Structural Engineering and Mechanics, Vol. 31, No. 3 (2009) 265-276
DOI: http://dx.doi.org/10.12989/sem.2009.31.3.265

Efficacy of pushover analysis methodologies: A critical evaluation

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(Received April 9, 2007, Accepted January 15, 2009)

Abstract. Various Pushover analysis methodologies have evolved as an easy as well as designers-friendly alternative of nonlinear dynamic analysis for estimation of the inelastic demands of structures under seismic loading for performance based design. In fact, the established nonlinear dynamic analysis to assess the same, demands considerable analytical and computational background and rigor as well as intuitive insight into inelastic behavior for judging suitability of the results and its interpretation and hence may not be used in design office for frequent practice. In this context, the simple and viable alternative of Pushover analysis methodologies can be accepted if its efficacy is thoroughly judged over all possible varieties of the problems. Though this burning issue has invited some research efforts in this direction, still a complete picture evolving very clear guidelines for use of these alternate methodologies require much more detailed studies, providing idea about how the accuracy is influenced due to various combinations of basic parameters regulating inelastic dynamic response of the structures. The limited study presented in the paper aims to achieve this end to the extent possible. The study intends to identify the range of applicability of the technique and compares the efficacy of various alternative Pushover analysis schemes to general class of problems. Thus, the paper may prove useful in judicial use of Pushover analysis methodologies for performance based design with reasonable accuracy and relative ease.

Keywords: capacity curve; demand curve; nonlinear static procedure; performance based design; performance point; Pushover analysis.

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1. Introduction

The nonlinear static procedure popularly known as Pushover analysis is a means of evaluation of performance based structural demand and is becoming increasingly popular to the investigators in the context of performance based seismic design because of its simplicity and easy applicability. On the other hand, nonlinear dynamic analysis can assess the inelastic behavior of structure with sufficient reliability and is a powerful tool in predicting the inelastic demands in every element of the structural system. However, use of the same requires considerable effort and time that makes it impractical and conceptually difficult for regular use by the practicing engineers.

Pushover analysis uses an equivalent single-degree-of-freedom system to estimate the dynamic response of multi-degree-of-freedom system as has been referred in the reports ATC-40 (ATC 1996) and FEMA-356 (BSSC 2000). In a multidegree of freedom system responding in multimodes during vibration, Pushover analysis can consider only the deformation to take place maintaining a fixed shape. Hence, the capacity curve providing a monotonic force-displacement relationship is obtained by continually pushing the structure maintaining an appropriate but single dominating shape, popularly the first mode shape. The capacity curve thus obtained through non-linear static analysis is expressed in acceleration-displacement response spectrum (ADRS) format. Response spectrum curves drawn with increased system damping to incorporate hysteretic energy dissipation in the equivalent sense are superimposed on such ADRS spectrum. The point of intersection of these two curves is supposed to yield the performance point yielding the inelastic demand in equivalent sense. Hence, there are two fold approximations in this process of replacing nonlinear dynamic analysis through linear dynamic analysis with increased equivalent damping and nonlinear static analysis. The first and fundamental approximation lies in the philosophy of replacing the accurate but complicated and rigorous method through a set of relatively simple and easily visualizable methods. On the other hand, the second approximation stems from the consideration of a single predominant mode to represent the response of a multimodal system. Some literatures (Chopra and Goel 2002, Chopra and Goel 2003) have studied well the implication of this second category of approximation and suggested improvement on this issue. On the other hand, the impact of inaccuracies introduced due to the first approximation, which is rather the approximation of the philosophy itself, seems to be yet an area initiating the need of a very detailed careful study, in the backdrop of a recent study (Chopra and Goel 1999) described later. In fact, such a study may help to establish the applicability or inapplicability of the Pushover analysis for various classes of problems very transparently and clearly. In fact, the research conducted so far (Krawinkler and Seneviratna 1998, Mwafy and Elnashai 2001) reveals that Pushover analysis performs well for short period systems whose response is dominated by first mode. However, it is needed to realize whether the short period or the dominance of the first mode plays most important role to regulate the accuracy. Another study (Chopra and Goel 1999) as mentioned earlier, though pointed that the inaccuracy involved in the philosophy itself may result in an error to the extent of 50%, a more specific study indicating the level of acceptability of various varieties of Pushover analysis methodologies for various ranges of lateral periods and extents of inelasticity are needed to be made. While a gap still remains here for throwing more light on the domain of acceptability of the classical philosophy of Pushover analysis methodologies, various improvements of such philosophy are attempted to be made in terms of proposing adaptive Pushover analysis (e.g., Gupta and Kunnath 2000, Papanikolaou and Elnashai 2005), exhibiting minor improvement (Antoniou and Pinho 2004) only, or clubbing the philosophy with that of incremental dynamic analysis (e.g., Vamvatsikos and Cornell 2002, Vamvatsikos and

Cornell 2005). However, such rapid advancement may more easily be visualized, understood and judged, if the gap in the fundamental level is filled up. The present paper is humble effort in this direction.

On the other hand, a recent study (Akkar *et al.* 2003) makes an attempt to critically judge the efficacy of Displacement coefficient method - a well known variety of Pushover Analysis methodology. This study shows that the method results in an underestimation in inelastic displacement demand particularly for structures with stiffer periods (period less than 1 sec.) and higher response reduction factor (even beyond a value greater than 2). In this context, present study attempts to make a critical performance evaluation of all varieties of Pushover analysis methodologies, viz., Procedure A, Procedure B, Procedure C and Displacement coefficient method. This is done through a set of single degree of freedom systems chosen for all possible combinations of basic parameters like natural period, response reduction factor etc., isolating their effects from that arising from complexity of modal combination in multi-degree of freedom system. The performance evaluation of displacement coefficient method, though studied earlier, is included to emerge a relative picture of performance of various methodologies as well as to confirm the trends observed in earlier study (Akkar *et al.* 2003).

2. Philosophies of various pushover analysis methodologies

The detailed formulations and steps behind the Pushover analysis methodologies are outlined in ATC-40 (ATC 1996). The basic methodology of the analysis involves plotting of load displacement curves in ADRS format (base shear versus roof displacement) and superposing the reduced demand spectrum in the same two-dimensional space to obtain the point of intersection of these two curves. The demand denoted through the abscissa of this point, terminologically known as performance point, is claimed to be an accurate enough prediction of inelastic seismic demand in report ATC-40 (ATC 1996). Three standard varieties of graphically iterative procedure are prescribed in the same report ATC-40 (ATC 1996) based on the above philosophy. These are referred to as Procedure A, B and C respectively in the present paper following relevant report (ATC 1996). The damping of the system, during its inelastic vibration, can be viewed as a combination of viscous damping inherent in the structure and hysteretic damping represented by the area of the loops of the plot of base shear against roof displacement. The literature (Chopra 1995) demonstrates that the hysteretic damping may be represented as equivalent viscous damping and may be used to estimate spectral reduction factors. Thus the elastic 5% damped response spectrum is reduced by multiplying its ordinates with the spectral reduction factors to plot the spectrum for increased effective damping. To evaluate the equivalent damping the energy dissipated by the structure is considered from a single idealized hysteresis loop applicable for ductile building and short duration earthquake. For structures undergoing number of load reversals causing rapid deterioration in its hysteretic energy dissipation capacity due to stiffness and strength deterioration as well as pinching, a factor k has been introduced to compute equivalent increased damping as has been discussed in the report ATC-40 (ATC 1996).

In fact, philosophically all these three procedures aims to arrive at a point which is associated with an inelastic displacement, d_{pi}, requiring the strength ordinate (expressed in terms of acceleration) available from the capacity curve exactly being same i.e., meeting the strength demand obtained from the ADRS spectrum drawn due to equivalent effective damping, β_{eff}, which is adequate for that particular inelastic displacement d_{pi}. Procedure A tends to converge to such a point through successive iterations. Procedure B calculates several upper and lower bound combinations of strength demand in terms of spectral acceleration S_a, β_{eff} and d_{pi} as possible demand indexes and ultimately tries to achieve the aim by seeing for which combination, strength demand (expressed in terms of acceleration S_a) becomes same as the available capacity expressed in terms of acceleration with same d_{pi}. Procedure C uses two such upper bound and lower bound combinations which lead to a cruder convergence as compared to Procedure B, in many cases. The issue may be well understood from the step-wise elaborations of these methods in the standard literature ATC-40 (ATC 1996).

The other standard method of Pushover analysis is Displacement coefficient method. This does not involve capacity and demand curves. On the other hand, this method uses elastic spectrum based on the equivalent lengthened period which can account for the large period behavior arising due to the inelastic excursions, even using an elastic spectral ordinate. Further, various factors are multiplied with the spectral abscissa corresponding to such empirically estimated lengthened period to incorporate the effect of changing modal displacement to roof displacement, the effect of continued plastic yielding, the effect of the shape of hysteresis model and that due to increased second order effect.

3. Procedure, modeling and parametric variations

A large number of single degree of freedom systems is studied by the four categories of Pushover Analysis as well as by rigorous nonlinear dynamic analysis using the standard elasto-plastic hysteresis behavior. The comparison of the results helps to judge the efficacy of the methods. To cover the feasible range of structures, a lateral period (T) range of 0.1 to 3.0 sec. has been considered with an interval of 0.1 sec. for short to medium period structures and an interval of 0.5 sec. for long period structures.

Responses are presented separately for short period range (0.1 sec. to 0.5 sec), medium period range (0.5 sec. to 1.0 sec.) and long period range (1.0 sec. to 3.0 sec.) for convenience of understanding. The response reduction factors for buildings and other structures may vary in a wide range of 1 to 8 depending on importance factors, redundancies of the system as well as other sociotechnical issues presented in the technical document FEMA-273 (BSSC 1997). Hence, the present study considered $R_\mu = 1$, 2, 4, 6 and 8 to quantify various extents of inelastic performance levels. For the purpose of rigorous nonlinear dynamic analysis, the nonlinear equations of motion are numerically solved in the time domain by Newmark's β-γ method. Newmark's parameters are chosen as $\beta = 0.5$ and $\gamma = 0.25$ to achieve unconditional stability. The time step for integration is taken less than T/1000 sec. This time step is found to be sufficiently small from sample convergence studies conducted in each case. The accuracy has been further improved by employing modified Newton- Raphson's technique in each incremental time step. 5% of critical damping is considered to evaluate the displacement demand by nonlinear dynamic analysis.

4. Ground motions

Two artificially generated earthquake time histories consistent with the design spectrum of the Indian earthquake code of practice, i.e., IS 1893-1984, are generated by a procedure detailed in the

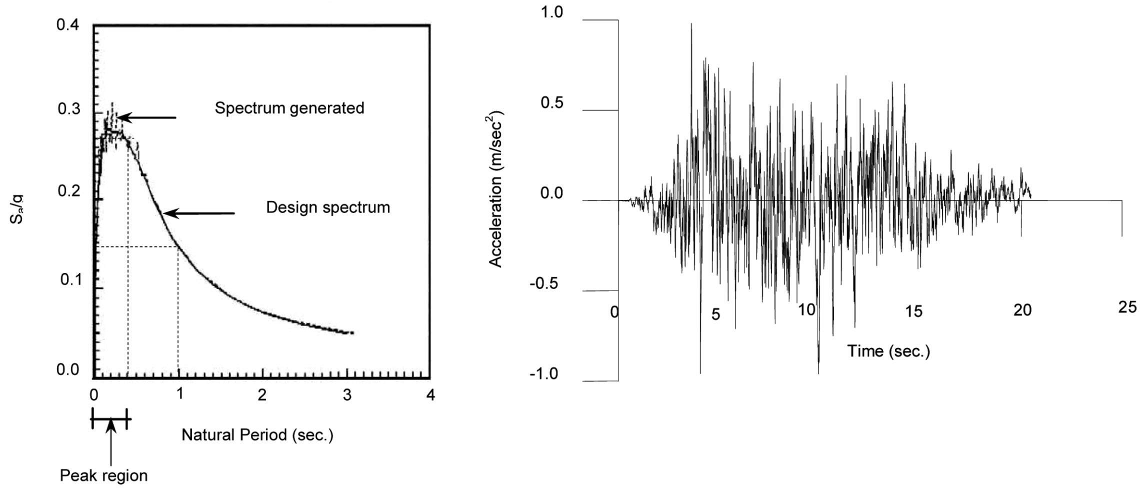

Fig. 1 Spectrum of simulated ground motion, design spectrum of IS: 1893-1984 corresponding to 5% damping and acceleration-time history

literature (Khan 1987). The design spectrum of the Indian earthquake code is derived from the well accepted design spectrum developed in reference (Housner 1959). The acceleration time histories as well as the response spectrum generated from it, superposed with the target codal design spectrum as presented in Fig. 1, which is extracted from the literature (Dutta *et al.* 2004), exhibits close resemblance. Thus, these time histories are expected to have the characteristics in terms of the frequency contents as intended through this well accepted spectrum and hence adopted in the present investigation likewise a few recent investigations (Dutta *et al.* 2004, 2005).

5. Results and discussion

To explain the methodologies followed for computation of inelastic demands through Procedure A, B, C and Displacement coefficient method, the solution of a numerical example with following details are presented in schematic form in Fig. 2 to Fig. 5.

An idealized SDOF system with mass equal to 1 kg having lateral stiffness of 157.91 N/m yielding lateral natural period of vibration T = 0.5 sec is considered. The elastic strength demand due to spectrum consistent ground motion from the time history analysis is computed as 0.852 N. Subsequently, yield strength of the system, relevant to a response reduction factor $R_\mu = 4$, is set to 0.213 N. 5% damped elastic response spectrum has been obtained from the referred code (IS 1893, 1984) directly. The performance points obtained for the example system, using Procedure A, Procedure B and Procedure C are 1.542489 cm, 1.542489 cm and 1.522 cm respectively. Target displacement of the same system obtained through Displacement coefficient method is 0.999867 cm, while the exact nonlinear dynamic analysis yields a displacement value equal to 1.75284 cm. For this particular case, Displacement coefficient method provides lower estimate of the target displacement in comparison to the exact inelastic dynamic analysis. In fact, a considerably lower bound value is yielded by Displacement coefficient method in most of the cases as observed through the results of the large number of systems studied and presented herein.

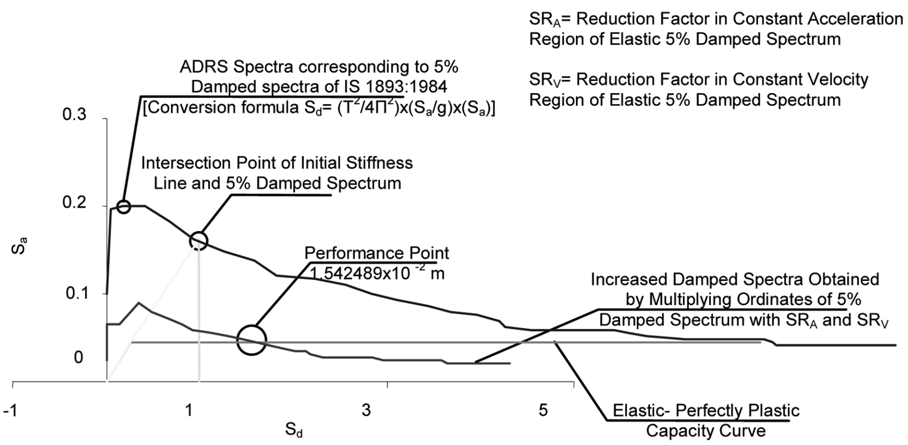

Fig. 2 Schematic representation of procedure A

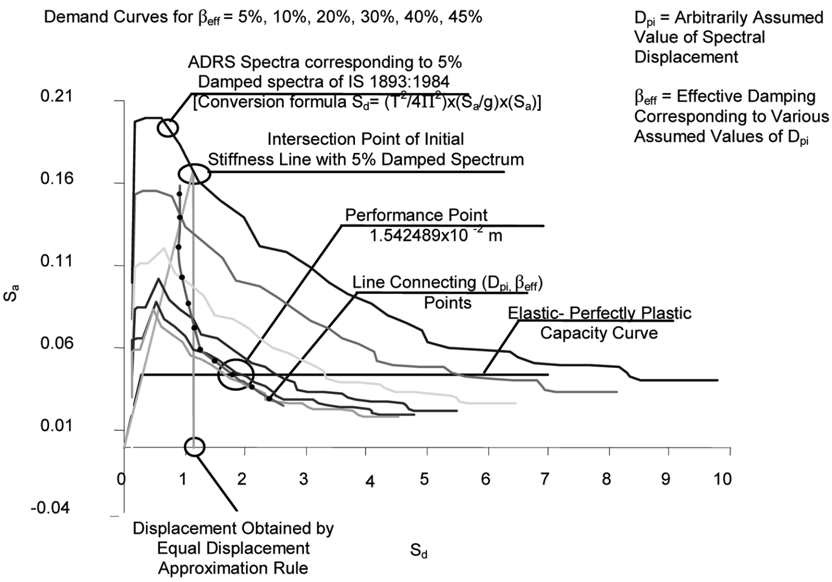

Fig. 3 Schematic representation of procedure B

For gauging the performance of various procedures of Pushover Analysis, the displacement is calculated by each of the four procedures and the detailed non-linear response history analysis. Response so obtained is plotted as a function of lateral period for the sake of comparison. Hence each plot contains five curves corresponding to five different procedures. The various curves corresponding to rigorous nonlinear analysis, Procedure A, Procedure B, Procedure C and Displacement coefficient method are designated by D_i, D_A, D_B, D_C and D_{DCM} respectively. Furthermore, five such plots are presented for five different response reduction factors for each of the three period ranges, namely 0.1 sec. to 0.5 sec., 0.5 sec. to 1.0 sec. and 1.0 sec. to 1.5 sec. The set of results corresponding to these three period ranges are depicted in Figs. 6, 7 and 8 respectively, each of which contains five plots each with five curves as described above.

Figs. 6 to 8 show that the performances of all Pushover Analysis methodologies are reasonably

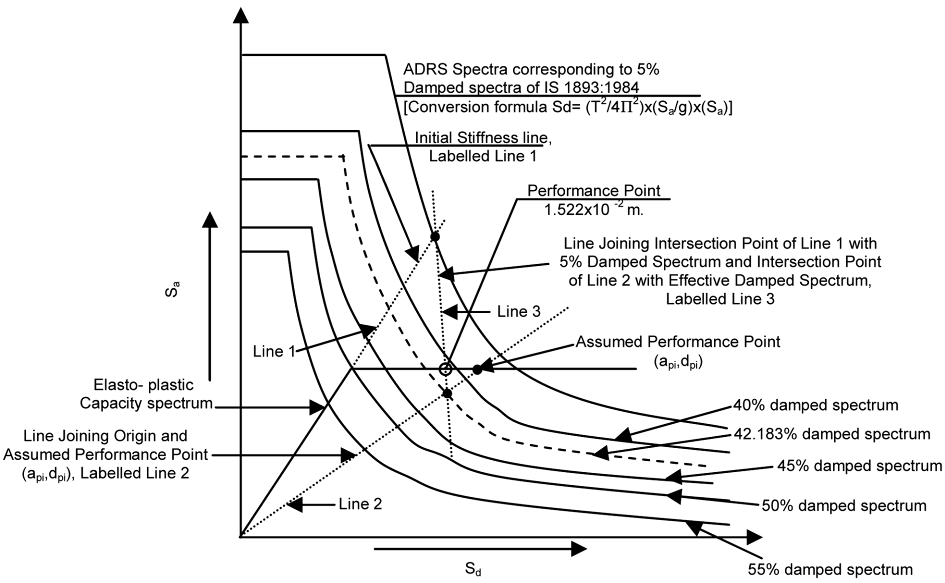

Fig. 4 Schematic representation of procedure C

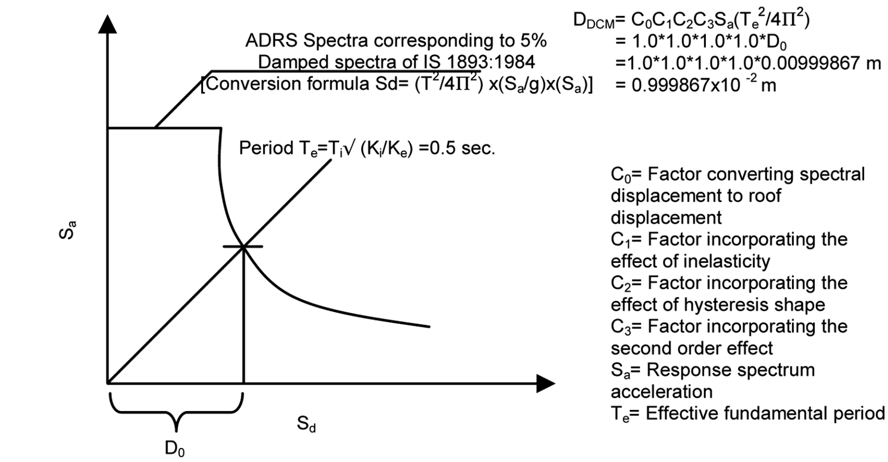

Fig. 5 Schematic representation of displacement coefficient method

accurate for systems with lower response reduction factor implying lesser inelastic action, which is physically intuitive. Figs. 6 and 7 show that Procedures A, B and C perform better for upto a response reduction factor, $R_\mu = 4$ for short to medium period systems. On the other hand, Fig. 8 shows that performance of Displacement coefficient method seems to be acceptable upto $R_\mu = 2$ for long period systems. For higher response reduction factors, namely, $R_\mu = 6$ and 8, most of the times, procedures A, B and C may yield an upper bound for very short period systems while a considerably lower bound for medium to long period systems.

Displacement coefficient method always provides a lower bound and in general performs worser than the others over the entire feasible range of natural periods of structures, for higher response reduction factors $R_\mu = 6$ and 8. This may be expected in view of the following. The factor C_1 is

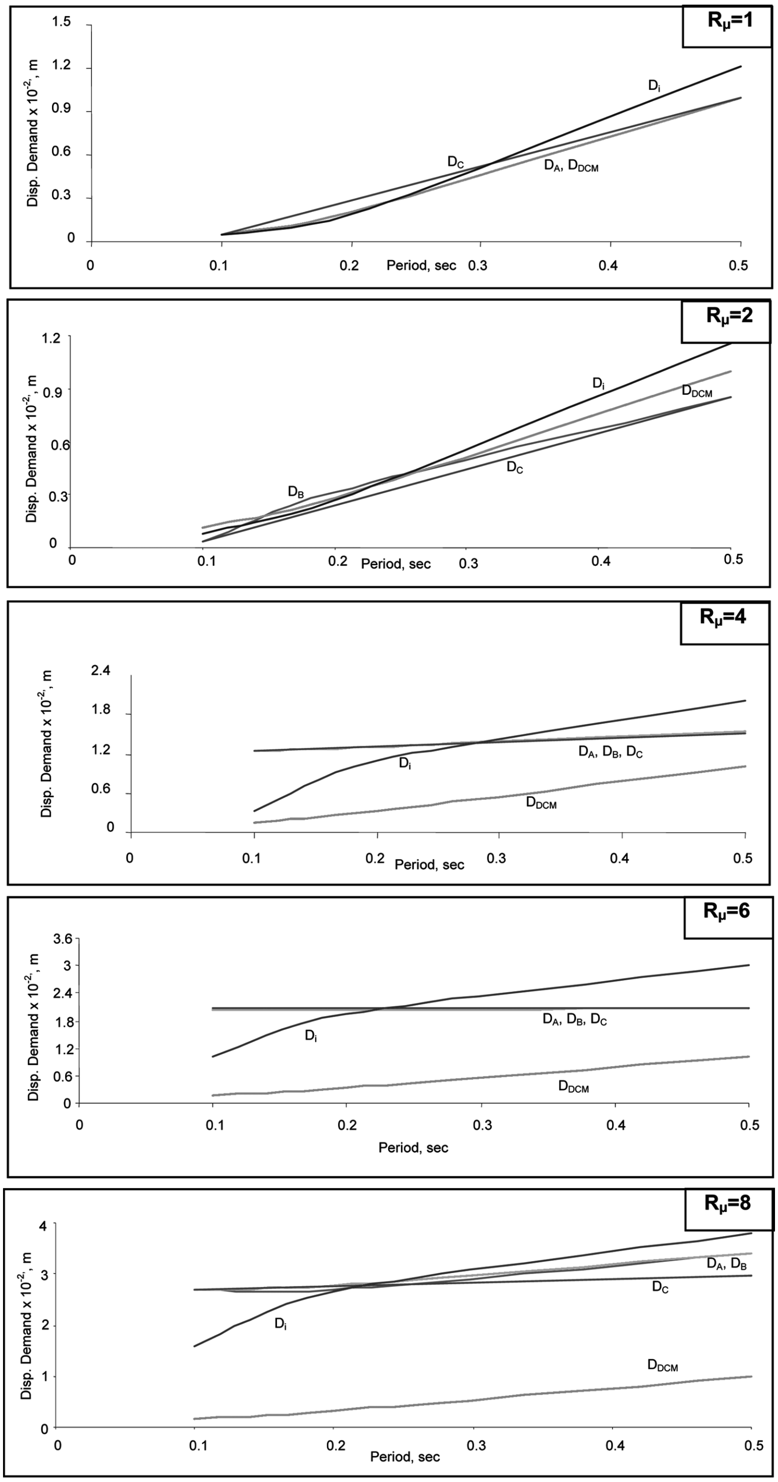

Fig. 6 Displacement demand obtained by various methods for short period systems

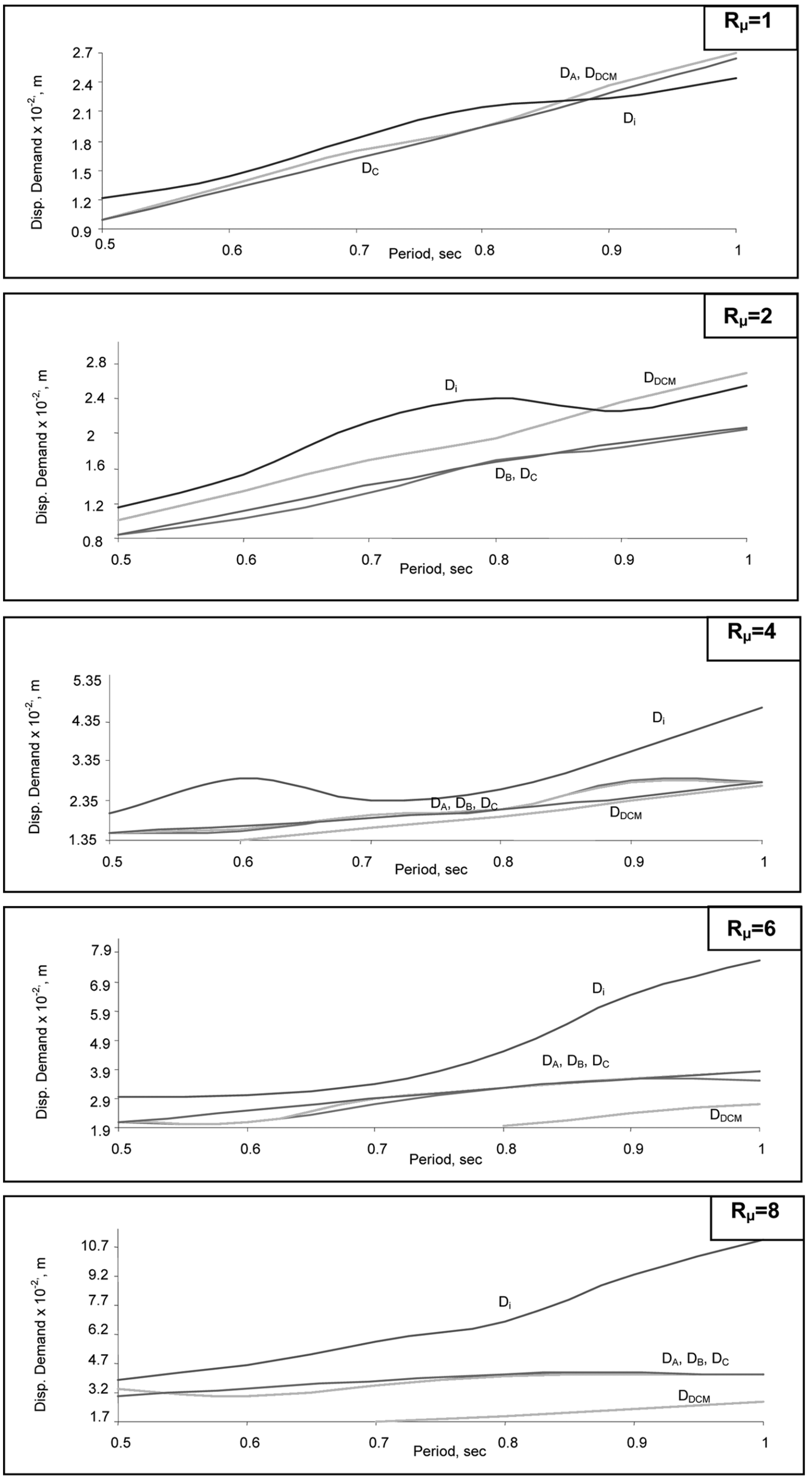

Fig. 7 Displacement demand obtained by various methods for medium period systems

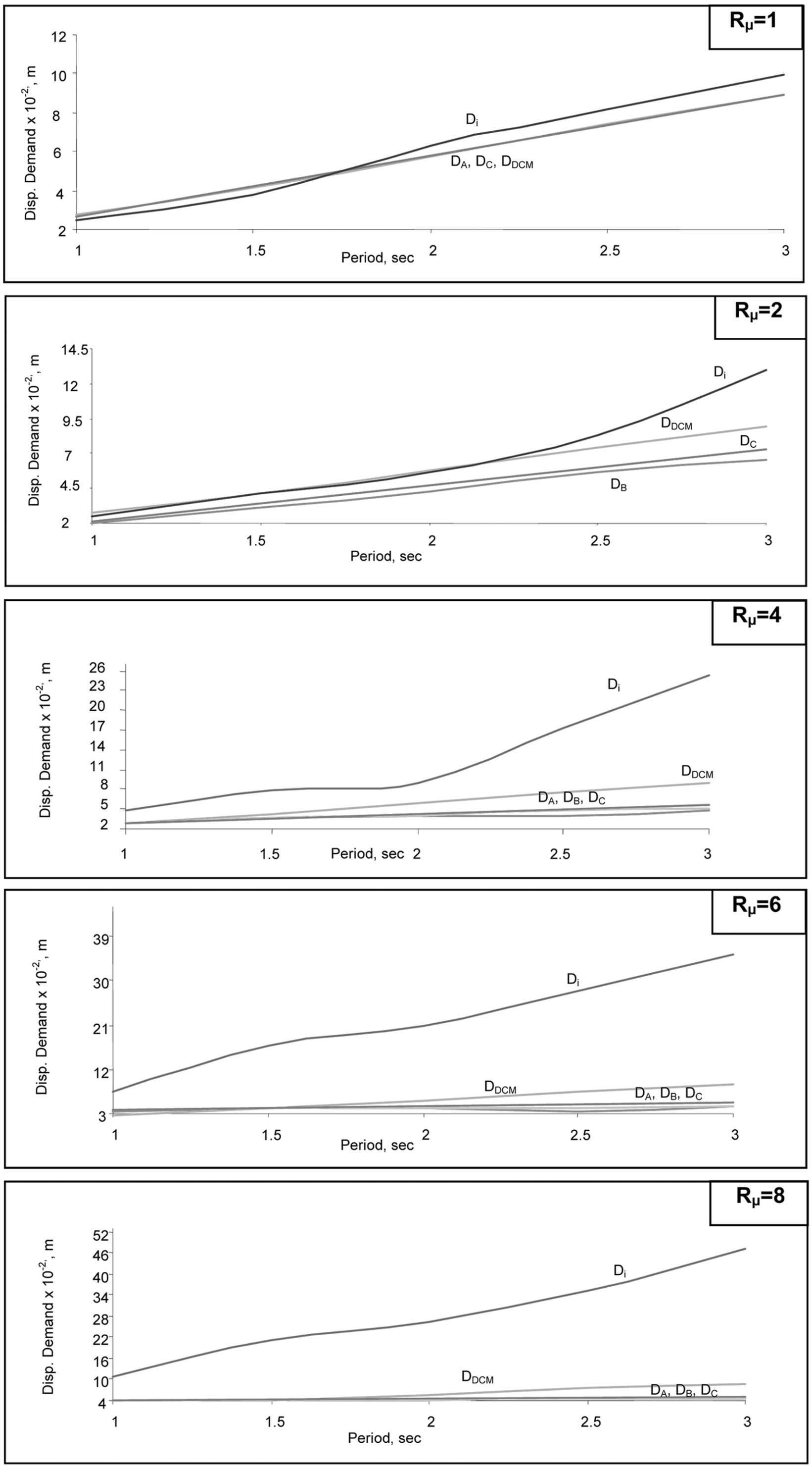

Fig. 8 Displacement demand obtained by various methods for long period systems

used in this method to incorporate the effect of inelasticity within an elastic spectral displacement ordinate and the formulation of C_1 is available in ATC 40 (ATC 1996). However, the formulation of the factor C_1 does not recognize the effect of higher inelasticity through adequate participation of response reduction factor R_μ over a large domain of effective fundamental period as is obtained from the previous report (ATC 1996).

Thus, if making an approximate analysis is the only option, then Procedure A, B, or C can only be used with proper caution, for systems with short period but higher response reduction factors. Curves presented in Fig. 6 may be of help to arrive at a wise judgement in this regard.

6. Conclusions

Pushover Analysis is an approximate and made-easy procedure of replacing rigorous nonlinear dynamic analysis through a drawn-down damped spectrum based linear analysis and a simplistic nonlinear static analysis for obtaining the inelastic demand of a structure. However, judging the sanctity of such methodology in a complete sense is an emerging prerequisite before encouraging its wide spread use for a performance based design. The limited study presented in this paper leads to the following broad conclusions in this regard.

1. The solution of simple numerical example is demonstrated by four standard procedures of Pushover Analysis for better understanding of the implementation of these methodologies. This also offers an impression of their relative performance to assess inelastic seismic demand.
2. All varieties of Pushover Analysis perform poorly for yielding the inelastic demands of the structures with higher response reduction factors.
3. For short to medium period systems, Procedure A, B or C can be used upto a response reduction factor $R_\mu = 4$.
4. For long period systems, the displacement coefficient method may yield reasonable results upto a response reduction factor $R_\mu = 2$. In fact, such observation is broadly in line with the observation in an earlier study (Akkar *et al.* 2003).
5. For higher response reduction factors, Procedures A, B or C may be used with adequate judgement to have a broad idea about the inelastic demands for some of the short period systems. The curves presented in Fig. 6 may be used for making a broad judgement in this regard.

Though a few earlier studies (e.g., Goel and Chopra 1999) has pointed out about the inaccuracies involved in Pushover analysis methodologies, the present paper may prove useful to choose the right combinations of lateral period and response reduction factor for which Pushover Analysis methodologies may be used with reasonable accuracy in performance based design.

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