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Technical Note

An efficient FE model for dynamic instability analysis of imperfect composite laminates

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1. Introduction

The dynamic instability analysis of composite laminates is of considerable importance in which the effect of shear deformation becomes very significant consideration mainly due to its laminated configuration. The problem becomes more involved if some inter-laminar imperfection is found in the form of weak bonding or otherwise. The instability of plates subjected to in-plane loads may occur below the critical load of the structure over a range of excitation frequencies. The well-known Hill's method of infinite determinants is used for solving a system of Mathieu-type equation in the present problem to predict the stability properties. Dynamic instability of plates under different in-plane loads has been investigated by a number of different investigators in case of perfect interface (Deolasi and Datta 1995, Chen and Yang 1990, Kwon 1991, Chattopadhay and Radu 2000). However, no such studies based on Refined Higher order Shear Deformation Theory (RHSDT) are found in the literature even in case of perfect composite laminates. In this paper attempt has been made for the first time to study the dynamic instability of imperfect composite laminates using an efficient finite element plate model based on RHSDT in combination with linear spring layer model (Chakrabarti and Sheikh 2007).

2. Formulation

In the general formulation to model RHSDT and linear spring layer theory in the FE analysis (Chakrabarti and Sheikh 2007), the element stiffness matrix $[k]$, element geometric stiffness matrix $[k_g]$ and element mass matrix $[m]$ are evaluated for all the elements and assembled together to form the overall stiffness matrix $[K]$, geometric stiffness matrix $[K_G]$ and mass matrix $[M]$ of the whole structure. With these matrices, the equation of equilibrium for an elastic system undergoing small displacements at the instant of buckling may be written as

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$$[M]\{\ddot{\partial}\}+[[K]-P[K_G]]\{\partial\} = \{0\} \tag{1}$$

In the above equation, the in-plane load factor P is periodic and may be expressed in the form

$$P = P_S + P_t \cos\Omega t \tag{2}$$

where P_S is the static portion of P, P_t is the amplitude of the dynamic portion of P with Ω as the frequency of excitation. The buckling load P_{cr} may be used to express P_S and P_t as follows

$$P_S = \alpha P_{cr}, \quad P_t = \beta P_{cr} \tag{3}$$

where α and β are static and dynamic load factors respectively. Using Eqs. (2) and (3) the equation of motion (1) may be expressed as series of algebraic equations for the determination of instability regions. Principal instability region, which is of practical importance leads to the dynamic instability equation

$$\left[[K]-\alpha P_{cr}[K_G] \pm \beta P_{cr}[K_G]-\frac{\Omega^2}{4}[M]\right]\{\gamma\} = \{0\} \tag{4}$$

The above eigenvalue solution give the value of Ω, which are the bounding frequencies of the instability regions for the given values of α and β. Before solving the above equations, the stiffness matrix $[K]$ is modified through imposition of boundary conditions (Chakrabarti and Sheikh 2007).

3. Numerical examples

Numerical examples on imperfect as well as perfect composite laminates under uniformly distributed in-plane edge loadings are solved in this section using the proposed finite element plate model.

3.1 Cross-ply square laminate simply supported at the four edges

This is a problem of simply supported cross-ply (0/90/90/0) square laminate having imperfection at the layer interfaces. The analysis is carried out by the proposed element using mesh sizes (full plate) 16 × 16 taking h/a = 0.01, 0.05, 0.10, 0.20 and 0.25. In this problem, all the layers are of same thickness and material properties ($E_1 = 40E$, $E_2 = E$, $G_{12} = G_{13} = 0.6E$, $G_{23} = 0.5E$ and ν_{12} = 0.25). The imperfections at the layer interfaces are defined by the parameters: $R_{11}^k = R_{22}^k = Rh/E$ and $R_{12}^k = R_{21}^k = 0.0$ where the non-dimensional parameter R is varied from 0.0 to 1.2 (R = 0.0 represents perfect interface). The factors α and β are varied to identify the lower and upper boundaries of the excitation frequency. The results obtained are presented in the form of excitation frequency parameter, $\Omega = \omega(a^2/h)\sqrt{\rho/E}$ in Table 1. The results presented in Table 1 show that the excitation frequency decreases rapidly with the increase in the imperfection parameter (R).

3.2 Simply supported square plate having three orthotropic layers

A simply supported three layered square plate ($h/a = 0.1$) having imperfection at the layer

Table 1 Excitation frequency parameters (Ω) of a simply supported square laminated composite plate (0/90/90/0) subjected to uniform in-plane edge loading (Bi-axial)

R	β	Excitation frequency parameters, Ω					
		α = 0.0		α = 0.2		α = 0.4	
		Upper	Lower	Upper	Lower	Upper	Lower
Thickness ratio (h/a) = 0.01							
0.0	0.3	40.357	34.696	36.680	30.341	32.591	25.245
	0.5	42.075	32.591	38.562	27.909	34.696	22.264
	0.8	44.528	29.150	41.225	23.801	37.633	16.830
0.2	0.3	40.207	34.567	36.544	30.228	32.470	25.151
	0.5	41.919	32.470	38.419	27.806	34.567	22.181
	0.8	44.362	29.042	41.072	23.713	37.493	16.767
	0.3	39.044	33.568	35.487	29.354	31.531	24.424
1.2	0.3	39.044	33.568	35.487	29.354	31.531	24.424
	0.5	40.707	31.531	37.308	27.002	33.568	21.540
	0.8	43.080	28.202	39.884	23.027	36.409	16.283
Thickness ratio (h/a) = 0.20							
0.0	0.3	23.220	19.963	21.104	17.457	18.752	14.525
	0.5	24.208	18.752	22.187	16.058	19.963	12.810
	0.8	24.335	16.772	23.719	13.694	21.653	9.683
0.2	0.3	18.563	15.959	16.872	13.956	14.991	11.612
	0.5	19.353	14.991	17.737	12.837	15.959	10.241
	0.8	20.481	13.408	18.962	10.948	17.310	7.741
1.2	0.3	11.218	9.645	10.196	8.434	9.060	7.018
	0.5	11.696	9.060	10.719	7.758	9.645	6.189
	0.8	12.378	8.103	11.459	6.616	10.461	4.678

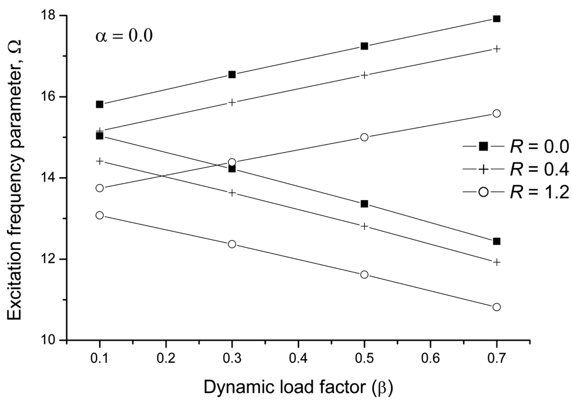

Fig. 1 Instability region of a simply supported laminated plate having orthotropic layers (α = 0.0)

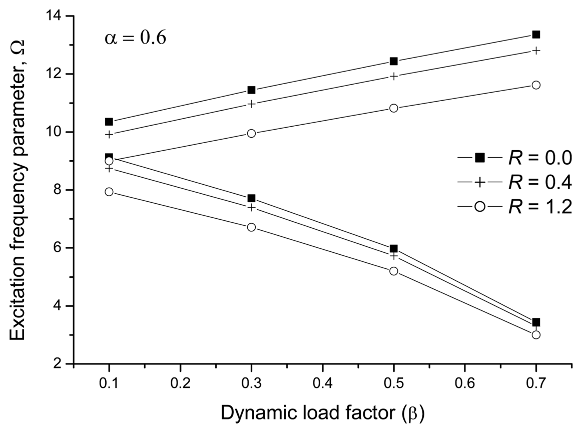

Fig. 2 Instability region of a simply supported laminated plate having orthotropic layers (α = 0.6)

interfaces and subjected to in-plane bi-axial loading is studied here. The central orthotropic layer has a thickness of $0.8h$ while each of the face layers is $0.1h$ thick. The material properties of the orthotropic face layers are taken as multiple (K_t) of those of the central layer/core where the value of K_t is taken as 5.0. The material properties used for the core are E_{22}/E_{11} = 0.543, G_{12}/E_{11} = 0.2629, G_{13}/E_{11} = 0.1599, G_{23}/E_{11} = 0.2668, ν_{12} = 0.3. The imperfections at the layer interfaces are defined by the parameters: $R_{11}^k = R_{22}^k = Rh/E_{11}$ and $R_{12}^k = R_{21}^k = 0.0$ where the non-dimensional parameter R is varied from 0.0 to 1.2 (R = 0.0 represents perfect interface); α and β are varied as before. The results obtained for the excitation frequency parameters, $\Omega = 100\omega\sqrt{\rho h^2/E_{11}}$ are presented in Figs. 1 and 2 for different values of α (0.0 and 0.6).

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