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Technical Note

Analytical method for main cable configuration of two-span self-anchored suspension bridges

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1. Introduction

Self-anchored suspension bridge provides an appealing architectural presence, and seems to become a symbol of the cities. Similar to conventional suspension bridges, self-anchored suspension bridges perform highly geometrically nonlinear behavior before attaining their equilibrium configuration during their construction.

One of the fundamental targets that engineers concern about in designing of a self-anchored suspension bridge is to determine the configuration of main cables. Analytical method has been utilized to search for the configuration of main cables of traditional earth-anchored suspension bridges. However, as for the self-anchored suspension bridges, analytical method on simulating the construction process is more complicated for the reason that the anchored points of the main cables at the ends of the stiffening girder will move under the action of horizontal component of cable force.

At present, most of the methods, which are used to analyze suspension bridges, are the discretization methods based on finite element theories or balance iteration method. Kim and Lee proposed 3-D elastic catenary element, which also requires balance iteration procedure. Tan and Zhang (2006) derived an analytical method for main cable configuration of two-tower, three-span symmetrical self-anchored suspension bridges. Further investigating this method, if the bridge is a two-span, single-tower unsymmetrical structure, the equations of the main span cable is different and more complicated. This paper proposes two displacement compatible equations by considering of the compressed deformation of the main girder and the tower, and then describes an analytical method for cable configuration of self-anchored suspension bridges.

The proposed method does not need discretization as the F. E. method. Fewer design parameters, such as spans, sags, properties of cables, characters of stiffening girders and towers are needed in calculation. And then the results, such as unstressed cable length, saddle and anchor displacement, can be calculated rapidly. This method is highly efficient, so that it takes only a few minutes from data preparation to solving out of all the results.

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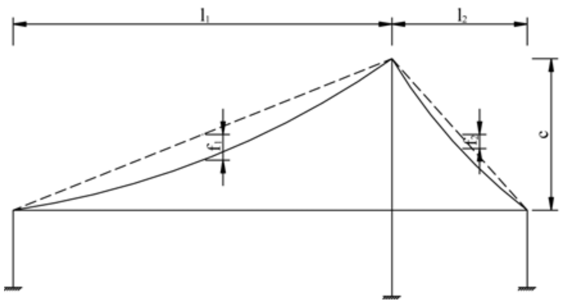

Fig. 1 Final state

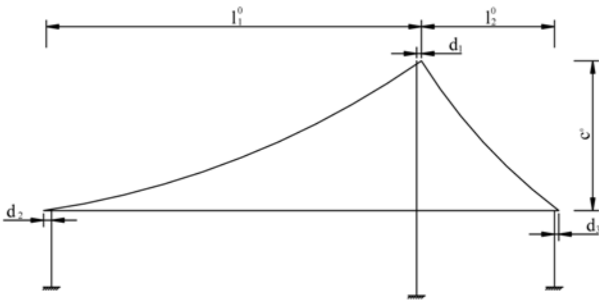

Fig. 2 Initial state

2. Analytical method for main cable configuration

2.1 Equations of the main cable

The equations of the main cables are based on the following assumptions: (1) The dead load is uniform and is carried by the cable alone. Therefore, under dead load alone the cable assumes a parabolic shape. (2) The cables are assumed to be perfectly flexible and not to undergo pressure and bending. (3) Hooke's law is applicable to the cable material. (4) The cross section of stiffening girder is isometric. (5) The hangers are continuously distributed.

The configuration of the main cable depends on the dead load that the main cable undergoes. In the final state, the equations of the main cables can be written as

$$z_i = \frac{4f_i x(l_i - x)}{l^2} + \frac{c}{l_i}x \qquad (i = 1, 2) \tag{1}$$

where l_1 and l_2 are the lengths of main span and side span respectively, f_1 and f_2 the sags of main span cable and side span cable respectively, c the height of the tower. Suppose that the horizontal component of the main span cable force equals that of the side span, we have $f_2 = f_1 \cdot l_2^2/l_1^2$. In the following paper, $i = 1, 2$ indicates the main span and the side span conditions respectively except for special statements.

In the initial state, the equations of main cables can be expressed as

$$\begin{gathered} z_i = \frac{H^0}{q}\left[\cosh\alpha_i - \cosh\left(\frac{2\beta_i x}{l_i^0} - \alpha_i\right)\right], \quad \beta_i = \frac{q l_i^0}{2H^0} \\ \alpha_i = \beta_i + \sinh^{-1}\frac{\beta_i(c^0/l_i^0)}{\sinh\beta_i} \qquad (i_i = 1, 2) \end{gathered} \tag{2}$$

In these equations, H^0 is the horizontal component of the cable force produced by self-weight, q the weight per unit length of the cables.

2.2 The length of main cable

The length of main cable can be obtained by the arc length integral formula. The expression for the cable length in the final state and under self-weight can be obtained respectively as

$$s_i = \frac{l_i^2}{16f_i}\left[\frac{4f_i-c}{l_i}\sqrt{1+\left(\frac{4f_i-c}{l_i}\right)^2}+\frac{4f_i+c}{l_i}\sqrt{1+\left(\frac{4f_i+c}{l_i}\right)^2}\right]$$

$$+\frac{l_i^2}{16f_i}\left[\ln\left|\frac{4f_i-c}{l_i}+\sqrt{1+\left(\frac{4f_i-c}{l_i}\right)^2}\right|-\ln\left|-\frac{4f_i+c}{l_i}+\sqrt{1+\left(\frac{4f_i+c}{l_i}\right)^2}\right|\right] \quad (i=1,2) \tag{3}$$

$$s_i^0 = \frac{H^0}{q}\left[\sinh\left(\frac{ql_i^0}{H^0}-\alpha_i\right)+\sinh\alpha_i\right] \quad (i=1,2) \tag{4}$$

2.3 The deformation compatible equations of main cable and the solution

Calculating the elastic deformation of the main cables that occurs during their erection, we have

$$\Delta s_i = \frac{H}{EA}\left(\frac{c^2}{l_i}+l_i+\frac{16f_i^2}{3l_i}\right)-\frac{n_gH_0}{4EAq_0}\left[2q_0l_i^0+H^0\sinh\left(\frac{2ql_i^0}{H^0}-2\alpha_i\right)+H^0\sinh2\alpha_i\right] \quad (i=1,2) \tag{5}$$

$$H = \frac{wl^2}{8f} \tag{6}$$

Where w is the total dead load per unit length along spans which supported by the main cables in the final state, H the horizontal component of the cable force produced by total dead loads; Δs_1 and Δs_2 are the elastic deformation of the main span cable and the side span cable respectively. E represents the elastic modular and A represents the cross sectional area of the main cables. The n_g equals 0 or 1 represents the unstressed cable length or the cable length under self-weighted state.

Since the deformation of the main cable can be calculated by the cable length of final state minus that of self-weighted state, the deformation compatible equation can be given as

$$\Delta s_i = s_i - s_i^0 \quad (i=1,2) \tag{7}$$

The saddle displacement and the displacement of the anchored points for the main girder are denoted by d_1, d_2 and d_3 respectively. Relationships for them are as the follows

$$l_1^0 = l_1+d_1+d_2, \quad l_2^0 = l_2-d_1+d_3 \tag{8}$$

$$d_2 = \frac{H\cdot l_1}{E_BA_B}, \quad d_3 = \frac{H\cdot l_2}{E_BA_B} \tag{9}$$

$$c^0 = c+\Delta c \tag{10}$$

Where, A_B represents the cross sectional area and E_B represents the elastic modular of the girder respectively. c^0 and c are the initial height and the design height of the tower respectively, Δc the deformation of the tower. Eqs. (7) to (10) are called the deformation compatible equations.

When solving these equations, at first, get the horizontal component of the cable force by Eq. (6). Then calculate the vertical displacement of the tower and horizontal displacement of the anchorages by Eqs. (10), and (9). The horizontal component of cable force H^0, and d_1, the pre-displacement of saddles are independent variables. H^0 and d_1 can be obtained by Eq. (7) with putting other equations into them. Solve the initial cable length of the main span s_1^0 and side span s_2^0. Finally, Eqs. (1) and (2) show the main cable configuration of final state and self-weighted state respectively. When H^0 in Eq. (5) is 0, the unstressed length of the main cable can be obtained.

3. Example

The LinFen Yingbin Bridge is a single-tower, two-span, asymmetrical concrete self-anchored suspension bridge with its main span of 70 m and side span of 25 m. Table 1 shows the parameters of this bridge. Table 2 shows the contrast of unstressed length obtained by the analytical method and finite element method.

The results of these two methods are closed, but the analytical method is much simpler and efficient.

In the analytical method, no F. E. model is needed. With predefining the essential parameters, such as spans, sags, properties of cables, characters of stiffening girders and towers, the cable configuration equations could be established, as well as the deformation compatible equations. The initial and final configuration of the cable, the unstrained cable length, and saddle displacement could be obtained rapidly by solving these equations. This method is highly efficient, so that it takes only a few minutes from data preparation to solving out of all the results.

Table 1 Structural parameters of the Yingbin Bridge

	A (m^2)	I (m^4)	E (N/m^2)	w (kN/m)
Main cable	0. 092278 × 2	0	1.9×10^{11}	9.346 × 2
Stiffening girder	17.75	12.60	3.5×10^{11}	536.99
Hangers	0.002809 × 2	0	1.9×10^{11}	0.776
Tower	6.3 × 2	4.725 × 2	3.5×10^{10}	157.5 × 2

Table 2 Comparison between the analytical method and F. E. M.

	S^0 (m)	S_1^0 (m)	Displacement of saddles (m)
Finite element method	67.124	31.585	0.067
Analytical method	67.189	31.584	0.070
Errors	0.065	–0.001	0.003

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