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Stochastic value index for seismic risk management of existing lifelines

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Abstract. This study proposes a certain measure or investment strategy for decision making associated with seismic retrofitting. This strategy reduces the risk of a large-scale malfunction such as water supply loss under seismic risks. The authors developed a stochastic value index that will be used in the overall evaluation of social benefit, income gain, life cycle costs and failure compensation associated with existing lifeline systems damaged by an earthquake during the remaining service period. Optimal seismic disaster prevention investment of deteriorated lifeline systems is discussed. Finally, the present study provides a performance-based design method for seismic retrofitting strategies of existing lifelines which are carried out using the target probabilities of value loss and structural failure.

Keywords: stochastic value index; seismic risk management; seismic investment; existing lifeline.

1. Introduction

Decision on seismic investment concerns on two parties: the designers and the financiers. And the most appropriate decision must be acceptable to both.

When a new construction or a retrofitting project is planned, the target performance is determined so that it is acceptable to all parties involved like stakeholders, project operation firms, design and construction contractors. In this case there are two target performances that can be identified. The first target performance (A) would come from stakeholders and the project operating firms and will naturally be in the monetary terms. The other target performance (B) would come from engineers of the project operating firms and the contractors which is expressed in terms of probability of failure or its equivalent measures of safety index, partial safety factor or factor of safety.

The target performance (A) and (B) must be expressed in mutually related measures which are defined and functionally formulated to include both monetary investment and safety measure variables.

Originally, performance-based design method was developed to provide such a measure and many

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decision measures have been proposed to satisfy the above conditions. Some of these measures are used in the financial markets. However these are insufficient to be applied to practical engineering problems because of very simplified structural modeling.

Current approaches (Wen 2001, Timashev 2001, Val 2004) such as life-cycle cost minimum criterion, cost benefit ratio criterion and net present value methods are developed to evaluate the cost optimality. Generally speaking, however, life cycle cost method and cost benefit ratio criterion provide a minimum cost estimate, but does not suggest whether this cost is really optimal for project feasibility. However it is necessary to provide a good balance between the cost estimate, the income gain and social benefit of a project during its service period to ascertain the feasibility of an infrastructure project. The net present value (NPV) method is widely used in assessing the financial viability of public or private projects. However for public projects such as a water supply network, the assessment must include the NPV of social benefit which is obtained through people's satisfaction for newly created public services.

Hoshiya (2005) proposed a stochastic net present value method, wherein he discussed the probability that the net value will be negative for various discount rates. However his approach cannot relate with the probability of structural failure. The NPV considering the project value was proposed by Porter *et al.* (2004) who discussed the uncertain net asset value of an investment opportunity considering market risk and seismic risk for private buildings. Their approach can be used for private buildings but cannot be applied directly to lifeline network systems.

In this case, a proposed stochastic value index is adopted for the overall evaluation of social benefit, income gain, life cycle costs and failure compensation associated with the existing lifeline systems damaged by an earthquake during the remaining service period.

Discussions are given for the optimal seismic disaster prevention investment for an existing deteriorated lifeline system which is always threatened by seismic hazards (JRA 1996) of the ground motion (EQ1) caused by the maximum operating earthquake (MOE) as well as the ground motion (EQ2) produced by the maximum credible earthquake (MCE), respectively. The optimal seismic investment or the minimum requirement of the value index is formulated in a probabilistic manner by introducing the probability distribution of value index, while various damage costs caused by seismic disaster are evaluated on the seismic risk analysis unique to the existing pipeline network system.

Finally, the present study can furnish a performance-based design method for seismic retrofit strategies of deteriorated lifelines which are carried out using the target probabilities of value loss and structural failure.

2. Investment decision making for existing lifeline retrofitting

2.1 Stochastic value index

A final balance of cost and income over the life cycle period should be a key factor in the evaluation of the feasibility of a project. This balance, which we call a value index, V_0, is a resulting value that can be produced by the project operation, and can be expressed in terms of

$$V_0 = B + I - E - (C_0 + C_M) \tag{1}$$

in which B, I, E, C_0 and C_M are the accumulated total amounts of social benefit, income gain, operational expense, initial cost and maintenance cost, respectively, during its service life.

It should be noted that the infrastructure such as a water supply network system provides daily services to consumers using the structural system established by the large-scale investments. While the private investment can provide any services to particular customers, it cannot supply any services to all the people. So the social benefit to be given to all the people must be included in the value estimation for any infrastructure projects such as water supply lifeline services.

When a seismic investment, C_S, is taken into consideration, this value index, V_1, can be represented by

$$V_1 = B + I - E - (C_0 + C_M + C_S) = V_0 - C_S \tag{2}$$

It should be noted that the seismic vulnerability of the lifeline system causes not only structural damages to the network links but also functional damages to the demand nodes in terms of serviceability loss. This loss in turn results in the loss of social benefit, ΔB and income gain, ΔI, respectively. The value index V_2 in this situation is given by

$$\begin{aligned} V_2 &= B + I - E - (C_0 + C_M + C_S) - (C_R + \Delta B + \Delta I) \cdot 1_{EQ}(t_{EQ}) \\ &= V_0 - \{C_S + C_{EQ} \cdot 1_{EQ}(t_{EQ})\} \end{aligned} \tag{3}$$

where C_R is the restoration cost and C_{EQ} is the seismic damage cost which is defined as

$$C_{EQ} = C_R + \Delta B + \Delta I \tag{4}$$

and

$$1_{EQ}(t_{EQ}) = \begin{cases} 1: \text{ an earthquake occurs over some time duration } t_{EQ} \\ 0: \text{ an earthquake does not occur over some time duration } t_{EQ} \end{cases} \tag{5a}$$

with

$$E[1_{EQ}(t_{EQ})] = P[EQ] \tag{5b}$$

Let us take a probability of value loss which is calculated as the sum of seismic event and non-seismic event as follows

$$P[V_2 < 0] = P[V_o < (C_S + C_{EQ})|EQ]P[EQ] + P[V_o < C_S|\overline{EQ}]P[\overline{EQ}] \tag{6}$$

Solving Eq. (6), the mean value of value index, V_0, can be given by

$$\mu_{V_o} = C_S + \mu_{C_{EQ}} - \sqrt{\sigma_{V_o}^2 + \sigma_{C_{EQ}}^2} \cdot F_{V_o|EQ}^{-1}\left\{\frac{P[V_2 < 0] - P[V_o < C_S|\overline{EQ}]}{P[EQ]} + P[V_o < C_S|\overline{EQ}]\right\} \tag{7}$$

where $F_{V_0|EQ}(v)$ is a probability distribution function of V_0 conditional on an EQ, while $\mu_{C_{EQ}}$ and $\sigma_{C_{EQ}}$ are the mean value and standard deviation of C_{EQ}, and σ_{V_0} is the standard deviation of V_0, while $P[EQ]$ is a probability of earthquake occurrence. This equation means that, when a seismic investment C_S is given, the minimum requirement of μ_{V_0} can be evaluated by Eq. (7), in which the statistic parameters on C_{EQ} and the probability of value losses are significantly important to obtain its refined estimation.

Once the target value for V_2 is given in terms of $p_{V_2}^{\text{Target}}$, on the other hand, the optimal C_S is also given by the following equation

$$C_S^{Optimal} = \mu_{V_o} - \mu_{C_{EQ}} + \sqrt{\sigma_{V_o}^2 + \sigma_{C_{EQ}}^2} \cdot F_{V_o|EQ}^{-1}\left\{\frac{p_{V_2}^{\text{Target}} - p_{V_1}}{P[EQ]} + p_{V_1}\right\} \tag{8}$$

where the probabilities of value loss for V_1 and V_2 are defined by

$$p_{V_1} = P[V_1 < 0|\overline{EQ}] \\ p_{V_2} = P[V_2 < 0] \tag{9}$$

And the probability of earthquake occurrence $P[EQ]$ that the T_R-return period earthquake occurs at least once in interval T_D-T_p is given (Ang 2007) by

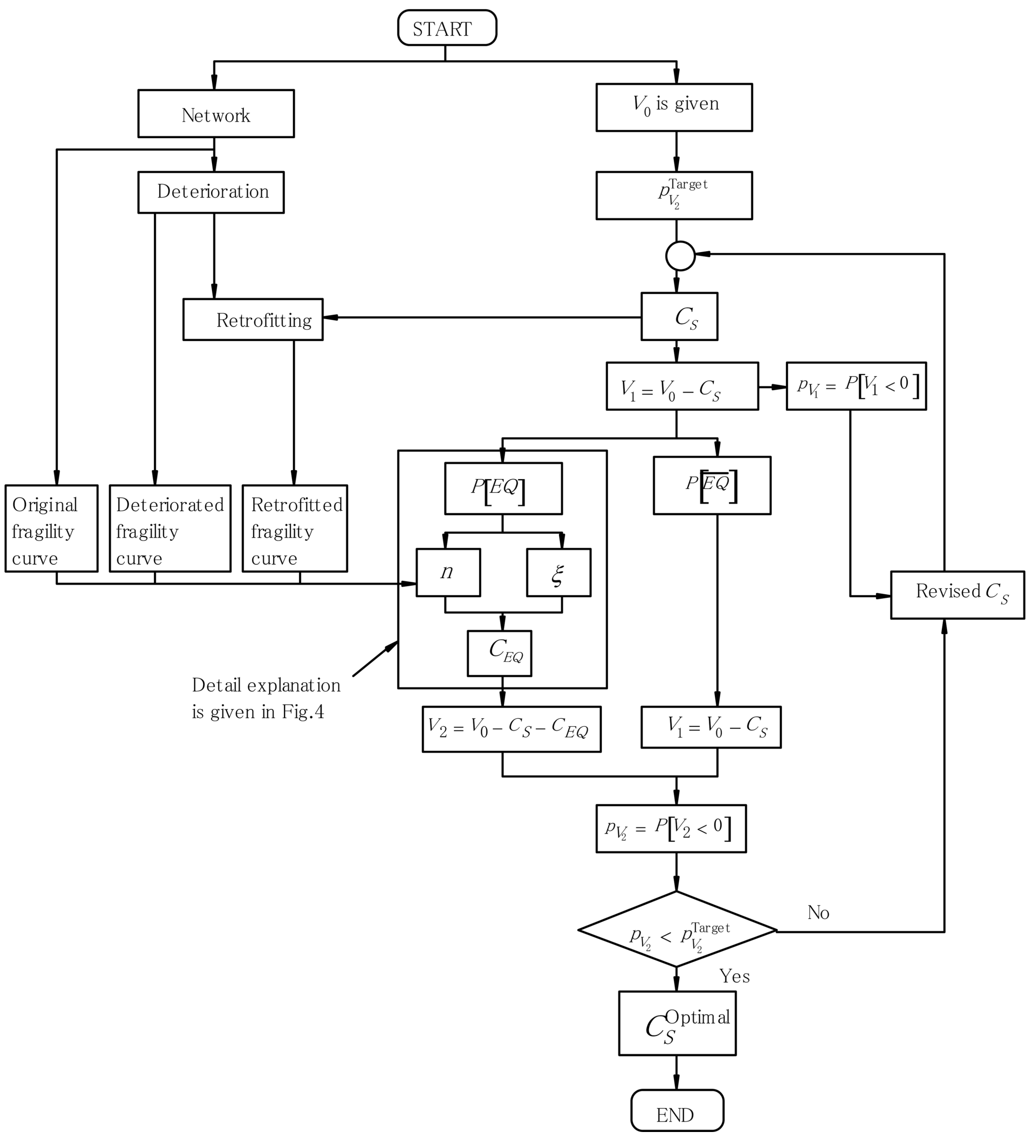


Fig. 1 Flow chart of the optimal seismic investment for the target probability of value loss

$$P[EQ] = 1 - \left(1 - \frac{1}{T_R}\right)^{T_D - T_p} \tag{10}$$

in which T_p and T_D are the present time point and the end point of service period, respectively.

Fig. 1 illustrates the flow chart to obtain the optimal seismic investment when a value index V_0 is assigned.

2.2 Probability distribution of value index of V_0

The present study aims to provide a methodology to obtain the optimal seismic investment or the minimum requirement of value loss which is related to the seismic risk. So the discussion on the seismic risk assessment is developed from an engineering basis, while the value loss of V_0 should be basically discussed from a financial basis considering the market risk. If the financial engineering approach is taken in this study, it may provide the good insight in evaluation of the statistical information on the value index V_0. In order to concentrate the discussion on the engineering including the seismic damage cost evaluation under the structural and functional damage conditions of the network system, however, the financial problems such as the effects of discount rate, tax payment, various kinds of insurance approaches, and so on, should be minimized and simplified. Once the seismic damage cost is evaluated from the engineering basis, additional consideration on those financial effects under seismic risk will be easily developed in the future study (Rackwiz 1999).

The project activity from the initial stage, T_0, to the present time, T_p, can be expressed using past data, while the future activity from the present point to the final stage, T_D, is constrained by many uncertain factors. So the value loss, V_0, will be controlled by many uncertain variables. To express this condition of value loss, the probability of value loss is introduced.

Annual balance denoted by $f(t)$ is defined as the social benefit less the initial cost, while annual income denoted by $g(t)$ is defined as the income less other costs as follows.

$$\begin{aligned} f(t) &= b(t) - c_o(t) \\ g(t) &= i(t) - e(t) - c_M(t) \end{aligned} \tag{11}$$

in which $b(t)$, $i(t)$, $e(t)$, $c_o(t)$ and $c_M(t)$ show the annual values at the t-th year for the variables of B, I, E, C_o and C_M.

Both of these functions are mutually independent because $f(t)$ relating to the long-term investment changes in the lifetime scale, but the $g(t)$ belonging to the business activity varies in the daily market scale.

The schematic profiles of these functions are shown in Fig. 2.

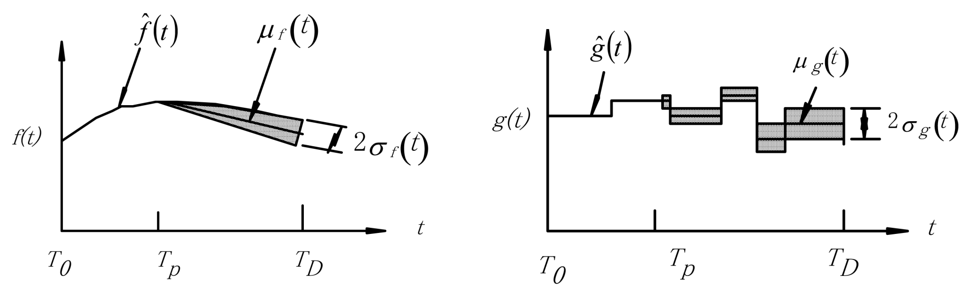


Fig. 2 Schematic profile of annual balance $f(t)$ and annual income $g(t)$

The shaded areas in Fig. 2 express the range of uncertain values for the function of $f(t)$ and $g(t)$ at any point t after the present point, T_p.

Now the following simplified assumption is introduced to describe the uncertain behaviors on the function of of $f(t)$ and $g(t)$

$$f(t) = \begin{cases} \hat{f}(t) & T_0 \le t \le T_p \\ N(\mu_f, \sigma_f) & T_p < t \le T_D \end{cases}$$
$$g(t) = \begin{cases} \hat{g}(t) & T_0 \le t \le T_p \\ N(\mu_g, \sigma_g) & T_p < t \le T_D \end{cases} \tag{12}$$

where $\hat{f}(t)$ and $\hat{g}(t)$ are fixed values of the functions of $f(t)$ and $g(t)$ in the past stage, while $N(\mu_X, \sigma_X)$ is the normal distribution of an uncertain variable, X, in the future stage whose mean is given by μ_X, and its standard deviation is denoted by σ_X.

One may divide the value index Vo at the present time into the past portion and the future portion. Then, the Eq. (1) can be rewritten as

$$V_0(T_p) = W_0(T_p) + U(T_p) \tag{13}$$

in which

$$W_0(T_p) = \int_{T_0}^{T_p} \{\hat{f}(t) + \hat{g}(t)\}\,dt$$
$$U(T_p) = \int_{T_p}^{T_D} \{f(t) + g(t)\}\,dt \tag{14}$$

The coefficient of variation $\delta_f(t)$ and $\delta_g(t)$ for the functions of $f(t)$ and $g(t)$ is assumed to depend on the elapsed time from the present point as shown in the following way

$$\delta_f(t) = c_f \cdot (t - T_p)^{\alpha} / (T_D - T_p)^{\alpha} \tag{15}$$

$$\delta_g(t) = c_g \cdot (t - T_p)^{\beta} / (T_D - T_p)^{\beta} \tag{16}$$

in which c_f, c_g, α and β are parameters for Eqs. (15) and (16).

Since $U(t)$ belongs to a future portion, this is a random variable. So the expected value and its standard deviation of the variable $U(t)$ are given by

$$E[U(T_p)] = E\left[\int_{T_p}^{T_D} \{f(t) + g(t)\}\,dt\right] = \int_{T_p}^{T_D} \{E[f(t)] + E[g(t)]\}\,dt = \int_{T_p}^{T_D} \{\mu_f(t) + \mu_g(t)\}\,dt \tag{17}$$

$$Var[U(T_p)] \approx \left[\frac{\{c_f \mu_f(T_p)\}^2}{2\alpha + 1} + \frac{\{c_g \mu_g(T_p)\}^2}{2\beta + 1}\right] \cdot (T_D - T_p) \tag{18}$$

From these discussion, the value index V_0 is given as a deterministic function in the past portion, while an uncertain characteristic of V_0 in the future portion can be described with the distribution function given by Eqs. (17) and (18). The function $f_{V_0}(v, t)$ is a probability density function of the value index $V_0(t)$, the statistic parameters of which are evaluated at the time point t. Fig. 3 shows the schematic profile of the probability density function of the value index $V_0(t)$.

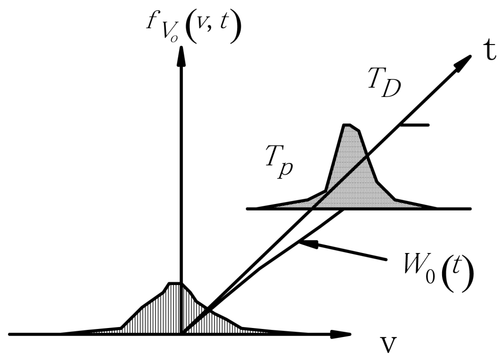


Fig. 3 Schematic profile of probability density function of the value index V_0

3. Seismic investment complying with the seismic performance requirement

3.1 Definitions of serviceability and structural damage states

A lifeline system is originally designed such that all demand nodes are serviceable after an earthquake. The levels of serviceability should be determined by the engineers operating the lifeline system. The system serviceability can be measured in terms of the ratio which is given by the number of serviceable nodes divided by that of the total nodes in the network system suffering any structural damage.

Table 1 provides the definitions of serviceability damage states and structural damage modes, respectively.

It should be noted that the resistance of structural components is controlled by the retrofitting effect due to the seismic investment C_S, the deterioration effect up to the present time T_p and any seismic damage by an earthquake occurring at the time t_{EQ}. The residual structural resistance for each structural damage criterion is denoted as R^{minor}, R^{moderate} and R^{major} for *minor, moderate* and *major* damage modes, respectively.

Let us define the performance functions for structural damage modes in the following way.

$$\begin{aligned} Z_{EQ}^{\text{minor}}(C_S, T_p, t_{EQ}) &= R^{\text{minor}}(C_S, T_p) - (D+L) - S\cdot 1_{EQ}(t_{EQ}) \\ Z_{EQ}^{\text{moderate}}(C_S, T_p, t_{EQ}) &= R^{\text{moderate}}(C_S, T_p) - (D+L) - S\cdot 1_{EQ}(t_{EQ}) \\ Z_{EQ}^{\text{major}}(C_S, T_p, t_{EQ}) &= R^{\text{major}}(C_S, T_p) - (D+L) - S\cdot 1_{EQ}(t_{EQ}) \end{aligned} \tag{19}$$

in which D, L and S are dead load, live load and seismic load, respectively.

Then the corresponding probabilities of structural damage mode at the structural components are defined as

$$p_{fi}^{Z} \equiv P[Z_{EQ}^{\text{minor}} < 0], \quad p_{fo}^{Z} \equiv P[Z_{EQ}^{\text{moderate}} < 0], \quad p_{fa}^{Z} \equiv P[Z_{EQ}^{\text{major}} < 0] \tag{20}$$

It should be noted that the water distribution system is composed of many pipelines, each link of which is also a series system of structural components. When the occurrences of structural damage at the structural components are assumed to follow the Poisson process along the link, the probability of structural link damage denoted by Z_{link} can be developed as follows:

Table 1 Definitions of serviceability damage states and structural damage modes

(1) Definitions of serviceability damage states

Serviceability damage state	Symbol	Definition
minor damage state	D_{EQ}^{minor}	the system serviceability is functional without any disruption for the ground motion (EQ1) produced by the maximum operational earthquake (MOE), and the probability exceeding the minor damage state is defined as p_{fi}^{D}.
moderate damage state	$D_{EQ}^{moderate}$	the system serviceability is functional after short repair disruption for the ground motion (EQ2) produced by the maximum credible earthquake (MCE), and the probability exceeding the moderate damage state is defined as p_{fo}^{D}.
major damage state	D_{EQ}^{major}	the system serviceability is functional after restoring disruption for the ground motion (EQ2) produced by the maximum credible earthquake (MCE), and the probability exceeding the major damage state is defined as p_{fa}^{D}.

(2) Definition of structural damage modes

Serviceability damage state	Symbol	Definition
minor damage mode	Z_{EQ}^{minor}	the elastic structural response S_1^Z by the ground motion (EQ1) exceeds the critical level S_a^Z (or *Rminor*), and the probability of minor damage occurrence is defined as p_{fi}^{Z}.
moderate damage mode	$Z_{EQ}^{moderate}$	the inelastic structural response ε_2^{Z*} by the ground motion (EQ2) exceeds the critical level ε_L^Z (or $R^{moderate}$) for the small leakage, and the probability of moderate damage occurrence is defined as p_{fo}^{Z}
major damage mode	Z_{EQ}^{major}	the inelastic structural response ε_2^{Z*} by the ground motion (EQ2) exceeds the critical level ε_U^Z (or R^{major}) for the large leakage, and the probability of major damage occurrence is defined as p_{fa}^{Z}.

$$
\begin{aligned}
P[Z_{link}^{\text{minor}}(l_i)|EQ_1] &= 1-\exp\left[-\int_0^{l_i} \nu_{\text{minor}}(x)dx\right] \\
P[Z_{link}^{\text{moderate}}(l_i)|EQ_2] &= 1-\exp\left[-\int_0^{l_i} \nu_{\text{moderate}}(x)dx\right] \\
P[Z_{link}^{\text{major}}(l_i)|EQ_2] &= 1-\exp\left[-\int_0^{l_i} \nu_{\text{major}}(x)dx\right]
\end{aligned} \tag{21}
$$

in which the damage rate n_{mz} per unit length which is defined at the point x is given by

$$
\nu_{mz}(x) = \nu_0 P[Z_{EQ}^{mz}(x) < 0|EQ] \tag{22}
$$

where the damage mode *mz* denotes *major*, *moderate* or *minor* in the structural damage mode, ν_0 is a number of pipe joints per unit length, and l_i is a stretching length of the *i*-th link in the distribution pipelines.

Eq. (22) is a fragility curve which is analytically formulated to estimate the structural damage of pipeline components. When the deteriorating effect or retrofitting effect is taken into consideration, such a fragility curve can be obtained from Eq. (22) by using the revised performance function in Eq. (19) which is replaced by a deteriorated residual strength or a newly retrofitted strength.

Using Eq. (21), probabilities of serviceability damage states are developed as follows.

$$p_{fi}^{D} \equiv P[\overline{D}_{EQ}^{\text{minor}}] = P[\overline{D}_{EQ}^{\text{minor}}|\overline{EQ_1}]\cdot P[\overline{EQ_1}] + P\left[\overline{D}_{EQ}^{\text{minor}}\middle|\bigcup_{i=1}^{NL} Z_{link}^{\text{minor}}(l_i)\right]\cdot P\left[\bigcup_{i=1}^{NL} Z_{link}^{\text{minor}}(l_i)|EQ_1\right]\cdot P[\overline{EQ_1}]$$

$$p_{fo}^{D} \equiv P[\overline{D}_{EQ}^{\text{moderate}}] = P[\overline{D}_{EQ}^{\text{moderate}}|\overline{EQ_2}]\cdot P[\overline{EQ_2}] + P\left[\overline{D}_{EQ}^{\text{moderate}}\middle|\bigcup_{i=1}^{NL} Z_{link}^{\text{moderate}}(l_i)\right]$$

$$\cdot P\left[\bigcup_{i=1}^{NL} Z_{link}^{\text{moderate}}(l_i)|EQ_2\right]\cdot P[\overline{EQ_2}] + P\left[\overline{D}_{EQ}^{\text{moderate}}\middle|\bigcup_{i=1}^{NL} Z_{link}^{\text{major}}(l_i)\right]\cdot P\left[\bigcup_{i=1}^{NL} Z_{link}^{\text{major}}(l_i)|EQ_2\right]\cdot P[\overline{EQ_2}]$$

$$p_{fa}^{D} \equiv P[\overline{D}_{EQ}^{\text{major}}] = P[\overline{D}_{EQ}^{\text{major}}|\overline{EQ_2}]\cdot P[\overline{EQ_2}] + P\left[\overline{D}_{EQ}^{\text{major}}\middle|\bigcup_{i=1}^{NL} Z_{link}^{\text{moderate}}(l_i)\right]$$

$$\cdot P\left[\bigcup_{i=1}^{NL} Z_{link}^{\text{moderate}}(l_i)|EQ_2\right]\cdot P[\overline{EQ_2}] + P\left[\overline{D}_{EQ}^{\text{major}}\middle|\bigcup_{i=1}^{NL} Z_{link}^{\text{major}}(l_i)\right]\cdot P\left[\bigcup_{i=1}^{NL} Z_{link}^{\text{major}}(l_i)|EQ_2\right]\cdot P[\overline{EQ_2}] \quad (23)$$

in which $D_{EQ}^{\text{minor}}, D_{EQ}^{\text{moderate}}$ and D_{EQ}^{major} mean the occurrence events of *minor*, *moderate* and *major* serviceability damage states, respectively, while the symbol of the upper bar denotes the complementary event of the serviceability damage state.

3.2 Probability of value loss

When the seismic investment C_S is applied to improve the seismic performance of structural components, the probability of value loss V_2 given in Eq. (3) is formulated for corresponding seismic damage modes, on the basis of which the seismic damage cost C_{EQ} is estimated

$$p_{V_2}^{\text{minor}} \equiv p_{V_2}(C_S, C_{EQ}^{\text{minor}}) = \{P[V_2<0|\overline{D}_{EQ}^{\text{minor}}]P[\overline{D}_{EQ}^{\text{minor}}|EQ_1] + P[V_1<0|D_{EQ}^{\text{minor}}]P[D_{EQ}^{\text{minor}}|EQ_1]\}P[EQ_1]$$

$$p_{V_2}^{\text{moderate}} \equiv p_{V_2}(C_S, C_{EQ}^{\text{moderate}})$$
$$= \{P[V_2<0|\overline{D}_{EQ}^{\text{moderate}}]P[\overline{D}_{EQ}^{\text{moderate}}|EQ_2] + P[V_1<0|D_{EQ}^{\text{moderate}}]P[D_{EQ}^{\text{moderate}}|EQ_2]\}P[EQ_2]$$

$$p_{V_2}^{\text{major}} \equiv p_{V_2}(C_S, C_{EQ}^{\text{major}})$$
$$= \{P[V_2<0|\overline{D}_{EQ}^{\text{major}}]P[\overline{D}_{EQ}^{\text{major}}|EQ_2] + P[V_1<0|D_{EQ}^{\text{major}}]P[D_{EQ}^{\text{major}}|EQ_2]\}P[EQ_2] \quad (24)$$

3.3 Statistical evaluation of the seismic damage cost

In order to obtain the optimal solution of C_S and V_o by Eqs. (7) and (8), the mean value and its variance of the seismic damage cost C_{EQ} in Eq. (4) must be estimated. Prior to calculating the seismic damage cost, its component C_R, ΔB and ΔI can be formulated in terms of statistical manner. The evaluation procedure of these statistics is shown in Fig. 4.

The restoration cost C_R^{mz} which is estimated as the summation over the NL links of the distribution network can be calculated as the unit restoration cost c_R^{mz} multiplied by the number n_i^{mz} of structural damage points in the damage mode mz for the i-th link which is given by

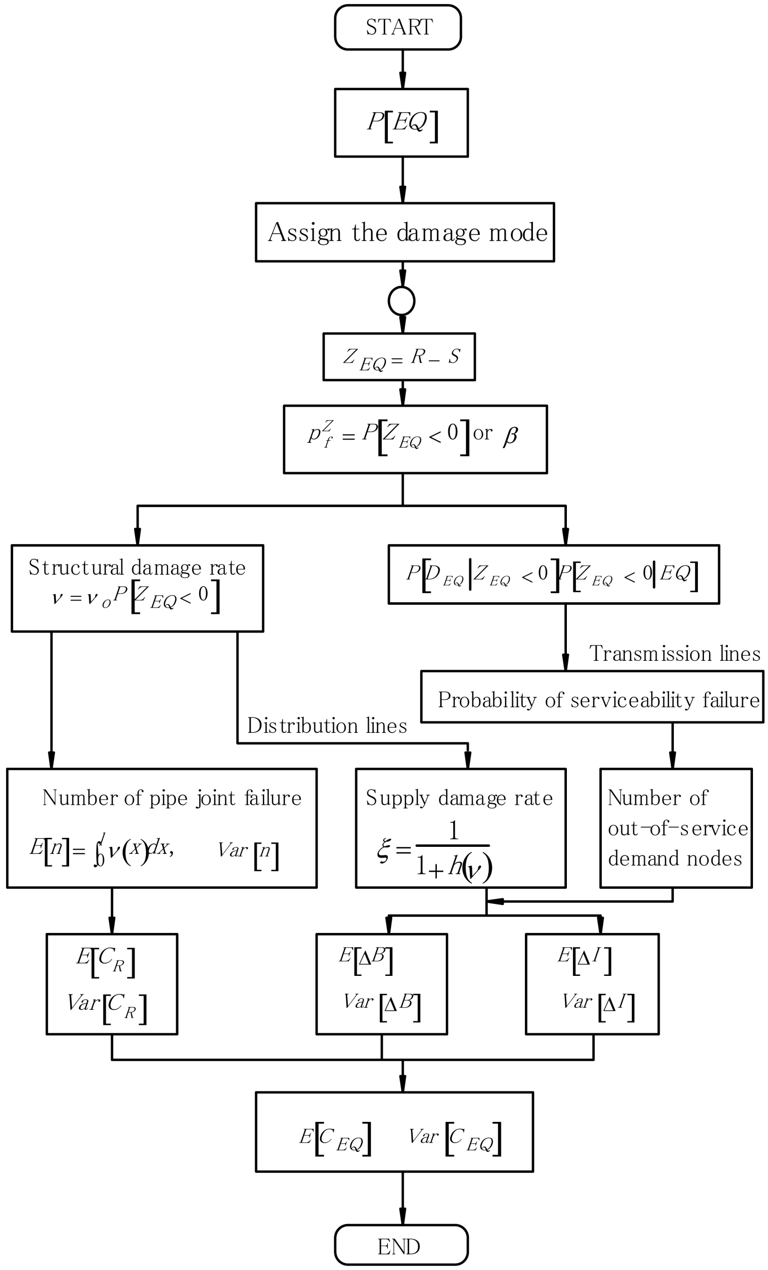


Fig. 4 Flow chart of estimation procedure of the seismic damage cost

$$n_i^{mz} = \int_0^{l_i} \nu_{mz}(x)dx \tag{25}$$

where the unit restoration cost c_R^{mz} is defined as the cost replacing a pipe in the damage mode *mz* by a new ductile cast iron pipe shown in Fig. 7. The mean and its standard deviation of the restoration cost are given by

$$E[C_R^{mz}] = \sum_{i=1}^{NL} c_R^{mz} n_i^{mz}, \quad \sqrt{Var[C_R^{mz}]} = \sqrt{\sum_{i=1}^{NL} (c_R^{mz})^2 n_i^{mz}} \tag{26}$$

where l_i and NL are the i-th stretching length of the distribution pipelines and a number of all links in the distribution network system in Fig. 6.

Kawakami (1996) introduced the supply damage rate or out-of-service rate in the major serviceability damage state of the water distribution network which is a simplified formulae using the structural damage rate $\nu_{major}(x)$.

$$\xi(x) = \frac{1}{1 + 0.0473\{\nu_{major}(x)\}^{-1.61}} \tag{27}$$

In this study, this supply damage rate is modified to comply with the serviceability damage state and the structural damage mode as follows.

$$\xi_{mz}^{md}(x) = \frac{1}{1 + h^{md}\{\nu_{mz}(x)\}} \tag{28}$$

in which *md* denotes *major, moderate* or *minor* in the serviceability damage state, and $h^{md}(\nu_{mz})$ is a function of the structural damage mode ν_{mz}.

Fig. 5 shows the conceptual illustration to describe the short disruption of lifeline service during the restoration period after the seismic damage occurrence. The loss of social benefit ΔB is drawn as the shadow zone.

Using this value, loss of social benefit ΔB is roughly estimated by

$$\begin{aligned} E[\Delta B_{mz}^{md}] &\approx b(T_-) \cdot \bar{\xi}_{mz}^{md} \cdot \int_0^{\Delta t} \{1 - f_B^{md}(t)\} dt \\ Var[\Delta B_{mz}^{md}] &\approx \{\delta_b \cdot E[\Delta B_{mz}^{md}]\}^2 \end{aligned} \tag{29}$$

where

$$\bar{\xi}_{mz}^{md} = \frac{\int_A \xi_{mz}^{md}(x) a(x) dx}{\int_A a(x) dx} \tag{30}$$

and $b(T_-)$ is a social benefit per year just before the earthquake occurrence time T, while A, $a(x)$, $f_B^{md}(t)$ and δ_b are the whole service area of the water distribution network, pipeline density per unit

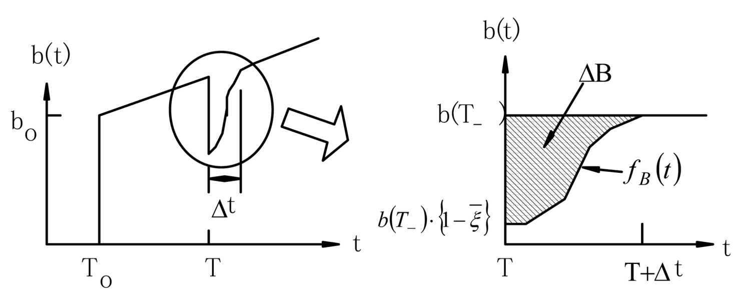


Fig. 5 Temporal trend of social benefit during the restoration process after the earthquake

area, restoration function in the serviceability damage state *md* and the coefficient of variation of $b(T_{-})$, respectively.

In the similar way, loss of income gain is also obtain as the

$$E[\Delta I_{mz}^{md}] \approx E\left[\frac{I}{B}\right] \cdot E[\Delta B_{mz}^{md}], \quad Var[\Delta I_{mz}^{md}] \approx \{\delta_b \cdot E[\Delta I_{mz}^{md}]\}^2 \tag{31}$$

Especially, in the distribution network, the probability of serviceability damage states in Eq. (21) is simply evaluated by using the supply damage rate $\bar{\xi}_{mz}^{md}$ as follows

$$P\left[\bar{D}_{EQ}^{md} \middle| \bigcup_{i=1}^{NL} Z_{link}^{mz}(l_i)\right] = \bar{\xi}_{mz}^{md} \tag{32}$$

In the transmission network, on the other hand, the probability of serviceability damage states should be estimated with the probability of network connectivity from the source nodes to the target demand nodes, the analytical approach of which is not described herein because this study is not focused on the transmission network system.

Using Eq. (23), the mean value of the seismic damage cost for the serviceability damage state is formulated as

$$E[C_{EQ}^{\text{minor}}] = (C_R^{\text{minor}} + \Delta B_{\text{minor}}^{\text{minor}} + \Delta I_{\text{minor}}^{\text{minor}}) \cdot P\left[\bar{D}_{EQ}^{\text{minor}} \middle| \bigcup_{i=1}^{NL} Z_{link}^{\text{minor}}(l_i)\right] \cdot P\left[\bigcup_{i=1}^{NL} Z_{link}^{\text{minor}}(l_i) \middle| EQ_1\right]$$

$$E[C_{EQ}^{\text{moderate}}] = (C_R^{\text{moderate}} + \Delta B_{\text{moderate}}^{\text{moderate}} + \Delta I_{\text{moderate}}^{\text{moderate}}) \cdot P\left[\bar{D}_{EQ}^{\text{moderate}} \middle| \bigcup_{i=1}^{NL} Z_{link}^{\text{moderate}}(l_i)\right] \cdot P\left[\bigcup_{i=1}^{NL} Z_{link}^{\text{moderate}}(l_i) \middle| EQ_2\right]$$

$$+ (C_R^{\text{major}} + \Delta B_{\text{major}}^{\text{moderate}} + \Delta I_{\text{major}}^{\text{moderate}}) \cdot P\left[\bar{D}_{EQ}^{\text{moderate}} \middle| \bigcup_{i=1}^{NL} Z_{link}^{\text{major}}(l_i)\right] \cdot P\left[\bigcup_{i=1}^{NL} Z_{link}^{\text{major}}(l_i) \middle| EQ_2\right]$$

$$E[C_{EQ}^{\text{major}}] = (C_R^{\text{major}} + \Delta B_{\text{moderate}}^{\text{major}} + \Delta I_{\text{moderate}}^{\text{major}}) \cdot P\left[\bar{D}_{EQ}^{\text{major}} \middle| \bigcup_{i=1}^{NL} Z_{link}^{\text{moderate}}(l_i)\right] \cdot P\left[\bigcup_{i=1}^{NL} Z_{link}^{\text{moderate}}(l_i) \middle| EQ_2\right]$$

$$+ (C_R^{\text{major}} + \Delta B_{\text{major}}^{\text{major}} + \Delta I_{\text{major}}^{\text{major}}) \cdot P\left[\bar{D}_{EQ}^{\text{major}} \middle| \bigcup_{i=1}^{NL} Z_{link}^{\text{major}}(l_i)\right] \cdot P\left[\bigcup_{i=1}^{NL} Z_{link}^{\text{major}}(l_i) \middle| EQ_2\right] \tag{33}$$

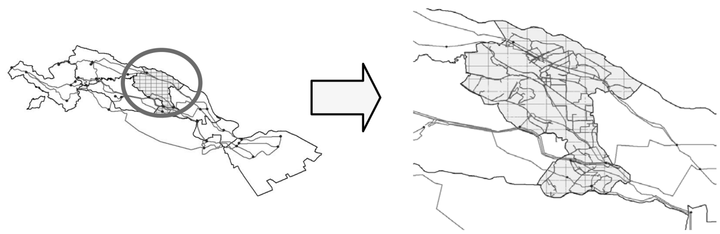
The variance of C_{EQ} can be developed in the similar manner.

4. Application to the seismic disaster prevention for lifeline systems

4.1 Analytical model of lifeline system

A water supply lifeline network system (Koike 2007) which is located in a metropolitan area of Japan is adopted as the sample model for this analysis. In this figure, the whole area is divided into several sub-zones which are the distribution districts to be controlled by the water supply authority of this city.

The water lifeline network system has a hierarchy system which is composed of transmission lines, distribution and service network systems as shown in Fig. 6. The transmission pipelines have been constructed and maintained with a high grade seismic performance level, while the distribution and service networks have partially old-fashion type of cast iron pipes with mechanical joints as



(1) Transmission pipelines (2) Distribution network

Fig. 6 The distribution network located in the transmission pipelines

Table 2 Pipeline profiles of the distribution network

	Pipe length (m)				
Diameter	Cast Iron Pipe	Ductile cast Iron Pipe		Welded Steel Pipe	
mm		old joint	seismic joint	poor quality	high quality
200	4738	20091	0	1390	0
250	52	53	0	0	0
300	3620	14898	0	925	0
350	2820	14049	0	2121	0
400	0	0	0	879	0
500	0	7000	5260	0	23023
Subtotal	11230	56091	5260	5315	23023
Total			100919		

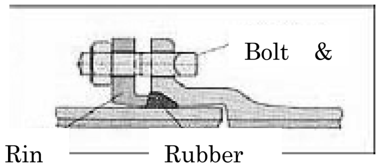
well as the steel and ductile cast iron pipes with more seismically reliable joints. In this situation, potential seismic damages will be triggered from the weakest joints of cast iron pipes. The main target of the seismic retrofit activity should be focused on the replacement of old fashion type of cast iron pipes to newly developed ductile cast iron pipes or arc-welded steel pipes of the distribution network system. Fig. 6(2) shows a sample profile of the distribution network system. Table 2 is a list of pipe materials and their total elongations for each diameter in the distribution network system.

4.2 Reinforcement and retrofitting of existing pipelines

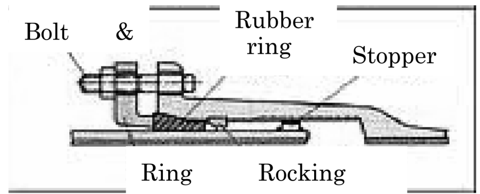
The water distribution pipeline in Japan (JWWA 1997) is generally composed of cast iron joint pipes and ductile cast iron joint pipes. Almost 80% of such distribution pipelines have old cast iron pipes, while the remaining 20% are replaced by new ductile cast iron pipes. Most typical joint models are shown in Fig. 7.

Old cast iron pipes and ductile cast iron pipes with old type joint are replaced with new ductile cast iron pipes, while deteriorated steel pipes are also changed to new pipes. Joint performance of old and new pipes are compared in Table 3.

The deteriorating behavior (Frangopol 2003) of old pipe joints under traffic load vibrations is assumed to be modeled by the following equation:



(1) Old type of mechanical joint



(2) New type of mechanical joint

Fig. 7 Illustrations of mechanical joints

Table 3 Seismic performance of pipe joints for various damage modes

Type	Joint of Resistance (mm)		
	Mean		
	Minor	Moderate	Major
Old joint	5	10	15
New joint	30	45	60
Type	COV		
	Minor	Moderate	Major
Old joint	0.2	0.25	0.3
New joint	0.05	0.05	0.1

Table 4 Deterioration factors for pipe joints

Deterioration factors		
Type	d_1	d_2
Old joint	0.5	0.25
New joint	0.1	0.05

$$\psi(t) = 1 - d_1\left(\frac{t}{T_D}\right)^{d_2} \tag{34}$$

in which d_1 and d_2 are parameters to describe the deteriorating behavior as shown in Table 4.

Two types of earthquakes are assumed to arrive in the Poisson process during the residual service period. The seismic loads due to these events correspond to the earthquake ground motions, EQ1 and EQ2, whose return periods are assumed to be $T_{R1} = 50$ years and $T_{R2} = 475$ years.

The structural failure of a pipe joint occurs when the relative displacement between adjacent pipes exceeds the allowable displacement of pipe joint. So the seismic relative displacement Δu is estimated by

$$\Delta u = U_h\left\{1 - \cos\left(\frac{2\pi\Delta l}{L}\right)\right\} \tag{35}$$

in which U_h, L and Δl are horizontal ground displacement, travelling wave length and interval between adjacent pipes, respectively. The ground response of Eq. (35) is given by

Table 5 Seismic responses at joint portion for the seismic responses of EQ1 and EQ2

Seismic Response (mm)		
Type	Mean	COV
EQ1	8	0.3
EQ2	37	0.4

Table 6 parameters for various damage modes of the supply damage rate

	Serviceability damage state		
	major	moderate	minor
a	0.0473	0.319	∞
b	−1.61	−1.18	0

$$U_h = \frac{2}{\pi^2} S_V(T_G) \cdot T_G \tag{36}$$

where T_G is the typical period of the surface ground and S_V is the response velocity spectrum which is given in JWWA design codes for EQ1 and EQ2, respectively. The seismic responses of buried pipe joints for these earthquake loads are summarized in Table 5 in which the coefficient of variation of each seismic load is simply assumed with a typical value over the entire area of the network.

Using ψ in Eq. (33), a seismic strength under a deteriorating process is estimated in the following way

$$R(C_S, T_p; x \in l_{ij}) = \psi(T_p - T_0) \cdot R(C_S, T_0; x \in l_{ij}) \tag{37}$$

One may assume that the supply damage rate given in Eq. (28) is formulated as

$$h^{md}(\nu_{mz}(x)) = a_{md}\{\nu_{mz}(x)\}^{b_{md}} \tag{38}$$

The parameters a_{md} and b_{md} for major damage are prepared by Kawakami (1996) on the basis of the observation data in 1995 Kobe Earthquake. Noting that there are not any appropriate data on the moderate damage, the following parameters are assumed for simplicity herein as shown in Table 6.

The restoration function $f_B^{md}(t)$ shown in Fig. 5 is simply assumed to have a linear curve in interval T and $T + \Delta t$.

4.3 Improved seismic performance by the joint replacement

When an existing pipeline is retrofitted based on the above-mentioned scheme, the old pipes can be replaced by new pipes, so the replacing trend can be measured by the following ratio

$$J_R(t) = \frac{L_{old}(t)}{L_{old}(t) + L_{new}(t)} \tag{39}$$

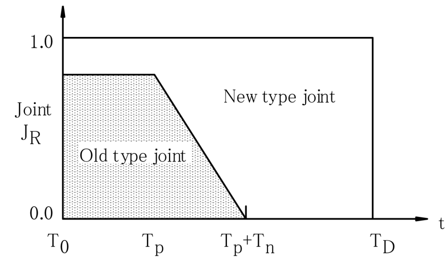


Fig. 8 Old pipe replacement in the time interval T_n

$$L_{old}(t) = \sum_{i=1}^{NL} l_i^{old}(t), \quad L_{new}(t) = \sum_{i=1}^{NL} l_i^{new}(t) \tag{40}$$

and NL, $l_i^{old}(t)$ and $l_i^{new}(t)$ are number of links, old joint pipe length and new joint pipe length of the i-th link, respectively.

4.4 Probabilities of value loss and serviceability damage for various seismic investments

Let us assume that the present time T_p is 30 years since the initial point, and the residual period from now to the terminal point is 70 years.

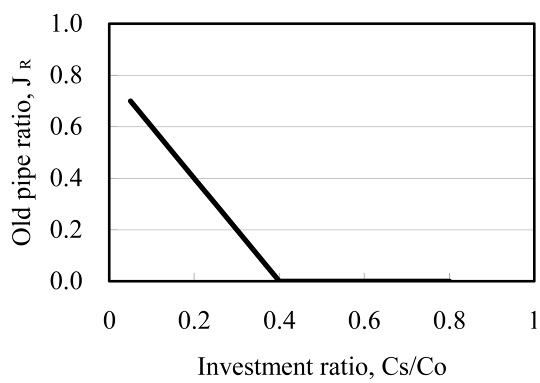


Fig. 9 Old pipe replacement for various investments

Table 7 Numerical conditions

Item	Symbol	Unit	Amount
Total pipe length	L	km	100
unit length of pipe	u_L	m	5
Repair cost ratio of old pipe per initial cost	C_{joint}^{old}	1/joint	0.0001
Repair cost ratio of new pipe per initial cost	C_{joint}^{new}	1/joint	0.0001
Restoration period	Δt	month	0.5
Return period of EQ1	T_{R1}	year	50
Return period of EQ2	T_{R2}	year	475
Duration period of pipe joint replacement activity	T_n	year	10,30,70

Fig. 9 shows the rate of joint replacement when a seismic investment is given to the deteriorated lifeline system under the numerical condition of Table 7. This figure means that the case of *Cs/Co* less than 30% cannot replace 20% of old joints, while the case of *Cs/Co* greater than 40% can replace the whole old joints.

Figs. 10 and 11 shows the probabilities of value loss and serviceability damage (or failure) of the pipeline network for various seismic investment strategies C_S when the retrofitting period T_n of the replacement activity is 30 years. Fig. 10 expresses the probability of value loss for various damage modes. Especially, this probability in the minor damage mode shows a sudden drop at the seismic investment ratio of C_S/Co = 0.3 to 0.4. This dropping trend is not clear in the other damage modes. The same dropping trend appears in Fig. 11 where the moderate damage mode decreases the probability of failure in the case of C_s/Co less than 0.3, the minor and major damage modes decrease the probability of failure in the case of C_s/Co greater than 0.4.

Fig. 12 shows the probabilities of value loss and failure for various retrofitting periods T_n under the same seismic investment cost. Both figures shows the same trend. Short retrofitting period (for

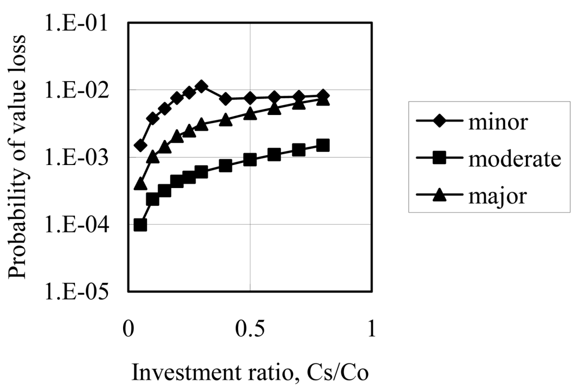


Fig. 10 Probability of value loss p_{V_2} for various seismic investments

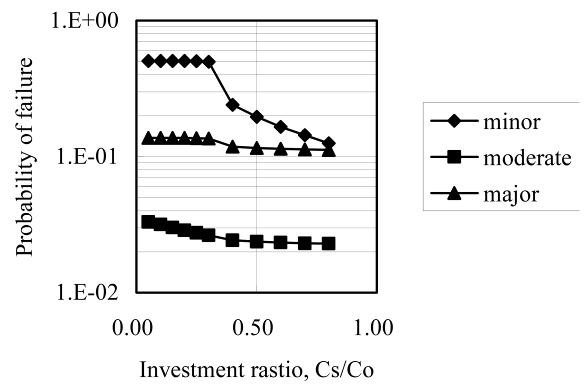
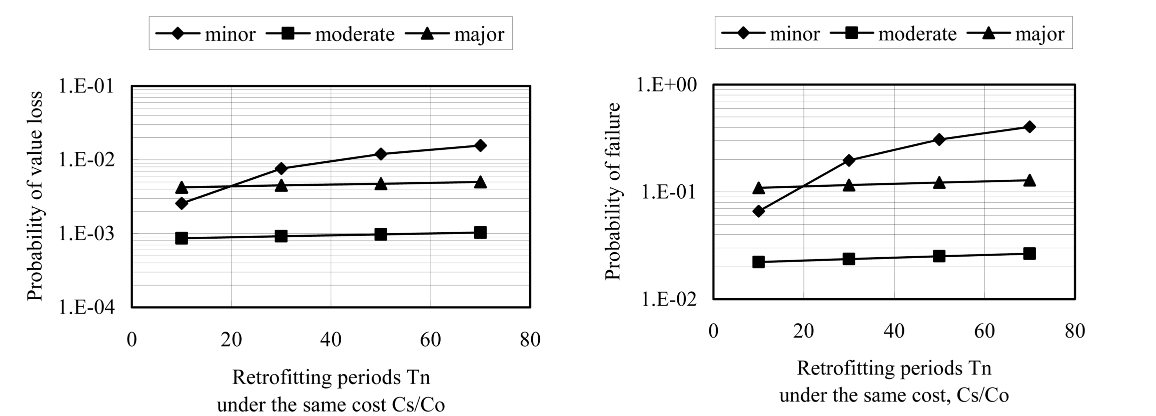


Fig. 11 Probability of serviceability damage p_f^D for various seismic investments



(1) Probability of value loss (2) Probability of serviceability damage (or failure)

Fig. 12 Probabilities of value loss p_{V_2} and serviceability damage p_f^D for various retrofitting periods under $C_S/C_0 = 0.5$

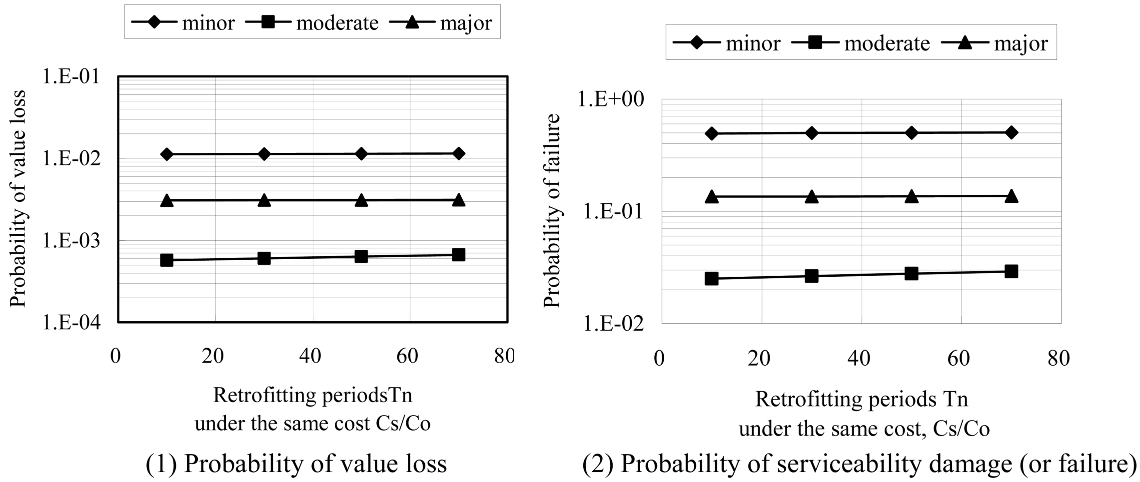


Fig. 13 Probabilities of value loss p_{V_2} and serviceability damage p_f^D for various retrofitting periods under $C_S/C_0 = 0.3$

instance $T_n = 10$) means rapid replacement by the concentrated investment, while long retrofitting period (for instance $T_n = 70$) reflects slow replacement by the low-level continuous investment.

In the case of $Cs/Co = 0.5$, all the old joints can be replaced by the new one within the residual service period according to Fig. 9. In this case, rapid replacement with $T_n = 10$ can improve the probabilities of value loss and failure, especially in the minor mode, while the slow replacement with $T_n = 70$ increase the probabilities of value loss and failure.

Fig. 13 shows the diagrams in the case of $Cs/Co = 0.3$. In this case, the old joints cannot be replaced by the new one within the residual service period because the seismic investment is not enough for the whole pipe joint replacement. These figures suggest that the insufficient investment is difficult to improve the probabilities of value loss and failure for any retrofitting period T_n.

5. Conclusions

This study proposes a certain measure or investment strategy for decision making associated with the seismic retrofitting. This strategy reduces the risk of a large-scale breakdown such as water supply loss under seismic risks. In this study, the authors developed a stochastic value index which is used in evaluating the total social benefit, income gain, life cycle costs and failure compensation associated with the existing lifeline systems that may be damaged by an earthquake during the remaining service period. Using this stochastic value index, seismic disaster prevention investment for existing deteriorating lifeline system is discussed.

Several important results can be summarized:

(1) For the water supply lifeline system which is closely related to social benefit which the infrastructure can provide to all the people, the stochastic value index introduced herein is more useful measure for the decision makers than current measures such as life-cycle cost or cost/benefit ratio.

(2) A new formulation to obtain the optimal seismic investment is developed in a probabilistic manner by introducing the probability distribution of value index, while various damage costs

caused by seismic disaster are evaluated on the seismic risk analysis unique to the existing pipeline network system.

(3) An analytical method unique to existing deteriorating lifeline networks is developed to estimate the seismic damage cost which includes restoration cost as well as value loss due to serviceability damages of the network system.

(4) The relationship between the value loss in the finance and the structural failure in the engineering is functionally combined and formulated in the probabilistic manner through the stochastic value index and by using the seismic risk analysis.

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