Structural Engineering and Mechanics, Vol. 30, No. 4 (2008) 481-500
DOI: http://dx.doi.org/10.12989/sem.2008.30.4.481

Characterization of elastic properties of pultruded profiles using model updating procedure with vibration test data

Jesiel Cunha†

Federal University of Uberlândia, PO Box 593, Uberlândia, Brazil

Emmanuel Foltête‡ and Noureddine Bouhaddi‡†

University of Franche-Comt, Besançon, France

(Received April 8, 2008, Accepted August 28, 2008)

Abstract. In this paper, a model updating technique in dynamics is used to identify elastic properties for pultruded GFRP-Glass Fiber Reinforced Plastic framed structural systems used in civil construction. Traditional identification techniques for composite materials may be expensive, while this alternative approach allows to identify several properties simultaneously, with very good precision. Furthermore, the procedure of a non-destructive type has a relatively simple implementation. Properties describing the mechanical behavior for beam and shell finite element modeling are identified. The used formulation is based on the minimization of eigensolution residuals. Important points concerning model updating procedures have been observed, such as the particular vibrational behavior of the test structure, the modeling strategies and the optimal placement of the sensors in the experimental procedure. Results obtained by experimental tests show the efficiency of the proposed procedure.

Keywords: composite materials; pultruded profiles; elastic properties; model updating; vibration.

1. Introduction

Used successfully in the automobile, aeronautical, naval, space and sports sectors, composite materials are also chosen by designers and engineers of the civil construction, considering their structural and constructive qualities. Facing more severe requirements, sometimes in aggressive environments, composite materials are an interesting alternative solution. Either for the repairing or design of new structures, they present some advantages with respect to the traditional materials of the civil construction, such as low weight, high strength and anti-corrosion properties. On the other hand, due to the great variety of materials and structural configurations, the understanding of the mechanical behavior and the intrinsic phenomena of composites can be a complex task.

† Ph.D., Associate Professor, Corresponding author, E-mail: jecunha@ufu.br
‡ Ph.D., Associate Professor, E-mail: emmanuel.foltete@univ-fcomte.fr
‡† Ph.D., Professor, E-mail: noureddine.bouhaddi@univ-fcomte.fr

Currently, one of the main modalities of composite materials used in civil construction structures consists of the pultruded profiles in Glass Fiber Reinforced Plastic (GFRP). A pultruded fiber glass structural shape is constituted of reinforcing fibers (glass) embedded in a polymer matrix (polyester, vinylester, epoxy or phenolic). Basically, the fiber reinforcement provides the strength and stiffness while the resin provides the resistance to the environment. The stiffness and strength of the material mainly depend on the type, the volume fraction and the stacking sequence of the reinforcements. The main advantages of the pultruded profiles as compared to traditional materials of civil construction (steel, wood and concrete) are the excellent strength and stiffness/density ratio; lightness, reducing the assembly and transport costs; corrosion resistance and chemical inertia; dimensional stability; reduced maintenance costs.

The anisotropic character, the diversity of the materials and the variety of the architectures make the mechanical behavior of the pultruded profiles very particular. In this context, the elastic characterization of the pultruded profiles can be made experimentally or by using micromechanics approach, where the layers properties are calculated as an equivalent homogeneous material from the constituents' properties (fiber and matrix). The number of properties necessary to characterize the mechanical behavior depends on the modeling technique. Considering symmetry of the layers, this number can vary from two, for the case of modeling with beam elements based on Kirchhoff theory, up to five, in the case of modeling with plate elements with First-order Shear Deformation Theory. The traditional experimental identification methods of composite materials, in particular for the pultruded profiles, are in general expensive in terms of time and required equipment. The majority of the techniques demand the accomplishment of a specific test for each individual elastic property. Moreover, these methods frequently exhibit precision and implementation problems.

The objective of this work is to identify the elastic properties of pultruded profiles used in framed structures of civil engineering. For this, a model updating technique in dynamics has been applied. By modifying the design variables represented by the elastic properties, a problem of parametric identification is formulated, pertaining to the class of inverse problems. From the mathematical point of view, we are inserted in the context of an optimization problem, with the search of the minimum of a cost function, formed by the residuals which express the distance between the numerical model and the real structure. The used formulation is based on the minimization of eigensolution residuals, which is a technique commonly called sensitivity method, i.e., an optimization method that makes use of sensitivities of the eigenvalues and eigenvectors for selected parameters.

The interest in the dynamic tests lies on the fact that several types of energy of different natures (membrane, bending, shear) intervene in the vibratory behavior of a structure. This fact is perfectly coherent with the character of multiaxial identification inherent to composites. It is thought that this technique allows simultaneous identification of several properties from a single test. For traditional techniques, the static behavior can be subject to the local phenomena, while dynamic response (modes and frequencies) represents in general the global behavior of the structure, which is interesting from the identification point of view. The technique can be applied to a wide range of profiles; it is of relatively simple implementation and is of non-destructive type.

As opposed to the previous studies dedicated to the subject (Deobald and Gibson 1986, Frederiksen 1994, Frederiksen 1998, Pedersen 1988, Bledzki *et al.* 1999, Sol 1986, Cunha and Piranda 1999, Ip *et al.* 1998, Gagneja *et al.* 2001), in which plate-like structures are considered, the present methodology is applied to a pultruded beam with I-shape cross-section. The identification of elastic properties in this type of structure by model updating technique in dynamics is relatively new.

Some theoretical and practical aspects of the model updating procedure in dynamics are approached. A general model updating tool is presented, allowing a large number of identification possibilities. The presented results demonstrate the efficiency of the proposed methodology.

2. Strategies for profile modeling

In the context of composite material theories, pultruded profiles can be considered as a laminated structure (Davalos and Qiao 1999, Sonti 1992, Clarke 1996). Analytical or numerical modeling of these profiles formed by unidirectional or mat plies is generally carried out by using beam or plate elements. The theory to be used and the associated number of elastic properties necessary to describe the mechanical behavior depend on the adopted modeling strategy.

In this work, two strategies for profile modeling have been applied: beam and plate FEM discretizations. These are the common approaches used for design process, considering ultimate and serviceability limit states. For beam modeling, the resulting framed structures are used to obtain the bending moment, shear force and normal force diagrams. For plate modeling, more detailed and specific applications can be developed, as stress concentration, failure mechanism and criteria analysis.

Fig. 1 shows the geometry of the pultruded beam with the global reference system, where u, v and w are the corresponding displacements.

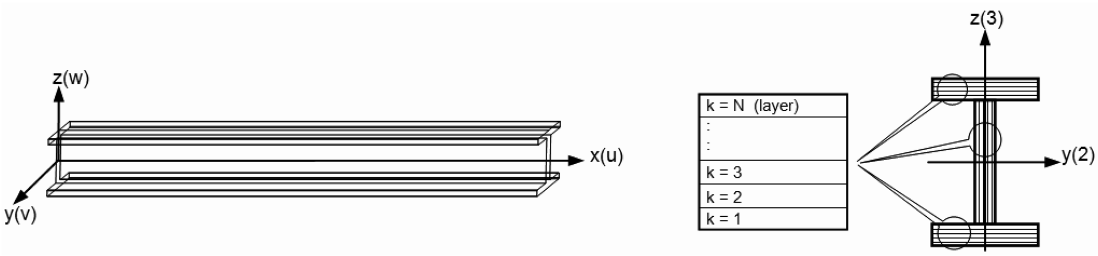

Fig. 1 Configuration of the pultruded I-beam

2.1 Modeling with plate elements

Assuming that the profiles are of open thin-walled cross section type, web and flange will be under plane-stress state. In this case, the mechanical behavior is described by the Kirchhoff-Love theory (Classical Laminated Plate Theory). Moreover, assuming that the axes of orthotropy of the material coincide with the geometrical axis of the beam, the stress-strain relationship is reduced to

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \gamma_{12} \end{bmatrix} \tag{1}$$

where Q_{ij} are the reduced stiffness constants. For orthotropic and transverse isotropic materials these constants are given by

$$Q_{11} = \frac{E_1}{1 - \nu_{12}\nu_{21}} \qquad Q_{22} = \frac{E_2}{1 - \nu_{12}\nu_{21}} \qquad Q_{12} = \frac{\nu_{21}E_1}{1 - \nu_{12}\nu_{21}} \qquad Q_{66} = G_{12} \tag{2}$$

In this case, the elastic characterization of the pultruded profile is made by four independent elastic properties: E_1 (Young's modulus in the longitudinal direction), E_2 (Young's modulus in the transverse direction), G_{12} (In-plane shear modulus) and ν_{12} (Poisson's ratio).

In the cases of thick web and/or flanges and for materials with a high Young's modulus/shear modulus ratio, the shear effect becomes important, which leads to the First-order Shear Deformation Theory or a Higher-order Shear Deformation Theory. The elastic properties for this case are five: E_1 (Young's modulus in the longitudinal direction), E_2 (Young's modulus in the transverse direction), G_{12} (In-plane shear modulus), G_{23} (Shear modulus in 23-plane) and ν_{12} (Poisson's ratio).

Concerning Poisson's ratio ν_{12}, this property does not influence the dynamic and static behavior of the pultruded beams used in framed structures (beams with high slenderness ratio) with respect to the calculation of stress and moment resultants. Thus, this parameter was not considered for identification in this study.

2.2 Modeling with beam elements

This modeling implies the transformation of the three-dimensional structure, which can possess a complex cross section, in a unidimensional structure with an equivalent homogeneous section. The homogenization is carried-out at material and geometrical levels. Thus, the originally orthotropic material is converted into an isotropic equivalent one. For the beam formulation it is usual to define the equivalent stiffnesses. Considering the Euler-Bernoulli (without shear), Timoshenko (with shear) and Vlasov (torsion with warping) theories, the following stiffnesses can be defined as: EA (axial stiffness); EI (bending stiffness); kGA (shear stiffness); GJ (torsional stiffness, from the Saint-Venant torsion solution); EI_w (warping stiffness, from the Vlasov torsion solution), where A is the cross section area, I is the moment of inertia, k is the shear correction coefficient, G is the shear modulus, J is the Saint-Venant torsion constant (torsion moment of inertia) and I_w is the warping moment of inertia.

Differently from thin-walled isotropic beams, for composite material beams, bending and torsional vibrations can be coupled due to material anisotropy, even for profiles with double symmetry. The equations of motion can be obtained from Hamilton's principle. If the shear effect is considered (Timoshenko theory), the equations of motion for a free isotropic I-beam expressed in function of the displacements u, v and w and rotations θ_x, θ_y and θ_z are

$$\begin{gathered}
EAu'' = \rho A\ddot{u} \\
k_y GA(v'' - \theta_z') = \rho A\ddot{v} \\
k_z GA(w'' - \theta_y') = \rho A\ddot{w} \\
EI_y\theta_y'' - k_z GA(w' + \theta_y) = \rho I_y\ddot{\theta}_y \\
EI_z\theta_z'' + k_y GA(v' - \theta_z) = \rho I_z\ddot{\theta}_z \\
GJ\theta_x'' - EI_w\theta_x^{iv} = \rho(I_p\ddot{\theta}_x - I_w\ddot{\theta}_x'')
\end{gathered} \tag{3}$$

where superscript primes and dots denote the differentiation with respect to axial coordinate x and time, respectively. This set of differential equations allows describing the general vibrational behavior of I-beams with relatively low slenderness ratio, including shear and warping effects. Thus, in the general case of beam vibration modeled with beam elements, two elastic properties are necessary to describe the mechanical behavior: E (Young's modulus) and G (Shear modulus, associated with shear correction coefficients k_y and k_z).

In the context of model updating technique, in the transition from plate modeling (Kirchhoff-Love theory) to beam modeling (Timoshenko theory), shear correction coefficients k_y and k_z can assume values which do not have necessarily the physical sense which characterizes them. In this case, these coefficients act simply as updating parameters to obtain the equivalent beam model.

3. General formulation of the eigensolution sensitivity method

The sensitivity method consists in the minimization of a residual based on eigensolutions, which are considered as output quantities (Fig. 2). In elastodynamics, the advantages of the sensitivity method when compared to other updating methods are, in a general sense, the following: neither expansion nor condensation are required; it is exploitable when the number of sensors is reduced and is therefore well adapted to large systems; it is robust with respect to the measurement noise; it enables to ensure physical meaning to the updating. Its disadvantages are related to the difficult convergence or the possibility of reaching local minima, thus requiring an initial finite element model which represents quite well the dynamic behavior of the real structure; the necessity of pairing modes; the utilization of generalized masses and the numerical problems in the presence of multiples or quasi-multiple eigenvalues. Nevertheless, most of these disadvantages can be dealt with using ad-hoc procedures (Cunha and Piranda 1999).

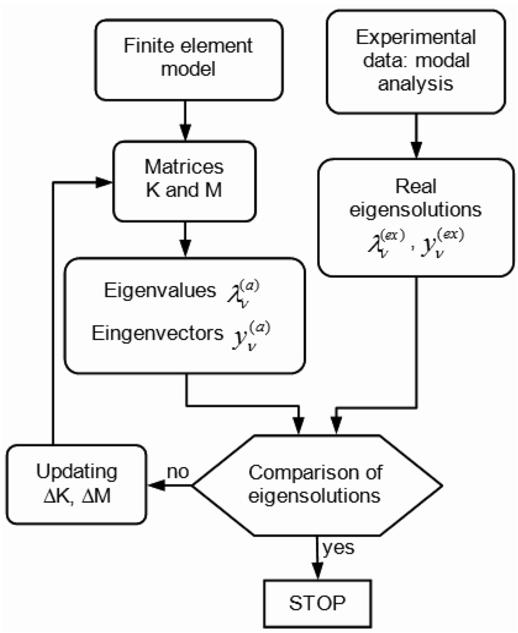

Fig. 2 General flowchart of the sensitivity method

In the updating procedure, corrections ΔK and ΔM for the stiffness and mass matrices of the model (a) related to the experimental (ex) are determined

$$K^{(ex)} = K^{(a)} + \Delta K \qquad M^{(ex)} = M^{(a)} + \Delta M \tag{4}$$

To do so, we suppose that the finite element model is composed by sub-domains called *macro-elements* which comprise elements depending on the same parameters

$$K = \sum_{e=1}^{r} K_e \in R^{N,N} \qquad M = \sum_{e=1}^{r} M_e \in R^{N,N} \tag{5}$$

where r is the number of elements in the macro-element, K_e and M_e are respectively the stiffness and mass matrices associated with the element e. The corrections are made by acting on the p stiffness macro-elements and q mass macro-elements as follows

$$K^{(a)} = \sum_{i=1}^{p} k_i K_i \qquad M^{(a)} = \sum_{j=1}^{q} m_i M_j \tag{6}$$

where k_i and m_j are unknown correction coefficients, $K^{(a)}$ and $M^{(a)}$ are the assembled stiffness and mass matrices. The residual is formed from the distances between the measured eigensolutions of the structure and those calculated from the finite element model evaluated on the c instrumented coordinates

$$\Delta y_v = y_v^{(ex)} \quad y_v^{(a)} \in R^{c,1} \qquad \Delta\lambda_v = \lambda_v^{(ex)} \quad \lambda_v^{(a)} \in R \qquad v = 1, \ldots.., L \tag{7}$$

In the sensitivity method, differences between the eigensolutions (eigenmodes y and eigenvalues λ) are expressed as functions of the increments of correction parameters. To do so, first order Taylor series expansions in the vicinity of paired eigensolutions of the model are used

$$y_v^{(ex)} = y_v^{(a)} + \sum_{i=1}^{p} \frac{\partial y_v^{(a)}}{\partial k_i} dk_i + \sum_{j=1}^{q} \frac{\partial y_v^{(a)}}{\partial m_j} dm_j \qquad \lambda_v^{(ex)} = \lambda_v^{(a)} + \sum_{i=1}^{p} \frac{\partial \lambda_v^{(a)}}{\partial k_i} dk_i + \sum_{j=1}^{q} \frac{\partial \lambda_v^{(a)}}{\partial m_j} dm_j \tag{8}$$

In matrix form we have

$$\begin{bmatrix} \Delta y_1 \\ \vdots \\ \Delta y_L \\ \hline \Delta\lambda_1 \\ \vdots \\ \Delta\lambda_L \end{bmatrix} = \left[\begin{array}{ccc|ccc} \frac{\partial y_1}{\partial k_1} & \cdots & \frac{\partial y_1}{\partial k_p} & \frac{\partial y_1}{\partial m_1} & \cdots & \frac{\partial y_1}{\partial m_q} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial y_L}{\partial k_1} & \cdots & \frac{\partial y_L}{\partial k_p} & \frac{\partial y_L}{\partial m_1} & \cdots & \frac{\partial y_L}{\partial m_q} \\ \hline \frac{\partial \lambda_1}{\partial k_1} & \cdots & \frac{\partial \lambda_1}{\partial k_p} & \frac{\partial \lambda_1}{\partial m_1} & \cdots & \frac{\partial \lambda_1}{\partial m_q} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial \lambda_L}{\partial k_1} & \cdots & \frac{\partial \lambda_L}{\partial k_p} & \frac{\partial \lambda_L}{\partial m_1} & \cdots & \frac{\partial \lambda_L}{\partial m_q} \end{array}\right] \begin{bmatrix} dk_1 \\ \vdots \\ dk_p \\ \hline dm_1 \\ \vdots \\ dm_q \end{bmatrix} \tag{9}$$

$$\underset{[L(c+1),\,1]}{\underset{\Delta Z}{\begin{bmatrix} \Delta y \\ \hline \Delta\lambda \end{bmatrix}}} = \underset{[L(c+1),\,p+q]}{\underset{S}{\left[\begin{array}{c|c} S_y^{(k)} & S_y^{(m)} \\ \hline S_\lambda^{(k)} & S_\lambda^{(m)} \end{array}\right]}} \underset{[p+q,\,1]}{\underset{\Delta p}{\begin{bmatrix} \Delta p^{(k)} \\ \hline \Delta p^{(m)} \end{bmatrix}}}$$

where L is the number of measured eigensolutions; c is the number of measured degrees of freedom (DOF). In the sensitivity matrix S, expressions of the first derivatives of eigensolutions with respect to the stiffness and mass parameters are obtained by deriving the modal equilibrium equation of the model and by taking into account the orthonormality relations (Cunha and Piranda 1999)

$$\frac{\partial y_v^{(a)}}{\partial k_i} = Y^{(a)} t_{\sigma v}^i = \sum_{\sigma=1}^{SB} y_\sigma^{(a)} t_{\sigma v}^i \qquad \frac{\sigma \lambda_v^{(a)}}{\partial k_i} = {}^T y_v^{(a)} K_i^{(a)} y_v^{(a)} \tag{10}$$

$$\frac{\partial y_v^{(a)}}{\partial m_j} = Y^{(a)} a_{\sigma v}^j = \sum_{\sigma=1}^{SB} y_\sigma^{(a)} a_{\sigma v}^j \qquad \frac{\sigma \lambda_v^{(a)}}{\partial m_j} = \lambda_v^{(a)T} y_v^{(a)} M_j^{(a)} y_v^{(a)} \tag{11}$$

with

$$t_{\sigma v}^i = \frac{{}^T y_\sigma^{(a)} K_i^{(a)} y_v^{(a)}}{\lambda_\sigma^{(a)} \lambda_v^{(a)}}, \sigma \neq v \qquad t_{vv}^i = 0 \tag{12}$$

$$a_{\sigma v}^i = \lambda_v^{(a)} \frac{{}^T y_\sigma^{(a)} M_j^{(a)} y_v^{(a)}}{\lambda_\sigma^{(a)} \lambda_v^{(a)}}, \sigma \neq v \qquad a_{vv}^j = \frac{1}{2} {}^T y_v^{(a)} M_j^{(a)} y_v^{(a)} \tag{13}$$

where $Y^{(a)} \in R^{N,SB}$ is the sub-base formed by SB eigenvectors of the model.

The optimization method used to calculate the solution Δp in Eq. (9) is of the gradient type with inequality constraints. The cost function is formed by the weighted differences between the eigensolutions

$$J(p) = {}^T\Delta y(p) W_y \Delta y(p) + {}^T\Delta\lambda(p) W_\lambda \Delta\lambda(p) \tag{14}$$

subject to inequality constraints $\Delta p_i^{\inf} \leq \Delta p_i \leq \Delta p_i^{\sup}$ and lateral constraints $p_i^{\inf} \leq p_i \leq p_i^{\sup}$, where $\Delta y = y^{(ex)} - y^{(a)} \in R^{Lc,1}$ is the vector of eigenvector differences; $\Delta\lambda = \lambda^{(ex)} - \lambda^{(a)} \in R^{L,1}$ is the vector of eigenvalue differences; $W_y \in R^{Lc, Lc}$, $W_\lambda \in R^{L, L}$ and $W_p \in R^{p+q, p+q}$ are weighting matrices chosen according to the specificity of the problem. In this work, weighting values for eigenvectors are same than for eigenvalues.

3.1 Optimal placement of sensors in view of model updating

The selection of sensor placements is a particularly important issue in model updating procedures. Indeed, although the finite element model possesses a large number of DOF's, in practice only a limited number of coordinates can be instrumented, hence the necessity of choosing adequately the location of the sensors in the structure. The optimal selection of sensors pursues one of the following objectives: improved of the structure's vibration modes, minimal errors in response expansion or model condensation, improved capability of structural modification localization, maximization of the numerical conditioning of the sensitivity matrix, etc (Foltête and Piranda 2001).

In this work, the objective of DOF's sensors selection is to obtain a projection basis that is the most orthogonal as possible, which will allow a matching between calculated and identified eigenvectors. In view of the model updating procedure, this goal can be considered as a technique for planning the experimental test. The method consists in exploiting the modal matrix with a minimum condition number. We propose constructing a submatrix $Y^{(a)} \in R^{c,n}$, $c > n$ successively line by line. The proposed procedure is defined by the following steps: for a number n of given basis vectors, the displacement vectors of each DOF i on n modes are formed as

$$y_i^{(a)} = [y_{i1}^{(a)}, y_{i2}^{(a)}, \ldots\ldots, y_{in}^{(a)}] \in R^{1,n} \tag{15}$$

where $y_{ij}^{(a)}$ is the displacement of the DOFi for the mode j. The first retained DOF ($y_1^{(a)}$) has to maximize the norm of $y_i^{(a)}$. Then we construct all matrices $Y_{2j}^{(a)} \in R^{2,n}$ with all j ($j \neq 1$) retained DOF's

$$Y_{2j}^{(a)} = [y_1^{(a)} \quad y_j^{(a)}]^T \tag{16}$$

The rank and the condition number of all matrices $Y_{2j}^{(a)}$ are determined. The second selected DOF maximizes the rank and minimizes the condition number of $Y_{2j}^{(a)}$. Finally, all matrices $Y_{pK}^{(a)} \in R^{p,n}$, with $K \neq 1$, $K \neq 2$, ... are constructed

$$Y_{pK}^{(a)} = [y_1^{(a)} \quad y_2^{(a)} \quad \ldots \quad y_K^{(a)}]^T \tag{17}$$

The conditioning and the rank of the matrices $Y_{pK}^{(a)}$ are evaluated. The p DOF's which maximize the rank and minimize the condition number of $Y_{pK}^{(a)}$ are selected. In the condition number minimization method, the selected DOF's depend on the choice of the first DOF or the imposed DOF's that can generate several families of solutions. Thus, the selection technique remains sub-optimal. This aspect reveals the essentially numerical character of the method. There are no explicit physical considerations which allow, for example, an interpretation in terms of kinetic or potential energies.

4. Vibrational behavior of the test structure

The studied pultruded profile has a cross section 'I' 150 × 74.5 × 5.5 mm, fabricated by Exel Composites (Utilo standard profile), according to E23 grade of EN 13706. The profile is constituted by polyester resin and glass fibers (continuous roving and continuous strand mat), with a polyester surface veil. The Young's modulus supplied by the manufacturer is $E = 28.6$ GPa. The measured density is 1860 kg/m^3. The calculated geometrical properties are: $A = 1.720 \times 10^{-3}$ m^2; $I_y = 5.98 \times 10^{-6}$ m^4; $I_z = 4.09 \times 10^{-7}$ m^4; $J = 2.42 \times 10^{-8}$ m^4; $I_w = 2.11 \times 10^{-9}$ m^6; $k_y = 0.48$; $k_z = 0.43$.

Characterization of dynamic behavior of test structure is made by performing FE computations. The free-free boundary condition is adopted for identification of elastic properties. Assuming that the principal directions of orthotropy coincide with the beam reference system, which actually is the case of pultruded profiles formed by continuous roving and continuous strand mat plies, modeling of the laminated structure layer to layer is not necessary. As depicted in Fig. 3, only a correct positioning of the local reference systems of the material in web and flange components is necessary. In this case, the profile is considered as an orthotropic material at a macroscale level.

The tested I-beam exhibits double cross-sectional symmetry. Moreover, elastic and geometric symmetry with respect to the layer stacking are present. For this type of section, the shear center and the centroid are coincident, thus bending and torsional motions are uncoupled. For model updating procedure, it is important that mode shapes are uncoupled, because the torsional motion will not be observed by vibration sensors. Only bending modes about y and z axis will be used to form the experimental modal basis.

For thin-walled profiles, local bending deformations of web and flanges occur, especially for modes about z axis. From a model updating point of view, this behavior can generate difficulties in the model-test comparison. Indeed, when the structure is modeled by beam elements, local

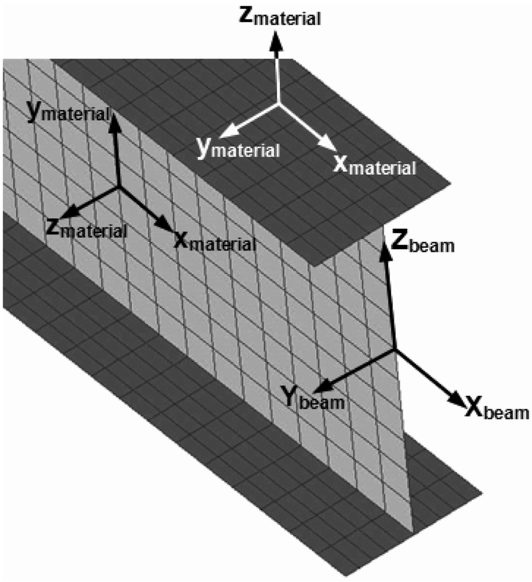

Fig. 3 Elastic and geometrical reference system for material modeling

deformations of the profile are not reproduced. Local effects are more significant for higher frequencies, which reduces the modal basis to be used in model updating. Moreover, considering that the number of sensors is limited, presence of local deformations makes more difficult the pairing between measured and calculated modes.

Besides the bending mode shapes, which will be those effectively used in model updating, torsional and cross-sectional opening type modes are present in the beam dynamic behavior, even for lower frequencies. From a model updating point of view, these configurations are undesirable, because such modes will not be experimentally identified. Thus, they must be eliminated in the process by an adequate pairing technique. The different typical mode shapes of the test structure are shown in Fig. 4.

5. Experimental procedure

The experimental modal basis has been identified on the tested specimen by using MIMO measurements associated to a specific identification technique called "Simulated Mode Appropriation" (Foltête and Piranda 2001). In classical MIMO measurements, the structure is excited by several forces produced by shakers connected to the chosen excitation DOF's, while the responses are recorded by a set of accelerometers mounted on the selected observation DOF's. The main advantages of this technique are: the use of stationary random forces allowing to record and average a large number of samples of the data, thus, minimizing the measurement noise; the control of the force amplitudes; the reliability of the measured DOF's (location and direction). Nevertheless, it presents also some disadvantages: structural perturbations introduced by shakers and accelerometers; force distortions caused by misalignments of the shakers or some parasite vibrations of the stingers connecting the shakers to the structure; the high number of recording channels.

In this study, the MIMO technique consists in mounting some accelerometers at the excitation

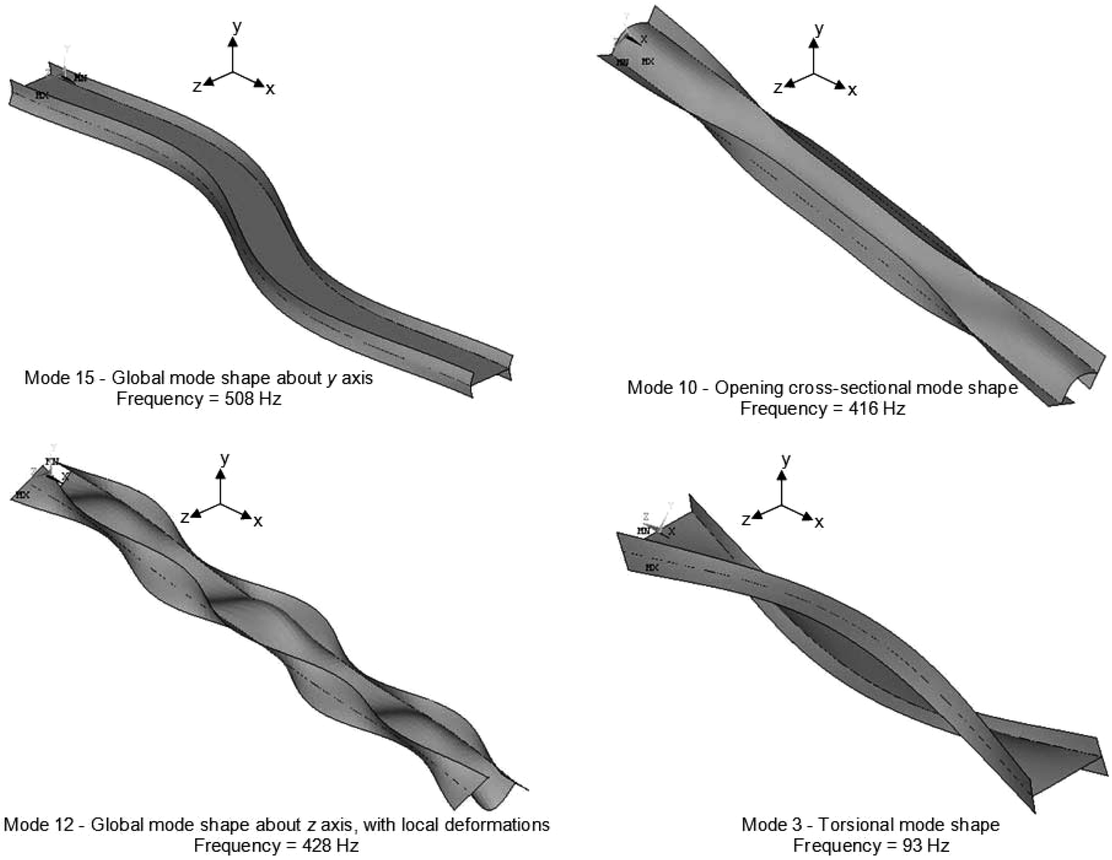

Fig. 4 Different mode shapes in vibrational behavior of the pultruded beam

DOF's and successively exciting the structure at the observation DOF's with an impact hammer. Once all measurements have been made, data are equivalent to those obtained by the classical MIMO measurements previously described. The main disadvantages of this alternative technique are inherent to the hammer excitation: uncertainties on the location and direction of the observation DOF's, no control of the excitation amplitude, small number of averages. On the other hand, it presents some interesting advantages: low number of accelerometers, minimization of the structural perturbation; the cost of the experiment and the time for its preparation; possibility of using a large number of observation DOF's; opportunity to apply MIMO identification techniques which are more reliable in particular when the structure exhibits coupled modes.

The used identification theory is derived from the "normal mode testing" techniques which were developed in the 1960's. The main idea is to calculate from MIMO FRF's the forces which would lead to the optimal phase resonance at each eigenfrequency of the analyzed band. It presents two important advantages: a phase resonance parameter allows a quasi-automatic selection of the modes (playing the same role as the stabilization diagram) even for strongly coupled modes; the method estimates the real eigenvalues and eigenvectors associated to the conservative associated system even if the damping is non proportional.

Before carrying-out the experimental measurements, optimal placement of sensors procedure has

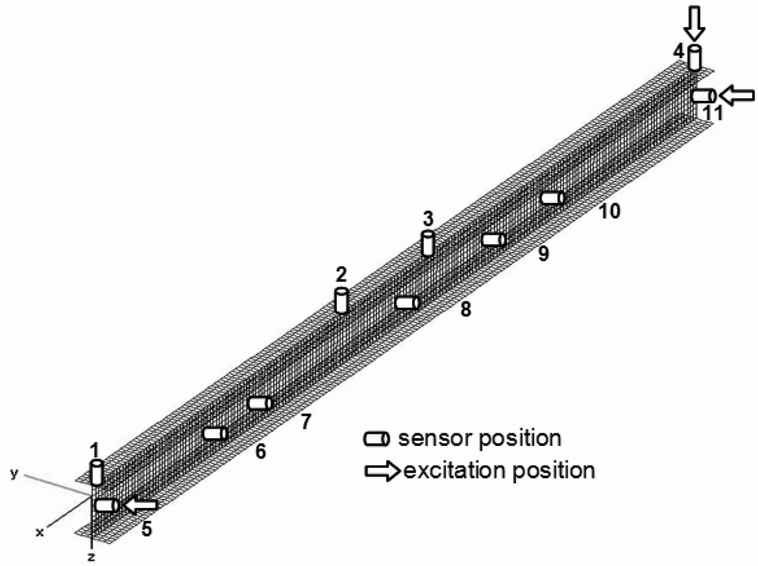

Fig. 5 Optimal placement of sensors and excitation points on the free-free beam

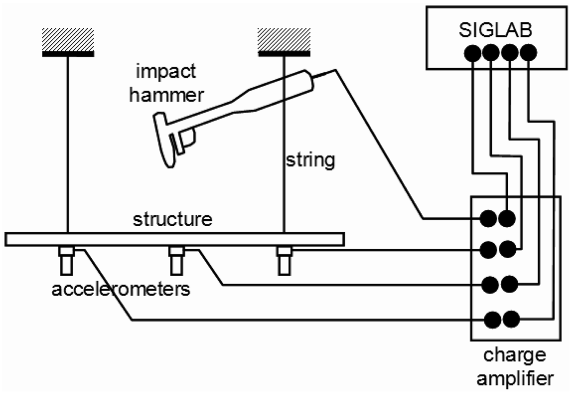

Fig. 6 Scheme of the experimental set-up

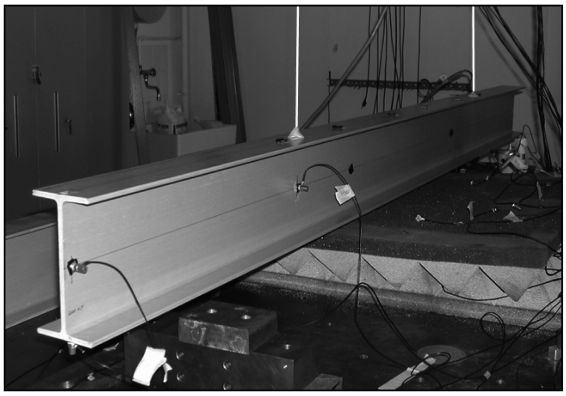

Fig. 7 Test structure and experimental apparatus

been performed, according to the technique described in section 3.1. Fig. 5 shows the results obtained.

Figs. 6 and 7 show the studied specimen and the experimental apparatus. Three lightweight accelerometers are mounted on the profile and connected to a four-channel acquisition system. The first channel is dedicated to the hammer force measurement. The studied frequency band is [0-1200 Hz] with 3200 spectral lines. The “Simulated Mode Appropriation” technique led to the identification of 12 modes in this band.

6. Identification of elastic properties

The identification of stiffness properties of composite materials by typical model updating methods in dynamics is relatively new. Some authors model the structure as global homogeneous, identifying the laminate engineering constants - E_x, E_y, G_{xy} and v_{xy} (Deobald and Gibson 1986, Frederiksen 1994, Frederiksen 1998, Pedersen 1988, Bledzki *et al.* 1999). This is, in fact, a sort of simplification of the problem and does not allow access to the layer engineering constants. On the other hand, layer engineering constants can be directly identified. In this case, the constants would be strongly nonlinear with respect to the eigenvectors and eigenvalues. Another approach consists in the identification of stiffness constants derived from the constitutive equation which expresses the resulting forces and moments as functions of extension and shear deformations and curvatures - matrices A, B and D (Sol 1986, Cunha and Piranda 1999, Ip *et al.* 1998, Gagneja *et al.* 2001).

More recently, new identification procedures have been proposed in order to improve the precision and the robustness of the results. Cugnoni *et al.* 2007 and Matter *et al.* 2007 utilize a measurement system with a contact-free excitation and a higher order composite plate/shell finite element model, assuring a better quality of the numerical and experimental responses. Daghia *et al.* 2006 use a modified strategy of identification, where the influence of the parameters on the natural frequencies is explicited, improving reliability and convergence of the estimation process. In this work, a higher order theory is used to model the thick plate. Baere *et al.* 2007 examine a fabric-reinforced composite material with a very small value of Poisson's ratio. The accurateness of the in-plane elastic properties is validated with static tensile tests. In addition to the in-plane properties, Shi *et al.* 2004 present identification of transverse shear moduli based on Timoshenko beam theory. In the same context, to facilitate the physical interpretation of the dynamic response, Lauwagie *et al.* 2008 identify elastic properties from the beam shaped layered specimens. Finally, as opposed to the traditional free boundary conditions, Lee and Kam 2006 propose identification of elastic properties in laminated composite plates supported by elastic restraints at both the edges and centers of the structure, which provides a large experimental data set.

In this work, the used model updating tool, named AESOP®-Analytical/Experimental Structural Optimization Platform (AESOP 2005), allows a general and flexible procedure of identification under several aspects (Fig. 8). AESOP interfaces with the external programs including finite element codes MSC Nastran™ and ANSYS®. Therefore, the used theory and the elastic constants to be identified depend only on the laminated theories available in these programs. In this work, elastic properties E_1, E_2 and G_{12} have been directly identified. It is considered that the flanges of the pultruded profile posses the same elastic properties as the web.

AESOP is built on the MATLAB® environment. The purpose of AESOP is to solve general optimization and finite element model updating problems and to offer the user a large variety of algorithms, including both local (conjugate gradient, constrained leas squares) and global (design of experiments, Monte Carlo simulation, genetic algorithms, neural networks, multiobjective optimization) methods. Furthermore, AESOP is designed to drive external analysis codes and to import the data necessary for evaluating a specified objective function. At the present time, AESOP benefits from as extensive interface with MSC Nastran and sophisticated management tools are provided to handle multiple solution sequences and superelement analyses (AESOP 2005). AESOP offers a wide range of features allowing to understand, predict and improve the behavior of the model, such as localization of dominant modeling errors, approximate reanalysis methods for model behavior and robustness analysis.

The main phases of the applied model updating procedure in AESOP are:

(a) Numerical modeling of the test structure: use of available theory in the commercial FEM codes. For plate modeling, a Kirchhoff-Love element type has been used - Nastran/CQUAD4 -

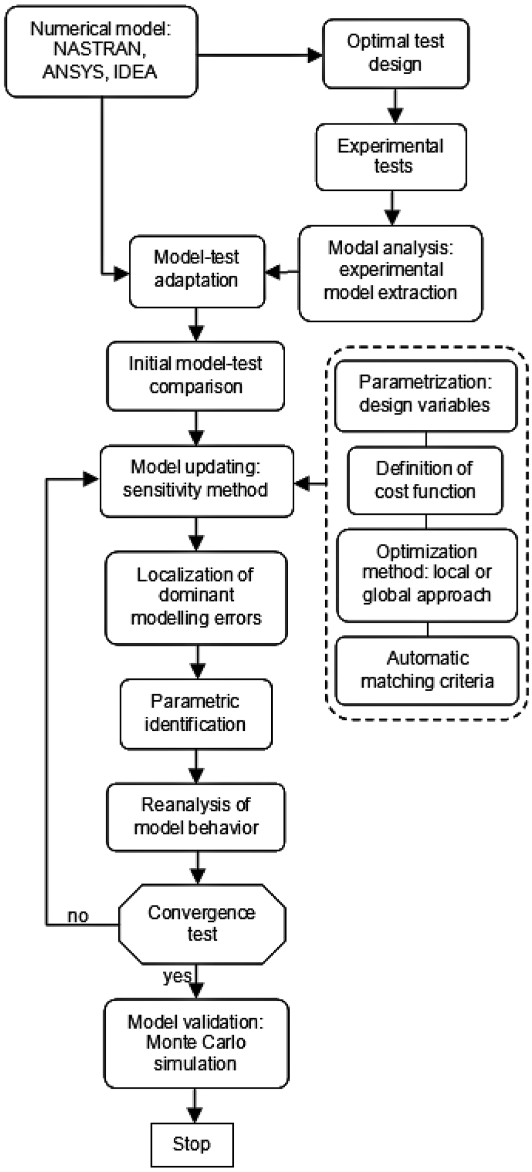

Fig. 8 Flow diagram of the model updating procedure by AESOP program

Quadrilateral Plate Element. For beam modeling, a Timoshenko theory element type has been used - Nastran/CBEAM - Beam Element;

(b) Computation of dynamic responses by solving eigenvalue problem;

(c) Test preparation with the optimal placement of sensors and excitation points: as shown in section 3.1;

(d) Modal identification and importation of experimental data using an universal file: as described in section 5. The universal file is formed by measured frequencies, mode shape amplitudes and damping;

(e) Linking between numerical and experimental models: model linking, as the name implies, establishes the geometrical transformations between two models, in order to compare them. Linking comprises two distinct transformations - one involving the difference in the global coordinate systems of each model and the other involving the differences in meshes;

(f) Correlation between the two sets of data by automatic matching of the modes: use of a MAC matrix, where MAC indicates the Modal Assurance Criterion, defined between two modes y_ν and z_μ by

$$MAC(\nu,\mu) = \frac{(y_\nu^T \times z_\mu)^2}{\|y_\nu\|_2^2 \times \|z_\mu\|_2^2} \tag{18}$$

(g) Definition of the design variables (parameters to be updated/identified): possibility of choice any parameter present on the numerical FEM model;

(h) Definition of control parameters for optimization algorithm (inequality and lateral constraints, cost function): as shown in section 6.1;

(i) Choice of the results to explore during and after updating process: several tools are available - evolution of errors, parameters, cost function, etc;

(j) Setting of the control parameters for iterative algorithm (number of iterations, convergence criterion, local or global optimization algorithm): as shown in section 6.1;

(k) Analysis of results.

6.1 Results for modeling with beam elements

The elastic properties to be identified for modeling with beam elements are: E, G, k_y and k_z. As explained in section 2.2, in reality the equivalent shear stiffness kGA will be identified. Some important parameters used in the updating procedure are: interval of lateral constraints of parameters p: E, G = [0.1;10]; k_yGA, k_zGA = [0.05; 2.5]; inequality constraints for each iteration: $\Delta p \leq 0.01$; cost function: frequencies and modes error (Eq. 14); automatic mode matching criteria: $MAC = 0.70$; number of sensor points: 11 (4 in z direction and 7 in y direction); local optimization method: Constrained Linear Least Squares; FE model: 200 elements of CBEAM - Nastran, resulting in 1413 DOF's. The lateral constraints of the parameters were chosen simply on the basis that the limits represent a large search space, where the physical sense remains. The choice of their initial values is based on the currently practical values provided by designers and manufacturers.

In order to better understand the influence of each property on eigenfrequencies in the model updating process, a Monte Carlo simulation has been performed. To obtain a set of parameters close to the updated solution it was assumed that the behavior of the system could be described by a uniform probability density function, which guarantees a convenient exploitation of the search space. Latin hypercube sampling has been applied with a sample size of 400. Correlation coefficients were the parameters used to indicate if variations of the eigenfrequencies are due to design variables with which eigenfrequencies have a linear relation. They are mathematical representations of clouds of points, indicating the normalized slope of the least square straight line. The correlation coefficient between the parameter p and the frequency f is defined by

$$r_{pf} = \frac{\sigma_{pf}}{\sigma_p \sigma_f} \tag{19}$$

where σ_{pf} is the covariance between the parameter p and the frequency f and σ_p and σ_f are the corresponding standard deviations. The values of the correlation coefficients range between-1 and 1.

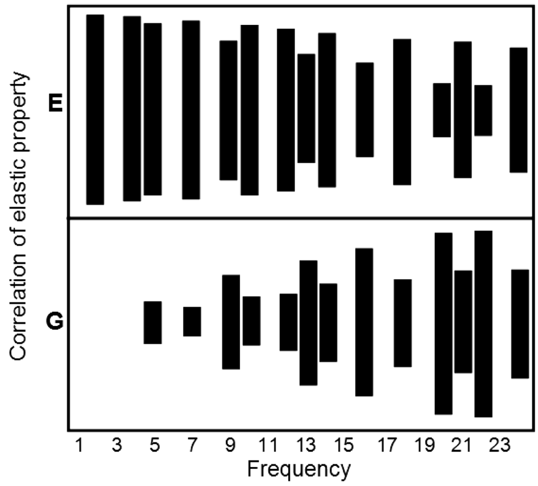

Fig. 9 Correlation between elastic properties and frequencies

A value near zero indicates that there exists no correlation between the variables.

Thus, considering that the correlation coefficients can be used to express the sensitivity of the eigenfrequencies with respect to the parameters, Fig. 9 shows that the lower frequencies are very sensitive to the Young's modulus, while the higher frequencies are more sensitive to the shear modulus. Therefore, the modal basis to be used in the model updating procedure should necessary consider lower and higher frequencies.

The chosen experimental basis has 6 bending modes of "beam type", i.e., when the local deformation effects in web and flanges become significant, the mode is discarded. These effects are particularly present for the bending modes about z axis. Fig. 10 and Tables 1 and 2 show that the results of the updating procedure are satisfactory. A good convergence of the cost function and elastic properties is observed. The final MAC values are close to 100% and the frequency errors are almost zero. The sensitivities of the eigenfrequencies and eigenvectors with respect to the elastic properties are globally balanced, with a more significant value for the Young's modulus. Instabilities in the evolution of sensitivities can be explained by the instabilities present in the pairing process. The large number of iterations required up to convergence can also be explained by this fact and by the intentional choice of a low increment of the solution in optimization procedure.

A greater experimental modal basis would allow a more confident identification, validating the updated model. In this case, only bending modes about y axis would be used, because higher bending modes about z axis present pronounced local effects.

6.2 Results for modeling with plate elements

The elastic properties to be identified for modeling with plate elements are: E_1, E_2 and G_{12}. Important parameters used in the updating procedure are: lateral constraints: $p = E_1$, E_2, $G_{12} =$ [0.1;10]; inequality constraints for each iteration: $\Delta p \leq 0.1$; cost function: frequencies and modes error (Eq. 14); automatic mode matching criteria: $MAC = 0.70$; number of sensor points: 11 (4 in z direction and 7 in y direction); local optimization method: Constrained Linear Least Squares; FE model: 4400 elements of CQUAD4 - Nastran, resulting in 27738 DOF's.

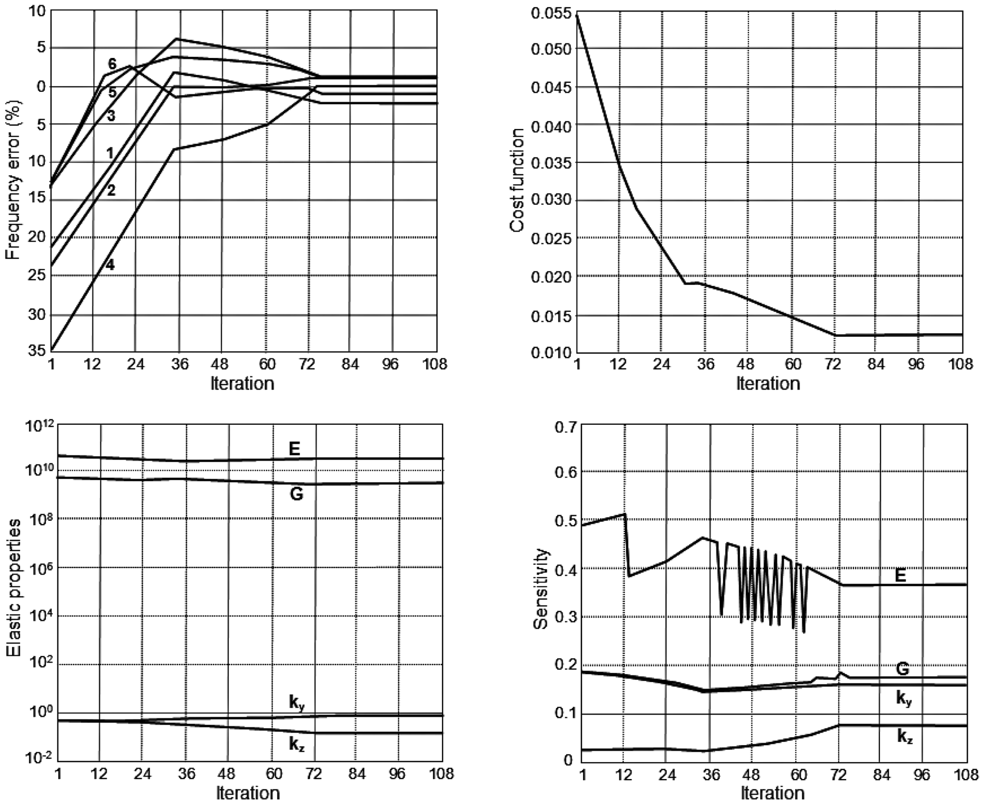

Fig. 10 Evolution of model updating parameters versus iterations - beam modeling

Table 1 Identified elastic properties using beam modeling

Property	Initial values	Updated values
E (N/m^2)	4×10^{10}	2.988×10^{10}
G (N/m^2)	5×10^{9}	2.901×10^{9}
k_yGA (N)	4.30×10^{6}	3.59×10^{6}
k_ZGA (N)	4.30×10^{6}	6.54×10^{5}

Table 2 Frequency errors for beam modeling

	Mode shape (nb of nodes)	Measured frequency (Hz)	Model frequency (Hz)	Model error (%)	Updated frequency (Hz)	Error after updating (%)
1	bending/z(2)	52	63	22	53.63	3.13
2	bending/z(3)	138	172	24	137.83	0.12
3	bending/y(2)	198	225	14	196.30	0.86
4	bending/z(4)	244	329	35	246.29	0.94
5	bending/y(3)	467	523	12	462.55	0.95
6	bending/y(5)	1050	1183	13	1063.36	1.27

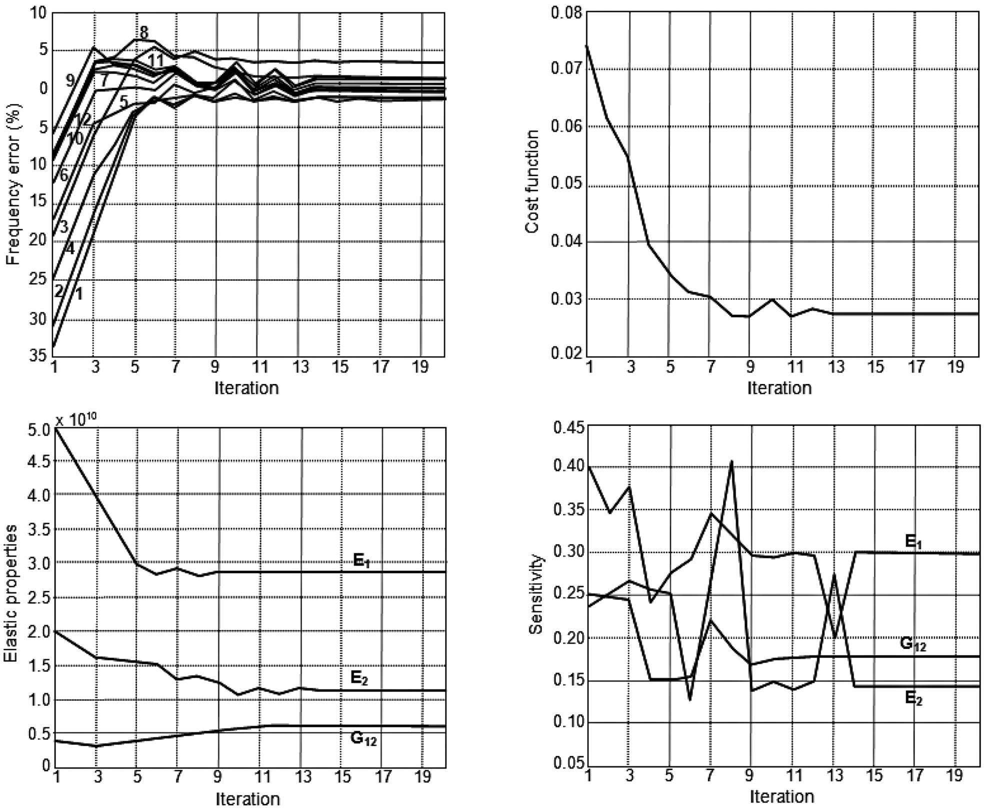

Fig. 11 Evolution of model updating parameters versus iterations - plate modeling

Table 3 Identified elastic properties using plate modeling

Property	Initial values	Updated values
E_1 (N/m^2)	5×10^{10}	2.860×10^{10}
E_2 (N/m^2)	2×10^{10}	1.105×10^{10}
G_{12} (N/m^2)	4×10^9	6.105×10^9

Table 4 Frequency errors for plate modeling

	Mode shape (nb of nodes)	Measured frequency (Hz)	Model frequency (Hz)	Model error (%)	Updated frequency (Hz)	Error after updating (%)
1	bending/z(2)	52	70	35	53	1.92
2	bending/z(3)	138	182	32	141	2.17
3	bending/y(2)	198	237	20	192	3.03
4	bending/z(4)	244	304	25	246	0.82
5	bending/z(5)	326	380	17	323	0.80
6	bending/z(6)	383	428	12	380	0.92
7	bending/z(7)	435	474	9	434	0.23
8	bending/y(3)	467	508	9	462	1.07
9	bending/z(8)	486	513	6	487	0.21
10	bending/z(9)	533	573	8	535	0.38
11	bending/z(10)	590	631	7	593	0.51
12	bending/z(11)	644	699	9	653	1.40

The used experimental basis has the first 12 bending modes. Differently from the beam modeling, all bending modes (about y and z axis) will be used to constitute the modal basis, because plate modeling can reproduce local deformation effects. Fig. 11 and Tables 3 and 4 show that the results of the updating procedure are very satisfactory. A good convergence of the cost function and elastic properties is noticed. MAC's are values close to 100% and the frequency errors are between 0 and 3%. Elastic properties sensitivities are balanced. As explained previously, instabilities in the evolution of the sensitivities are due to the variations in the pairing of modes.

7. Conclusions

The evolution of the composite material applications is largely dependent, among other aspects, on the accurate knowledge of the phenomena governing their mechanical behavior. In this sense, this study contributes to develop standard test methods and design recommendations to the overall characterization of the mechanical behavior of pultruded framed structures.

The employed model updating technique showed to be very interesting for identification of elastic properties of pultruded profiles. Obtained results for a real structure show the effectiveness of the sensitivity method. For all procedures, different starting points have been chosen (initial numerical models). For all the cases, a same final solution was observed (Cunha 2005).

Identification of the elastic properties using plate modeling allows us to consider a larger modal basis, which is important in the model updating process. The numerical modeling of the structure by plate and beam elements was important to understand the complex vibrational behavior of the pultruded profile. In reality, pultruded beams present a mechanical behavior of three-dimensional nature. For beam modeling, local deformation effects present on the mode shapes are not reproduced, reducing the modal basis to be considered.

It is important to understand that in a model updating process, the variables to be identified assume the necessary values to obtain equivalent responses for experimental and numerical models. They act as parameters to adjust the mathematical model. Thus, a comparison between identified values obtained from the two modeling strategies (beam and plate) is not obvious, because mathematical requirements can prevail over the physical sense of the problem. For this kind of problem, the assertion "identified parameters" must be understood in a large sense.

The identification technique from dynamic tests is very interesting, because it can be applied to a wide range of structures. This procedure of a non-destructive type has a relatively simple implementation. Only three accelerometers have been used in the experimental setup. The used approach is well adapted to the anisotropic character of the composite materials. As compared to other identification techniques, the method has the advantage of considerably simplify the tests since several different properties are identified simultaneously. The computational time used in the algorithm of identification was on the order of 30 minutes for plate modeling. Another advantage lies in the fact that the identification procedure can be applied directly to the structure (plates, profiles, tubes or more complex geometrical shapes).

The consideration of different practical aspects of the model updating technique allowed an efficient approach of the problem. Some details of the procedure have been intentionally omitted. The main objective of this work has been to show the feasibility and efficiency of the proposed methodology.

Acknowledgements

The authors thank the support of CAPES (Coordenação de Aperfeiçoamento de Pessoal de Nível Superior) and COFECUB (Comité Français d'Évaluation de la Coopération Universitaire avec le Brésil), within the framework of the cooperation agreement CAPES/COFECUB (Brazil/France).

References

AESOP - *Analytical/Experimental Structural Optimization Platform*. (2005), Version 5.0, Laboratoire de Mécanique Appliquée Raymond Chaléat - LMARC, Besançon - France.

Baere, I., Van Paepegem, W., Degrieck, J., Sol, H., Van Hemelrijck, D. and Petreli, A. (2007), "Comparison of different identification techniques for measurement of quasi-zero Poisson's ratio of fabric-reinforced laminates", *Composites: Part A*, **38**, 2047-2054.

Bledzki, A.K., Kessler, A., Rikards, R. and Chate, A. (1999), "Determination of elastic constants of glass/epoxy unidirectional laminates by the vibration testing of plates", *Compos. Sci. Technol.*, **59**(13), 2015-2024.

Clarke, J.L. (1996), *Structural design of polymer composites - Eurocomp Design Code and Handbook*, E & FN SPON, London.

Cugnoni, J., Gmür, T. and Schorderet, A. (2007), "Inverse method based on modal analysis for characterizing the constitutive properties of thick composite plates", *Comput. Struct.*, **85**, 1310-1320.

Cunha, J. (2005), *Caractérisation, modélisation, validation et optimisation des assemblages de structures de construction civile en matériaux composites*, Rapport post-doctorat, LMARC - Université de Franche-Comté.

Cunha, J. and Piranda, J. (1999), "Application of model updating techniques in dynamics for the identification of elastic constants of composite materials", *Composites: Part B*, **30**, 79-85.

Daghia, F., Miranda, S., Ubertini, F. and Viola, E. (2007), "Estimation of elastic constants of thick laminated plates within a Bayesian framework", *Comp. Struct.*, **80**, 461-473.

Davalos, J.F. and Qiao, P. (1999), "A computational approach for analysis and optimal design of FRP beams", *Comput. Struct.*, **70**, 169-183.

Deobald, L.R. and Gibson, R.F. (1986). "Determination of Elastic Constants of Orthotropic Plates by a Modal Analysis/Rayleigh-Ritz Technique", *Proceedings 4th IMAC*, Los Angeles, February, 682-690.

Foltête, E. and Piranda, J. (2001), "Transforming complex eigenmodes into real ones based on an appropriation technique", *J. Vib. Acoustics*, **123**, 92-97.

Frederiksen, P.S. (1994). "Estimation of elastic moduli in thick composite plates by inversion of vibrational data", *Proceedings of the Second Int. Symposium on Inverse Problems*, Paris, 111-118.

Frederiksen, P.S. (1998), "Parameter uncertainty and design of optimal experiments for the estimation of elastic constants", *Int. J. Solids Struct.*, **35**(12), 1241-1260.

Gagneja, S., Gibson, R.F. and Ayorinde, E.O. (2001), "Design of test specimens for the determination of elastic through-thickness shear properties of thick composites from measured modal vibration frequencies", *Compos. Sci. Technol.*, **61**, 679-687.

Ip, K.H., Tse, P.C. and Lai, T.C. (1998), "Material characterization for orthotropic shells using modal analysis and Rayleigh-Ritz models", *Composites-Part B*, **29**, 397-409.

Lauwagie, T., Lambrinou, K., Patsias, S., Heylen, W. and Vleugels, J. (2008), "Resonant-based identification of the elastic properties of layered materials: application to air-plasma sprayed thermal barrier coatings", *NDT&E Int.*, **41**, 88-97.

Lee, C.R. and Kam, T.Y. (2006), "Identification of mechanical properties of elastically restrained laminated composite plates using vibration data", *J. Sound Vib.*, **295**, 999-1016.

Matter, M., Gmür, T., Cugnoni, J. and Schorderet, A. (2007), "Improved modal characterization of the constitutive parameters in multilayered plates", *Compos. Sci. Technol.*, **67**, 1121-1131.

Pedersen, P. (1988), *Laminates: analysis, sensitivity, optimal design, identification of material parameters*. Notes for Lectures.

Shi, Y., Sol, H. and Hua, H. (2005), "Transverse shear modulus identification by an inverse method using

measured flexural resonance frequencies from beams", *J. Sound Vib.*, **285**, 425-442.
Sol, H. (1986), *Identification of Anisotropic Plate Rigidities using Free Vibration Data*. PhD Thesis, Vrije Universiteit Brussel.
Sonti, S.S. (1992), *Stress analysis of pultruded structural shapes*. MSc Dissertation, West Virginia University.