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Exact solution for in-plane displacement of redundant curved beam

Lili Zhu†

School of Mechanical Engineering, Dalian Jiaotong University, Dalian 116028, P.R. China

Yinghua Zhao

Institute of Road and Bridge Engineering, Dalian Maritime University, Dalian 116026, P.R. China

Guangxin Wang

School of Mechanical Engineering, Dalian Jiaotong University, Dalian 116028, P.R. China

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1. Introduction

Early theoretical research of curved beams dates back to 1980s (Zhao *et al.* 2006). Analytical solutions of internal forces and deformations were derived (Heins 1981, Yao 1989) for plane curved beams with constant curvature under out-of-plane loads, based on traditional mechanics of structures, in which the coupling of flexure and torsion was taken into account. Further, Tufekci and Dogruer (2006) obtained the exact solution of out-of-plane problems of a plane arch with varying curvature and cross section. Along with the development of the study, some theoretical results for plane curved beams have been applied in the design of curved bridges (Sun 1995). Nevertheless, the in-plane problems of curved beams have not been studied widely. Much less research could be found in related Criterion and Standards. In order to investigate the reason of in-plane damage happened in curved bridges in recent years, Li *et al.* (2007) and Li and Zhao (2008) presented the solution of the displacement of the plane curved beams subjected to an in-plane load and a changed temperature by using structural mechanics method. The method brings some errors for application without considering the effects of the transverse shear deformation and torsion-related warping. The purpose of this study is to give the exact solution of curved beams stressed by the centripetal force of vehicles running along the bridge in plane, including the contribution of transverse shear deformation effect. The accuracy and efficiency of the theory are demonstrated by comparing the analytical solutions with the results obtained by structural mechanics method. Finally particular

† Ph.D., Corresponding author, E-mail: xiaozhuzhu_1979@163.com

emphasis is placed upon influence of curvature, load position and boundary condition on the beam deformation.

2. Solution of in-plane problems of redundant curved beams

Using the explicit solution of internal forces and deformations of the spatial warping curved beams under complicated loads (Zhu and Zhao 2008), the exact solution for the in-plane displacement u_ξ of 1 redundant curved beam (Fig. 1) can be obtained as

When $\beta < \theta$

$$u_\xi = -\left[\frac{X_1}{2}\left(\frac{1}{EA_s}+\frac{r^2}{EI_\eta}\right)+\frac{z_1 r^2}{EI_\eta}\left(\cos\beta - 1+\sin\beta\tan\frac{\phi}{2}\right)+\frac{X_2 EI_\eta + X_3 A_s G r^2 + X_4 GI_\eta}{4A_s G^2}\right]$$

$$Pr\bigg/\left[z_2 - \frac{2EI_\eta(\phi - \sin\phi)}{G}\right]$$

When $\beta > \theta$

$$u_\xi = -\left[\frac{X_5}{2}\left(\frac{1}{EA_s}+\frac{r^2}{EI_\eta}\right)+\frac{z_1 r^2}{EI_\eta}\left(\cos\beta - 1+\sin\beta\tan\frac{\phi}{2}\right)+\frac{X_6 EI_\eta + X_7 A_s G r^2 + X_8 GI_\eta}{4A_s G^2}\right]$$

$$Pr\bigg/\left[z_2 - \frac{2EI_\eta(\phi - \sin\phi)}{G}\right] \tag{1}$$

where $\boldsymbol{e}_s$, $\boldsymbol{e}_\xi$ and $\boldsymbol{e}_\eta$ are the tangential, two principal axes' unit vectors of the curved beam (Fig. 2), respectively; r is the radius of the axis of curved beam; A_s is the cross-sectional area and I_η is the area moment of inertia around η axis, E and G are respectively Young's and shear moduli; z_1, z_2, $X_1 \ldots X_8$ can be referred in Zhu and Wang (2008).

It shows that the two are equivalent to each other by comparing Eq. (1) omitted the items describing the effect of the transverse shear deformation with the solution (Li and Zhao 2008) obtained by structural mechanics method, which can testify the accuracy of the theory.

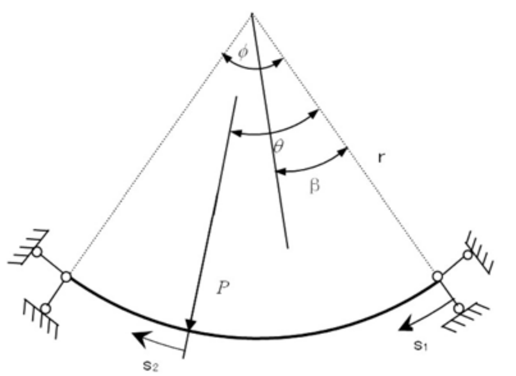


Fig. 1 The diagram of 1 redundant curved beam under in-plane bending

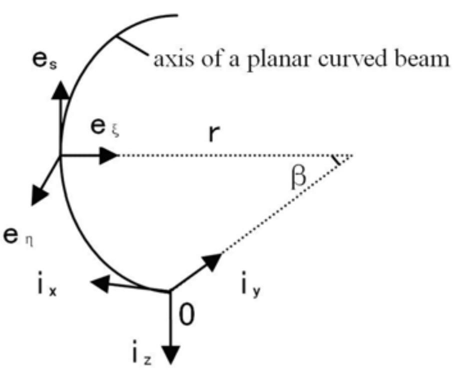
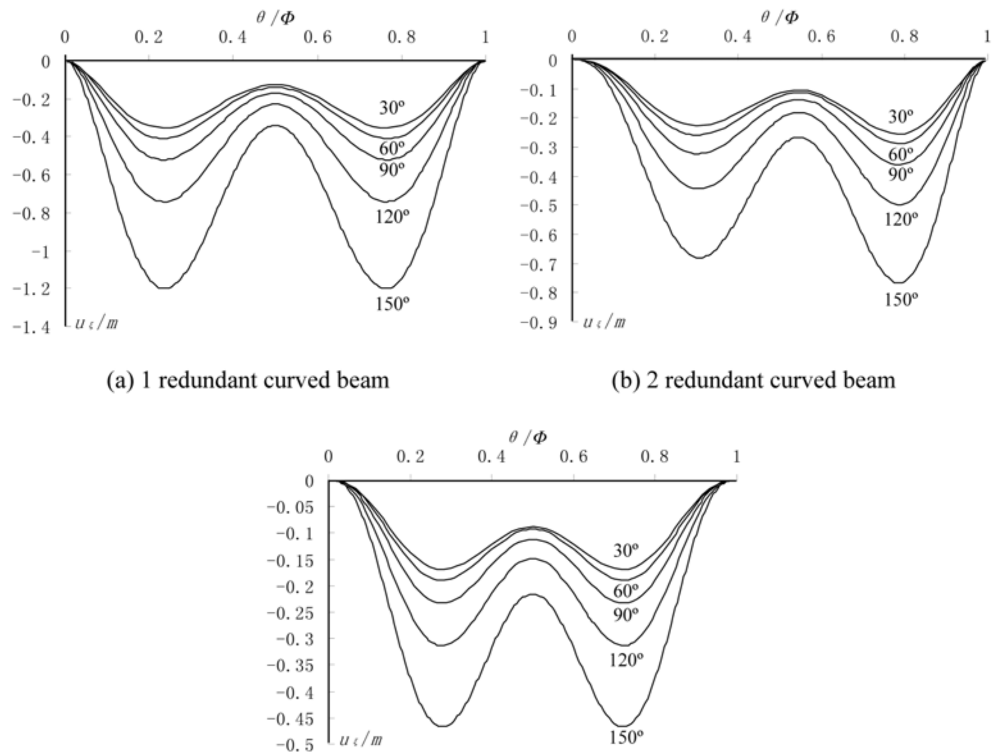


Fig. 2 Axis of a planar curved beam

In the same way, substituting the different restraint conditions at the two ends of the beam into the analytical solution of the warping spatial curved beam (Zhu and Zhao 2008), the exact solutions for the in-plane displacement of 2 and 3 redundant curved beams can be obtained conveniently.

3. Effects of curvature, load position and boundary condition

Fig. 3 shows the comparison of displacements u_ξ of 5 kinds of curved beams ($\phi = 30^o$, 60^o, 90^o, 120^o, 150^o) at different load positions with different boundary conditions, respectively. The span length of the beam L = 100 mm, the point load P = 10000N, and other data of material property and geometry can be referred in Zhu and Wang (2008). It is seen that the maximum displacements happen when the point load stresses at the 1/4 to 1/3 position adjacent to the end of the curved beam, while the smaller displacements appear at the area near the midpoint of the curved beam. Fig. 4 shows the comparison of u_ξ on 1, 2 and 3 redundant curved beams, with constant load position $\theta = \phi/4$ but varying ϕ ($\phi = 30^o$, 60^o, 90^o, 120^o). It can be observed that for the 3 kinds of restraint conditions, u_ξ increases as ϕ increases, that is to say the in-plane displacement increases as the curvature goes up if the span length L remains constant. Furthermore, as one might expect, the in-plane displacements decrease as the constraints of the boundary conditions increase, from hinged-hinged to hinged-fixed to fixed-fixed revealed in the Figs. 3 and 4.

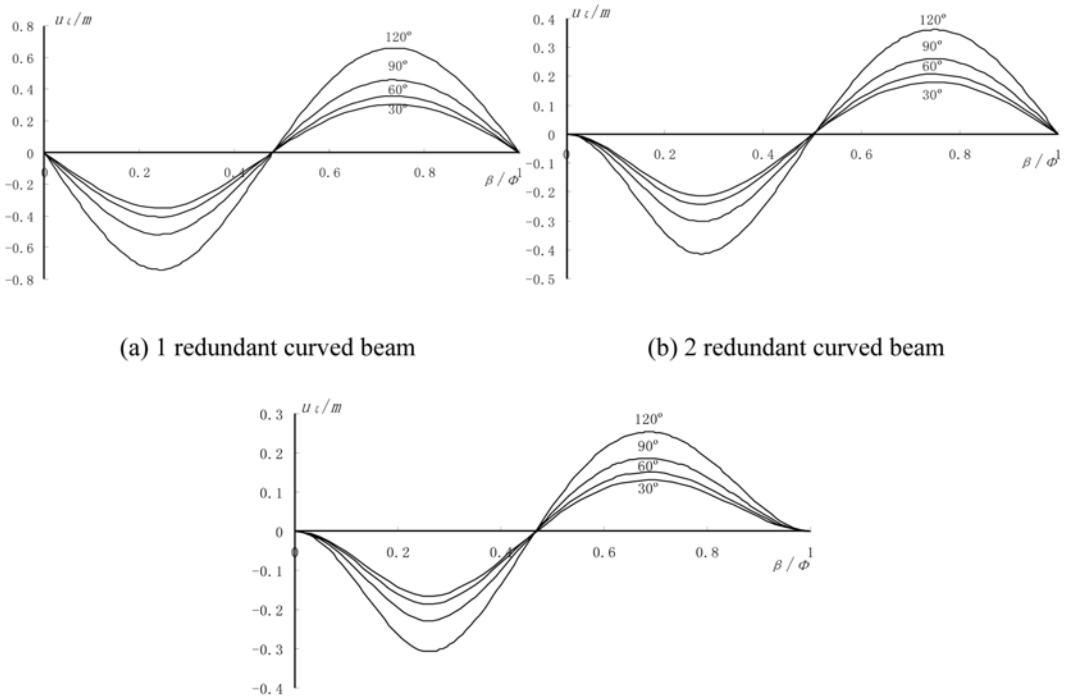


(a) 1 redundant curved beam

(b) 2 redundant curved beam

(c) 3 redundant curved beam

Fig. 3 Comparison of displacements in the position of the load



(a) 1 redundant curved beam (b) 2 redundant curved beam

(c) 3 redundant curved beam

Fig. 4 Effect of curvature to the in-plane displacement of curved beam

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