminate, b (mm)	Cutout type	Cutout size[*] c (mm) × d (mm)	c/b	d/b
279	Square	39 × 39	0.14	0.14
279	Square	78 × 78	0.28	0.28
279	Square	117 × 117	0.42	0.42
279	Rectangular	78 × 39	0.28	0.14
279	Rectangular	117 × 39	0.42	0.14

[*]c and b are dimensions of cutout along x- and y-axis, respectively.

cases, the in-plane boundary conditions are identical and in-plane shear load is applied on the edges $x = b$; where, b refers to the width of the square plate. Most of the results of this investigation refer to the boundary condition BC1 (all edges simply supported) except those in the section dealing with the effects of boundary conditions. Results for failure loads and corresponding deflections are presented in the following non-dimensionalized forms:

In-plane shear load $N_{xy}b^2/E_2h^3$

Maximum transverse deflection w_{max}/h

Here, E_2 is the transverse elastic modulus of a lamina; h is the thickness of the laminate; b is the width of the square plate; N_{xy} is the in-plane shear loads per unit width of the plate; and, w_{max} is the maximum transverse deflection.

3. Verification of results

The accuracy of the developed program was checked by comparing the results for buckling and transverse displacements in postbuckling range with those of Kosteletos (1992). Results apply to square $(\pm 45)_s$ laminate without cutout with clamped edges and subjected to in-plane shear loads (positive and negative). The computations were performed for 913C-XAS, a low cure epoxy carbon fiber composite [as used in Kosteletos (1992)] with material properties: $E_1 = 150$ GPa, $E_2 = E_3 =$ 9.5 GPa, $G_{12} = G_{13} = 1.07$ GPa, $\nu_{12} = \nu_{13} = 0.263$. The values of nondimensionalized (i.e., $N_{xy}b^2/E_2h^3$) buckling load given by Kosteletos (1992) for positive and negative shear are 40.0 and 126.0, respectively, and the corresponding values obtained in the present study are 37.84 and 118.0. The values of maximum transverse displacement in nondimensionalized form (i.e., w_{max}/h) obtained in present study for positive shear loads (i.e., $N_{xy}b^2/E_2h^3$) of 45 and 50 are 1.58 and 2.08, respectively, and the corresponding values reported by Kosteletos (1992) are 1.60 and 2.25. For negative shear loads of 140 and 150, the maximum transverse displacements obtained in the present study are 3.09 and 3.73, respectively, which are very close to the values (i.e., 3.00 and 3.80) reported by Kosteletos (1992). Good agreement can be seen in the results obtained by the developed program and those reported in literature.

4. Results and discussion

4.1 Response and failure under in-plane shear load

The postbuckling and the progressive failure responses of $(+45/-45/0/90)_{2s}$, $(+45/-45)_{4s}$ and $(0/90)_{4s}$ laminates with square cutout, c/b = 0.14 were carried out. The results in terms of the non-dimensional load versus the maximum transverse deflection under the positive shear are shown in

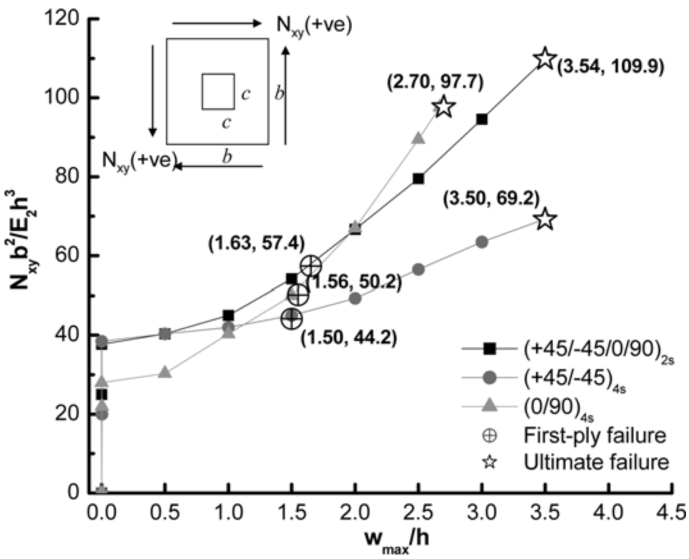

Fig. 2 Load-deflection responses of various laminates with a square cutout (c/b = 0.14) under positive shear

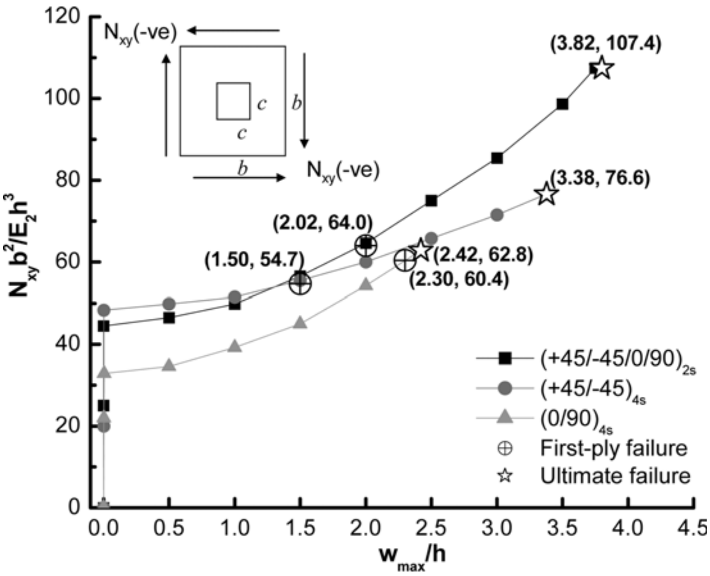

Fig. 3 Load-deflection responses of various laminates with a square cutout (c/b = 0.14) under negative shear

Fig. 2. It is noted that the buckling load of the $(0/90)_{4s}$ laminate is lower than that of the other two laminates. However, its load carrying capacity is increased in the advanced stages of the postbuckling deformation. It is observed that the in-plane normal stresses transverse to the fiber direction are the primary causes of the first-ply failure in $(+45/-45/0/90)_{2s}$ and $(+45/-45)_{4s}$ laminates, but the in-plane shear stress is the cause for $(0/90)_{4s}$ laminate. The cause of ultimate failure is the onset of delamination in the case of $(+45/-45/0/90)_{2s}$ and $(+45/-45)_{4s}$ laminates, while the widespread in-plane shear mode of failure is responsible for the ultimate failure of the $(0/90)_{4s}$ laminate. Fig. 3 shows the corresponding load-deflection diagram for the negative shear load, which is similar to that for the positive shear especially in the case of $(+45/-45/0/90)_{2s}$ and $(+45/-45)_{4s}$ laminates, however, a significant difference is observed in the response of the $(0/90)_{4s}$ laminate for the two directions of the shear load. There is no change in the mode of first-ply failure due to the change in the direction of shear loads for all laminates.

Similar investigations were carried out for larger square cutouts of sizes, c/b = 0.28 and 0.42 in terms of the load versus maximum transverse deflection. It is to be noted that there is no significant change in the trend of response due to change in the direction of shear loads for all laminates. As shown in Table 3, the critical locations of the first-ply failure of the $(+45/-45/0/90)_{2s}$ laminate lie at the corners of the cutout of size c/b = 0.14; for larger cutouts these lie near the corners of the laminate under the positive shear while under the negative shear, corners of the cutout are more susceptible to failure for all sizes of the cutouts. For the $(0/90)_{4s}$ laminate, the first-ply failure locations lie at the corners of the laminate while for the $(+45/-45)_{4s}$ laminate these are at the corner of the cutout, irrespective of the size of the cutout and directions of the shear load. No appreciable change in the failure mode is observed with an increase in the cutout size irrespective of the direction of shear load. It is to be noted that the onset of delamination is the predominant mode of ultimate failure for laminates with cutouts.

Table 3 Mode of first-ply and ultimate failures, and location of the first-ply failure for various laminates with square cutouts of different sizes

Laminate	Square cutout size	Mode of first-ply failure	Mode of ultimate failure	Location of first-ply failure
$(+45/\text{-}45/0/90)_{2s}$	0.14	MF^{*}_{22} $(MF_{22})^{\#}$	DELMN*** (DELMN)#	$CC^{\Theta\Theta}(CC)^{\#}$
	0.28	MF_{22} (MF_{22})	DELMN (DELMN)	$CL^{\otimes}$ (CC)
	0.42	MF_{22} (MF_{22})	DELMN (DELMN)	CL (CC)
$(+45/\text{-}45)_{4s}$	0.14	MF_{22} (MF_{22})	DELMN (DELMN)	CC (CC)
	0.28	MF_{22} (MF_{22})	DELMN (DELMN)	CC (CC)
	0.42	MF_{22} (MF_{22})	DELMN (DELMN)	CC (CC)
$(0/90)_{4s}$	0.14	MF^{**}_{12} (MF_{12})	$WSMF^{\Theta}_{12}$ $(WSMF_{12})$	CL (CL)
	0.28	MF_{12} (MF_{12})	$WSMF_{12}$ $(WSMF_{12})$	CL (CL)
	0.42	MF_{12} (MF_{12})	$WSMF_{12}$ $(WSMF_{12})$	CL (CL)

#Parameter in the parentheses refers to the negative shear loading whereas outside the parentheses refers to the positive shear loading; *MF_{22} refers to matrix failure due to in-plane normal stresses transverse to fiber direction; **MF_{12} refers to in-plane shear failure; ***DELMN refers to onset of delamination; $^{\Theta}WSMF_{12}$ refers to widespread in-plane shear failure; $^{\Theta\Theta}$CC refers to corner of the cutout; $^{\otimes}$CL refers to corner of the laminate

Fig. 4 and Fig. 5 show the load versus deflection responses of $(+45/\text{-}45/0/90)_{2s}$ laminate without cutout and with square cutouts of different sizes under positive and negative shear loads, respectively. It is observed that the buckling, first-ply, and ultimate failure loads of laminates are lower in the presence of cutout and decrease with an increase in the cutout size for both directions of the shear loading. However, in the advance stages of postbuckling deformation under positive shear load, the laminate with a cutout of size, $c/b = 0.42$ shows higher strength than the corresponding value for the cutout size, $c/b = 0.28$. This behaviour is attributed to the increase in the

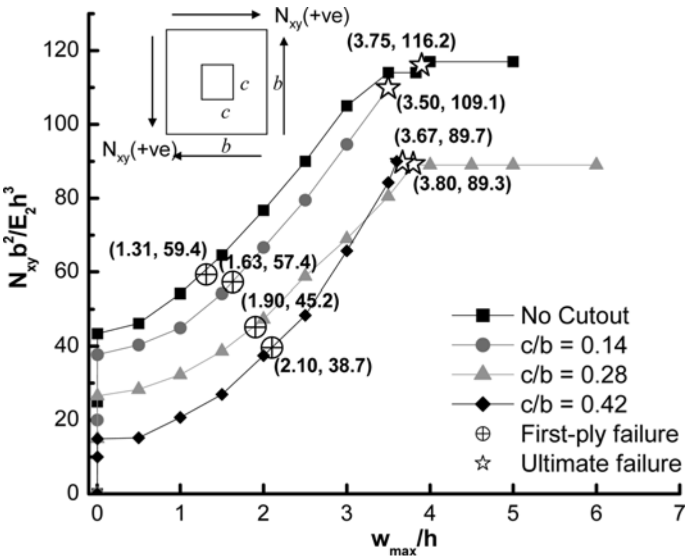

Fig. 4 Load-deflection responses of $(+45/\text{-}45/0/90)_{2s}$ laminate without and with square cutouts of different sizes under positive shear load

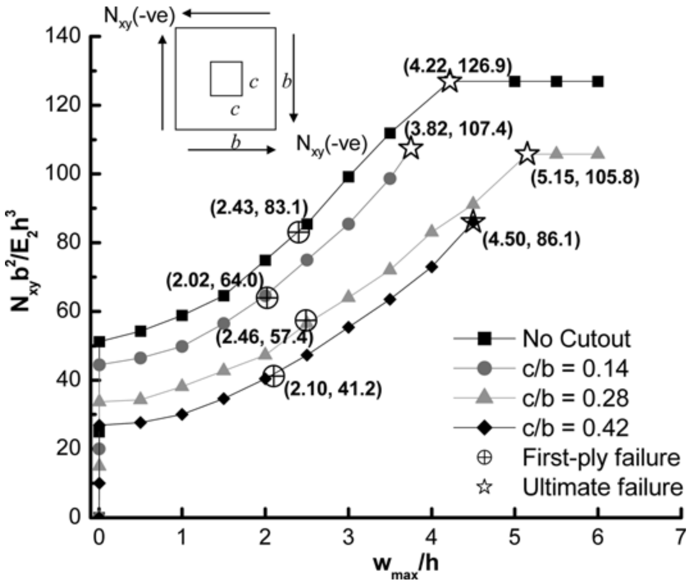

Fig. 5 Load-deflection responses of $(+45/-45/0/90)_{2s}$ laminate without and with square cutouts of different sizes under negative shear load

axial prebuckling stiffness of the laminate. However, this characteristic is not observed under negative shear (Fig. 5). Similar responses were observed in other laminates, except in cross-ply laminate $(0/90)_{4s}$. In the $(0/90)_{4s}$ laminate, the first-ply and ultimate failure loads are higher in the presence of small cutout under both directions of shear loading in comparison to that without cutout. With the increase in the cutout size, the first-ply failure load of the $(0/90)_{4s}$ laminate decreases and becomes smaller than that with no cutout in the case of positive shear while it is larger under the negative shear, irrespective of the size of the cutout. The enhanced strength of the $(0/90)_{4s}$ laminate is attributed to the change in mode of the first-ply failure, i.e. the mode of first-ply failure with no cutout is matrix failure (due to in-plane normal stresses transverse to the fiber direction) while it is the in-plane shear mode which is responsible for failure in the laminate with a cutout (Table 3). Also, the ultimate failure load of the cross-ply laminate is larger for all cutout sizes in case of positive shear and for $c/b = 0.14$ in the case of negative shear in comparison to that with no cutout. It is attributed to the facts that in the presence of the cutout, the first-ply failure of laminate is delayed and also, the ultimate failure due to the onset of delamination is precluded.

The effect of cutout aspect ratios (c/b) with constant $d/b = 0.14$, on the buckling and postbuckling responses in terms of the buckling and failure loads, and the maximum transverse deflection corresponding to the first-ply failure for the $(+45/-45/0/90)_{2s}$ laminate under positive and negative shear loads are shown in Table 4. It is to be noted that there is a monotonic decrease in the buckling and first-ply failure load with aspect ratio of the cutout, irrespective of the direction of the shear load. For positive shear loading, it is observed that the first-ply failure occurs at the corners of the laminate for aspect ratios $c/b \geq 0.28$, while for aspect ratios $c/b < 0.28$, it occurs at the corners of the cutouts. After the first-ply failure, the net section of the cutout becomes the most critical section for progressive failure. It is also observed that for cutouts of aspect ratios $c/b \leq 0.28$, the ultimate failure is caused due to the onset of delamination while for $c/b > 0.28$, the large transverse deflection is the cause of ultimate failure. The behaviour of the laminate with cutout of different

Table 4 Effect of the cutout aspect ratio (with constant d/b = 0.14) on failure characteristics of square $(+45/-45/0/90)_{2s}$ laminate under positive and negative shear loads

Cutout aspect ratio (c/b)	Buckling load		First-ply failure load		Ultimate failure load		$(w_{max}/h)^*$	
	Positive	Negative	Positive	Negative	Positive	Negative	Positive	Negative
No Cutout	43.46	51.20	59.38	83.05	116.18	126.94	1.31	2.43
0.14	37.72	44.47	57.37	63.97	109.87	107.44	1.63	2.02
0.21	34.78	41.60	54.15	59.88	106.03	106.14	1.70	2.03
0.28	32.63	39.45	53.07	58.09	114.03	109.73	1.83	2.17
0.35	30.12	37.30	50.20	56.30	100.41	112.60	1.87	2.29
0.42	28.33	35.14	47.69	53.07	96.82	105.43	1.92	2.33

*Non-dimensionalized maximum transverse deflection associated with the first-ply failure.

aspect ratios under the negative shear is noted to be similar to that observed for the positive shear except that the first-ply failure location lies at the corners of the cutouts of sizes c/b < 0.42 and at the corners of the laminate with a cutout c/b = 0.42. No change in the mode of the first ply-ply failure is viewed with an increase in the aspect ratio of the cutout for both directions of the shear load. It is worth mentioning that for cutouts of sizes $c/b \leq 0.06$, the onset of delamination occurs before the first-ply failure at the net section of the cutout and hence the full strength of laminate cannot be utilized for very small cutouts of sizes $c/b \leq 0.06$.

4.2 Effect of edge boundary conditions

The buckling, first-ply and ultimate failure loads, and maximum transverse deflections (associated with failure loads) of the $(+45/-45/0/90)_{2s}$ quasi-isotropic laminate with the square cutout (c/b = 0.14 & d/b = 0.14) and without a cutout are shown in Tables 5, 6 for the positive and the negative shear loads, respectively, for different edge boundary conditions, i.e., BC1, BC2 and BC3. As expected, the buckling and first-ply failure loads of a laminate with a cutout are lower than those without cutout for all boundary conditions, irrespective of the nature of (i.e., positive/negative) the shear

Table 5 Failure Characteristics of $(+45/-45/0/90)_{2s}$ laminate with and without cutout for different edge boundary conditions# under positive shear load

Load	Without cutout			With cutout		
	BC1	BC2	BC3	BC1	BC2	BC3
Buckling load	43.46	65.40	83.48	37.72	56.80	64.98
First-ply failure load	59.38	80.03	95.96	57.37	69.71	73.15
Ultimate failure load	116.18	86.49	98.54	109.87	101.10	97.25*
$(w_{max}/h)^{f\$}$	1.31	1.19	0.98	1.63	1.27	1.15
$(w_{max}/h)^{u\$}$	3.75	1.79	1.22	3.54	3.13	7.01

#BC1 refers to a plate with all edges simply supported; BC2 refers to a plate with two longitudinal edges (y = 0 and $y = b$) simply supported and the other two edges clamped; and BC3 refers to a plate with all edges clamped; *wide spread in-plane shear failure following fiber failure alongwith τ_{xz} shear failure causes ultimate failure of laminate; $\$(w_{max}/h)^f$ and $(w_{max}/h)^u$ are the non-dimensionalized maximum transverse deflections corresponding to first-ply failure and ultimate failure, respectively.

Table 6 Failure Characteristics of $(+45/-45/0/90)_{2s}$ laminate with and without cutout for different edge boundary conditions# under negative shear load

Load	Without cutout			With cutout		
	BC1	BC2	BC3	BC1	BC2	BC3
Buckling load	51.20	77.03	94.24	44.47	67.12	80.89
First-ply failure load	83.05	91.65	105.00	63.98	78.78	88.65
Ultimate failure load	126.94	97.25	110.20	107.44	103.00	118.35
$(w_{max}/h)^{f\,\$}$	2.43	1.40	1.01	2.02	1.33	0.87
$(w_{max}/h)^{u\$}$	4.22	1.80	1.63	3.82	2.53	2.21

#BC1, BC2 and BC3 have the same meanings as defined in Table 5.
$ $(w_{max}/h)^f$ and $(w_{max}/h)^u$ have the same meanings as defined in Table 5.

loads. A similar observation is true in the case of ultimate load for the BC1 boundary condition. However, it is worth noting that the ultimate loads of the laminate with a cutout are larger for BC2 and BC3 boundary conditions than those without cutout for both the positive and negative shear loads. This is attributed to the fact that early onset of delamination, which occurs in the laminate with no cutout with these two boundary conditions (i.e., BC2 & BC3), is precluded due to the introduction of cutout.

5. Conclusions

Based on the results and discussion, the following observations are made:

- The corners of the cutouts are the critical first-ply failure locations for a simply supported quasi-isotropic $(+45/-45/0/90)_{2s}$ laminates with smaller size square cutouts (i.e., $0.06 \le c/b \le 0.14$) and rectangular cutouts (with aspect ratios, $c/b < 0.28$ & $d/b = 0.14$), irrespective of the direction of shear load. In case of quasi-isotropic laminates with larger cutouts (i.e., square cutouts, $c/b > 0.14$ and rectangular cutouts with aspect ratios, $c/b \ge 0.28$ & $d/b = 0.14$), the first-ply failure occurs at the corners of the laminates and at the corners of the cutouts under positive and negative shear, respectively; except for $c/b = 0.42$ & $d/b = 0.14$, in which case the failure under negative shear occurs at the edge, $x = 0$.
- The first-ply failure locations for the $(0/90)_{4s}$ and $(+45/-45)_{4s}$ laminates are at the corners of the laminate and corner of the cutout, respectively, irrespective of the size of the cutout and directions of the shear load.
- The matrix failure due to in-plane normal stresses transverse to the fiber direction is the primary mode of the first-ply failure in $(+45/-45)_{4s}$ and $(+45/-45/0/90)_{2s}$ laminates while in-plane shear mode of failure is the primary mode of failure of $(0/90)_{4s}$ laminate, irrespective of directions of the shear load and size of the cutout.
- The ultimate failure of the simply supported $(+45/-45/0/90)_{2s}$ laminates depends on the geometry and size of the cutout. The onset of the delamination is the cause of ultimate failure of the quasi-isotropic laminate with square cutouts of all sizes and rectangular cutouts of sizes $c/b \le 0.28$ & $d/b = 0.14$, irrespective of the nature of shear load. In the case of laminate with rectangular cutouts of larger sizes (i.e., $c/b > 0.28$ & $d/b = 0.14$), the large transverse deflection is the cause of the ultimate failure, irrespective of direction of shear loads.

- The onset of delamination in $(+45/-45/0/90)_{2s}$ laminate occurs even before the first-ply failure for very small size of cutouts ($c/b \leq 0.06$), irrespective of directions of the shear load.
- There is a monotonic decrease in buckling and postbuckling strengths with an increase in the aspect ratio of the cutout in simply supported $(+45/-45/0/90)_{2s}$ laminate.
- The modes of ultimate failure in simply supported $(+45/-45)_{4s}$ and $(0/90)_{4s}$ laminates are the onset of delamination and widespread in-plane shear, respectively, for both directions of the shear load.
- Ultimate failure of $(0/90)_{4s}$ laminate due to the onset of delamination is precluded due to presence of the cutout.
- Buckling and failure loads of anisotropic, $(+45/-45)_{4s}$ and quasi-isotropic, $(+45/-45/0/90)_{2s}$ laminates with cutouts are less than those without cutout, irrespective of directions of the shear load. However, in the case of cross-ply $(0/90)_{4s}$ laminate, the buckling load is less, but failure loads depend on the size of the cutout and the direction of the shear load.
- Buckling and first-ply failure loads of $(+45/-45/0/90)_{2s}$ laminate with a square cutout (c/b = 0.14) are less than those without cutout for all the three edge boundary conditions. However, the ultimate loads of the laminate with two parallel edges simply supported & the other two clamped (BC2) and the one with all edges clamped (BC3) are larger in the case of laminate with cutout for both directions of the shear load.

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Notations

The following symbols are used in this paper:

b	: in-plane dimensions of the square plate in x- and y-direction;
c and d	: dimensions of cutout in x- and y-direction, respectively;
E_1, E_2 and E_3	: principal Young's moduli in fiber direction and other two transverse directions, respectively;
F_i, F_{ij}	: strength tensors;
G_{12}, G_{13} and G_{23}	: shear moduli associated with planes 1-2, 1-3 and 2-3, respectively;
h	: thickness of the square plate;
N_{xy}	: applied in-plane shear stress;
R, S and T	: shear strengths of lamina in planes 2-3, 1-3 and 1-2, respectively;
w_{max}	: maximum transverse deflection;
X (X_t or X_c)	: normal strength (tensile or compressive, respectively) of lamina in fiber direction-1;
Y (Y_t or Y_c)	: normal strength (tensile or compressive, respectively) of lamina in direction transverse to the fiber direction-1;
Z (Z_t or Z_c)	: normal strength (tensile or compressive, respectively) of lamina in principal material direction-3, i.e., perpendicular to plane of lamina;
ν_{12}, ν_{13} and ν_{23}	: Poisson's ratios associated with planes 1-2, 1-3 and 2-3, respectively;
σ_1, σ_2, σ_3	: normal stress components in principal material directions 1, 2 and 3, respectively (the subscript 1 referring to the fiber direction);
σ_{DN}	: peel strength equal to the tensile normal transverse strength of lamina;
σ_{DS}	: interlaminar shear strength equal to transverse shear strength corresponding to the plane 1-3 of lamina;
θ	: fiber orientation with respect to x-direction;
τ_{12}, τ_{13} and τ_{23}	: shear stress components in principal material planes 1-2, 1-3 & 2-3, respectively; and
τ_{xz}	: transverse shear stress corresponding to x-z plane.

Appendix

(a) Tsai-Hill Criterion

In this criterion a considerable interaction exists among failure strengths of the lamina as against the other non-interactive criteria such as Hashin (1980) and Tsai's (1984) tensor polynomial criteria. Tensor polynomial form of the Tsai-Hill criterion can be obtained from the following most general polynomial failure criterion of Tsai (1984) at failure state.

$$F_1\sigma_1 + F_2\sigma_2 + F_3\sigma_3 + 2F_{12}\sigma_1\sigma_2 + 2F_{13}\sigma_1\sigma_3 + 2F_{23}\sigma_2\sigma_3 + F_{11}\sigma_1^2 +$$
$$F_{22}\sigma_2^2 + F_{33}\sigma_3^2 + F_{44}\tau_{23}^2 + F_{55}\tau_{13}^2 + F_{66}\tau_{12}^2 + \ldots \geq 1$$

Wherein, F_i, F_{ij} are the strength tensors of the second and fourth rank; $\sigma_1, \sigma_2, \sigma_3$ are the normal stress components in principal material directions 1, 2 and 3, respectively (the subscript 1 referring to the fiber direction); $\tau_{12}, \tau_{13}, \tau_{23}$ are the shear stress components in the planes 1-2, 1-3 and 2-3, respectively. In Tsai-Hill criterion the following strength tensors are used in the above expression.

$$F_1 = F_2 = F_3 = 0\,; \quad F_{11} = \frac{1}{X^2}; \quad F_{22} = \frac{1}{Y^2}; \quad F_{33} = \frac{1}{Z^2}; \quad F_{44} = \frac{1}{R^2}; \quad F_{55} = \frac{1}{S^2}; \quad F_{66} = \frac{1}{T^2}$$

$$F_{12} = -\frac{1}{2}\left(\frac{1}{X^2} + \frac{1}{Y^2} - \frac{1}{Z^2}\right); \quad F_{13} = -\frac{1}{2}\left(\frac{1}{Z^2} + \frac{1}{X^2} - \frac{1}{Y^2}\right); \quad F_{23} = -\frac{1}{2}\left(\frac{1}{Y^2} + \frac{1}{Z^2} - \frac{1}{X^2}\right)$$

In the above expressions *X, Y* and *Z* are the normal strengths (tensile or compressive, depending upon the sign of $\sigma_1, \sigma_2, \sigma_3$) along principal material directions 1, 2 and 3, respectively; *R, S* and *T* are the shear strengths of lamina in planes 2-3, 1-3 and 1-2, respectively.

(b) Interlaminar Failure Criterion

As per the interlaminar failure criterion, the onset of delamination takes place when the interlaminar transverse stress (calculated by integration of equilibrium equations) components satisfy the following expression:

$$\left(\frac{\sigma_3}{\sigma_{DN}}\right)^2 + \frac{\tau_{13}^2 + \tau_{23}^2}{\sigma_{DN}^2} \geq 1$$

Where, σ_3 is the transverse normal stress component; τ_{13}, τ_{23} are the transverse shear stress components in principal material planes 1-3 and 2-3, respectively; σ_{DN} is the peel strength and σ_{DS} is the interlaminar shear strength; these are taken equal to the tensile normal transverse strength and transverse shear strength (corresponding to the plane 1-3) of lamina, respectively.