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Detection of delamination damage in composite beams and plates using wavelet analysis

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Abstract. The effectiveness of wavelet transform in detecting delamination damages in multilayered composite beams and plates is studied here. The damaged composite beams and plates are modeled in finite element software ABAQUS and the first few mode shapes are obtained. The mode shapes of the damaged structures are then wavelet transformed. It is observed that the distribution of wavelet coefficients can identify the damage location of beams and plates by showing higher values of wavelet coefficients at the position of damage. The effectiveness of the method is studied for different boundary conditions, damage location and size for single as well as multiple delaminations in composite beams and plates. It is observed that both discrete wavelet transform (DWT) and continuous wavelet transform (CWT) can detect the presence and location of the damaged region from the mode shapes of the structures. DWT may be used to approximately evaluate the size of the delamination area, whereas, CWT is efficient to detect smaller delamination areas in composites.

Keywords: Wavelet transform; composite beams and plates; delamination damage.

1. Introduction

Damage or minor cracks present a serious threat to the performance of structures. For this reason, there has been a growing interest to establish a nondestructive methodology to identify the existence of any crack in structures and to assess its location and severity. Wavelet transform of static deflection profile or mode shapes of damaged structures appears to be an efficient damage detection tool (Wang and Deng 1999, Kim and Melhem 2003, Zou *et al*. 2000) as the prior states of stress are not required in this technique.

Chang and Chen (2003) employed Gabor wavelet function to transform the mode shapes of a damaged cantilever Timoshenko beam. It was found that wavelet coefficients of scales 4 and higher can identify the local perturbation due to damage by showing a peak at the position of the crack. This study was later extended (Chang and Chen 2005) for the detection of location and estimate the damage size of a double cracked cantilever Timoshenko beam. Further, the fundamental vibration mode shapes of a cracked cantilever beam and a simply supported plate with allover part-through crack parallel to one edge are analyzed using continuous wavelet transform (CWT) by Douka *et al*.

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(2003, 2004). The position of the crack is located by a sudden change in the spatial variation of the wavelet coefficients. To estimate the size of the crack, an intensity factor is defined which relates the size of the crack to the coefficients of the wavelet transform. The efficiency of wavelet transform in detecting localized damages in a cracked cantilever beam from its static deflection profile was studied by Rucka and Wilde (2006). Quek *et al*. (2001) examined the sensitivity of Haar and Gabor wavelet transform functions in locating different cracks in simply supported and clamped beams. Crack characteristics studied include the length, orientation and width. The method was found to be sensitive to the curvature of the deflection profile and the support condition. For detection of discrete cracks, Haar wavelet exhibited superior performance. Li *et al*. (2006) employed Gabor wavelet function to extract damage information from experimental/finite element flexural wave data of a cracked cantilever beam under an impact at its free end. The effect of inclined crack was also examined. Most recently, Chang and Chen (2004), Loutridis *et al*. (2005), Kim *et al*. (2006) and Rucka and Wilde (2006) employed two dimensional wavelet transform for detection of crack location and crack depth for isotropic plates.

It may be noted that most of the available research works have been dealt with the detection of transverse cracks in isotropic beams and plates from its global response. However, inplane cracks or delamination damages, which present a major threat to composite structures have been sparsely treated in the open literature. Most recently, Wei *et al*. (2004) and Yan and Yam (2004) investigated active detection of delamination for multilayer composites using a combination of modal analysis and wavelet transform.

It is observed from the existing literature that the damage detection of structures using wavelet transform is a subject of ongoing research and further studies are still required to establish an accurate methodology to predict the delamination location as well as delamination size for damaged composite beams and plates. In the present paper, the possibility of detecting delamination damages from the global response of multilayered composite beams and plates is investigated. The damaged beams and plates are modeled in finite element software ABAQUS and the first few mode shapes are obtained. The mode shapes of the damaged structures are then wavelet transformed to examine the damage information. The effectiveness of different wavelet functions in detecting the location and extent of damage is studied for different boundary conditions, damage positions, sizes and numbers of delamination in composite beams and plates.

2. Wavelet analysis

Wavelet transform complex-valued maps a time function into two dimensional function of a (scale parameter) and b (shift parameter). Let $f(t)$ be a signal of interest in the time domain $(-\infty, \infty)$. Let $\psi(t)$ be a complex-valued function localized in both time and frequency domains. $\psi(t)$ is called as the mother wavelet. The daughter wavelets $\psi_{a,b}(t)$ are generated from the mother wavelet by translation and dilation, as defined below

$$\psi_{a,b}(t) = |a|^{-p}\psi\left(\frac{t-b}{a}\right) \tag{1}$$

where p is usually taken as 1/2. The daughter wavelet $\psi_{a,b}(t)$ oscillates at a frequency a^{-1} and is positioned at time b. When a is very small, the interval in which the wavelet is nonzero contracts around the point b.

2.1 Discrete wavelet transform

In discrete wavelet transform (DWT), wavelets with integer parameters are often used in wavelet transform and, for example, can be generated from the mother wavelet according to

$$\psi_{j,k}(t) = 2^{j/2}\psi(2^j t - k) \tag{2}$$

where integers j and k are the dilation (scale) and translation (position) indices, respectively. The mother wavelet must satisfy the admissibility condition given by

$$\int_{-\infty}^{\infty} |\psi^*(\omega)|^2 \frac{d\omega}{\omega} < \infty \tag{3}$$

where $\psi^*(\omega)$ is the Fourier transform of $\psi(t)$

$$\psi^*(\omega) = \int_{-\infty}^{\infty} \psi(t) e^{-i\omega t} \tag{4}$$

with $i = \sqrt{-1}$. A consequence of eq. (3) is that the mother wavelet must have zero mean value. The wavelet transform (wavelet coefficients) for the signal then can be determined with the integration limits running from 0 to ∞, that is

$$C_{j,k} = \int_0^{\infty} f(t)\overline{\psi_{j,k}(t)}dt \tag{5}$$

The simplest wavelets were discovered by Haar (Haar 1910) long before the wavelet theory was formally established. The mother Haar wavelet is defined by

$$\psi(t) = \begin{cases} 1, & 0 \le t < 1/2 \\ -1, & 1/2 \le t < 1 \\ 0, & otherwise \end{cases} \tag{6}$$

Although wavelets are usually used to analyze signals in the time domain, spatially distributed signals can be equally analyzed with wavelets. This can be done by simply replacing time t with a spatial coordinate. Without loss of generality, consider a spatial signal $f(x)$ distributed over [0, 1], where x represents a spatial coordinate. This signal (x) may be a displacement or strain measurement over a region of interest for a structure under static or dynamic loading. Region of interest can always be normalized to occupy [0 1]. The wavelet transform (wavelet coefficients) for the signal then can be determined with the integration limits running from 0 to 1, that is

$$C_{j,k} = \int_0^1 f(x)\overline{\psi_{j,k}(x)}dx \tag{7}$$

The wavelet transforms $C_{j,k}$ may be performed with various scaled versions of the mother wavelet function, local perturbations in a signal $f(x)$ may be reflected in the fine scale wavelets (those with relatively large j values) that are positioned (as indicated by the k value) at the locations of the perturbations. This is the *multiresolution* property of the wavelet transform. In other words, local perturbations can be determined by performing the wavelet transform to obtain wavelet coefficients, and by examining the variations of the wavelet coefficients with position. For signals distributed in two dimensions, the two dimensional wavelet transform may be used.

2.2 Continuous wavelet transform

The continuous wavelet transform is a mapping from a function $f(x)$ depending upon time or space to a function $C[f, \psi(a, b)]$ depending upon dilation parameter a and temporal or spatial translation parameter b. It is defined as

$$C[f, \psi(a,b)] = \frac{1}{\sqrt{a}}\int_{-\infty}^{+\infty} f(x)\,\psi^{*}\left(\frac{x-b}{a}\right)dx \tag{8}$$

where $(\;)^{*}$ denotes the complex conjugates. The function $\psi(x)$ is called the analyzing wavelet. The methodology of damage detection using wavelet transform is described in the next section.

3. Application of wavelet transform in structural damage detection

The wavelet transformation may be used for structural damage detection if some spatially distributed signal containing local perturbations caused by the presence of damage over the region is collected. Such signals may be obtained from surface or internal measurements of displacement, strain, or any other quantity whose values can be disturbed due to presence of damage. The procedure for this damage detection method is as follows:

1. First collect spatially distributed signals (displacements) over an area of interest. A number of measurement techniques can be used for this purpose, such as surface mounted or embedded network of sensors, fiber optics, computer vision, and scanning techniques. However, in the present work, ABAQUS finite element software is used to obtain the free vibration mode shapes of damaged composite beams and plates.
2. Perform the wavelet transform to the mode shapes using both discrete wavelets transform (DWT) and continuous wavelet transform (CWT) with different wavelet scales.
3. For each level of wavelet transformation, plot the value of the wavelet coefficients in the region of interest. A sudden change (e.g., peak) in the wavelet coefficients indicates a strong local perturbation in the signal in that region.
4. If a detected perturbation is not caused by a known source, such as a known geometric or material discontinuity, then it will be attributed to the presence of damage.

The effectiveness of the above method is studied here for composite beams and plates having single or multiple delaminations. Four different problems are taken up for investigation: (1) a cantilever composite beam with delamination of different lengths (2) A clamped composite plates with single delamination, (3) A clamped composite plate with two delaminations (4) A cantilever composite plate with two delamination damages.

4. Damage detection in cantilever composite beam

A damaged cantilever composite beam of length 300 mm and crosssectional area 20 mm × 20 mm is taken up for investigation (Fig. 1). The composite beam is made with four layers of graphite/epoxy laminates with stacking sequence [0°/90°/90°/0°]. A delamination damage between the second and third laminates of length d (d = 10 mm) at a distance of 140 mm from the clamped end is considered. The material properties of the graphire/epoxy composite considered in the present

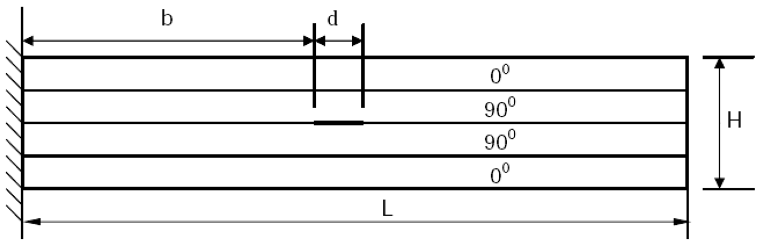

Fig. 1 A cantilever composite beam with delamination

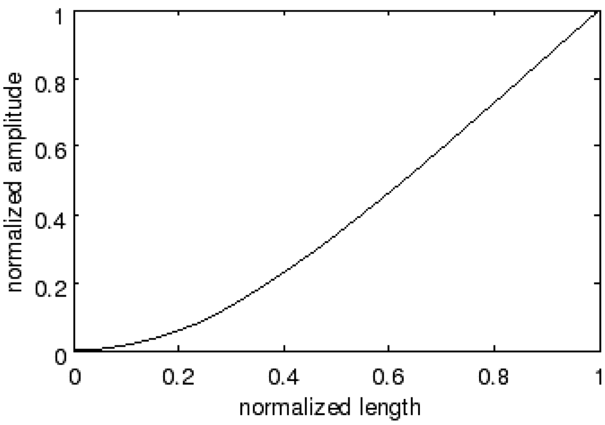

Fig. 2 Fundamental mode shape of cantilever composite beam with delamination of length d=10 mm at a distance of x=140 mm from the clamped end

analysis are: E_1 = 181 GPa, E_2 = 10.3 GPa, E_3 = 10.3 GPa, ν_{12} = 0.25, ν_{13} = 0.25, ν_{23} = 0.33, G_{12} = 7.17 GPa, G_{13} = 7.17 GPa, G_{23} = 2.87 GPa, and ρ = 1578 Kg/m^3.

The damaged composite beam is modeled in finite element software ABAQUS with 8 noded plain stress quadratic elements (element type CPS8R). Delamination damage is modeled by creating two layers of nodes at the interface between second and third laminas in the damaged region. The first mode shape of the damaged composite beam is plotted in Fig. 2. It is observed that the presence of delamination can not be detected from the mode shape. The mode shape is wavelet transformed using DWT (Haar wavelet) and CWT (Sym4 wavelet). For that purpose, the mode shape data is collected at 30 equidistant points on the beam.

4.1 Damage detection using DWT

The fundamental mode shape of cantilever composite beam is wavelet transformed using Haar wavelet. Wavelet coefficients are obtained from Eq. (7) for the wavelet scales 1-10. The values of wavelet coefficients are plotted over the region along the x axis for various wavelet scales, as shown in Fig. 3. It is observed that the wavelets of scales 5 and below are not fine enough to detect local perturbation induced by the delamination. Wavelets of scale 6 and higher are able to identify the local perturbations by showing peaks at the position of the delamination. It is further observed from Fig. 3 that the length of the discontinuity in the distribution of wavelet coefficients approximately reveals the delamination length.

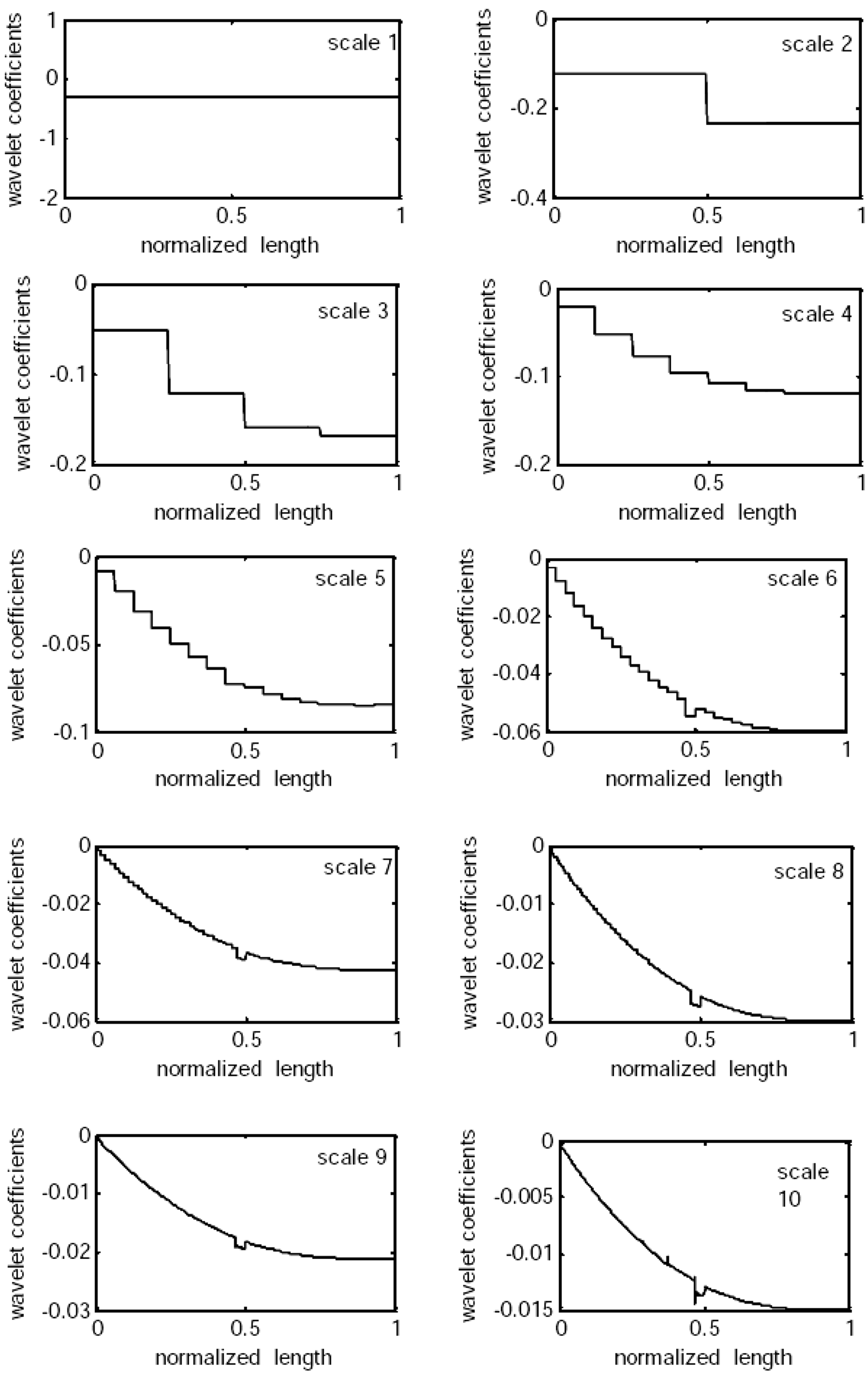

Fig. 3 Distribution of Haar wavelet coefficients for different wavelet scales (The fundamental mode shape of damaged cantilever composite beam is wavelet transformed)

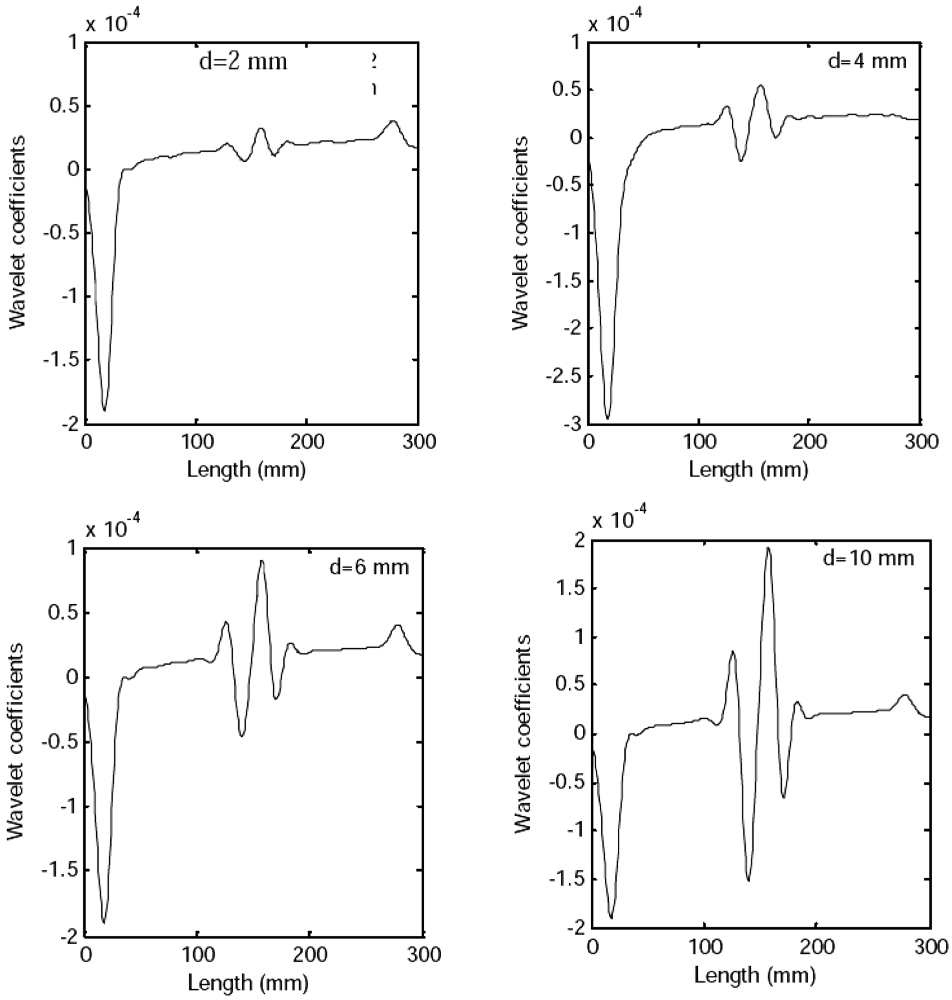

Fig. 4 Wavelet coefficients of the first mode shape of damaged cantilever composite beam for different delamination lengths (*d*)

4.2 Damage detection using CWT

Delamination of different lengths at a distance of 140 mm from the clamped end is considered here. The fundamental mode shape of cantilever composite beam is wavelet transformed using Sym4 wavelet. Wavelet coefficients are obtained from Eq. (8). Fig. 4 shows plot of wavelet coefficients (scale 15) for different delamination of length d=2, 4, 6, 10 mm at a distance 140 mm from clamped end. It has been observed that delamination of 2 mm and more can be detected using sym4 wavelet. With increase in delamination length, wavelet coefficient values increase. As more than one peaks are observed at the crack location, exact relation between delamination length and wavelet coefficient values can not be established. Wavelet coefficients are also high near the clamped end, which may be attributed to geometric discontinuity at the clamped end. Effect of wavelet scales on damage detection is studied in Fig. 5 for a delamination damage of 10 mm at a distance of 140 mm from the clamped end. It is found that the wavelet coefficients increases with

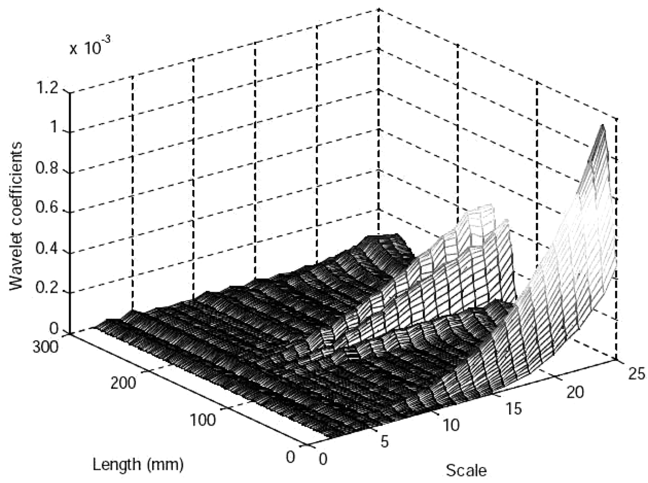

Fig. 5 Effect of wavelet scales on the wavelet coefficients of the first mode shape of damaged can tilever beam (d=10 mm.)

the increase in wavelet scales, but the pattern remains same.

It is observed that a minimum of 30 data points are necessary to define the mode shape of the beam in order to enable wavelet transformation to detect the delamination damage.

5 Damage detection in composite plates

The proposed wavelet based damage detection method for composite plates is demonstrated in this section. A four layered graphite/epoxy [0º/90º/90º/0º] rectangular composite plate with dimensions $L=300$ m, $W=300$ mm, $H=10$ mm is considered here (Fig. 6). The material of Graphite/epoxy composite used in the present analysis are: $E_1=181$ GPa, $E_2=10.3$ GPa, $E_3=10.3$ GPa, $\nu_{12}=0.25$, $\nu_{13}=0.25$, $\nu_{23}=0.33$, $G_{12}=7.17$ GPa, $G_{13}=7.17$ GPa, $G_{23}=2.87$ GPa and $\rho=1578$ Kg/m^3. The plate with damage is divided into m subregions in the x direction and n subregions in y directions. Damage region is located at $x=R_x$, $y=R_y$ and dimensions of the damaged region are $E_l\times E_w$.

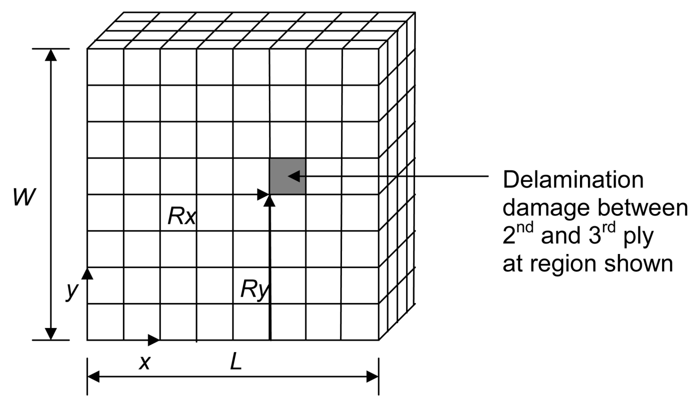

Fig. 6 A composite plate with internal delamination

Damage is modeled as a delamination between second and third plies. The composite plate is modeled with $300\times300\times8$ mesh of 20 noded quadratic isoparametric solid element in ABAQUS (element type C3D20R). The mode shapes of composite plate with delamination damage are obtained by using ABAQUS and are analyzed using wavelet transform to examine if local perturbations can be observed at the damage position.

5.1 Wavelet analysis

In case of plates, consider a 2D spatial signal $f(x, y)$ which distributes over $[0, L_x]$ in the x direction and $[0, L_y]$ in the y direction. The wavelet coefficients for the signal can be written as

$$C[f,\ \psi(a,b)] = \int_0^{L_y}\left(\int_0^{L_x} f(x,y)\,\psi_{(xa,yb)}(x)dx\right)\psi_{(ya,yb)}(y)dy \tag{9}$$

For simplification, two simplified forms are used to calculate the wavelet coefficients for the 2D signal in this study,

$$C[f,\ \psi(a,b,y)] = \int_0^{L_x} f(x,y)\,\psi_{(xa,yb)}(x)dx, \tag{10}$$

$$C[f,\ \psi(a,b,x)] = \int_0^{L_y} f(x,y)\,\psi_{(xa,yb)}(y)dy, \tag{11}$$

Eq. (10) is used in the wavelet analysis in x direction and Eq. (11) is used in the wavelet analysis in y direction. Morlet wavelet which can detect smaller damage regions has been chosen and used as a analyzing wavelet.

5.2 Clamped plate with single delamination

Delamination in the composite plate is modeled at the interface of the second and third laminas from bottom at 150-170 mm in x direction and 150-170 mm in y direction. The first mode shape of

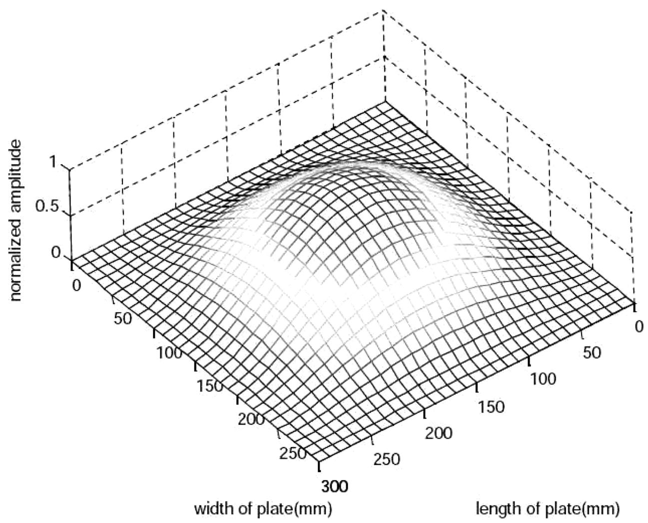

Fig. 7 The first mode shape of the clamped composite plate with a delamination damage at 150-170 mm in x direction and 150-170 mm in y direction

the damaged composite plate, obtained from finite element analysis in ABAQUS is shown in Fig. 7. In order to detect damage position (delamination), the first mode shape of the composite plate is wavelet transformed with Morlet wavelet function. The values of wavelet coefficients, according to wavelet analysis in the *x* and *y* directions are plotted along the length as shown in Fig. 8. It is observed that the values of the wavelet coefficients at the damaged region are higher than those of the remaining region. Similar to the case of cantilever beam, higher values of wavelet coefficients are also observed near the clamped ends, which may be attributed to geometric discontinuity and the corresponding local perturbations in deflection profile is also detected by the wavelet analysis. The second and third mode shapes of the clamped composite plate are also wavelet transformed and the corresponding wavelet coefficients are presented in Figs. 9 and 10 respectively. It is observed that wavelet coefficients of higher modes also can detect the presence and location of the delamination damage.

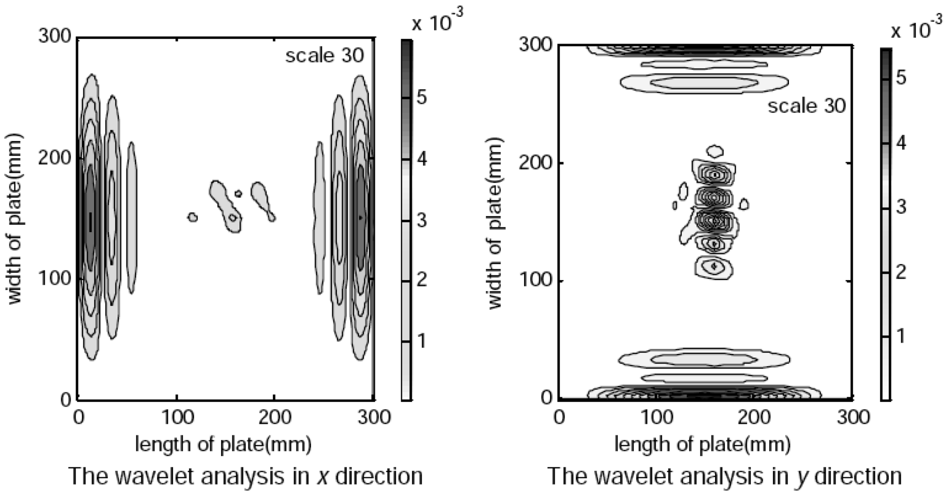

Fig. 8 Distribution of wavelet coefficients of the first mode shape of clamped composite plate

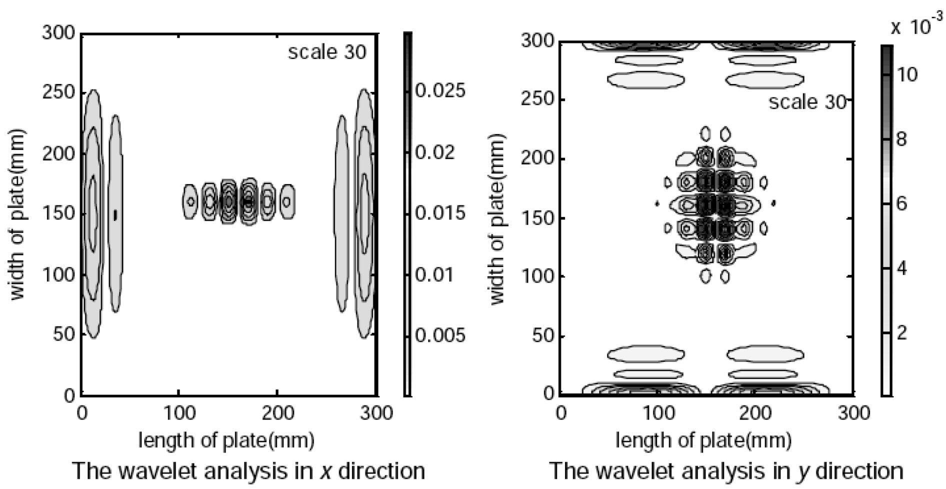

Fig. 9 Distribution of wavelet coefficients of the second mode shape of clamped composite plate

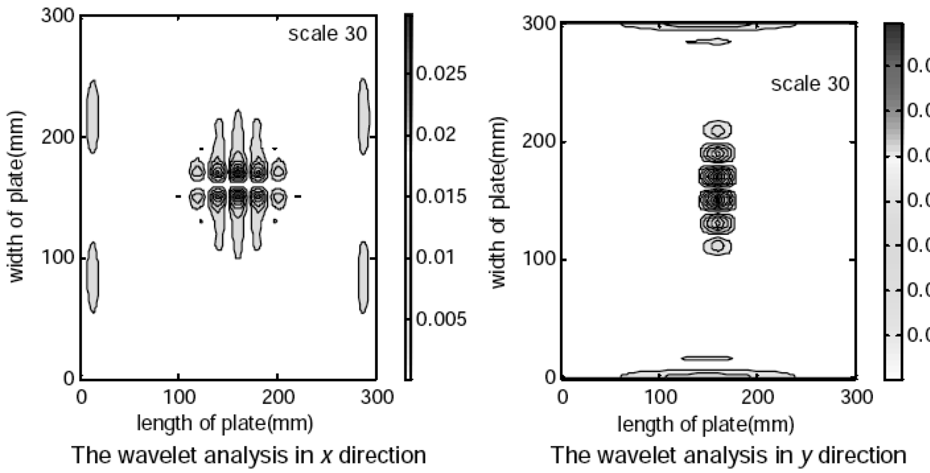

Fig. 10 Distribution of wavelet coefficients of the third mode shape of clamped composite plate

Wavelet analysis is repeated with different scales of Morlet wavelet and it is found that in case of clamped composite plates higher scales (30 or higher) of wavelet transform are required to detect the delamination location. With this study of wavelet transformation of first three modes of a clamped composite plate, it may be noted that delamination damage in composite plates can be detected using wavelet transform.

5.3 Clamped composite plate with two delamination locations

The proposed methodology is now tested for a clamped composite plate with two delamination damage areas of size 20 mm × 20 mm each. The location of the first damaged area is assumed at $R_{x1} = 150$ mm and $R_{y1} = 150$ mm, while the second delamination is considered near the support at $R_{x2} = 150$ mm and $R_{y2} = 20$ mm. The first and second mode shapes of the damaged plate are obtained using ABAQUS finite element software and 30 × 30 data points are collected to define the mode shape for further wavelet analysis. The mode shapes are wavelet transformed and the corresponding wavelet coefficients for the first and second modes are shown in Figs. 11 and 12 respectively. It is observed that both the damaged areas can be detected from Figs. 11 and 12. Here it may be pointed out that, when wavelet transformation is done in y direction, higher values of wavelet coefficients are observed near the clamped edge because of combined perturbation effect of delamination damage and support discontinuity.

5.4 Damage detection in cantilever composite plate

A cantilever composite plate with two delamination areas of size 20 mm × 20 mm each are considered here. First delamination location is at 150-170 mm in x direction and 150-170 mm in y direction and second delamination location is at 150-170 mm in x direction and 20-40 mm in y direction. The wavelet coefficients of the first and second mode are presented in Figs. 13 and 14 respectively. It is observed that the wavelet transformation of the first mode shape clearly detects both the delamination areas, whereas the second mode shape detects only one delamination location which is near to edge. Delamination at the mid portion of the plate can not be detected in this case

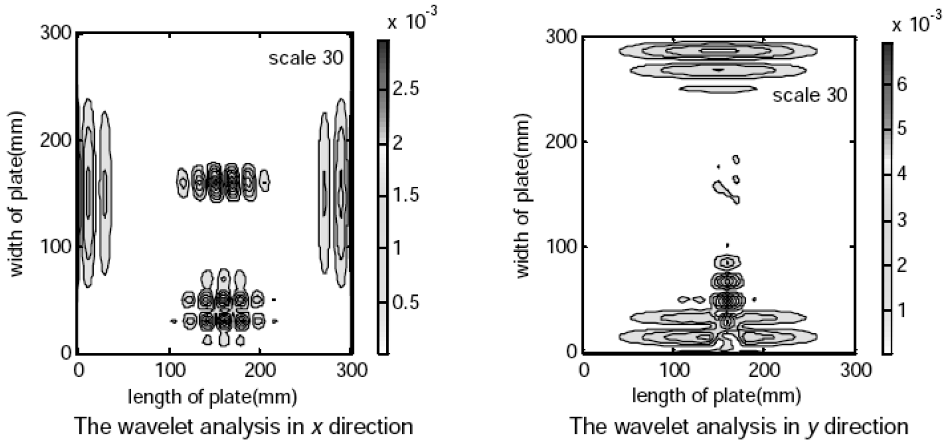

Fig. 11 Distribution of wavelet coefficients of the first mode shape of a clamped composite plate with two delamination locations (R_{x1}=150 mm, R_{y1}=150 mm, R_{x2}=150 mm, R_{y2}=20 mm)

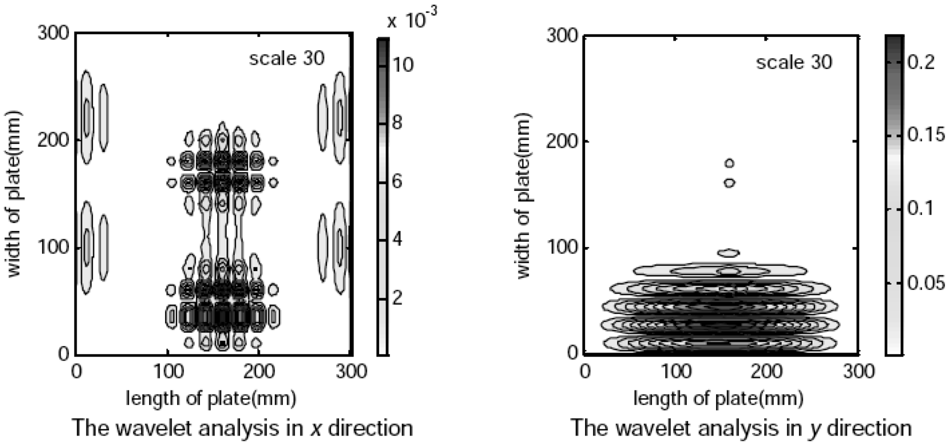

Fig. 12 Distribution of wavelet coefficients of the second mode shape of a clamped composite plate with two delamination locations (R_{x1}=150 mm, R_{y1}=150 mm, R_{x2}=150 mm, R_{y2}=20 mm)

because the variation in deflection profile is less at the mid portion. The study is repeated with smaller delamination areas and it is observed that delamination damage as minimum as 1% of total surface area can be detected by wavelet transformation of its theoretical mode shape. However, the efficiency of the methodology will depend on the accuracy of the experimentally measured mode shapes of the damaged plates.

6. Conclusions

The effectiveness of the wavelet transform methods in detection of delamination damages in multilayered composite beams and plates is studied here. ABAQUS finite element software is employed to obtain the free vibration mode shapes of damaged composite beams and plates and a MATLAB program is used for wavelet analysis of the mode shapes. It is observed that both discrete

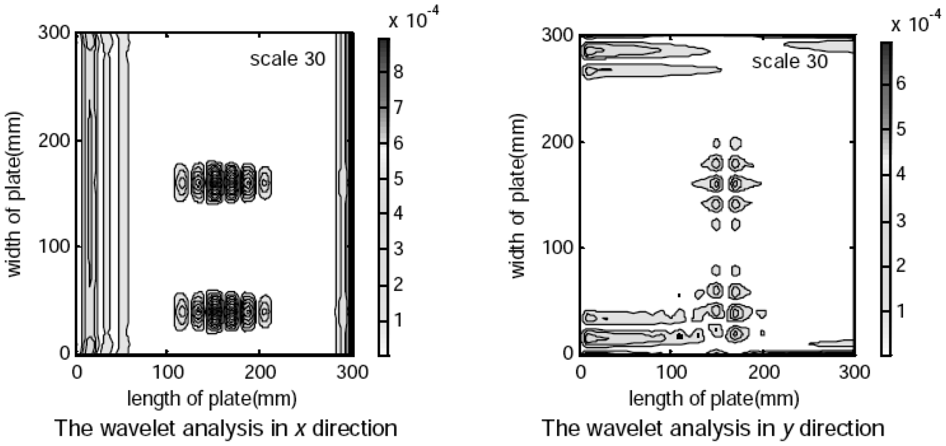

Fig. 13 Distribution of wavelet coefficients of the first mode shape of a cantilever composite plate with two delamination location (R_{x1}=150 mm, R_{y1}=150 mm, R_{x2}=150 mm, R_{y2}=20 mm).

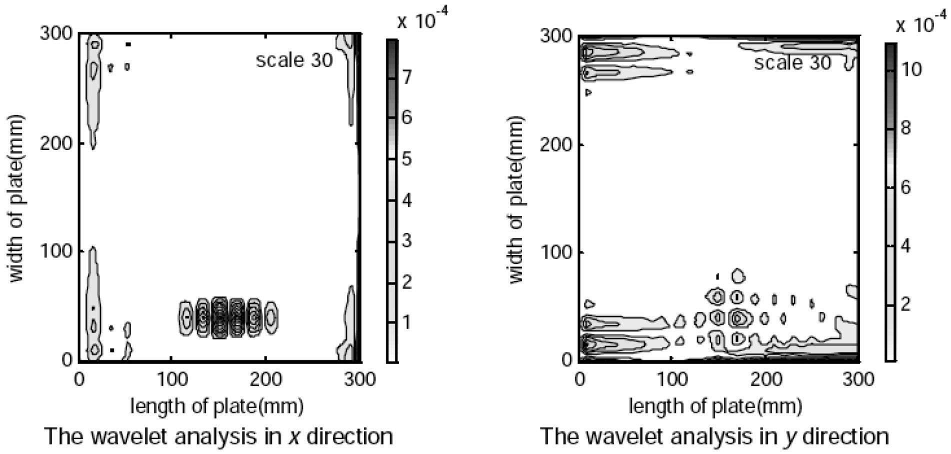

Fig. 14 Distribution of wavelet coefficients of the second mode shape of a cantilever composite plate with two delamination location (R_{x1}=150 mm, R_{y1}=150 mm, R_{x2}=150 mm, R_{y2}=20 mm).

wavelets transform (DWT) and continuous wavelet transform (CWT) can detect the presence and location of the damaged region from the mode shape of the structures. DWT may be used to evaluate the approximate size of delamination area, while CWT is efficient to detect smaller delamination areas. This method is effective if the damage location is outside a distance of 10% from the support. This method based on structural vibration can be implemented in practice using a few piezoelectric patch sensors and actuators embedded in the composite beams and plates.

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