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Probabilistic shear strength models for reinforced concrete beams without shear reinforcement

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Abstract. In order to predict the shear strengths of reinforced concrete beams, many deterministic models have been developed based on rules of mechanics and on experimental test results. While the constant and variable angle truss models are known to provide reliable bases and to give reasonable predictions for the shear strengths of members with shear reinforcement, in the case of members without shear reinforcement, even advanced models with complicated procedures may show lack of accuracy or lead to fairly different predictions from other similar models. For this reason, many research efforts have been made for more accurate predictions, which resulted in important recent publications. This paper develops probabilistic shear strength models for reinforced concrete beams without shear reinforcement based on deterministic shear strength models, understanding of shear transfer mechanisms and influential parameters, and experimental test results reported in the literature. Using a Bayesian parameter estimation method, the biases of base deterministic models are identified as algebraic functions of input parameters and the errors of the developed models remaining after the bias-correction are quantified in a stochastic manner. The proposed probabilistic models predict the shear strengths with improved accuracy and help incorporate the model uncertainties into vulnerability estimations and risk-quantified designs.

Keywords: Bayesian parameter estimation; epistemic uncertainty; model errors; error analysis; probabilistic models; reinforced concrete beams; shear strength.

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1. Introduction

The flexural behavior of reinforced concrete (RC) members has been well understood such that their flexural strengths are predicted with reasonable accuracy over a wide range of cases. By contrast, it is difficult to predict the shear strengths of RC members accurately due to the uncertainties in the shear transfer mechanism, especially after cracks are initiated. For more accurate prediction of the shear strengths, many sophisticated approaches have been proposed based on mechanical or physical models of structural behavior/failure, fracture mechanics, and nonlinear finite element analyses. The constant and variable angle truss models are known to provide reliable bases and to give reasonable results for the shear strengths of members with shear reinforcement. For members without shear reinforcement, however, even advanced models may show lack of accuracy despite their complicated procedures, and often lead to fairly different predictions than other similar models. Since there is yet no agreement on such approaches among researchers, prediction of shear strengths of RC beams is still considered an active open research field with important recent publications.

As a result, most design code provisions use empirical models developed based on simplified rules of mechanics and/or regression analyses of experimental data. The number of experimental observations used then for developing such models was often limited. These deterministic models exhibit uncertain biases and errors that prevent accurate predictions over a wide range of input parameter values. This uncertainty is due to imperfect descriptions of shear transfer mechanism, missing parameters, and insufficient amount of the test data. Recently, research efforts have been made to develop shear strength models for RC beams without stirrups that have improved accuracy and reduced uncertainties (Bazant and Yu 2005, Russo *et al*. 2005, Choi *et al.* 2007, Choi and Park 2007). The models were developed based on the improved shear transfer mechanisms and/or on the consideration of the most significant parameters. The developed models were then calibrated by experimental data, and sometimes further simplified for practical uses.

This paper envisions a stochastic *framework* for developing shear strength models based on existing shear strength models and experimental observations. The framework improves the accuracy of predictions, helps incorporate the model uncertainties into vulnerability estimations and risk-quantified designs, and provides better understanding of shear transfer mechanisms and the roles of various parameters on shear strengths. In particular, this paper develops probabilistic shear strength models for RC beams without shear reinforcement (stirrup) by use of a Bayesian method (Gardoni *et al.* 2002) based on widely-used, state-of-the-art deterministic shear strength models and an extensive database of observed shear strengths (Reineck *et al.* 2003). First, the overall bias and scatter of each of the selected deterministic model is quantified as constant terms that are independent of input parameters. This is to later evaluate the overall performance of the models developed in this paper. Second, probabilistic shear strength models are developed by identifying the biases in the deterministic models as algebraic functions of input parameters and by estimating the uncertain errors that remain after the bias correction. Based on understanding of the shear transfer mechanism, a set of explanatory terms are selected for describing biases of deterministic models. Explanatory terms with significant contributions are identified during Bayesian updating through a systematic removal process. The performance of the developed probabilistic models is confirmed by comparison with the actual test results. This study also attempts to calibrate the parameters in the deterministic models based on experimental observations, and to propose a general method that can construct probabilistic models directly from experimental data when reliable deterministic models are not available or influencing input parameters are not clearly identified.

This paper focuses on the shear strengths of RC members *without* shear reinforcement. While structural concrete members such as slabs and footings are being designed without such reinforcement, many members such as beams and columns typically have shear reinforcement. The shear behaviors of RC members with shear reinforcement are considerably different from those without. However, even in the case of designing a member with shear reinforcement, it is essential to accurately predict the shear strength of an RC member that has no shear reinforcement. This is because most shear design provisions require estimating the shear strength contributed by concrete (V_c) in order to check whether shear reinforcement is necessary or not.

2. Shear strengths of reinforced concrete beams without shear reinforcement

2.1 Shear transfer mechanisms

Due to the complex stress redistribution after cracking, the shear transfer mechanisms of RC beams have not been clearly understood yet. Fig. 1 visualizes the basic shear transfer mechanisms in accordance with the findings of the state-of-the-art reports by joint ASCE-ACI Committees 426 (1973, 1974) and 445 (1998, 2000). The important shear transfer mechanisms include (1) the shear in the uncracked compression zone, (2) the dowel action of the longitudinal reinforcement, (3) the interface shear transfer due to the aggregate interlocks or the surface roughness of the cracks, and (4) the residual tensile stresses across the cracks.

Since the shear resistance provided by the uncracked compression zone is limited by the depth of the zone, its contribution to shear strength in a relatively slender beam with no axial compression is insignificant. Local roughness in a crack plane results in the interface shear transfer, which decreases as the crack opening increases or the aggregate size decreases. However, this is not necessarily true in high strength concrete since cracks can pass through aggregates and its relatively

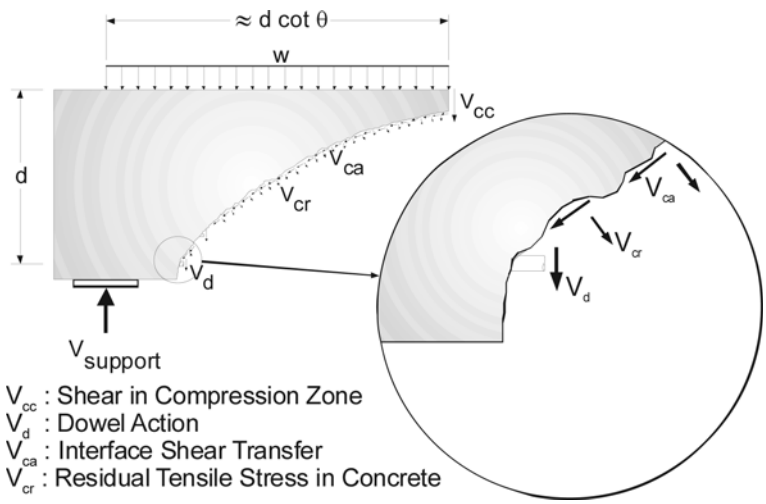


Fig. 1 Shear transfer mechanisms contributing to shear strength of reinforced concrete beam without shear reinforcement

smooth crack planes reduce interface shear transfer. The contribution of the dowel action of the longitudinal bars is affected by the amount of concrete cover beneath the bars. The residual tensile stresses in concrete also contribute to the shear transfer as they directly transmit it across cracks. When a crack opening is small, the resistance provided by the residual tensile stresses is significant while less significant in large beams. For deep beams, arch action is the most dominant shear transfer mechanism.

2.2 Predictive parameters influencing shear strength

The aforementioned shear transfer mechanisms help identify predictive parameters that may affect the shear strength of a RC beam, such as (1) the concrete compressive strength, (2) the beam depth, (3) the shear span-to-depth ratio, (4) the amount of longitudinal reinforcement, and (5) the axial forces.

The shear strength of an RC beam increases as the concrete material strength increases. The concrete tensile strength is known to have a great influence on the shear strength, but the concrete compressive strength f_c' is used instead in most shear strength formulas. This is because tensile tests are more difficult to conduct and usually show greater scatter than compression tests.

The shear strength decreases as the depth of a beam increases. This is often called "size effect". It has been effectively demonstrated by the experimental studies by Kani (1967) and Shioya *et al.* (1989). Shioya *et al.* tested members with depths ranging from 100 mm to 3000 mm. They observed that the ultimate shear stress of the largest member was only one-third of that of the smallest one. Bazant and Kim (1984) explained this effect based on fracture mechanics, while Collins and Kuchma (1999) and Reineck (1990) explained it by the reduction of the interface shear transfer due to the larger crack widths that occur in members with relatively large depth.

The shear span-to-depth ratio is the ratio of the distance (a) between the support and the loading point to the effective depth (d) of the beam, i.e., a/d. The shear strength increases as the shear span-to-depth ratio decreases. This phenomenon is quite significant in "deep beams" with the ratio less than about 2.5 because a portion of shear is transmitted directly to the support by an inclined strut ("arch action"). As strut-and-tie models consider this direct shear flow in a concrete member, it is more appropriate to use strut-and-tie models than sectional design approaches for these deep beams. All the test data considered in this paper have shear span-to-depth ratios greater than 2.4. Therefore, the probabilistic models developed in this paper should be used within the range.

For a given magnitude of loading, as the longitudinal reinforcement ratio decreases, flexural stresses and strains in concrete increase. Thus the crack widths increases and the shear strength is reduced. In addition, dowel action is weakened due to the low reinforcement ratio.

When members are subjected to axial tension, the shear strengths decrease. By contrast, axial compression increases the shear strengths since it increases the depth of the uncracked compression zone as well as the interface shear transfer. For members subjected to significant axial compression, however, brittle failures often occur. This paper does not account for the effect of axial forces in developing probabilistic shear strength models as all the beams in the shear database were tested with no axial forces applied.

2.3 Existing shear strength models

Researchers have proposed various shear strength models by imposing different levels of relative importance to the aforementioned shear transfer mechanisms. Strength models such as tooth model (Kani 1964), strut-and-tie model, and truss model are proposed based on the understanding of structural behavior and failure. Some models are developed based on fracture mechanics theories that explain the stress concentration at the crack tip and the decrease of tensile stress on the crack plane. Mitchell and Collins (1974) and Vecchio and Collins (1982) evolved the variable-angle truss model by proposing the compression field theory (CFT) that considers compatibility and stress-strain relationship in cracked concrete in addition to equilibrium conditions. However, the CFT can be used for reinforced concrete members with shear reinforcement only because it does not account for the tensile stress in cracked concrete. For this reason, Vecchio and Collins (1986) developed the modified compression field theory (MCFT) that can take into account the influence of tensile stresses on the post-cracking shear behavior, which thus can be used for predicting service load behavior and ultimate strength in reinforced concrete members with and without shear reinforcement. Isenberg (1993) and other researchers predicted shear strengths by nonlinear finite element analyses, and Vecchio and Palermo (2002) also utilized the MCFT in their nonlinear finite element analyses.

Despite their accuracy, these sophisticated approaches are difficult to follow in some practical situations. Therefore, most shear design codes use empirical or semi-empirical models for predicting shear strengths. As an attempt to achieve models that are not only accurate but also useful in practice, this paper develops probabilistic shear strength models by identifying biases in eight existing deterministic models and quantifying the remaining errors in a probabilistic manner. Table 1 lists the shear strength models that are used as base models in developing probabilistic models in this paper.

The equation 11-3 in ACI 318-08 (ACI 2008) is the simplest and the most commonly used model. Since it accounts for the influence of concrete strength only, it shows significant biases in its predictions against other influencing parameters. This model tends to overestimate the strengths of beams having relatively large depths. It overestimates the shear strength as the longitudinal reinforcement ratio ρ decreases while it provides conservative estimates for heavily reinforced

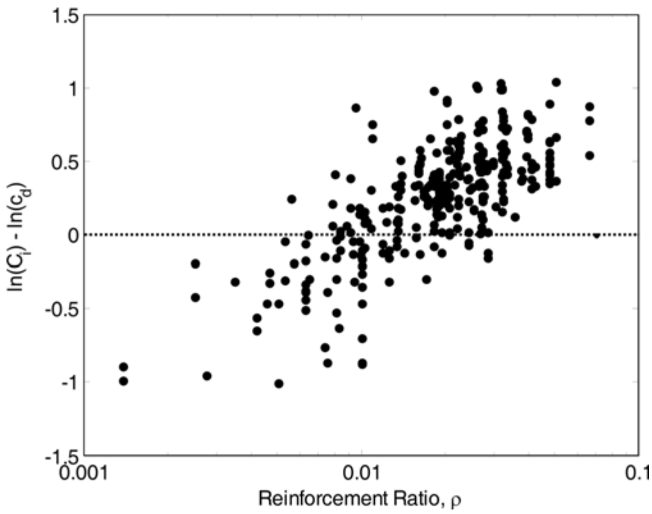


Fig. 2 Errors of ACI 11-3 model versus longitudinal reinforcement ratio before bias-correction

beams. This bias is demonstrated in Fig. 2, which plots the difference between the natural logarithms of observed shear strengths C_i in a shear database (See the following subsection) and the strengths c_d predicted by the ACI equation 11-3. The equation 11-5 in ACI 318-08 (ACI 2008) accounts for the influence of longitudinal reinforcement ratio in addition. However, it still tends to overestimate the shear strengths for lightly reinforced beams. The overall accuracy of the equation 11-5 is considered to be similar to that of the equation 11-3 although it is more complicated.

Table 1 Shear strength models used for developing probabilistic models

Model	Equation
ACI 11-3 (ACI 2008)	$V_c = \frac{1}{6}\sqrt{f_c'}\,b_w d$
ACI 11-5 (ACI 2008)	$V_c = \left(0.158\sqrt{f_c'} + 17\rho\frac{V_u d}{M_u}\right)b_w d \le 0.3\sqrt{f_c'}\,b_w d$ where $V_u d/M_u \le 1.0$
Eurocode draft (2003)	$V_c = 0.12k(100\rho f_c')^{1/3} b_w d$ where $k = 1 + \sqrt{200/d} \le 2.0,\ \rho \le 0.02$
Tureyen and Frosch (2003)	$V_c = \frac{5}{12}\sqrt{f_c'}\,b_w c$ where $c = kd$, $k = \sqrt{2\rho n + (\rho n)^2} - \rho n$ and $n = E_s/E_c$
Zsutty (1971)	$V_c = 2.2\left(f_c'\rho\frac{d}{a}\right)^{1/3} b_w d$
Okamura and Higai (1980)	$V_c = 0.2\frac{(100\rho)^{1/3}}{(d/1000)^{1/4}}(f_c')^{1/3}\left(0.75 + \frac{1.40}{a/d}\right)^{1/3} b_w d$
Bazant and Yu (2005)	$V_c = 1.1044 \cdot \rho^{3/8} b_w\left(1 + \frac{d}{a}\right)\sqrt{\frac{f_c' d_0 d}{1 + d_0/d}}$ where $d_0 = \kappa(f_c')^{-2/3}$ and $\kappa = 693.7623\sqrt{d_a}$
Russo *et al.* (2005)	$V_c = 1.13\xi\left[\rho^{0.4}(f_c')^{0.39} + 0.5\rho^{0.83} f_y^{0.89}\left(\frac{a}{d}\right)^{-1.2-0.45(a/d)}\right]b_w d$ where $\xi = \frac{1 + \sqrt{5.08/d_a}}{\sqrt{1 + d/(25d_a)}}$

Notations: V_c (N): shear strength; f_c' (MPa): concrete compressive strength; b_w (mm): web width; d (mm): effective depth; $\rho = A_s/b_w d$: longitudinal reinforcement ratio in which A_s is the amount (area) of longitudinal reinforcement; V_u (N): factored shear force; M_u (N·mm): factored moment; $E_s = 2.0 \times 10^5$ (MPa): elastic modulus of reinforcement; $E_c = 4700\sqrt{f_c'}$ (MPa): elastic modulus of concrete; a (mm): shear span length; d_a (mm): the maximum aggregate size; and f_y (MPa): the yielding strength of the longitudinal reinforcement.

The model in the Eurocode draft (2003) considers the size effect, but its predictions tend to be conservative in most cases. The model by Tureyen and Frosch (2003) looks similar to the ACI equations, but its conceptual basis is fairly different as it neglects the shear transfer in cracked concrete. This model provides more accurate predictions than ACI equations, but it tends to overestimate for large beams. The model by Zsutty (1971) is more accurate than the ACI models despite its simple form. However, it often overestimates the strengths for deep beams since it does not account for the size effect. The model by Okamura and Higai (1980) contains all the aforementioned influencing parameters except axial forces, and provides good estimations without any severe biases in most cases.

Recently, Bazant and Yu (2005) proposed a shear strength formula that can account for the "size effect." This model was theoretically derived based on fracture mechanics and calibrated by a large number of experimental test data. The effects of primary influencing parameters were considered. The size effect law was verified utilizing dimensional analysis and cohesive crack model in which the shear strength was inversely proportional to the square root of the depth, $\sqrt{d}$. They proposed two models: the "design" model with embedded conservatism and the "fit" model for accurate prediction. In this paper, the "fit" model was selected to develop a probabilistic model that can account for the size effect based on facture mechanics theory.

Russo *et al.* (2005) proposed models for the shear strength of RC beams without shear reinforcement by utilizing beam and arch action and by determining necessary unknown parameters included in their formula from experimental data collected from the literature. They modified the shear formula to yield the best prediction uniformity of shear strength by minimizing the coefficient of variation (c.o.v.) of shear strength ratios, i.e., the measured to calculated shear strength. They also developed two models: "proposed design formula" (with embedded conservatism) and "shear strength formula" (accurate prediction). The latter model was used in this study.

2.4 Database for shear strengths of reinforced concrete beams

Existing design code provisions usually do not provide uniform levels of safety. This is mainly because only small sets of existing test results were used to develop shear strength models. In order to overcome this, Reineck *et al.* (2003) developed an extensive database for the shear strengths of RC beams without shear reinforcement. The database has 439 shear strengths observed for a wide range of structural parameters. This database has been checked against various selection criteria discussed by the ACI-ASCE Committee 445, including the minimum concrete compressive strength and the minimum size of beams, and assurance of shear failure. Removing some test beams cast with light-weight concrete and the tests for which maximum aggregate size is not available, this paper uses the reduced database of 341 tests to develop probabilistic strength models based on the deterministic models listed in Table 1.

Most members (93%) of the reduced database have rectangular cross-sections while the rest of them are T-shaped beams. Fig. 3 provides an overview on the distributions of four influencing parameters (f_c', d, ρ and a/d) and four other notable ratios (E_s/E_c, d_a/d, d/h and b_w/h where h denotes the height of the beam) that are used for deterministic and probabilistic shear strength models in this paper. 230 beams have concrete compressive strengths lower than 40 MPa. The vast majority of the members have the effective depth between 200 to 300 mm while a few members are about 2 meters deep. More than half of test specimens have the longitudinal reinforcement ratios greater than 2.0% while 55 beams have less than 1.0% reinforcement ratios. 135 members have the shear-span-to-

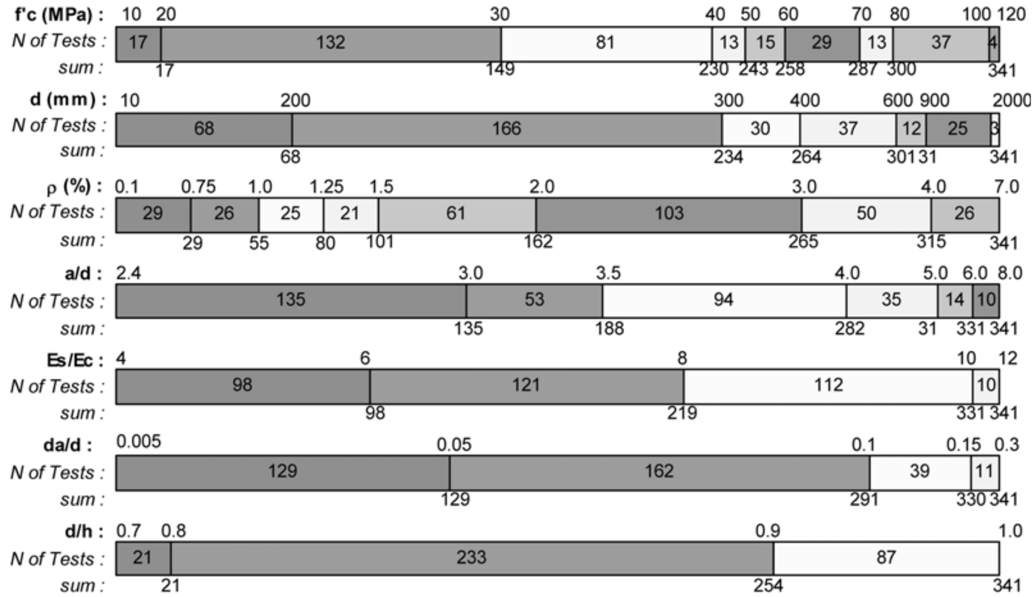


Fig. 3 Distributions of parameter values of reinforced concrete beams in shear database (341 test results)

depth ratios 2.4~3.0, and the ratios of some slender members are about 6.0. Since the experiments were conducted for the purpose of research, some cases may not represent the realistic conditions of existing structures necessarily. However, the comprehensiveness of the database enables us to investigate the shear strengths of members over a wide range of parameter values. The database lacks uniformity in the distribution of key parameters or explanatory terms since it is a collection of numerous test results in the literature. This may prevent models developed based on the database from achieving uniform accuracy over a wide range of parameter values. Therefore, this study will examine the errors of the developed models in sub-ranges of key parameters to confirm its uniform accuracy.

3. Probabilistic shear strength models

3.1 Bayesian methodology for probabilistic models

We aim to develop probabilistic models for the shear strengths of RC beams by identifying the biases in the base deterministic models and quantifying the uncertainty in the remaining errors. We adopt a Bayesian methodology originally developed for constructing probabilistic models for the capacities of RC columns (Gardoni *et al*. 2002) and later for the seismic demands of RC bridges (Gardoni *et al.* 2003). The probabilistic shear strength models predict the shear strength C in the form

$$C(\mathbf{x}, \Theta) = c_d(\mathbf{x}) + \gamma(\mathbf{x}, \theta) + \sigma\varepsilon \tag{1}$$

where $\mathbf{x}$ is the vector of input parameters measured during tests on RC beams, e.g., ρ, f_c', b_w, a and d; $\Theta = (\theta, \sigma)$ denotes a set of model parameters introduced to fit the model to the test results;

$c_d(\mathbf{x})$ is an existing deterministic model; $\gamma(\mathbf{x}, \boldsymbol{\theta})$ is the correction term for the bias inherent in the deterministic model that is expressed as a function of the input parameters $\mathbf{x}$ and uncertain model parameters $\boldsymbol{\theta} = [\theta_1, \theta_2, \ldots, \theta_p]^T$; ε is the normal random variable with zero mean and unit variance; and σ is a model parameter that represents the magnitude of the model error that remains after the bias-correction.

This model is based on the following two assumptions: (1) the model variance is independent of input parameters $\mathbf{x}$ ("homoskedasticity" assumption). Note that for given $\mathbf{x}$, $\boldsymbol{\theta}$ and σ, the variance of the model $C(\mathbf{x}, \boldsymbol{\theta})$ is σ^2 that is not the function of $\mathbf{x}$; and (2) ε has the normal distribution ("normality" assumption). In order to justify these assumptions, we often need to employ a suitable transformation of the quantity of interest. Diagnostic plots of the observed data versus the model predictions are often used to verify the suitability (Rao and Toutenburg 1997).

Since the true form of the bias-correction function $\gamma(\mathbf{x}, \boldsymbol{\theta})$ is unknown, one can try to express it using a suitable set of p "explanatory" functions $h_i(\mathbf{x})$, $i = 1, \ldots, p$, in the form

$$\gamma(\mathbf{x}, \boldsymbol{\theta}) = \sum_{i=1}^{p} \theta_i h_i(\mathbf{x}) \tag{2}$$

Based on mechanics theories or expert opinions, one may try some functions of influencing input parameters as explanatory functions. Applying the natural logarithms to satisfy the homoskedasticity assumption, we develop probabilistic models using the form

$$\ln[C(\mathbf{x}, \boldsymbol{\Theta})] = \ln[c_d(\mathbf{x})] + \sum_{i=1}^{p} \theta_i h_i(\mathbf{x}) + \sigma\varepsilon \tag{3}$$

A probabilistic shear strength model is constructed by finding the values of the model parameters $\boldsymbol{\Theta} = (\boldsymbol{\theta}, \sigma)$ that makes the model in Eq. (3) fit the test results best. We use Bayesian parameter estimation method for this purpose. Suppose $p(\boldsymbol{\Theta})$ is the joint probability density function (PDF) of a "prior" distribution reflecting the knowledge about $\boldsymbol{\Theta}$ prior to obtaining objective observations such as test results. The Bayesian approach updates this prior distribution to the "posterior" distribution $f(\boldsymbol{\Theta})$ based on the test results. The well-known Bayesian updating rule (Box and Tiao 1992) is

$$f(\boldsymbol{\Theta}) = \kappa L(\boldsymbol{\Theta}) p(\boldsymbol{\Theta}) \tag{4}$$

where $L(\boldsymbol{\Theta})$ is the "likelihood" function representing the likelihood of the test results; and $\kappa = [\int L(\boldsymbol{\Theta}) p(\boldsymbol{\Theta}) d\boldsymbol{\Theta}]^{-1}$ is the normalizing factor. We consider the posterior means of the posterior function, i.e., $\mathbf{M}_{\Theta} = \int \boldsymbol{\Theta} f(\boldsymbol{\Theta}) d\boldsymbol{\Theta} = [\mu_{\theta_1}, \mu_{\theta_2}, \ldots, \mu_{\theta_n}, \mu_{\sigma}]^T$ as the best-fitting parameters and substitute them into $\boldsymbol{\Theta} = [\theta_1, \theta_2, \ldots, \theta_p, \sigma]^T$ of Eq. (3) to construct a probabilistic shear strength model. It is not straightforward to compute the multifold integrals for obtaining the normalizing factor κ, the posterior mean vector $\mathbf{M}_{\Theta}$, and covariance matrix $\Sigma_{\Theta\Theta} = \int \boldsymbol{\Theta}\boldsymbol{\Theta}^T f(\boldsymbol{\Theta}) d\boldsymbol{\Theta} - \mathbf{M}_{\Theta}\mathbf{M}_{\Theta}^T$. Among various methods for this computation including Markov Chain Monte Carlo sampling (MCMC), we use an importance sampling method employing the sampling density function centered at the maximum likelihood point (Gardoni 2002).

When there is no available information that may be used for the prior distribution of parameters, one can employ a "non-informative" prior that has little influence on the posterior distribution. Box and Tiao (1992) showed that the non-informative prior distribution of $\boldsymbol{\theta}$ is locally uniform such that $p(\boldsymbol{\Theta}) \cong p(\sigma)$. The non-informative prior of σ then takes the form

$$p(\sigma) \propto \frac{1}{\sigma} \tag{5}$$

The likelihood function $L(\Theta)$ is defined as a function proportional to the conditional probability of the observations for given input parameters $\mathbf{x}$. One of three kinds of observations can be made from the i-th test: (1) "failure datum": the demand C_i is measured at the instant of the failure, i.e., $C_i = c_d(\mathbf{x}_i) + \gamma(\mathbf{x}_i, \theta) + \sigma\varepsilon$; (2) "lower bound datum": the component does not fail up to the demand level C_i, i.e., $C_i < c_d(\mathbf{x}_i) + \gamma(\mathbf{x}_i, \theta) + \sigma\varepsilon$; and (3) "upper bound datum": the component has failed under a lower demand than measured C_i, i.e., $C_i > c_d(\mathbf{x}_i) + \gamma(\mathbf{x}_i, \theta) + \sigma\varepsilon$. Under the assumption of statistically independent tests and the normality assumption of ε, the likelihood function is derived (Gardoni *et al.* 2002) as

$$\begin{aligned} L(\Theta) \propto & \prod_{\text{failure data}} \left\{ \frac{1}{\sigma} \varphi\left[\frac{C_i - c_d(\mathbf{x}_i) - \gamma(\mathbf{x}_i, \theta)}{\sigma}\right] \right\} \times \\ & \prod_{\text{lower bound data}} \Phi\left[-\frac{C_i - c_d(\mathbf{x}_i) - \gamma(\mathbf{x}_i, \theta)}{\sigma}\right] \times \\ & \prod_{\text{upper bound data}} \Phi\left[\frac{C_i - c_d(\mathbf{x}_i) - \gamma(\mathbf{x}_i, \theta)}{\sigma}\right] \end{aligned} \tag{6}$$

where $\varphi(\cdot)$ and $\Phi(\cdot)$ respectively denote the PDF and the cumulative distribution function (CDF) of the standard normal distribution. Note that the likelihood function can use even the information of lower and upper bound data in constructing a probabilistic model while these are usually neglected during model development.

This Bayesian approach is particularly useful when (1) a database has non-failure data, i.e., lower or upper bound data or (2) a small number of data are used while subjective information is available for informative prior distribution. However, the shear database used in this paper contain "failure datum" only, and non-informative prior distribution is used. Therefore, the results from the Bayesian approach would converge to those by classical statistical parameter estimation methods. The main reason for using the aforementioned Bayesian approach in this paper is to utilize its stepwise procedure (Gardoni *et al.* 2002) for removing insignificant explanatory terms. When a parameter has the largest posterior coefficient of variation (c.o.v.) after a Bayesian updating, its corresponding explanatory function is considered the least informative function and then dropped from the bias-correction function $\gamma(\mathbf{x}, \theta)$. A Bayesian updating is performed again using this bias-correction function with fewer terms. This process is repeated until such a removal increases the posterior mean of σ by an unacceptable amount. This removal process of the Bayesian methodology allows researchers to not only try any explanatory functions they consider important but also identify informative terms in a systematic manner without performing numerous regression analyses with all possible combinations of explanatory terms.

3.2 Overall biases and scatters of deterministic shear strength models

Before developing probabilistic models, we first evaluate the overall biases and scatters of the eight models in Table 1 through comparison with the shear database. This is to judge the overall performance of the probabilistic models in this paper during their development. For this purpose, we

Table 2 Overall bias and scatter of base models

Model	Posterior means	
	θ (bias)	σ (scatter)
ACI 11-3	0.257	0.382
ACI 11-5	0.165	0.335
Eurocode Draft	0.456	0.223
Tureyen and Frosch	0.287	0.245
Zsutty	0.0261	0.244
Okamura and Higai	0.116	0.176
Bazant and Yu	0.0142	0.166
Russo *et al.*	0.00120	0.156

use the aforementioned Bayesian methodology with the following model employing a constant bias-correction term $\gamma(\mathbf{x}, \theta) = \theta$

$$\ln[C(\mathbf{x}, \Theta)] = \ln[c_d(\mathbf{x})] + \theta + \sigma\varepsilon \tag{7}$$

The natural logarithms are applied to the strength quantities for consistency with the probabilistic shear strength models developed later in this paper. Since the error terms, θ and $\sigma\varepsilon$ are independent of the input parameters in $\mathbf{x}$, the posterior means of θ and σ respectively estimate the overall bias and scatter of each deterministic model. Since ε is assumed to be the standard normal random variable, the error measure $\ln(C/c_d) = \ln(C) - \ln(c_d)$ is a normal random variable whose mean and standard deviation are the posterior means of θ and σ, respectively if the uncertainties in $\mathbf{\Theta}$ are neglected.

Table 2 lists the posterior means of θ and σ of the eight base models. Both ACI models are known to exhibit significant amount of bias and scatter, which is confirmed by the large values of the posterior means. Due to the embedded conservatism, the models by Eurocode draft and Tureyen and Frosch show significant biases on the conservative side. However, they can predict the strengths with smaller scatter than the ACI models. Zsutty's model shows small bias overall and its scatter is comparable to those by Eurocode. Tureyen and Frosch, Okamura and Higai, Bazant and Yu, and Russo *et al*. predict the shear strength with the smallest scatter among the considered models. In particular, the last two models show significantly reduced overall biases and scatters. This is due to their rigorous derivations based on fracture mechanics theories and calibration of parameters using extensive database of experimental observations.

3.3 Development of probabilistic shear strength models

As demonstrated in Fig. 2, the biases of deterministic shear strength models depend on given input parameter values. Therefore, we aim to develop probabilistic shear strength models with bias-correction term that is given as a function of input parameters. We use the Bayesian methodology employing the form in Eq. (3) with explanatory functions of input parameters selected based on the aforementioned shear transfer mechanisms and influencing parameters. The posterior mean of σ represents the error due to the effects that are not captured completely by the selected deterministic model or the explanatory functions. Therefore, we can confirm the effects of a selected bias-

Table 3 Posterior means of σ after Bayesian updating

Model	Posterior mean of σ		
	Constant bias (Table 2)	H_1	H_2
ACI 11-3	0.382	0.222	0.165
ACI 11-5	0.335	0.218	0.177
Eurocode Draft	0.223	0.172	0.165
Tureyen and Frosch	0.245	0.178	0.167
Zsutty	0.244	0.185	0.168
Okamura and Higai	0.176	0.159	0.157
Bazant and Yu	0.166	0.156	0.154
Russo *et al.*	0.156	0.146	0.146

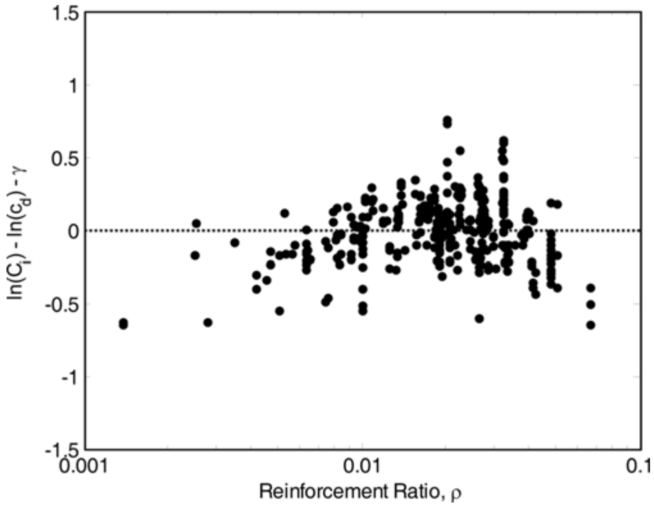


Fig. 4 Errors of ACI 11-3 shear strength model versus longitudinal reinforcement ratio after bias-correction by H_1

correction term by how much reduction can be made for the posterior mean of σ.

We try a set of explanatory functions, $H_1 = \{2, \rho, a/d, E_s/E_c, d_a/d, d/h, b_w/h\}$. The constant $h_1(\mathbf{x}) = 2$ is selected to detect the bias that is independent of the variables in $\mathbf{x}$. The explanatory functions $h_2(\mathbf{x}) = \rho$, $h_3(\mathbf{x}) = a/d$, $h_4(\mathbf{x}) = E_s/E_c$ and $h_5(\mathbf{x}) = d_a/d$ are selected to capture the biases related to the input parameters that mainly affect the shear transfer mechanisms. These explanatory functions or influencing parameters appear in the formulas shown in Table 1 as well. $h_6(\mathbf{x}) = d/h$ is selected to detect the effect of dowel action of the longitudinal bars that depends on the amount of concrete cover beneath the bars. $h_7(\mathbf{x}) = b_w/h$ is selected for the possible effect of the sectional aspect ratio. Note that these explanatory functions are made dimensionless for consistency with the dimensionless residual, $\ln[C(\mathbf{x}, \boldsymbol{\Theta})] - \ln[c_d(\mathbf{x})]$.

The posterior means of σ are listed in the third column of Table 3. For all the base models considered, the mean values are significantly reduced from the previous results by the form in Eq. (7) (shown in the second column). This indicates that significant portions of the errors that are

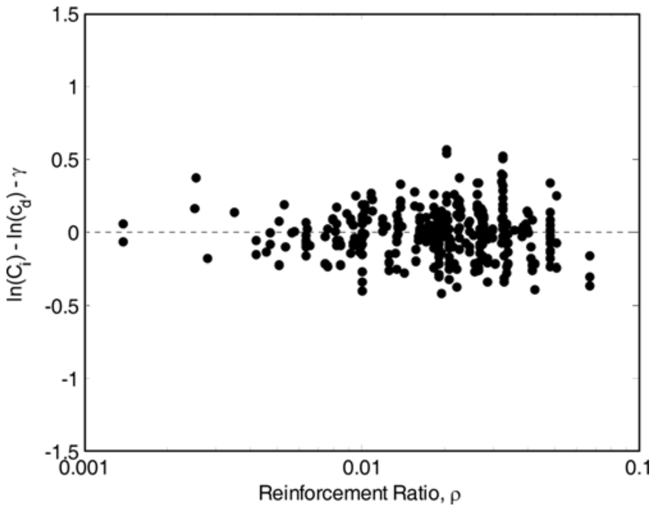


Fig. 5 Errors of ACI 11-3 shear strength model versus longitudinal reinforcement ratio after bias-correction by H_2

Table 4 Posterior means of θ_i after Bayesian updating

Model	Posterior means of corresponding θ_i					
	ln2	$\ln\rho$	$\ln\left(\frac{a}{d}\right)$	$\ln\left(\frac{E_s}{E_c}\right)$	$\ln\left(\frac{d_a}{d}\right)$	$\ln\left(\frac{d}{h}\right)$
ACI 11-3	2.53	0.395	–0.451	0.466	0.175	–1.68
ACI 11-5	3.09	0.336	–0.275	0.194	0.233	
Eurocode	0.95	0.122	–0.380	0.228		–2.38
Tureyen and Frosch	1.61		–0.414		0.184	–1.45
Zsutty	1.13	0.0638	–0.0954		0.193	–1.20
Okamura and Higai			–0.298	0.154		–1.35
Bazant and Yu	0.80		–0.242	0.118	–0.048	–1.17
Russo *et al.*	0.48	–0.0565		0.186		–1.162

previously identified as scatters are now captured as parameter-dependent biases. For example, Fig. 4 plots the differences between the natural logarithm of the observed shear strengths C_i and $\ln[c_d(\mathbf{x})] + \gamma(\mathbf{x}, \mathbf{M}_\theta)$, i.e., the strengths predicted by the probabilistic model developed based on the ACI equation 11-3. A comparison with Fig. 2 shows that the bias against the longitudinal reinforcement ratio is significantly reduced.

In order to improve the bias-correction, we try another set of explanatory functions, H_2 which is obtained by applying natural logarithm to each of the explanatory terms in H_1. This enables us to correct the deterministic model by multiplying the explanatory functions in H_1, i.e., $c_d(\mathbf{x})\cdot[2^{\theta_1}\cdot\rho^{\theta_2}\cdot(a/d)^{\theta_3}\ldots(b_w/h)^{\theta_7}]$. As shown in the fourth column of Table 3, the posterior means of σ are further reduced by this set of explanatory terms, which confirms that the biases are captured

Table 5 Means (divided by the base model $c_d(\mathbf{x})$) and coefficients of variation (c.o.v.) of predictions by the developed probabilistic strength models

Base model	Mean/$c_d(\mathbf{x})$	c.o.v.
ACI 11-3	$(5.85)\cdot\rho^{0.395}\cdot\left(\frac{a}{d}\right)^{-0.451}\cdot\left(\frac{E_s}{E_c}\right)^{0.466}\cdot\left(\frac{d_a}{d}\right)^{0.175}\cdot\left(\frac{d}{h}\right)^{-1.68}$	0.166
ACI 11-5	$(8.65)\cdot\rho^{0.336}\cdot\left(\frac{a}{d}\right)^{-0.275}\cdot\left(\frac{E_s}{E_c}\right)^{0.194}\cdot\left(\frac{d_a}{d}\right)^{0.233}$	0.178
Eurocode draft	$(1.96)\cdot\rho^{0.122}\cdot\left(\frac{a}{d}\right)^{-0.380}\cdot\left(\frac{E_s}{E_c}\right)^{0.228}\cdot\left(\frac{d}{h}\right)^{-2.38}$	0.166
Tureyen and Frosch	$(3.10)\cdot\left(\frac{d}{a}\right)^{-0.414}\cdot\left(\frac{d_a}{d}\right)^{0.184}\cdot\left(\frac{d}{h}\right)^{-1.45}$	0.168
Zsutty	$(2.22)\cdot\rho^{0.0638}\cdot\left(\frac{a}{d}\right)^{-0.0954}\cdot\left(\frac{d_a}{d}\right)^{0.193}\cdot\left(\frac{d}{h}\right)^{-1.20}$	0.169
Okamura and Higai	$(1.01)\cdot\left(\frac{a}{d}\right)^{-0.298}\cdot\left(\frac{E_s}{E_c}\right)^{0.154}\cdot\left(\frac{d}{h}\right)^{-1.35}$	0.158
Bazant and Yu	$(0.805)\cdot\left(\frac{a}{d}\right)^{-0.242}\cdot\left(\frac{E_s}{E_c}\right)^{0.118}\cdot\left(\frac{d_a}{d}\right)^{-0.0484}\cdot\left(\frac{d}{h}\right)^{-1.17}$	0.154
Russo *et al.*	$(0.478)\cdot\rho^{-0.0565}\cdot\left(\frac{E_s}{E_c}\right)^{0.186}\cdot\left(\frac{d}{h}\right)^{-1.16}$	0.146

more effectively. To demonstrate this improved bias-correction visually, the errors of the probabilistic strength model based on ACI 11-3 model are plotted in Fig. 5, which confirms that the bias against the reinforcement ratio has been further reduced. Table 4 shows the posterior means of θ_i for the H_2 explanatory terms that survived the stepwise removal process. The explanatory term $\ln(b_w/h)$ does not survive for any of the deterministic models considered, which implies the effects of the sectional aspect ratio are already being considered by the base models or its effect on shear strength is not significant according to the shear database.

Applying exponential functions to Eq. (3), the probabilistic shear strength model takes the form

$$\begin{aligned} C(\mathbf{x}) &= c_d(\mathbf{x})\cdot\exp\left[\sum_{i\in S}^{p}\mu_{\theta_i}h_i(\mathbf{x})\right]\cdot\exp(\mu_\sigma\varepsilon) \\ &= \tilde{c}_d(\mathbf{x})\cdot\exp(\mu_\sigma\varepsilon) \end{aligned} \quad (8)$$

where S denotes the set of indices of the explanatory terms that survive the stepwise removal process. If the uncertainties of the model parameters in $\mathbf{\Theta}$ are neglected, the normal random variable ε is the only random variable in the model. Hence, the shear strength follows the lognormal distribution while the mean and c.o.v. of the strength are derived as $\tilde{c}_d(\mathbf{x}) \cdot \exp(\mu_\sigma^2/2)$ and $[\exp(\mu_\sigma^2) - 1]^{1/2}$ respectively (Ang and Tang 2006). In the case $\mu_\sigma << 1$, the mean and c.o.v are closely approximated by $\tilde{c}_d(\mathbf{x})$ and μ_σ. Table 5 lists the means (divided by the base deterministic model $c_d(\mathbf{x})$ and c.o.v.'s of the probabilistic models developed by using the explanatory terms in H_2. The developed probabilistic models provide the mean values as unbiased predictions on the shear strengths while quantifying significantly reduced scatters by the c.o.v.'s.

For given input parameters **x**, the lognormal PDF of the shear strength prediction is given as

$$f_C(c|\mathbf{x}) = \frac{1}{\sqrt{2\pi}\mu_\sigma c}\exp\left[-\frac{1}{2}\left(\frac{\ln c - \ln c_d(\mathbf{x}) - \gamma(\mathbf{x}, \mathbf{M}_\theta)}{\mu_\sigma}\right)^2\right], \quad 0 < c \tag{9}$$

When the input parameters **x** have significant uncertainty due to measurement errors or intrinsic aleatory uncertainties, the PDF of the shear capacity should be obtained by total probability theorem, that is

$$f_C(c) = \int_{-\infty}^{\infty} f_C(c|\mathbf{x}) \cdot f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x}, \quad 0 < c \tag{10}$$

where $f_{\mathbf{X}}(\mathbf{x})$ denotes the joint PDF of **x**. Using this PDF of the shear strength, one can systematically incorporate the uncertainties of strength models and input parameters into risk-quantified designs or structural vulnerability/fragility estimations.

4. Bayesian methodologies for model calibration and construction

4.1 Calibration of deterministic models by adjusting model parameters

One can use the Bayesian parameter estimation method also for calibrating parameter values in existing deterministic models. Consider a form

$$\ln[C(\mathbf{x}, \Theta)] = \sum_{i=1}^{p} \theta_i h_i(\mathbf{x}) + \sigma\varepsilon \tag{11}$$

Note that this does not contain a deterministic model $c_d(\mathbf{x})$. Instead we select the parts constituting a deterministic model as explanatory terms so that the strength model in Eq. (11) will take the same expression as the deterministic model but with the model constants (such as 0.5 in ACI 11-3 model, $V_c = (1/6)(f_c')^{0.5} b_w s$) replaced by θ_i's. We perform only one Bayesian updating without the removal process so that the dimension of the shear strength is not changed by this calibration. The posterior mean of σ can be compared to those by the forms in Eqs. (3) and (7) to check the improvement by the calibration.

Table 6 lists the means and the c.o.v's of the calibrated models. Without any additional explanatory terms, this parameter calibration achieves significant bias-correction. This is confirmed by the c.o.v's that are smaller than the posterior means of σ shown in Table 2. However, the c.o.v.'s of these calibrated models are larger than those by the probabilistic models developed using additional explanatory terms, which are shown in Table 5. Comparable c.o.v.'s are obtained only for Zsutty, Okamura and Higai, Bazant and Yu and Russo *et al.* models. Note that these four models use the terms f_c', ρ, (a/d), b_w and d, which may be considered as critical terms that shear strength

Table 6 Means and coefficients of variation (c.o.v.) of strength models by model parameter calibration

Base model	Mean	c.o.v.
ACI 11-3	$(9.26) \cdot f_c'^{0.397} \cdot b_w^{0.809} \cdot d^{0.586}$	0.259
ACI 11-5	$(1.38 \times 10^{-7}) \cdot f_c'^{-28.0} \cdot b_w^{-28.0} \cdot d^{-28.9} + (55.7) \cdot \rho^{0.443} \cdot \left(\frac{V_u d}{M_u}\right)^{0.326} \cdot b_w^{1.06} \cdot d^{0.651}$	0.214
Eurocode draft	$(0.330) \cdot k^{2.19} \cdot \rho^{0.334} \cdot f_c'^{0.293} \cdot b_w^{0.984} \cdot d^{1.08}$	0.182
Tureyen and Frosch	$(3.84) \cdot f_c'^{0.471} \cdot b_w^{0.967} \cdot k^{0.869} \cdot d^{0.688}$	0.184
Zsutty	$(21.5) \cdot f_c'^{0.280} \cdot \rho^{0.376} \cdot \left(\frac{d}{a}\right)^{0.374} \cdot b_w^{0.985} \cdot d^{0.692}$	0.166
Okamura and Higai	$(11.4) \cdot \frac{\rho^{0.377}}{d^{0.581}} \cdot f_c'^{0.281} \cdot \left(0.75 + \frac{1.40}{a/d}\right)^{1.21} \cdot b_w^{0.993} \cdot d^{1.26}$	0.163
Bazant and Yu	$(1.13) \cdot \rho^{0.378} b_w^{0.945} \left(1 + \frac{d}{a}\right)^{2.15} \left(\frac{f_c' d_0 d}{1 + d_0/d}\right)^{0.501}$	0.156
Russo *et al.*	$(0.459) \cdot \xi^{0.888} \left[\rho^{0.352} (f_c')^{0.342} + 0.5 \rho^{0.664} f_y^{0.543} \left(\frac{a}{d}\right)^{1.07}\right] b_w^{0.976} d$	0.150

model should incorporate. Also note that the calibration of ACI 11-5 model leads to unusually small values for some of the terms. This is due to the limit of calibration of a deterministic model that has inaccurate form or that lacks important parameters or terms. Therefore, this model calibration should be considered only for improvement of deterministic models that contain critical parameter terms in a reasonable form.

4.2 Construction of probabilistic models using influencing parameters

One can construct a probabilistic model using the Bayesian method even when reliable deterministic models are not available (Clearly this is not the case for the shear strengths of reinforced concrete beams without shear reinforcement). In this case, a Bayesian parameter estimation with a form such as Eq. (11) is performed by employing parameters and explanatory terms selected based on the understanding of physics, test results and expert opinions. The Bayesian parameter estimation method helps identify influencing parameters and explanatory functions thereof through a step-wise removal process. The only restriction is that we select the explanatory functions such that the dimension of the bias correction function is the same as that of the quantity of interest. One possible way to assure this is to select a set of explanatory terms such that the sum of the terms has the same dimension as the term on the other side of Eq. (11), and add as many dimensionless explanatory functions as needed.

Although we have good deterministic models to be used as a base model for developing probabilistic shear strength models of RC beams (as demonstrated by eight base models above), we hereby try developing shear strength models following this approach to explore its feasibility in other problems. In order to make the bias-correction term have the same dimension as the shear strength of the beam, we first select three explanatory functions, $H_3 = \{\ln f_c', \ln b_w, \ln d\}$ so that the sum of these three explanatory functions have the same dimension as the natural logarithm of the shear strength. Now we can add many dimensionless explanatory functions as we wish. In this example, we use all the dimensionless explanatory functions in H_2. After Bayesian updating with successive deletion process for the dimensionless terms only, we derive the mean and the c.o.v. of the strength using the posterior means of the model parameters. The mean of the strength (in force unit) is derived as

$$\mu_{C(\mathbf{x})} = (1.01) f_c'^{0.688} \rho^{0.378} \left(\frac{d}{a}\right)^{0.381} \left(\frac{E_s}{E_c}\right)^{0.820} b_w^{0.991} d^{0.688} \tag{12}$$

and the c.o.v is 0.165. Approximating $b_w^{0.991}$ by b_w and dividing by $b_w d$, the strength is predicted in stress unit as

$$\mu_{C(\mathbf{x})/b_w d} = (1.01) f_c'^{0.688} \rho^{0.378} \left(\frac{d}{a}\right)^{0.381} \left(\frac{E_s}{E_c}\right)^{0.820} \left(\frac{1}{d}\right)^{0.312} \tag{13}$$

Note that the strength model obtained by this approach looks similar to Zsutty's formula, which is one of the deterministic models with the smallest overall bias according to the results in Table 2. The c.o.v. is comparable to those of the probabilistic models developed in this study.

As seen in Table 1, some deterministic models are made as the product of the sums. In order to construct a probabilistic shear strength model in such a form, we try a form

$$\ln[C(\mathbf{x}, \boldsymbol{\Theta})] = \sum_{i=1}^{p} \theta_i h_i(\mathbf{x}) + \ln\left[\prod_{i=p+1}^{p+q} h_i(\mathbf{x})^{\theta_i} + \prod_{i=p+q+1}^{p+q+r} h_i(\mathbf{x})^{\theta_i}\right] + \sigma\varepsilon \tag{14}$$

We first use explanatory functions in H_3 in the summation to guarantee that the expression on the right hand side has the same dimension as the natural logarithm of the shear capacity. Then, we use the dimensionless explanatory terms in H_1 for each of the two product terms. As a result, the mean of the strength is derived as (in force unit)

$$\mu_{C(\mathbf{x})} = (1.01) f_c'^{0.565} \cdot \left[\rho^{0.345} \cdot \left(\frac{d}{h}\right)^{-1.29} \cdot \left(\frac{E_s}{E_c}\right)^{0.455} + \rho^{0.381} \cdot \left(\frac{a}{d}\right)^{-3.79} \cdot \left(\frac{E_s}{E_c}\right)^{1.911}\right] \cdot b_w^{0.984} \cdot d^{0.734} \tag{15}$$

The c.o.v. of the strength is 0.153, which is comparable to that of the probabilistic models developed in this study. Approximating $b_w^{0.984}$ by b_w and dividing by $b_w d$ the strength is predicted in stress unit as

$$\mu_{C(\mathbf{x})/b_w d} = (1.01) f_c'^{0.565} \cdot \left[\rho^{0.345} \cdot \left(\frac{d}{h}\right)^{-1.29} \cdot \left(\frac{E_s}{E_c}\right)^{0.455} + \rho^{0.381} \cdot \left(\frac{a}{d}\right)^{-3.79} \cdot \left(\frac{E_s}{E_c}\right)^{1.911}\right] \cdot \left(\frac{1}{d}\right)^{0.266} \tag{16}$$

Encouraged by this result, Kim *et al.* (2007, 2008) used this model construction approach to develop shear strength/behavior models for RC beam-column connections under lateral earthquake load. The proposed approach successfully identified and modeled the contributions of various key parameters including concrete compressive strength without being limited by the specific descriptions of existing models.

5. Performance of probabilistic strength models

5.1 Comparison with experimental observations

In order to confirm the performances of the developed probabilistic models relative to experimental observations, we plot the observed strengths and the predictions by each deterministic model relative to the distribution by the probabilistic model developed based on the deterministic model (See Fig. 6). First, the test results in the database are rearranged in increasing order of the mean shear strengths $\mu_{C(\mathbf{x})}$ predicted by the probabilistic model. The mean curve is plotted together with a shaded area representing mean $\pm$ 1 standard deviation (SD) interval, which covers approximately 70% of the probability distribution of the strength. The observed strengths and the predictions by the corresponding deterministic model are shown by circles and x-marks, respectively.

The mean curves of all probabilistic models based on eight deterministic base models successfully represent the central tendencies of the observed shear strengths. This means that the developed probabilistic models achieve unbiased predictions successfully. It is also seen that the majority of

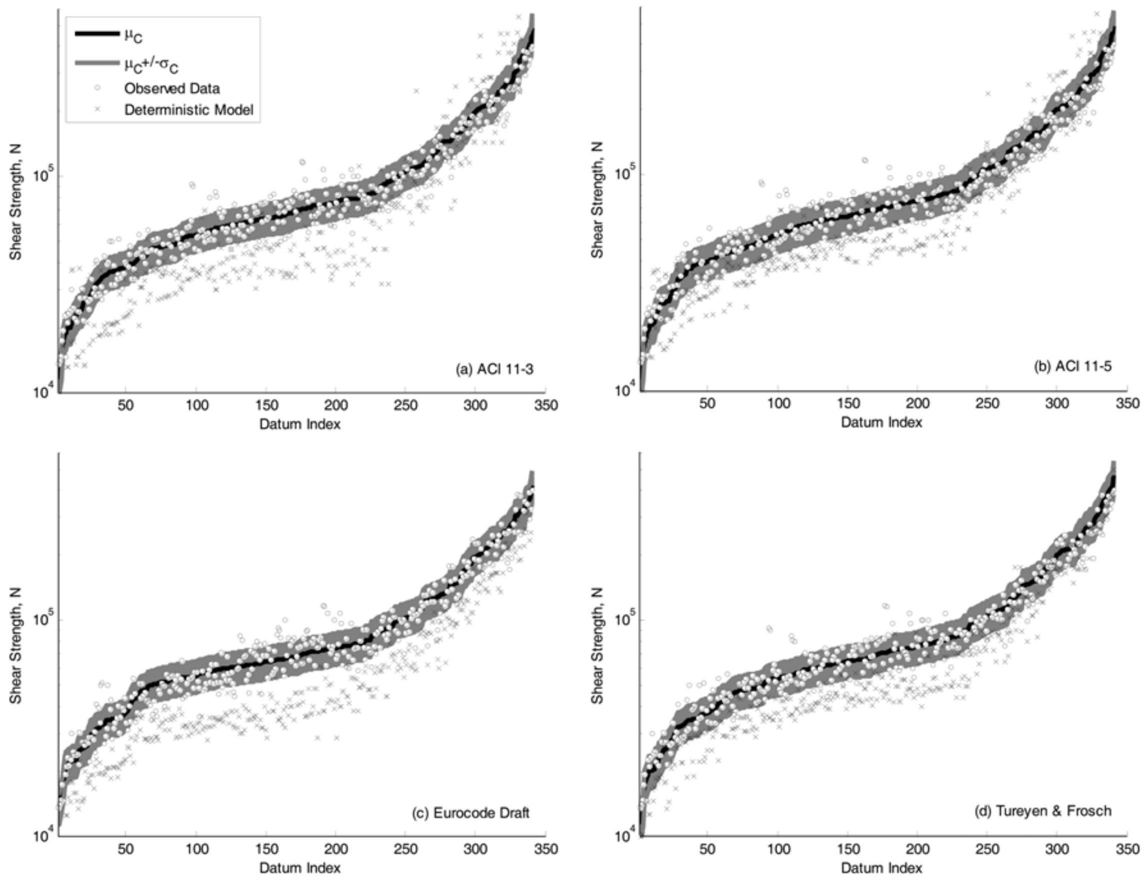


Fig. 6 Performance of probabilistic models developed based on (a) ACI 11-3, (b) ACI 11-5, (c) Eurocode, (d) Tureyen and Frosch, (e) Zsutty, (f) Okamura and Higai, (g) Bazant and Yu, and (h) Russo *et al.*

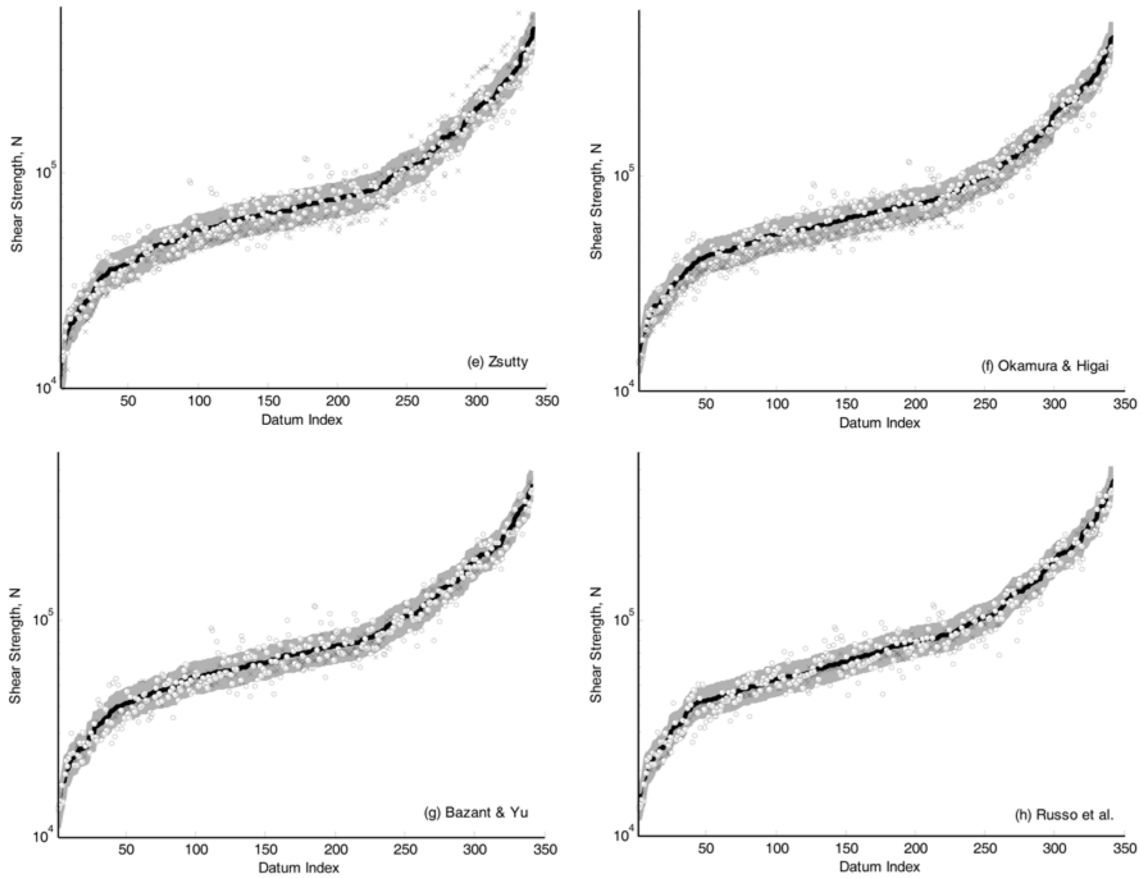


Fig. 6 Continued

the observed strengths fall within the mean ± 1SD intervals for all the models. By contrast, the deterministic models show significant scatter and/or biases. Most of the biases are observed on the conservative side especially in the ACI models, Eurocode draft and Tureyen and Frosch. The models by Bazant and Yu and Russo *et al.* show good performance in terms of both bias and scatter, which has been slightly further improved by developing the probabilistic models. Although not shown in this paper, the probabilistic models developed without using a deterministic model in Eqs. (12) and (15) matched the experimental observations successfully.

5.2 Uniform performance of the developed models

As seen in Fig. 3, the experimental database does not show uniform distribution in terms of key parameters. When models are developed based on a database with such non-uniform distribution, there is a risk that the models perform poorly in the ranges with fewer data points. An ideal solution is to partition the database and develop different models for predetermined ranges. For usefulness in practice, however, this paper develops probabilistic models that cover the entire range of the database instead of providing a set of range-specific models. In order to confirm the uniform performance of the developed models, the errors of the developed models were checked for selected

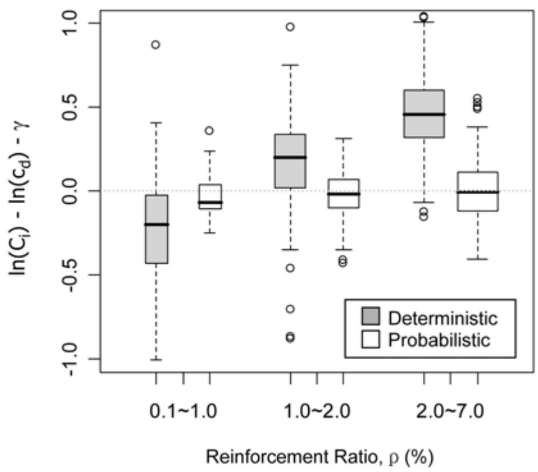
Table 7 Ranges of parameters selected for box plots of errors

Parameter	Selected ranges
f'_c	10~30 (low strength), 30~70 (medium), 70~120 MPa (high)
d	100~300 (small), 300~900 (ordinary), 900~2000 mm (large)
ρ	0.1~1.0 (lightly reinforced), 1.0~2.0 (moderately reinforced), 2.0~7.0 % (heavily reinforced)
a/d	2.4~3.0 (short shear span length), 3.0~4.0 (medium), 4.0~8.0 (slender)

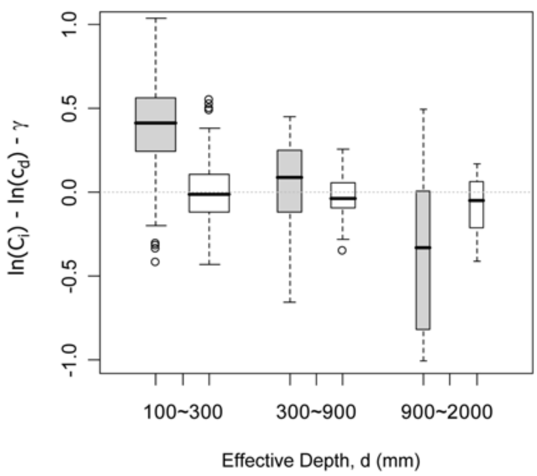
ranges of key parameters by use of boxplots (Navidi 2007). Table 7 shows the ranges of key parameters selected for the boxplots.

In a given boxplot, the lower and upper edges of the rectangle correspond to the first and third quartile values (25- and 75-percentiles) respectively while the horizontal line inside indicates the median value (50-percentile). Therefore, the location and vertical length of the rectangle in a box plot respectively visualize the bias and scatter of the prediction of a model in the given range of a parameter. The dotted lines ("whiskers") are extended from the top and bottom of the box to the maximum and minimum data points within a distance of 1.5×(the vertical length of the rectangle). All the extreme data points outside these whiskers, "outliers" are indicated by hollow circles. The horizontal length of the rectangle is given proportional to the number of data in the given range.

For example, Figs. 7 and 8 present the boxplots of the errors by ACI 11-3 and Tureyen and Frosch models and their corresponding probabilistic models developed in this paper. Fig. 7(a) shows that the ACI 11-3 model tends to underestimate the shear strengths for heavily reinforced members and to overestimate for lightly reinforced members. Fig. 7(b) shows that ACI 11-3 model fails to account for the size effect as significant biases are observed depending on the given effective depth. The probabilistic model developed based on ACI 11-3 ("ACI 11-3" in Table 5) corrects these biases uniformly in all the considered ranges.

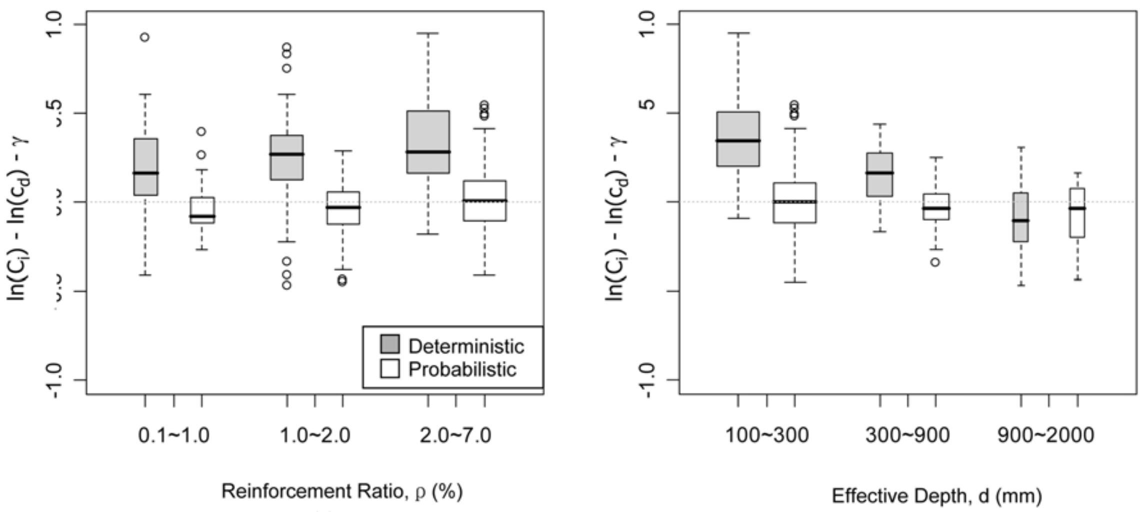


(a)



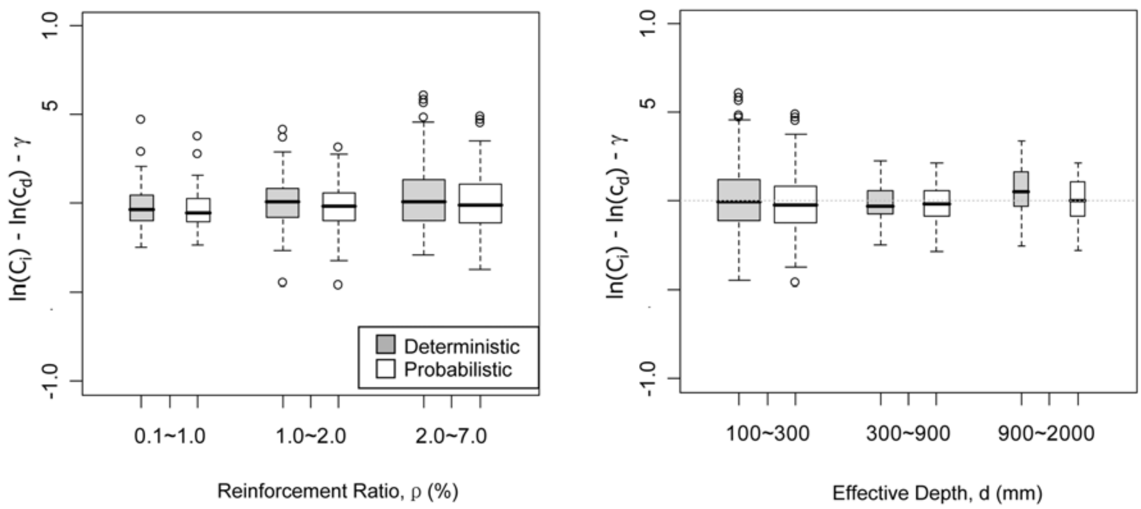
(b)

Fig. 7 Box plots of the errors of the ACI 11-3 model and its corresponding probabilistic model for selected ranges of (a) reinforcement ratio, and (b) effective depth



(a) (b)

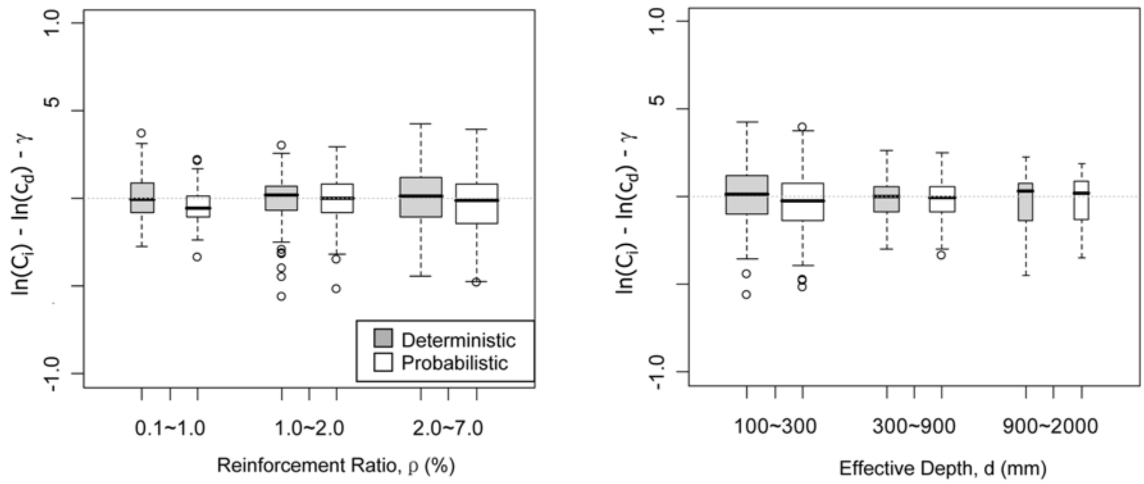
Fig. 8 Box plots of the errors of Tureyen and Frosch model and its corresponding probabilistic model for selected ranges of (a) reinforcement ratio, and (b) effective depth



(a) (b)

Fig. 9 Box plots of the errors of Bazant and Yu model and its corresponding probabilistic model for selected ranges of (a) reinforcement ratio; and (b) effective depth

As seen in Fig. 8, Tureyen and Frosch model has biases with similar trends. However, the magnitudes of the biases are smaller than those for ACI 11-3. This is because Tureyen and Frosch model indirectly considers the effect of longitudinal reinforcement ratio through the neutral depth and accounts for the shear strength contribution of larger uncracked concrete section for members with larger depth. As shown by the boxplots centered near zero, these biases have been successfully corrected by the developed probabilistic model ("Tureyen and Frosch" in Table 5) over the entire



(a) (b)

Fig. 10 Box plots of the errors of Russo *et al.* model and its corresponding probabilistic model for selected ranges of (a) reinforcement ratio, and (b) effective depth

range of key parameters. The boxplots of all the eight probabilistic models in Table 5 have been created for the selected ranges of the key parameters in Table 7. Although not shown due to the limited space of the paper, these boxplots confirmed that the developed probabilistic models perform uniformly well over the entire ranges of the selected key parameters despite the non-uniform distributions of parameters in the database.

As shown by the results in Table 2, the models by Bazant and Yu, and Russo *et al.* have good overall accuracy relative to the experimental database used in this paper. In order to check their uniform performance in different ranges of parameters, the boxplots of the errors by the original models and their corresponding probabilistic models developed in this paper have been created (Figs. 9 and 10). The boxplots confirm that these models provide accurate predictions uniformly over the entire ranges as well even in their original forms. No significant improvement either in bias correction or scatter reduction has been made by the probabilistic models developed in this paper. However, it is also noteworthy that the proposed method upgraded the accuracy of the other relatively simpler models to the level of these two sophisticated models without losing their simplicity and convenience in practical applications.

6. Conclusions

Probabilistic shear strength models are developed for reinforced concrete beams without shear reinforcement using a Bayesian methodology based on deterministic shear strength models and an extensive database of observed shear strengths. The overall biases and the scatters of eight shear strength models are first identified independently of input parameters so that the performance of newly developed models in this paper is assessed appropriately. Then, probabilistic models are developed by identifying the biases of the deterministic models in terms of explanatory functions of input parameters and by quantifying the errors remaining after the bias-correction. The Bayesian methodology allows researchers to try any explanatory functions they consider important, and then to identify informative terms based on experimental observations. It is observed that nonlinear transformation of explanatory terms may improve the bias-correction. Using the mean and coefficient of variation of the shear strengths provided by the developed probabilistic models, one can predict the shear strengths with reduced bias with model errors quantified. The probabilistic distributions of the strengths are derived to help incorporate the model uncertainties into risk-quantified designs and vulnerability estimations. A general procedure is proposed for calibrating model parameters in existing models without using additional explanatory terms. This calibration helps improve the accuracy of the deterministic models that employ critical terms in appropriate forms. The calibration of deterministic shear strength models revealed critical terms of shear strength models: longitudinal reinforcement ratio, concrete compressive strength, shear span-to-depth ratio, web width and effective depth. Also proposed is a method that would enable construction of probabilistic models in case reliable models do not exist or when influencing input parameters are not clearly identified. The improved performance of the developed probabilistic models and its uniform performance over different ranges of key parameters are confirmed successfully by comparison between predicted and observed strengths and by boxplots of model errors in selected ranges of key parameters.

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