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Technical Note

Estimation of short-term deflection in two-way RC slab

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1. Introduction

Deflection control generally governs the concrete slab thickness in order to satisfy serviceability requirements. However, unlike the case of one-way slabs, there is presently no simple procedure available in design codes as well as in the published literature to estimate maximum deflections of reinforced concrete (RC) two-way slabs. The present study attempts to address this problem, with regard to estimation of short-term deflections in RC two-way slabs using finite element analysis with appropriate stiffness modifiers (flexural and torsional). A modified Branson's formulation is proposed to account for the complex effects of flexural cracking and tension stiffening, and a simple modification is proposed to account for torsional cracking. The parameters in the proposed formulation are determined empirically, and the procedure validated by 23 experimental load-deflection data sets reported in the literature for uniformly loaded rectangular panels with different support conditions.

2. Proposed formulation

For an RC two-way slab subjected to bi-directional bending and torsion, the stiffness changes

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throughout the slab due to cracking, depending on the magnitude of the principal tensile stress. The cracking introduces anisotropic behaviour which varies from point to point. In the proposed approach, the idea is to retain the assumed isotropic behaviour in the elasticity, but also incorporate modifications in the sectional properties, by introducing the concept of effective flexural and torsional rigidities. For convenience, these stiffness properties are assumed to be uniform throughout the slab. The software package SAP2000 has been used for finite element analysis. In situations where hogging moments are also encountered at the slab edges, the flexural stiffness modifiers are suitably adjusted considering a weighted average of the effective flexural rigidity at the mid-span and at the supports.

2.1 Modification of flexural rigidity

The behaviour of two-way RC slabs is assumed to be linearly elastic till the slab cracks, and subsequently the flexural rigidity decreases on further loading. In order to accommodate the effects of flexural cracking in the evaluation of short-term deflections, suitable flexural stiffness modifiers are proposed to be applied in the short span ($\eta_x = D_{eff,x}/D_{gr}$) and long span ($\eta_y = D_{eff,y}/D_{gr}$) directions in the finite element analysis input.

The effective flexural rigidity ($D_{eff,x}$ and $D_{eff,y}$) can be conveniently modelled as a weighted average of the gross and fully cracked flexural rigidities, using the concept originally proposed by Branson (1977) for beams and adopted by ACI 318 (2005). However, a comparison (Govind 2006) of the code-based prediction of deflections in two-way wall supported slabs (using Branson's expression of effective moment of inertia) with experimental load-deflection data reveals an unsatisfactory fit in the post-cracking region. Nevertheless, the trends suggest that a modification of Branson's formulation may result in theoretical estimates of short-term deflection that agree closely with the experimental data. The following expressions for effective flexural rigidity ($D_{eff,x}$ and $D_{eff,y}$) are proposed accordingly

$$D_{eff,x} = \left(\frac{M_{cr}}{M_{xx}}\right)^{\mu} D_{gr} + \left(1 - \left(\frac{M_{cr}}{M_{xx}}\right)^{\mu}\right) D_{cr,x} \tag{1a}$$

$$D_{eff,y} = \left(\frac{M_{cr}}{M_{yy}}\right)^{\mu} D_{gr} + \left(1 - \left(\frac{M_{cr}}{M_{yy}}\right)^{\mu}\right) D_{cr,y} \tag{1b}$$

where, $D_{gr} \equiv$ flexural rigidity of the gross section, $D_{cr} \equiv$ flexural rigidity of cracked section, $M_{cr} \equiv$ cracking moment per unit width, M_{xx}, $M_{yy} \equiv$ applied maximum moment per unit width (obtainable from finite element analysis with gross stiffness), and $\mu \equiv$ a factor to be determined empirically. The relevant expressions for D_{gr}, D_{cr} and M_{cr} can be obtained from literature (Pillai and Menon 2003)

When the two-way slab panel extends beyond the edge supports (which may be wall supports or beam supports), or when there is rotational fixity provided at the edges, hogging bending moments are generated near the support regions. A weighted average of flexural rigidities at mid section and support sections is proposed, in line with ACI 318 recommendation for beams

$$D_{eff,avg} = 0.5 D_{eff,mid} + 0.25(D_{eff,1} + D_{eff,2}) \tag{2}$$

where, $D_{eff,avg} \equiv$ average effective flexural rigidity, $D_{eff,mid} \equiv$ effective flexural rigidity at mid section, $D_{eff,1}$ and $D_{eff,2} \equiv$ effective flexural rigidity at support sections. This average value of D_{eff} should be computed separately along X- and Y- directions.

2.2 Modification of torsional rigidity

In two-way slabs, twisting moments M_{xy} act along with bending moments M_{xx} and M_{yy}, and their values are readily available in the finite element analysis output. The loss in torsional stiffness on account of torsional cracking can be incorporated by modifying the torsional rigidity, $D_{eff, xy} = \eta_{xy} \times D_{gr, xy}$, where $D_{eff, xy}$ is the effective torsional rigidity and $D_{gr, xy}$ is the torsional rigidity based on the gross section.

The reduction in torsional stiffness is usually very significant in RC flexural members on account of torsional cracking. It is assumed the torsional stiffness reduction does not occur when the principal moment (obtainable as M_1 and M_2 in finite element analysis) does not exceed M_{cr}. The reduction is assumed to be 90 percent (Pillai and Menon 2003) when the principal moment approaches an ultimate value M_u. For simplicity, a linear variation of the torsional stiffness modifier η_{xy} is assumed between the un-cracked and ultimate stages

$$\eta_{xy} = 1 - 0.9\left[\frac{M_{xy} - M_{cr}}{M_u - M_{cr}}\right] \tag{3}$$

2.3 Empirical evaluation of parameter μ

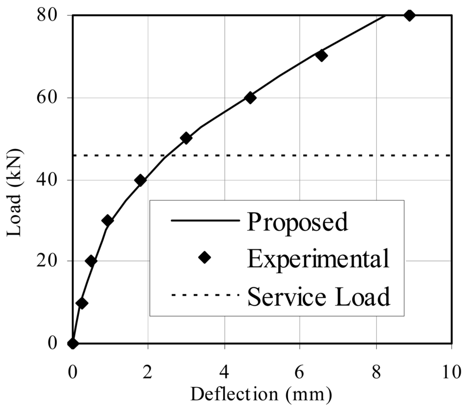
In order to arrive at an approximate value of the coefficient μ in Eq. (1), it is necessary to make comparisons of the deflection predictions with available experimental data. Load-deflection characteristics of 23 experimental results for two way slab with different support conditions (8 simply supported, 4 fixed, 8 beam supported and 3 continuous slab panels) are used for empirical determination of the parameters proposed for the estimation of deflection of two-way RC slabs. Govind (2006) presents details of all the experiments used in this study. Different values of the parameter μ [Eq. (1)] were tried so as to obtain good correlation with the experimental results. In order to converge on the best fit for the value of μ, the concept of least squares was used for deflection values up to service load (10 load-deflection points in each case). Based on these studies, an optimum value of μ = 6 was arrived at to be used in Eq. (1) for $D_{eff, x}$ and $D_{eff, y}$.

3. Comparison of proposed theory with experimental data

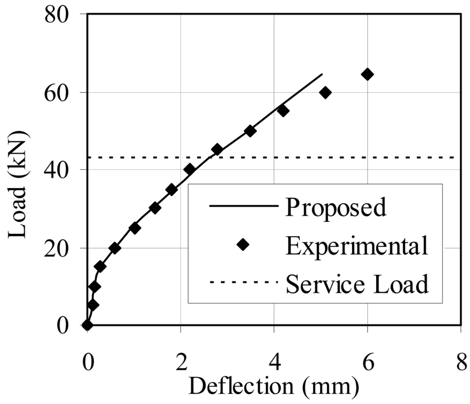
Four-noded isoparametric thin plate elements were used to model the slabs in finite element software SAP2000. This software has a facility called 'stiffness modifier', whereby the flexural and torsional rigidities can be modified for any element by multiplying with the specified stiffness modifiers.

Initially, the slab model is analyzed with stiffness modifiers equal to unity (i.e., with gross values of D_x, D_y and D_{xy}) and the maximum force resultants (M_{xx}, M_{yy} and M_{xy}) are obtained. Using these maximum values, the corresponding values of $D_{eff, x}$ and $D_{eff, y}$ and $D_{eff, xy}$ are calculated, and a weighted average of D_{eff} considered when hogging moments are encountered at the slab edges [refer Eq. (2)]. Accordingly, the values of the stiffness modifiers η_x, η_y and η_{xy} are obtained and the analysis is run again with the revised stiffnesses.

The theory is found to compare closely with the experimental results, particularly in the regions up to service load (taken as 0.7 times the ultimate load) in all of the 23 experimental results (8



(a)



(b)

Fig. 1 Typical load-deflection curve (a) for a slab on unyielding simple support, (b) for a slab on unyielding fixed support

simply supported, 4 fixed, 8 beam supported and 3 continuous slab panels). Fig. 1 shows the typical plots to demonstrate comparison of the predicted and experimental load-deflection behaviour.

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