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Technical Note

Measuring material length parameter with a new solution of microbend beam in couple stress elasto-plasticity

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1. Introduction

It is proved that the size effects exist for the material on microscale. Classic elasto-plasticity cannot explain the size effects because there is no length scale in the constitutive equations. Recently, couple stress/strain gradient theories (Fleck *et al.* 1994, Wang *et al.* 2003), including the material length parameter ℓ entering the constitutive equations, have been developed and successfully applied to predict the size effects.

For its simplicity, the beam model on microscale can be chose to measure the material length parameter ℓ. For example, Papargyri-Beskou *et al.* (2003) applied strain gradient theory into dynamic response of the beam on microscale; Pradhan and Sarkar (2009) used nonlocal theories to predict the size effects in the tapered fgm beams. Both of them adopted the elastic beam models, however, it is easy to enter plastic state for the microscale beam.

Stolken and Evans (1998), Haque and Saif (2003) successively designed the plastic microbend test where the material length parameter ℓ can be determined according to the relationship between the moment M and the surface strain ε_b. This test has been widely used to measure ℓ. Based on couple stress/strain gradient plasticity, the analytical solutions (Stolken and Evans 1998, Wang *et al.* 2003) have been developed successively under small deformation assumption. However, all of the solutions include an assumption that the beam is in the rigid-plastic state, which may cause some errors in analysis.

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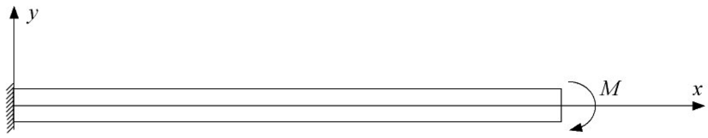

Fig. 1 Coordinate system (x, y) on a pure bending beam

2. Pure bending solutions in couple stress elasto-plasticity

Compared with strain gradient theories, couple stress is simpler and therefore is chosen to deduce the solution in this paper. The microscale beam with the thickness h and the width b is studied, whose Cartesian coordinate system (x, y) is shown in Fig. 1.

For a pure bending thin beam under plane strain deformation, the displacement fields can be given by

$$u = \kappa xy; \quad v = -\frac{\kappa}{2}\left(x^2 + \frac{\nu}{1-\nu}y^2\right) \tag{1}$$

and therefore the nonvanished components of strain tensor and curvature tensor are (Wang *et al.* 2003)

$$\varepsilon_x = \kappa y; \quad \varepsilon_y = -\frac{\nu}{1-\nu}\kappa y \tag{2}$$

$$\chi_{zx} = -\kappa \tag{3}$$

where, κ is the constant curvature. For simplicity, we adopt $\kappa > 0$ in this paper.

2.1 Solution in the elastic region

Substituting Eqs. (2) and (3) into the elastic constitutive equations, the nonvanished stresses σ_{ij} and couple stresses m_{ij} are be calculated as

$$\sigma_x = \frac{E}{1-\nu^2}\kappa y; \quad m_{zx} = -2\ell^2 G\kappa \tag{4}$$

and both of them contribute to the moment M at the across section (Chen and Wang 2001), i.e.

$$M = 2b\int_0^{h/2}(\sigma_x y - m_{zx})dy \tag{5}$$

Inserting Eqs. (4) into (5), the moment can be obtained

$$M = \frac{Ebh[h^2 + 12(1-\nu)\ell^2]}{12(1-\nu^2)}\kappa \tag{6}$$

On the elasto-plastic interface $y = y_{ep}$, the Tresca yield condition must be satisfied as

$$\Sigma\big|_{y=y_{ep}} = \left.\sqrt{(s_x - s_y)^2 + 4s_{xy}^2 + \frac{\chi_{zx}^2 + \chi_{zy}^2}{\ell^2}}\right|_{y=y_{eo}} = \Sigma_0 \tag{7}$$

where, s_{ij} is the deviatoric part of the stress tensor; Σ is the generalized effective stress. According to Eqs. (4) and (7), the vertical coordinate of the elasto-plastic interface can be determined

$$y_{ep} = \frac{1-\nu}{E\kappa}\sqrt{(1+\nu)^2\Sigma_0^2 - E^2\kappa^2\ell^2} \tag{8}$$

2.2 Solution in the plastic region

For the deformation theory, the following equations must be satisfied

$$\sigma_{kk} = 3K\varepsilon_{kk} \tag{9}$$

$$\frac{s_x}{e_x} = \frac{s_y}{e_y} = \frac{m_{zx}}{\ell^2\chi_{zx}} \tag{10}$$

where e_{ij} is the deviatoric part of the strain tensor. Substituting $\sigma_y = 0$ and $\varepsilon_z = 0$ into Eqs. (9) and (10) results in

$$\varepsilon_y = \frac{\sigma_x - \sqrt{\sigma_x^2 - 3K\sigma_x\varepsilon_x + 9K^2\varepsilon_x^2}}{3K} \tag{11}$$

For a power-law hardening material $\Sigma = \Sigma_0(\xi/\xi_0)^m$ where m is the plastic work hardening exponent and $\xi = \sqrt{(e_x-e_y)^2 + 4e_{xy}^2 + \ell^2(\chi_{zx}^2+\chi_{zy}^2)}$ is the generalized effective strain, the following equations can be eventually obtained

$$\varepsilon_y = \frac{\sigma_x - \sqrt{\sigma_x^2 - 3K\sigma_x\varepsilon_x + 9K^2\varepsilon_x^2}}{3K}; \quad \sigma_z = 3K\varepsilon_x - \sqrt{\sigma_x^2 - 3K\sigma_x\varepsilon_x + 9K^2\varepsilon_x^2} \tag{12-1}$$

$$m_{zx} = -\ell^2\kappa\frac{\sigma_x}{\varepsilon_x - \varepsilon_y}; \quad f = \Sigma - \Sigma_0\left(\frac{\xi}{\xi_0}\right)^m = 0 \tag{12-2}$$

According to Eqs. (12), it can be known that the function $f(\sigma_x)$ changes monotonously with the increase of σ_x, which means only one solution exists in $f(\sigma_x) = 0$. After solving out σ_x and m_{zx}, the moment can be obtained eventually as follows

$$M = \frac{2Eby_{ep}[y_{ep}^2 + 3(1-\nu)\ell^2]}{3(1-\nu^2)}\kappa + 2b\int_{y_{ep}}^{h/2}(\sigma_x y - m_{zx})dy \tag{13}$$

3. Measuring material length parameter

Firstly, a pure bending beam is investigated to predict the size effects with the solution in this paper. A power-law hardening material is considered here, with the elastic modulus E = 220 GPa, Poisson's ratio ν = 0.31 and the initial yield stress Σ_0 = 97 MPa.

Fig. 2 shows the changes of the normalized moment $M/\Sigma_0 bh^2$ with the strain ε_b at the upper surface for generalized Tresca yield criterion. In case of $h > 10\ell$, the corresponding curves in Fig. 2 will be very close to those for classical elasto-plasticity. When the thickness h is comparative with ℓ, the size effects become significant and the material turns stiffer as ℓ increases.

Moreover, Fig. 3 compares couple stress theory with classical theory for a cantilever beam. The dashed line means that the plasticity appears only in the region $x \leq 0.325L$ where L is the beam

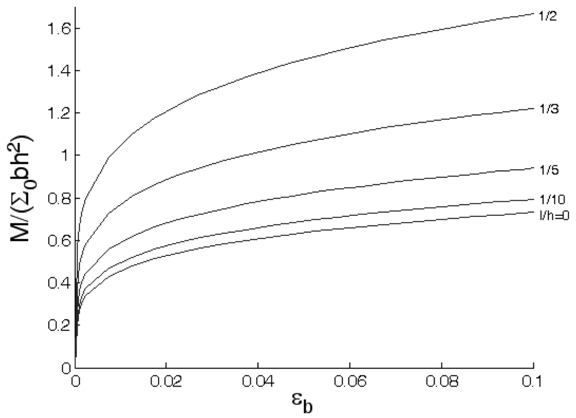

Fig. 2 The normalized moment $M/\Sigma_0 bh^2$ versus the surface strain $\varepsilon_b = \kappa h/2$ with $m = 0.2$

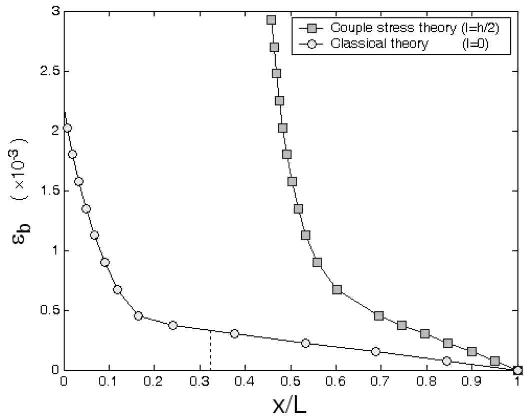

Fig. 3 The surface strain ε_b along the axial direction x for a cantilever beam. The beam thickness is $h = 1$ μm, the force applied at the free end is $F = 0.6$N, and the plastic work hardening exponent is $m = 0.1$

length. In this case, large deformation may be avoided. The size effects are also significant when the material length parameter is $\ell = h/2$.

Together with the solution in this paper, Fig. 2 and Fig. 3, respectively, can be applied to determine the material length parameter in couple stress theory for the pure bending beam and the cantilever beam.

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