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Wooden framed structures with semi-rigid connections: Quantitative approach focused on design needs

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Abstract. Mechanical connections are recognized as extremely important elements in the aspect of strength and structural safety. However, classical structural model does not consider the connection stiffness properties, and are based on models with pinned or rigid joints only. In fact, mechanical connections are deformable and behave not linearly, affecting the whole structure and inducing nonlinear behavior as well. The quantification of this effect, however, depends on the description of the working of the connectors and the wood response under embedment. The theoretical modeling of wood structures with semi-rigid connections involves not only the structural analysis, but also the modeling of both single and grouped moment resisting connectors and the study of the wood properties under embedment. The proposal of this paper is to approach these aspects, and to quantitatively study the influence of the moment resistant connection in wooden framed structures. Comparisons between rigid and semi-rigid connections and between linear and nonlinear analysis lead to quantitative results.

Keywords: wood structures; framed structures; matrix methods; structural models; semi-rigid connections.

1. Introduction

Wood is employed in a wide variety of structural systems. In spite of the limited dimension of wood pieces, its workability allows the building of creative and functional architectural arrangements. The improvement experienced by connections leads progressively to new structural systems. Today, the technology on joints allows connecting pieces up to two meters in depth (Kessel 1995). The needs are to avoid the splitting of wood and to provide resistance and ductility as well.

Wooden framed structures have many applications, with spans ranging up to 100 m (Kessel 1995). They are commonly plane structures where the joints have to support large efforts. Usually, joints are built with a group of pin-shaped connectors.

In general, although the strength of mechanical connections are specially considered in the design, the classical structural design of framed structures, based on models with pinned or rigid joints only, does not consider the deformation of the connections. In fact, mechanical connections are

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deformable and behave not linearly, affecting the whole structure and inducing nonlinear behavior as well. This effect is not clearly quantified in wooden structures, also considering that codes do not fully provide criteria for characterization of moment resisting connections.

The analysis of framed structures with semi-rigid connections involves aspects related to the modeling of structural behavior as well to the modeling of connection behavior itself.

The first aspect is the computation of the connections as deformable elements in the structural analysis and the construction of the non-linear governing equation. The connection deformation is theoretically included by means of computing its virtual work into the Principium of Virtual Work (PVW) equation. On the basis of the matrix model for framed structures (Gere and Weaver 1965), the computation of deformations of connections may be accounted through the summation, into the global matrix equation, of "connection elements" matrix equations, relating the internal efforts and the relative displacements between the two connected pieces. The approach based on "connections elements", used in this paper, was considered by Li *et al.* (1995) for structures with semi-rigid connections as a straight forward method to consider the deformation of the connections although it is not the only method for that - that effect often is considered through the modification of the bar elements (Gesualdo 1987, Pinheiro and Silveira 2005).

The second aspect of the analysis is the relationship between efforts and the relative displacements between connected pieces, i.e., the relationship between **a** and **u**, where $\mathbf{a}^T = \{M_i, N_i, V_i, M_j, N_j, V_j\}$ and $\mathbf{u}^T = \{\theta_i, u_i, v_i, \theta_j, u_j, v_j\}$. Fig. 1 shows a structure with connection element and the graphical representation of internal efforts M_i, N_i, V_i, M_j, N_j and V_j and the displacements θ_i, u_i, v_i, θ_j, u_j and v_j of the connection. Foschi (1977) proposed an analytical model for toothed metal plates, according to which the "internal" virtual work of the group of deforming connectors may be calculated and equaled to the "external" virtual work of the loads and the displacements in the bulk connection element. The internal virtual work of each deforming connector is usually derived from its load *vs* slip relationship, in such a way that the relationship $\mathbf{a} = \mathbf{f}(\mathbf{u})$, where **f** is a vector function of **u**, would be better written as $\mathbf{a} = \mathbf{f}(\mathbf{u}(\Delta^{(k)}))$, where $\Delta^{(k)}$ is the slip of each connector k, with k varying from 1 to the number of connectors. Jensen and Larsen (1998) developed relations for dowel type connections according to that analytical approach.

Besides the effect of deformations of connections on the structural behavior, the study of moment

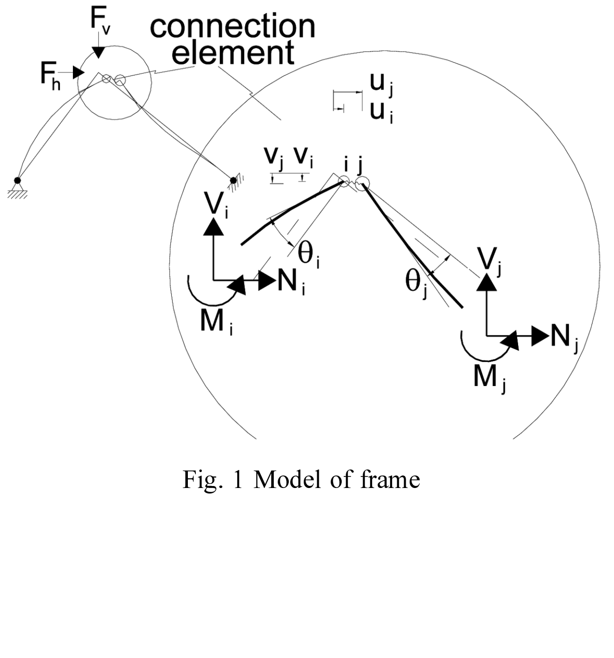

Fig. 1 Model of frame

resisting connections relates to the connections design, what is closely related to the design of the structure and its reliability. Komatsu, Karube and Harada (1996) proposed criteria for the design of moment resisting connection related specially to the spacing between fasteners. Hygroscopic characteristics, orthotropic symmetry and time-dependent mechanical behavior of wood are main facts that introduce complexities to the design of connections. A large number of works have been published in the field of wooden moment resisting connections and good references are given by Jensen and Lersen (2000). Recent progresses have been achieved to the understanding of damage and fracture of wooden connections. Jensen (2005) presented an analysis of splitting failure of connections according to fracture mechanics. In this work, we use the model referred by Jensen and Larsen (2000) as Plane Group of Mehanical Fasteners (PGMF) model, whose bases were described above, firstly proposed by Foschi (1977).

In a moment resisting connection, the stiffness and strength of each connector depends on the direction of the load in relation to the directions of wood fibers in both pieces connected.

Thus, the third aspect to be considered in the study of wooden framed structures with semi-rigid connections is the modeling of a single connector mechanical behavior. According to a simplified analytical model, the mechanical behavior is expressed by a relationship between the load acting in the connector (F) and the relative displacement (slip) between pieces in the direction of the load (Δ). Fig. 2 shows the graphical representation of the relative displacement in the plane of the single connector (perpendicular to the plane of the moment resisting connection).

This type of modeling has started in the middle of the 20^{th} century. Kuenzi (1955) developed a linear relationship founded in the theory of elastic beams on elastic foundation. Foschi (1974) extended the model and developed a nonlinear relationship founded considering the elastoplastic behavior of materials. The behavior of the connector depends on the width of the wooden pieces and the stiffness properties of wood in the direction of the embedment load - besides the connector diameter and steel properties.

Foschi’s works might be considered a landmark work in the analytical approach of single fastener elastoplastic behavior. The model of beam on elastic foundation used by Kuenzi (1955) and elastoplastic foundation used by Foschi (1974) combines accuracy and simplicity and are usually applied in analytical approaches of connections. Some works have been accomplished for fitting parameters for these models.

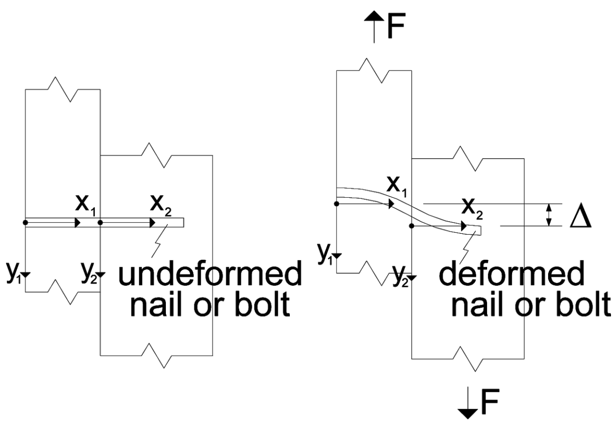

Fig. 2 Load and relative displacement

Thus, the forth and last aspect is the study of the behavior of wood under embedment, i.e., the behavior of wood as a foundation for the connector. Wilkinson (1971) has extended the studies developed by Kuenzi (1955) on elastic foundation. Recent and important progress was made by Racher and Bocquet (2005), who used a finite element analysis to model the non linear elastoplastic deformation of the foundation (wood) around the dowel in a dowel-type connection.

A large number of works have been developed on one or more of the four aspects presented above. The proposal of this research is to approach all the above aspects of the analysis of wooden framed structures with semi-rigid connections together, and as a result, to quantify the influence of the deformation of rotational connections in wooden frames. This work intends to contribute to answer how to calculate the design stiffness of connectors, and what are the parameters influencing the final results of structures with semi-rigid connections.

Several computer programs and spreadsheets were implemented to that, and experimental results were achieved. The results evidenced a great clarification about the behavior of wood structures with semi-rigid connections and the aspects to be considered in its design.

2. Matrix analysis

Computation of the connections as deformable structural elements may be made according to usual procedure of matrix analysis, similarly computation of solid bars. Consider the vectors **a** and **u**, where $\mathbf{a}^T = \{M_i, N_i, V_i, M_j, N_j, V_j\}$ and $\mathbf{u}^T = \{\theta_i, u_i, v_i, \theta_j, u_j, v_j\}$ (see Fig. 1).

For bar or beam elements, the relationship between the displacements u_1 and u_2 of any point of the element axis and the displacements at its ends (joined in **the** vector **u**), may be derived from theory of axially loaded bars and from theory of bending of beams, and are given by

$$u_1(x_1) = \mathbf{n}_u \cdot \mathbf{u} \tag{1a}$$

$$u_2(x_1) = \mathbf{n}_v \cdot \mathbf{u} \tag{1b}$$

Where

$$\mathbf{n}_u^T = \left\{1-\frac{x_1}{l}, 0, 0, \frac{x_1}{l}, 0, 0\right\} \tag{2a}$$

And

$$\mathbf{n}_v^T = \left\{0, 1-3\frac{x_1^2}{l^2}+2\frac{x_1^3}{l^3}, x_1-2\frac{x_1^2}{l}+\frac{x_1^3}{l^2}, 0, 3\frac{x_1^2}{l^2}-2\frac{x_1^3}{l^3}, -\frac{x_1^2}{l}+\frac{x_1^3}{l^2}\right\} \tag{2b}$$

The relationship between the deformation in any point of the beam element volume and the displacements of the point of its axis in the same section is given by

$$\varepsilon_{11}(x_1, x_2) = \frac{du_1(x_1)}{dx_1} - x_2\frac{d^2u_2(x_1)}{dx_1^2} \tag{3}$$

By defining the operator

$$\mathbf{B} = \frac{d}{dx_1}\mathbf{n}_u - x_2\frac{d^2}{dx_1^2}\mathbf{n}_v \tag{4}$$

And recalling Eq. (1a), Eq. (1b) and Eq. (4), Eq. (3) may be rewritten as

$$\varepsilon_{11}(x_1, x_2) = \mathbf{B} \cdot \mathbf{u} \tag{5}$$

Also, the stresses are related to strain by means of a single relation:

$$\sigma_{11} = E_1 \varepsilon_{11} \tag{6}$$

Thus, the Principium of Virtual Work in the beam element volume may be written as

$$\int_{V_e} E_1(\mathbf{B} \cdot \mathbf{u})(\mathbf{B} \cdot \delta\mathbf{u}) dV_e = \mathbf{a} \cdot \delta\mathbf{u} \tag{7}$$

Or, with j as free index ranging from 1 to 6, as six equations as follows

$$\left[\int_{V_e} E_1(\mathbf{B}_j \mathbf{B}) \cdot \mathbf{u} dV_e\right] \delta u_j = a_j \delta u_j \tag{8}$$

Once the quantities δu_j are arbitrary, they can be suppressed from both sides of equation above, and then, it may be rewritten as

$$\int_{V_e} E_1(\mathbf{B}_j \mathbf{B}) \cdot \mathbf{u} dV_e = a_j \tag{9}$$

Or

$$\mathbf{Ku} = \mathbf{a} \tag{10}$$

Where

$$\mathbf{K}_{ij}^e = \int_{V_e} E_1(\mathbf{B}_j \mathbf{B}_i) dV_e \tag{11}$$

For connection elements, the displacements at their ends, joined in the vector **u** are related to the relative displacements of the connectors, as will be shown in the next section. This relationship may be written as a matrix equation and has nonlinear form, written as

$$\mathbf{a} = \mathbf{f}(\mathbf{u}) \tag{12}$$

In order to be assembled in the global matrix equation, the equation above has to be approximated by a linear iterative for

$$\mathbf{K}^t(\mathbf{u}^k) \cdot \Delta\mathbf{u}^{k+1} = \mathbf{a} \cdot \mathbf{f}(\mathbf{u}^k) \tag{13a}$$

where

$$K_{ij}^t(\mathbf{u}^k) = \frac{\partial f_i(\mathbf{u}^k)}{\partial u_j} \tag{13b}$$

And

$$\mathbf{u}^{k+1} = \mathbf{u}^k + \Delta\mathbf{u}^{k+1} \tag{14}$$

The summation of the elemental equations leads to a global matrix equation. If the connections are supposed to behave linearly, then Eq. (12) takes the form of Eq. (10) and linearization is not necessary.

The matrix analysis as described above was implemented by the authors through a computer program in C++ language (Santana 2002). The construction of tangent stiffness matrix for connections – see Eq. (13a) and Eq. (13b) - was implemented according to the model for connection element described in the following item. However, the program allows analysis for linear semi-rigid behavior and rigid behavior for connections as well.

Not all the finite element programs contemplate the analysis of framed structures with nonlinear semi-rigid moment resisting connections. Comparable results in the linear range were obtained with the software ANSYS (ANSYS Inc. 1995). We used elements BEAM23 and MATRIX27 elements. MATRIX27 defines a general finite element with force (or moment) and displacement (or angular displacement) relationships referred to a global coordinate system. Its behavior is defined by a 3D stiffness matrix with 27 different components. We neglected the components other than K_x, K_y and K_z, corresponding to $K_{11}(=K_{44}=-K_{14}=-K_{41})$, $K_{22}(=K_{55}=-K_{25}=-K_{52})$ and $K_{33}(=K_{66}=-K_{36}=-K_{63})$, that is, we considered only relationships between force (or moment) and displacements (or angular displacements) in the same direction. The results coincided with those obtained with a linear matrix analysis made with the program implemented by the authors (mentioned above) and also agreed with experimental result, as will be shown later.

3. The connection element for moment resisting connections

The connection element approach is very favorable for computational programming. Considering this type of modeling, each connection element has two nodes (ends), each of them able to be connected to one node (end) of a beam element. At initial configuration, both nodes of connection element have the same position in plane. At deformed configuration, due to relative displacements of connectors, each of the nodes displaces to a different position. These positions are described by a set of six (three for each node) displacements joined in a vector **u**.

Fig. 3 shows the scheme of the general form of the connection element and their forces and displacements coordinates and the representation of the force and displacement in a real generic connector *k*.

In accounting for the connection deformation in the matrix analysis, the nodal displacements and

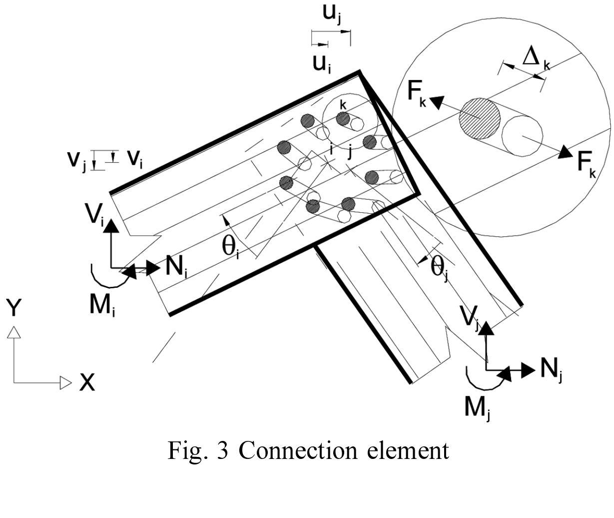

Fig. 3 Connection element

loads as external and the connectors slips and loads as internal. Before beginning to derive the equations, it is required the general relationship $F = f(\Delta)$, where f is a scalar function from Δ to F, nonlinear in general. This relationship is supposed to have different parameters following the direction of load in relation to the fibers direction. For practical reasons, this generic relationship may be determined in two perpendicular global reference directions, say x and y. From these relations, $F^{(k)} = f(\Delta^{(k)})$ for each connector in the direction of the load and relative displacement (which are supposed to be the same) may be compounded.

Be u_i, i ranging from 1 to 6, the nodal displacements of the joint, and a_i the corresponding nodal loads. Also be $\Delta^{(k)}$ the slip of a generic connector k of the joint, and $F^{(k)}$ the corresponding load.

The equation of the Principle of the Virtual Work (PVW) may be written as:

$$\sum_{i=1}^{6} a_i \delta u_i = \sum_{k=1}^{n} F^{(k)} \delta\Delta^{(k)} \cos(\alpha^{(k)} - \phi^{(k)}) \tag{15}$$

where n is the number of connectors, δ means "virtual", $\alpha^{(k)}$ is the angle between the connector load $F^{(k)}$ and the reference direction and $\phi^{(k)}$ is the angle between the virtual slip $\delta\Delta^{(k)}$ and the reference direction.

From geometric relations only, the slips may be written as a function of the nodal displacements, such that

$$\Delta x^{(k)} = \mathbf{Q}_x^{(k)} \cdot \mathbf{u} = Q_{xi}^{(k)} u_i \tag{16a}$$

$$\Delta y^{(k)} = \mathbf{Q}_y^{(k)} \cdot \mathbf{u} = Q_{yi}^{(k)} u_i \tag{16b}$$

and

$$\Delta^{(k)} = \sqrt{(\Delta x^{(k)})^2 + (\Delta y^{(k)})^2} \tag{16c}$$

where

$$\mathbf{Q}_x^{(k)} = \{-1 \;\; 0 \;\; r^{(k)}\sin\beta^{(k)} \;\; 1 \;\; 0 \;\; -r^{(k)}\sin\beta^{(k)}\} \tag{17a}$$

and

$$\mathbf{Q}_y^{(k)} = \{0 \;\; 1 \;\; -r^{(k)}\cos\beta^{(k)} \;\; 0 \;\; -1 \;\; r^{(k)}\cos\beta^{(k)}\} \tag{17b}$$

Also

$$\cos\alpha^{(k)} = \frac{\Delta x^{(k)}}{\Delta^{(k)}} \tag{18a}$$

$$\sin\alpha^{(k)} = \frac{\Delta y^{(k)}}{\Delta^{(k)}} \tag{18b}$$

These same relations are valid for the virtual quantities, with $\alpha^{(k)}$ replaced by $\phi^{(k)}$ in Eq. (18a) and Eq. (18b). The parameters $r^{(k)}$ and $\beta^{(k)}$ are the polar coordinates of the connectors in relation to a reference system with origin in the rotational center aligned with the global reference system.

These relations include the hypothesis of small displacements and the simplification that the deformation of wood in the region of the joint is very small, compared to the deformation that occurs around the connectors (embedment).

Eq. (15) may be written as

$$\mathbf{a} \cdot \delta\mathbf{u} = \sum_{k=1}^{n} \frac{F^{(k)}}{\Delta^{(k)}} [(\mathbf{Q}_x^{(k)} \cdot \mathbf{u})(\mathbf{Q}_x^{(k)} \cdot \delta\mathbf{u}) + (\mathbf{Q}_y^{(k)} \cdot \mathbf{u})(\mathbf{Q}_y^{(k)} \cdot \delta\mathbf{u})] \tag{19}$$

In the equation above, all the terms are function of $\mathbf{u}$ and $\delta\mathbf{u}$.

Since δu_i are arbitrary values, we can obtain a set of six equations with 6 unknowns, which are the nodal displacements, u_j. Thus, we obtain a nonlinear equation, introduced before by Eq. (12), which, in terms of components may be written as

$$a_j = f_j(\mathbf{u}) = \sum_{k=1}^{n} \frac{F^{(k)}}{\Delta^{(k)}} [(\mathbf{Q}_{xj}^{(k)} \mathbf{Q}_x^{(k)} + \mathbf{Q}_{yj}^{(k)} \mathbf{Q}_x^{(k)}) \cdot \mathbf{u}] \tag{20}$$

where j is a free index varying from 1 to 6.

Eq. (12) together with Eq. (20) describes the mechanical behavior of connection.

If linear behavior is adopted for connectors, then the ratio $F^{(k)}/\Delta^{(k)}$ in Eq. (20) is constant and we find a linear equation for the rotational connection. The stiffnesses (or initial stiffnesses, in the case of nonlinear behavior) K_x, K_y and K_z are specially important to the design of rotational connections. They control the distribution of internal efforts (moment, shear force and normal force) among connectors, as shown by Santana and Mascia (2002) and Racher (1995). In summary, the forces are distributed among the connectors proportionally to the initial stiffness of each of them.

Initial rotational stiffness, K_z, is an important property of rotational connections and is given by

$$K_z = \sum_{k=1}^{n} K_\alpha^{(k)} r^{(k)2} \tag{21}$$

where K_α is the connector's stiffness in the direction of the force due to moment only.

4. Behavior of pin-shaped connectors

The behavior of connectors is usually represented by a relationship between the load transferred by the connector (F) and its relative displacement or slip (Δ). For connections with linear behavior, it is

$$F = K\Delta \tag{22}$$

where K is the stiffness of the connector, or slip modulus of the connector, dependent on the direction of loading in reference to directions of fibers in both pieces connected. Given its importance in the design of connections, the Eurocode 5 (CEN 1995) and other codes give specifications for its determination, in two levels of load (service loads and ultimate loads), since real connections do not behavior linearly.

Foschi (1974) proposed a model according to the connector behaves like a perfect elastoplastic beam on nonlinear elastoplastic foundation. The author developed a finite element model to obtain the displacements (deformed shape) of the connector and also the relative displacement (Δ) for a given load (F).

For connections with nonlinear behavior, Foschi and Bonac (1977) proposed a nonlinear general form, given by

$$F = [p_o + p_1\Delta]\left[1 - e^{-\frac{k_o\Delta}{p_o}}\right] \tag{23}$$

where F is the load per unit length along connector axis, Δ is the foundation displacement along connector axis and p_o, p_1 and k_o are parameters. The general form given by Eq. (23) was used by these authors to fit sets of numerical values of F and Δ obtained from computational simulations.

The procedure of analysis described above was implemented by the authors through another computer program in C++ language (Santana 2002), separated from main matrix analysis program mentioned before and used for characterization of individual connectors only. The input parameters are the material and geometrical parameters of connection, remarkably the properties of wood under embedment in the direction of the load in reference to the fibers directions, in both pieces connected. The output parameters are pairs of numerical values of F and Δ. From these pairs of numerical values, we find the parameters of Eq. (23). For practical reasons, this procedure must be accomplished for two perpendicular reference directions, assuming that for parameters for other directions, Hankinson's formula applies.

5. Behavior of wood under embedment of pin-shaped connector

The embedment strength and stiffness properties of wood are conventional properties, which describe the performance of wood, in general terms, from the displacement read under load applied through a very stiff pin, in the well known embedment test. If the load is divided by the specimen thickness, the parameter K_o, the initial slope of the curve load against displacement, defines the initial wood response or initial stiffness.

For connection with linear response, the stiffness expressed in this way is usually named *foundation modulus*. The foundation modulus is an important property of wood in the scope of study of connections. In this work, it will be denoted generally by k.

Trayer (1932), cited by Kuenzi (1955), investigated the strength and stiffness of wood under embedment. He proposed that the foundation modulus could be estimated by $k = E_c d/z_w$, where E_c is the modulus of elasticity in compression in the parallel/perpendicular direction to fibers and z_w is the foundation depth (usually not well defined). This expression seems to derive from the consideration of the portion of wood under the connector as simply compressed.

Later, Wilkinson (1971) experimentally founded a correlation of the foundation modulus with diameter of the connector used in the embedment test, and proposed a property, the *wood bearing elastic capacity*, attributed exclusively to wood and given by $c_o = k/d$. Wilkinson found experimentally a correlation between the wood density and the wood bearing elastic capacity, and found a correlation between this last and the diameter of the connector used in the embedment test. Both works cited above are landmark references for elastic foundation and present essential concepts to be considered in design criteria for stiffness of connections according to the basis used currently. However, codes do not include this behavior, and some of then give the embedment strength – a failure parameter - only.

The discussion above applies to linear relationship between load and displacement, what in fact may be assumed up to a certain load level. Some rules such as Eurocode 5: 1995 (CEN 1995) prescribe values for the embedment stiffness and the embedment strength. However, wood has a nonlinear behavior, which, together with the nonlinear bending of connectors, are responsible for the nonlinear behavior of connection. Foschi (1974) proposed a general relationship, given by

$$p(x) = [p_o + p_1 v(x)]\left[1 - e^{-\frac{k_o v(x)}{p_o}}\right] \tag{24}$$

where $p(x)$ is the load per unit length along connector axis, $v(x)$ is the foundation displacement along connector axis and P_o, P_1 and K_o are parameters.

We consider important to detach that in a rotational connection, wood is loaded in arbitrary direction in reference to the direction of fibers, in such way that we propose that expressions for other directions other than parallel and perpendicular to fibers be more investigated and developed.

6. Theoretical and experimental results for a simple structure

In the following, we presented tests accomplished in connections subjected to bending moments. The scheme of the tests is shown in Fig. 4.

Characterization of parameters for individual connectors (see Eq. (23)), in two perpendicular reference directions, from consistent results of embedment tests for wood and the computational program for characterization of individual connectors implemented by the authors (see previous section "Behavior of pin-shaped connectors"), lead to k_o = 8204 kN/m, p_o = 1.64 kN and p_1 = 888 kN/m. These parameters have the same values in both reference directions because connections forming an angle of 90 degrees are approximately isotropic.

The third function of vector function in Eq. (20) - relationship between ending moment and angular displacement, with all five others degrees of freedom fixed - was implemented trough an electronic sheet. Input data are the parameters k_o, p_o and p_1; the direction of the longitudinal axes of

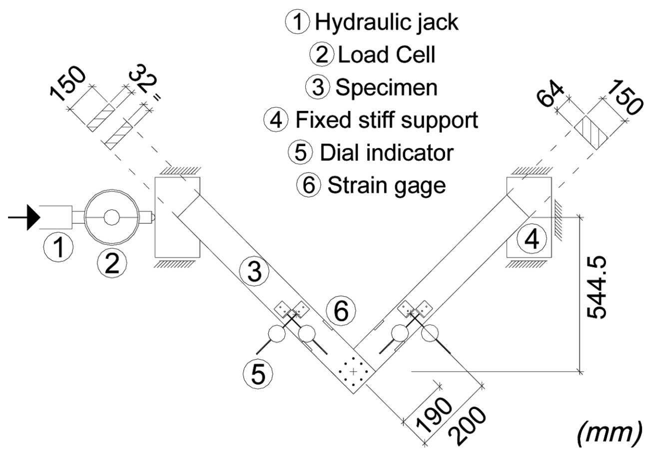

Fig. 4 Scheme of the test

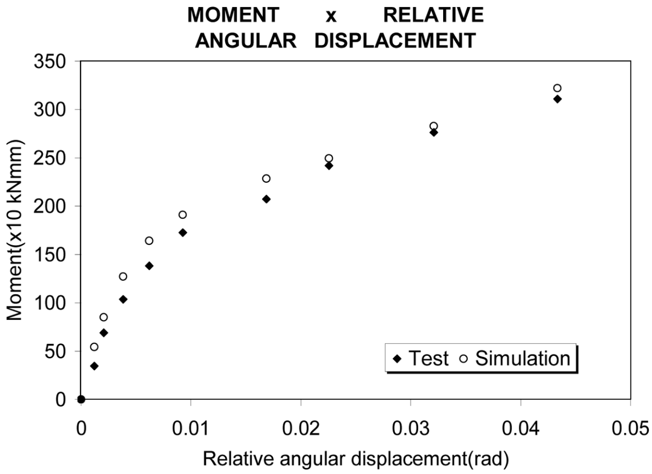

Fig. 5 Test results

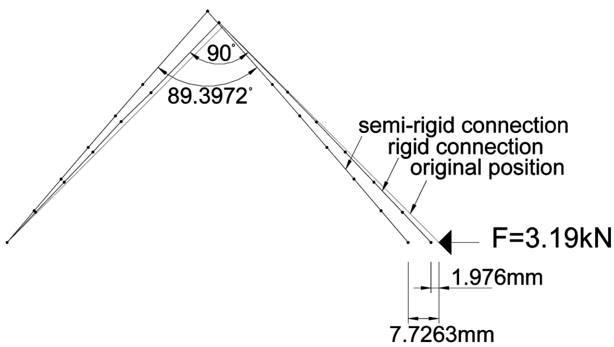

Fig. 6 Simulation results

wooden pieces with respect to a global reference direction; and the connectors' polar coordinates in the global reference system with origin in a rotational center.

Then, from a set of stepped displacements **u**, computation according to the presented model gives, as output data, the corresponding forces **a** (considering only angular displacements, we choose θ_z assuming increasing stepped values and corresponding values of M_z where obtained). We used six pairs of θ_z and M_z to build a curve. The results were shown in Fig. 5. An additional simple simulation showed that consideration of translational displacements would lead to small differences in the resulting forces, for this case.

Additionally, we used the parameters k_o for two reference directions and the Hankinson's formula to obtain $K_{\alpha,i}$ (initial connector stiffness) for each connector i and to build the parameter K_z (initial rotational stiffness) - see Eq. (21). We inserted this parameter in the software ANSYS (ANSYS Inc, 1995) by means of the MATRIX27 element. The result of the simulation is given in Fig. 6. Length of each moment arm is 544.5 mm. For a load of 3.19 kN, we have a bending moment in the connection of 1737 kNmm. Experimentally, this bending moment corresponds to a relative angular displacement of 0,01 *rad* (see Fig. 5). From the simulation with ANSYS we obtained a relative angular displacement of 90° minus 89.3972°, that is, 0,6028° (see Fig. 6) or 0,0105 *rad*.

7. Application

In this section we present an analysis of framed structures with semi-rigid connections with nonlinear behavior. We considered a frame as shown in Fig. 7 with span of 6.5 m, height of 3.5 m and ultimate load of 15 kN/m. In the same figure, the schematic bending moment diagram is shown. We considered E = 7180 MPa, A = 0.1 m^2 and I = 0.0020833 m^4 for both members, and width of 200 mm. We used columns with double rectangular section and beam with single rectangular section. After solving the structure through conventional analysis, we obtained the results shown in Table 1.

We designed the connections by calculating the forces over each connector due to internal efforts (M, N and V) as well as the resultant force, and comparing it to the resistance of the connector, calculated according NBR7190:1997 (ABNT 1997). The most critical bolt is not necessarily the most loaded because forces acts in different directions in relation to wood fibers direction in both pieces connected. For the design, we used a spreadsheet implement by the authors (Santana and

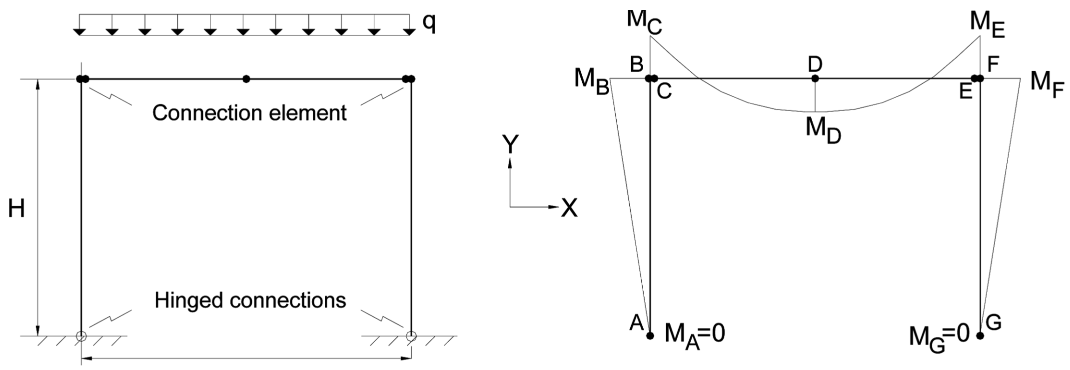

Fig. 7 Geometrical definitions

Table 1 Results obtained from conventional analysis

Section	N (kN)	V (kN)	M (kNm)
A	−48.75	−11.09	0
B	−48.75	−11.09	−38.81
C	−11.09	48.75	−38.81
D	−11.09	0	40.41

Table 2 Geometrical parameters of the distributions of bolts

Connection	Diameter of bolt (mm)	Radius of outer circular row (mm)	Total number of bolts (double shear)	Initial rotational stiffness (kNm)	Ultimate bending moment per shear (kNm)
C10	10	220	42	7740	23.60
C15	15	240	27	8080	23.60
C20	20	260[a]	18	8640	26.40
C25	25	270[a]	14	7870	26.90

[a] Radius may be considered impracticable.

Table 3 Estimated embedment parameters of wood – See Eq. (24)

	Parallel to fibers	Perpendicular to fibers
Parameter K_o (kN/m^2)	110×10^4	64×10^4
Parameter P_o (kN/m)	2.68×10^2	1.24×10^2
Parameter P_1 (kN/m^2)	0.03×10^4	0.19×10^4

Mascia 2002), by which the efforts are distributed among connectors according to their initial stiffnesses and distance from rotational center.

Several connections can be found, as shown in Table 2. However, not all could be considered practicable, because of the limited available space (height of section).

We found that the strength of connectors where highly limited by the low wood embedment properties (stiffness and strength) in the direction perpendicular to fibers direction. These properties were estimated with criteria from Brazilian standard NBR 7190:1997 (ABNT 1997) - specially safety factors. The embedment strength in the perpendicular direction to the fibers direction was estimated as one quarter of parallel direction. Initial stiffness of connectors were estimated with Kuenzi's model presented in a previous section, with foundation modulus obtained from tests results from Santana (2002) and given in Table 3 (parameter K_o).

Once the connections were designed, we proceeded to the semi-rigid connections analysis. We found the parameters describing the nonlinear behavior of the bolts – Eq. (23) - by means of the computational program for characterization of individual connectors, implemented by the authors as described before. We considered the embedding parameters of *Eucalipytus grandis* as shown in Table 3 (parameters K_o, P_o and P_1), fitted to results of embedding tests from Santana (2002). Data of steel were Young Modulus 21000 kN/cm^2 and Design Yield Strength 54.5 kN/cm^2.

For each of the bolt diameters considered, we obtained the parameters shown in Table 4. The parameters in both directions are the same because angle between members is 90 degrees. They are valid only for the connection with that particular geometrical configuration (members width and angle between members), and of course, for the wood species considered.

With the bolt parameters obtained and shown in Table 4, we performed the nonlinear analysis by means of the computational program for matrix analysis including connection elements, implemented by the authors as described before. This computational program allows linear analysis as well, through the consideration of initial response of bolts only (parameter k in Table 4), what leads to a linear rotational stiffness.

For each connection, we performed a linear and a nonlinear analysis. Results of internal efforts are shown in Table 5, with "NL" for nonlinear analysis and "L" for linear analysis. The displacement in the middle span varies from 9.8 mm from rigid connections analysis to 12 mm from linear analysis and 13 mm from nonlinear analysis.

It is important to notice that these results are dependent of the wood embedment behavior

Table 4 Parameters of the connectors used in the connections – See Eq. (23)

	d = 10 mm	d = 15 mm	d = 20 mm	d = 25 mm
Parameter k (kN/m)	55×10^2	79×10^2	93×10^2	100×10^2
Parameter p_o (kN)	3.4	5	5.5	5.5
Parameter p_1 (kN/m)	6×10^2	5×10^2	6×10^2	7×10^2

Table 5 Internal efforts obtained from nonlinear (NL) and linear (L) analysis

Connection(1)	Cross section	N(kN)		V(kN)		M(kN·m)	
		L	NL	L	NL	L	NL
C-10	A	−48.75	−48.75	-9.10	-8.51	0	0
	B	−48.75	−48.75	−9.10	−8.51	−31.83	−29.80
	C	−9.10	−8.51	48.75	48.75	−31.83	−29.80
	D	−9.10	−8.51	0	0	47.38	49.42
C-15	A	−48.75	−48.75	−9.17	−8.46	0	0
	B	−48.75	−48.75	−9.17	−8.46	−32.08	−29.60
	C	−9.17	−8.46	48.75	48.75	−32.08	−29.60
	D	−9.17	−8.46	0	0	47.14	49.62
C-20	A	−48.75	−48.75	−9.27	−8.41	0	0
	B	−48.75	−48.75	−9.27	−8.41	−32.44	−29.44
	C	−9.27	−8.41	48.75	48.75	−32.44	−29.44
	D	−9.27	−8.41	0	0	46.78	49.78
C-25	A	−48.75	−48.75	−9.12	−7.91	0	0
	B	−48.75	−48.75	−9.12	−7.91	−31.92	−27.69
	C	−9.12	−7.91	48.75	48.75	−31.92	−27.69
	D	−9.12	−7.91	0	0	47.30	51.53

(1) For rigid connection, see Table 1.

modeling and connector behavior modeling, which governs the nonlinear behavior of structure. Although we used estimated parameters for wood from some tests results and other experimental criteria (Table 3), results are consistent with a real range of wood properties. Besides, more important than considering the values individually, is to consider the comparison between nonlinear and linear analysis, as well as between analysis with rigid and semi-rigid connections.

8. Discussion

The comparison of the rigid connections' with semi-rigid connections' calculations demonstrates that the semi-rigid connections lead to a redistribution of bending moment diagram and that different connections cause different redistributions. If we designed the connection for the new bending moment, we would obtain a new redistribution. However, we can consider suitable the first results obtained, in favor of connection safety, since new results would lead to less moment in the connection.

The connections' parameters are the slip modulus and strength of individual connectors for purpose of rotational connection design, and the nonlinear parameters for embedment behavior of wood for purpose of determination of connectors' parameters for matrix analysis.

The slip modulus is not promptly available in most codes for both parallel and perpendicular as well as criteria for two members oriented in two different directions. The embedment stiffness of wood is not available as well. We considered a particular functional form firstly proposed by Foschi (1974), with three parameters, but other nonlinear behavior functions may be considered.

Nonlinear parameters may be determined by models as shown in this paper. The comparison of the semi-rigid linear and nonlinear calculations shows that the results are very similar inside the limit imposed by the failure of connection as. Possibly, the linear analysis is sufficient for the design of frames with semi-rigid connections.

The initial stiffness parameter of connectors (slip modulus) strongly affects the behavior of the connection and the structure by consequence. It is necessary that the slip modulus be normalized and well spread among designers. It is important to notice that, according to the LRFD (Load and Resistance Factor Design), the stiffness is usually minored to consider factors as deformations along time and others. However, the lower the stiffness, the lower will be the resulting moment in the connection, and the higher will be the moment in the middle span. Thus, the design will not be necessarily in the safe side.

9. Conclusions

The solution of framed structures involves several aspects not usually treated in the conventional design of wooden structures. The analysis involves several parameters, which, due to wood orthotropy and the variety of geometry of connections, has to be treated with care. In this work, we highlight the interrelations among all these aspects, in which extents the design may depend on experimental tests or in analytical models and material properties. These aspects are: the properties of wood as a foundation, the connector behavior, and the construction of the rotational stiffness behavior.

The consideration of structures with semi-rigid connections may be accomplished with a matrix method, widely available inside finite element analysis commercial programs. Linear analysis may be reasonably used for the design of semi-rigid connections, according to a linear or bilinear behaviour of connections. In spite of that, nonlinear analysis applies to the modeling of the real behavior of the connections. Anyway, the effect of connections deformation is significant, especially for displacements and moment distribution.

We consider that the aspects which lacks normalization are both resistance and stiffness of individual connectors in service and ultimate limit; method for construction of the behavior of rotational connection; properties of wood under embedment; criteria for the solution of the structure (design of connections). The technology of connections shall lead for stiffer connections as well as ductile.

Effects such as second order effects and shear effects were not considered in the comparative analysis between analysis rigid and semi-rigid connections.

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Notation

a	: vector of actions in the nodes
a_i	: component i of the vector **a**
A	: area
A_i	: angular coordinate defining the position of the connector i in the connection
B	: matrix relating the nodal displacements and the deformation at a point
d	: connector diameter
E	: modulus of elasticity in longitudinal direction
F	: force applied in a single connector
F_x	: force in the x-direction
F_y	: force in the y-direction
I	: moment of inertia
k, k_o	: wood foundation modulus
K_o	: slip modulus of connector
K_x	: stiffness of connection in reference direction x
K_y	: stiffness of connection in reference direction y
K_z	: rotational stiffness of connection
M	: bending moment
N	: normal force; number of nodes of the structure
$\mathbf{Q}_x$	: vector relating nodal displacement in the x-direction with slip of connectors
$\mathbf{Q}_y$	: vector relating nodal displacement in the y-direction with slip of connectors
S	: stiffness matrix
$\mathbf{S}^t$	: tangent stiffness matrix
u	: displacement in the x-direction
v	: displacement in the y-direction
V	: shear force
θ_z	: angular displacement around z-direction