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Force transfer mechanisms for reliable design of reinforced concrete deep beams

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Abstract. In this paper, a strut-and-tie model approach has been proposed to directly calculate the amount of reinforcements in deep beams, and the force transfer mechanisms for this approach were investigated using linear finite element analysis. The proposed strut-and-tie model provides quite similar force transfer mechanisms to the results of linear finite element analysis for the 28 deep beams. The load-carrying capacities calculated from the proposed method are both accurate and conservative with little scatter or trends for the 214 deep beams. The deep beams have different concrete strengths including high-strength, various combinations of web reinforcements, and wide range of and a/d ratios. Good accuracy was also obtained using VecTor2, nonlinear finite element analysis tool based on the Modified Compression Field Theory. Since the proposed method provides a safe and reliable means for design of deep beams, this can serve to improve design provisions in future adjustments and development of design guidelines.

Keywords: strut-and-tie model; shear strength; force transfer mechanism; reinforced concrete; deep beams.

1. Introduction

Numerous studies have been carried out over the last fifty years to evaluate the capacity of reinforced concrete deep beams (Aguilar *et al.* 2002, Clarke 1951, Kong *et al.* 1970, Foster and Gilbert 1996, Oh and Shin 2001, Park and Kuchma 2007, Rogowsky *et al.* 1986, Smith and Vantsiotis 1982, Tan *et al.* 1995, Guan 2005). Due to the small shear span-depth ratio, less than 2.5 for a deep beam, a large portion of applied loads are directly transmitted to supports. Therefore, the shear strength of deep beams is significantly greater than that of slender beams. The experiments have shown that diagonal splitting and concrete crushing are two common failure modes for supported deep beams with small shear span-to-depth ratios (Aguilar *et al.* 2002, Kong *et al.* 1970, Rogowsky *et al.* 1986, Smith and Vantsiotis 1982). Concrete crushing is typically observed in the compression zone at the head of the inclined crack and in the region adjacent to the loading plate. The disturbed stress distribution in the diagonal strut of deep beams induces transverse bursting forces and leads to the possibility of diagonal splitting failure. Thus, there is a complex state of strain in deep beams, and traditional sectional design approaches that are based on plane sections theory and utilize a parallel chord truss model are not applicable for the design of deep beams.

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The strut-and-tie model is an emerging and rational approach for design of deep beams. Provisions for the design of deep beams by the strut-and-tie models have been included in several codes of practice, including AASHTO LRFD, ACI 318-05, CEB-FIP, and the Canadian Code. This has been applied by many researchers to solve practical design problems (Marti 1985, Schlaich *et al.* 1987, Schlaich and Schäfer 1991, Schäfer 1996, Hwang and Lee 1999, 2000, Hwang *et al.* 2000a, 2000b, 2001, Foster and Malik 2002). This method directly considers the idealized flow of forces or internal load-carrying system of the discontinuity region (D-region). A strut-and-tie model consists of struts, ties, and nodes where struts are the compression members, ties are the tension members, and nodes are the meeting regions for the struts and ties.

Although statically determinate strut-and-tie models are usually adequate for most common design situations, statically indeterminate strut-and-tie models are also used to refine the load-carrying system of D-region. Designers are free to choose the geometry and dimensions of the load-resisting truss (strut-and-tie model) that will carry the imposed loads through the D-region to its supports. There is no single design solution as more than one strut-and-tie model is usually applicable. Many designers are uncomfortable with the flexibility provided by the strut-and-tie models which is often called an approximate approach with undetermined accuracy. Since the model can be selected based on engineering judgment, a number of guidelines have been established for choosing an optimal way of carrying the loads and the dimensions of the components of a strut-and-tie model. Thus, it is important for the research community to further evaluate the strengths and limitations of strut-and-tie design procedures, so that reliable design guidelines can be proposed.

This study proposes a strut-and-tie model approach that can improve design provisions. The force transfer mechanisms of the proposed method were analyzed via linear finite element analysis. The method considers the constitutive laws of cracked concrete and the softening effect induced by the bursting force perpendicular to the strut. The commercial software VecTor2 was employed to evaluate the capacity of nonlinear finite element analysis based on the Modified Compression Field Theory (MCFT) for the 28 deep beams.

2. Force transfer mechanisms

2.1 Finite element analysis

The force transfer mechanisms calculated from the proposed method were compared with those obtained by linear finite element analysis for 28 deep beams tested to failure in laboratories. The details of the 28 deep beams are presented in Table 1. The Fig. 1 shows the typical principal stress flows of compressive and tensile stresses of deep beams. Fig. 2 shows the strut-and-tie model selected in this study and the member forces of strut and tie components. Linear finite element analysis can provide reasonable estimation for the force transfer mechanism even for statically indeterminate structures (Schlaich *et al.* 1987, Foster 1992).

To estimate the horizontal and vertical forces, the tensile forces of concrete elements and smeared reinforcements resulting from finite element analysis for each direction are as follows

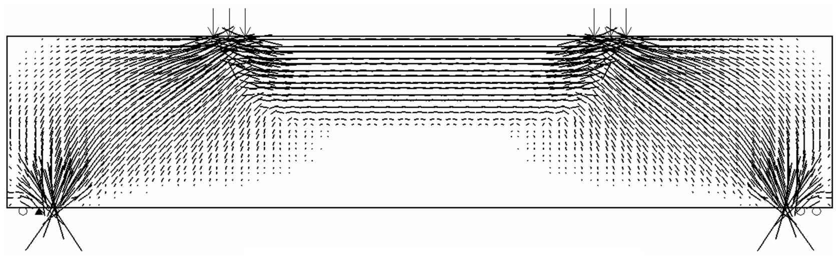
$$F_h' = f_{1\max}(\sin^2\alpha)A_{1-1} + f_{sh}A_{sh} \tag{1}$$

$$F_v' = f_{1\max}(\cos^2\alpha)A_{2-2} + f_{sv}A_{sv} \tag{2}$$

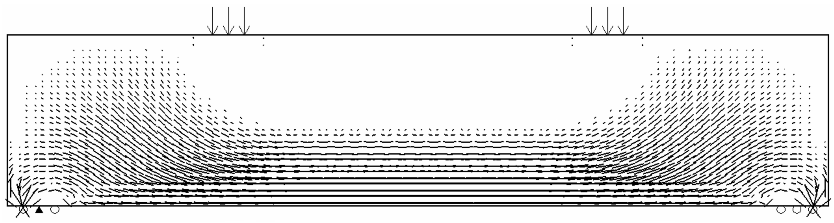
Table 1 Details of the selected 28 deep beams

Deep beam ID	f'_c (MPa)	b (mm)	h (mm)	a (mm)	d (mm)	a/d	ρ_h (%)	ρ_v (%)	ρ (%)	V_{test} (kN)	V_{fea} (kN)	V_{stm} (kN)	$\frac{V_{test}}{V_{fea}}$	$\frac{V_{test}}{V_{stm}}$
0A0-44	20.5	101.6	356	305	305	1	0	0	1.94	139.5	127.5	122.2	1.09	1.14
1A6-37	21.1	101.6	356	305	305	1	0.91	0.28	1.94	184.1	168.0	125.8	1.10	1.46
0B0-49	21.7	101.6	356	368	305	1.21	0	0	1.94	149.0	112.5	107.5	1.32	1.39
1B6-31	19.5	101.6	356	368	305	1.21	0.91	0.24	1.94	153.4	148.5	96.2	1.03	1.59
0C0-50	20.7	101.6	356	457	305	1.5	0	0	1.94	115.7	96.0	82.4	1.20	1.40
1C6-16	21.8	101.6	356	457	305	1.5	0.91	0.18	1.94	122.3	128.3	87.0	0.95	1.41
0D0-47	19.5	101.6	356	635	305	2.08	0	0	1.94	73.4	58.5	55.8	1.25	1.32
4D1-13	16.1	101.6	356	635	305	2.08	0.23	0.42	1.94	87.4	72.0	45.5	1.21	1.92
5-30	18.6	76	762	254	724	0.35	0.61	0.61	0.52	240.0	185.3	143.9	1.30	1.67
5-25	19.2	76	635	254	597	0.43	0.61	0.61	0.62	208.0	178.5	132.0	1.17	1.58
5-20	20.1	76	508	254	470	0.54	0.61	0.61	0.79	173.0	153.0	118.2	1.13	1.46
5-15	21.9	76	381	254	343	0.74	0.61	0.61	1.09	127.0	128.3	90.3	0.99	1.41
5-10	22.6	76	254	254	216	1.18	0.61	0.61	1.73	78.0	77.0	54.6	1.01	1.43
A1-1	24.6	203.2	457.2	914.4	390.4	2.34	0	0.38	3.10	222.5	213.8	155.8	1.04	1.43
B1-1	23.4	203.2	457.2	762	390.4	1.95	0	0.37	3.10	278.8	256.5	177.2	1.09	1.57
B2-1	23.2	203.2	457.2	762	390.4	1.95	0	0.73	3.10	301.1	267.8	176.3	1.12	1.71
C1-1	25.6	203.2	457.2	609.6	390.4	1.56	0	0.34	2.07	277.7	285.0	253.0	0.97	1.10
C2-1	23.6	203.2	457.2	609.6	390.4	1.56	0	0.69	2.07	290.0	315.0	232.3	0.92	1.25
D1-3	24.5	203.2	457.2	457.2	390.4	1.17	0	0.46	1.63	256.6	342.0	294.8	0.75	0.87
D4-1	23.1	203.2	457.2	457.2	390.4	1.17	0	1.22	1.63	312.2	360.0	293.9	0.87	1.06
N4200	23.7	130	560	425	500	0.85	0	0	1.56	265.2	247.5	301.2	1.07	0.88
H4100	49.1	130	560	250	500	0.5	0	0	1.56	642.2	467.5	375.7	1.37	1.71
H4300	49.1	130	560	625	500	1.25	0	0	1.56	337.4	297.5	278.9	1.13	1.21
H45C2	49.1	130	560	1000	500	2	0.43	0.34	1.56	235.3	237.5	174.3	0.99	1.35
H41A3	50.7	120	560	250	500	0.5	0.94	0.13	1.29	454.8	500.0	454.0	0.91	1.00
H43A3	50.7	120	560	625	500	1.25	0.94	0.13	1.29	291.0	300.0	216.5	0.97	1.34
U42A2	73.6	120	560	425	500	0.85	0.47	0.13	1.29	417.6	337.5	322.1	1.24	1.30
U45A2	73.6	120	560	1000	500	2	0.47	0.13	1.29	213.6	201.9	136.9	1.06	1.56
Average													1.08	1.38
Coefficient of Variation													0.13	0.18

where F'_h and F'_v are the summations of tensile forces in horizontal and vertical directions estimated at the section 1-1 and section 2-2 shown in Fig. 2, respectively. $f_{1\,max}$ is the maximum principal tensile stress acting at the section 1-1 in Eq. (1) or at the section 2-2 in Eq. (2); α is the angle of inclination of principal compressive stress. A_{1-1} and A_{2-2} are the cross-sectional areas of section 1-1 or section 2-2; f_{sh}, f_{sv}, A_{sh}, and A_{sv} are the tensile stresses and cross-sectional areas of smeared reinforcements for horizontal and vertical directions, respectively. Principal tensile stresses of the elements below the sectional centroid of the beam were not included to compute the horizontal tensile force F'_h because a larger portion of the horizontal tensile forces of the elements below the section centroid is induced by the beam action than transverse bursting force. Fig. 3(a) shows the force ratios of Eq. (1) and Eq. (2) divided by applied vertical force for the 28 deep beams.



(a) Principal compressive stress flows



(b) Principal tensile stress flows

Fig. 1 Principal stress flows in deep beams

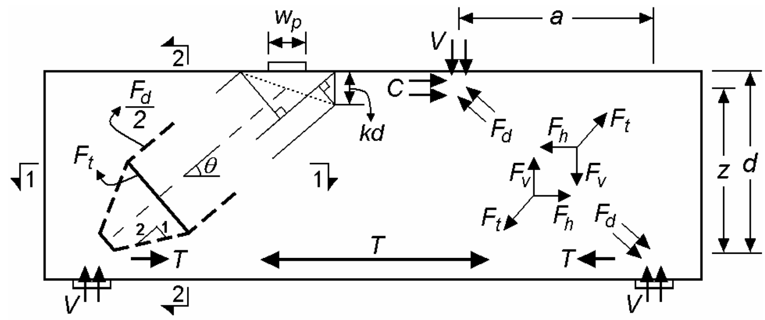


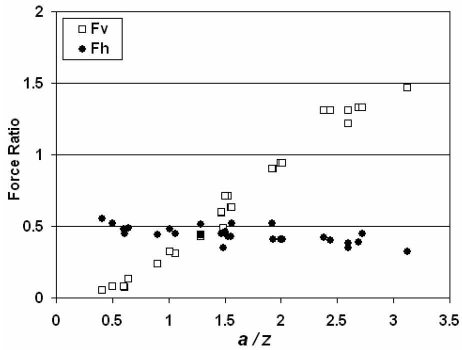
Fig. 2 Strut-and-tie model for deep beam

2.2 Strut-and-tie model

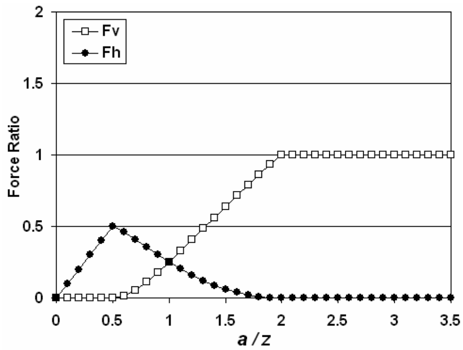
Softened truss models by other researchers have been proposed to determine the shear strengths of beam-column joints, deep beams, corbels, squat walls, and dapped-end beams (Hwang and Lee 1999, 2000, Hwang *et al.* 2000a, 2000b, 2001). In their approach, the strut-and-tie model is composed of diagonal, horizontal and vertical mechanisms. Since the strut-and-tie model is statically indeterminate to the second degree, they introduced two additional equations to define the stiffness ratios among the resisting mechanisms. The fraction of diagonal compression transferred by the horizontal tie in the absence of vertical tie is defined as

$$\gamma_h = \frac{2\tan\theta - 1}{3} \tag{3}$$

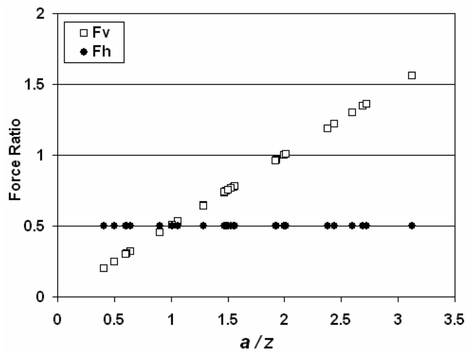
The fraction of diagonal compression transferred by the vertical tie in the absence of horizontal tie is defined as



(a) Finite element analysis



(b) Eq. (8) and Eq. (9)



(c) Proposed strut-and-tie model

Fig. 3 Force transfer ratios according to (a) Finite element analysis; (b) Eq. (8) and Eq. (9); (c) Proposed strut-and-tie model

$$\gamma_v = \frac{2\cot\theta - 1}{3} \tag{4}$$

where θ is the inclination angle of the diagonal strut with respect to the horizontal axis. The Eq. (3) and Eq. (4) were defined based on the study of the principal stress pattern from a linear elastic finite element analyses and the suggestions of Schäfer (1996). It was assumed that the entire shear is carried by the horizontal mechanism for $\theta \geq \tan^{-1}(2)$ and that the entire shear is carried by the vertical mechanism for $\theta \leq \tan^{-1}(1/2)$.

The ratios resisted by the diagonal, horizontal and vertical mechanisms are defined by Hwang and Lee (1999) using the Eq. (3) and Eq. (4) as follows

$$R_d = \frac{(1-\gamma_h)(1-\gamma_v)}{1-\gamma_h\gamma_v} \tag{5}$$

$$R_h = \frac{\gamma_h(1-\gamma_v)}{1-\gamma_h\gamma_v} \tag{6}$$

$$R_v = \frac{\gamma_v(1-\gamma_h)}{1-\gamma_h\gamma_v} \tag{7}$$

Normalizing the vertical shear force creates horizontal and vertical tensile forces shown in Eqs. (8) and (9).

$$F_h = \frac{1}{\tan\theta} R_h \tag{8}$$

$$F_v = R_v \tag{9}$$

Fig. 3(b) shows the force ratios of Eq. (8) and Eq. (9) for the 28 deep beams.

The strut-and-tie models directly consider the flow of forces in reinforced concrete deep beams. Schlaich and Schäfer (1991) observed that the shape of the concrete strut is bowed. As a result, transverse tensile forces exist within the strut. The disturbed stress distribution in the strut induces transverse bursting forces and leads to the possibility of diagonal splitting failure. The strut-and-tie model with "bottle-shaped" struts as shown in Fig. 2 is able to eliminate hidden dangers when the strut-and-tie model is too simple. Because this type of strut-and-tie model is statically determinate, member forces can be calculated from equilibrium equations only. This can be a primary advantage for the authors' model for design practice. When a statically indeterminate model is used, it is necessary to assume a stiffness ratio between components.

Equilibrium provides the following equations

$$D = \frac{V}{\sin\theta} \tag{10}$$

$$C = \frac{V}{\tan\theta} \tag{11}$$

$$F_t = \frac{V}{4\sin\theta} \tag{12}$$

where D, C, and F_t are the compressive forces in the diagonal and horizontal concrete struts, and the bursting tensile force in the tie of the strut-and-tie model is shown in Fig. 2. The compressive force in the strut is at a 2:1 slope as indicated in the ACI Code Section A.3.3. (ACI 2005). The horizontal and vertical components of the tie force can be obtained from equilibrium as follows

$$F_h = \frac{V}{4} = \frac{D\sin\theta}{4} \tag{13}$$

$$F_v = \frac{V}{4\tan\theta} = \frac{D\cos\theta}{4} \tag{14}$$

where F_h and F_v are the horizontal and vertical components of the tie force, respectively.

The forces of Eq. (13) and Eq. (14) are shown in Fig. 3(c) giving very similar results to linear finite element analysis for the 28 deep beams with different concrete strengths, various combinations of web reinforcements, and a/d ratio ranging from 0.35 to 2.34. However, Eq. (8) and Eq. (9) deviate from those results. The horizontal forces are almost constant regardless of the inclination of diagonal strut because of the vertical component of applied shear force. Thus, the proposed method provides good representation of the force transfer mechanisms for reinforced concrete deep beams.

3. Strut-and-tie mechanism for deep beams

Since a large portion of the supported loads are directly transmitted to supports due to the small shear span-to-depth ratio, the shear strengths of deep beams were significantly increased than those of slender beams. Experiments have shown that simply supported deep beams with a shear span-to-depth ratio (a/d) less than about 2.5 failed mainly from two common failure modes. These were diagonal splitting and concrete crushing. However, many of the specimens which were selected in this study may be more accurately described as having a combined bending and shear failure.

A strut-and-tie method was previously presented by Park and Kuchma (2007) based on secant stiffness formulation which has been used for the nonlinear finite element analysis of structural concrete by Vecchio (1989). Although this approach presents a systematic process for calculating the load-carrying capacity of deep beams, it is not able to directly compute the required amount of shear reinforcements. This study proposed an iterative strut-and-tie model approach to determine the amount of shear reinforcements. The method considers failure due to nodal crushing at the top of diagonal concrete strut, and diagonal splitting or concrete crushing of the strut as well as yielding of the longitudinal reinforcement. The details of the proposed approach are presented below.

3.1 Effective depths of struts and nodes

The effective depth of the top horizontal concrete strut was

$$w_c = kd \tag{15}$$

where d is the effective depth of deep beam and k was derived from the classical bending theory for a single reinforced beam section as

$$k = \sqrt{(n\rho)^2 + 2n\rho} - n\rho \tag{16}$$

where n is the ratio of steel to concrete elastic moduli; ρ is the longitudinal reinforcement ratio. The effective depth of the diagonal concrete strut was

$$w_d = \frac{a}{2}\sin\theta + kd\cos\theta \tag{17}$$

where $a/2$ should not be less than the length of loading plate, kd is the depth of the compression zone at the section, and the inclination angle of the diagonal strut with respect to the horizontal axis, θ can be obtained from

$$\tan\theta = \frac{d - kd/2}{a} = \frac{z}{a} \tag{18}$$

The notations for obtaining the effective depth w_d of Eq. (17) and strut angle θ of Eq. (18) are given in Fig. 2.

The effective width of the top node in the face of horizontal concrete strut was taken as a quarter of the overall height of deep beam based on the suggestion of Paulay and Priestley (1992) for the depth of the flexural compression zone of the elastic column as

$$c = \left(0.25 + 0.85\frac{N}{A_g f_c'}\right)h_c \tag{19}$$

where N is axial force; A_g is gross area; h_c is overall height of the column in the direction considered. The effective width of the top node in the face of the diagonal concrete strut was

$$w_{dn} = w_p \sin\theta + \frac{h}{4}\cos\theta \quad (20)$$

where w_p is the width of loading plate.

It is important to account for the different behavior of high-strength concrete deep beams with no web reinforcement or very light amounts of web reinforcement as they exhibited more brittle failures than normal strength concrete beams with similar levels of web reinforcement (Foster and Gilbert 1996, Oh and Shin 2001, Rogowsky *et al.* 1986, Quintero-Febres *et al.* 2006). In a recent investigation into the strength of high-strength concrete deep beams (Oh and Shin 2001) at *a*/*d* greater than about 1.0, web reinforcement restrained sudden shear failure. This indicates that if the web of a deep beam is heavily reinforced, the failure will be controlled by strut crushing. however, if sufficient web reinforcement is not provided, failure can occur suddenly due to the splitting of concrete struts. The splitting failure becomes more evident as concrete strength increases. To consider this brittleness of high-strength concrete, the area reduction factors presented by Park and Kuchma (2007) have been adopted as follows

$$\phi_{c1} = \rho_h + 0.75 \quad \text{for } a/d \le 075 \quad (21)$$

$$\phi_{c2} = \rho_v + 0.75 \quad \text{for } a/d \ge 1.0 \quad (22)$$

where ρ_h is the ratio of horizontal web reinforcement in %; ρ_v is the ratio of vertical web reinforcement in %.

The purpose of the area reduction factors accounted for the brittle failure of deep beams with high-strength concrete for the case that the concrete strength is greater than 42 MPa or when there is insufficient web reinforcement which was selected to be less than 0.25% in each direction. The strut area is reduced by horizontal web reinforcement when *a*/*d* is less than 0.75, and by vertical web reinforcement when *a*/*d* is greater than 1.0. This approach is supported by experimental test data from which it has been generally observed that horizontal web reinforcement is more effective than vertical web reinforcement when *a*/*d* is less than 0.75, and that vertical shear reinforcement is more effective than horizontal shear reinforcement when *a*/*d* ratios are greater or around 1.0 (Rogowsky *et al.* 1986, Kong *et al.* 1970). The use of area reduction factors leads to better agreement with the 214 test results examined in this study. Thus, the effective area of the diagonal concrete strut is expressed by

$$A_d = \phi_{c1}\phi_{c2} w_d b \quad (23)$$

3.2 Constitutive laws

Cracked reinforced concrete can be treated as an orthotropic material with its principal axes corresponding to the directions of the principal average tensile and compressive strains. Cracked concrete subjected to high tensile strains in the direction normal to compression was softer than concrete in a standard cylinder test (Hsu and Zhang 1997, Vecchio and Collins 1982, 1986, 1993). This strength and stiffness reduction is commonly referred to as compression softening. Applying this softening effect to the strut-and-tie model, the tensile straining perpendicular to the strut will

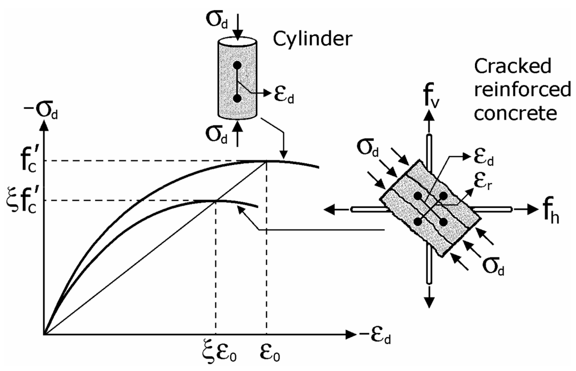


Fig. 4 Constitutive laws for cracked concrete

reduce the capacity of the concrete strut to resist compressive stresses. The stress of the concrete strut is determined from the following compression softening model proposed by Hsu and Zhang (1997) as shown in Fig. 4.

$$\sigma_d = \xi f_c'\left[2\left(\frac{\varepsilon_d}{\xi\varepsilon_0}\right)-\left(\frac{\varepsilon_d}{\xi\varepsilon_0}\right)^2\right] \quad \text{for} \quad \frac{\varepsilon_d}{\xi\varepsilon_0}\le 1 \tag{24}$$

$$\sigma_d = \xi f_c'\left[1-\left(\frac{\varepsilon_d/(\xi\varepsilon_0)-1}{2/\xi-1}\right)^2\right] \quad \text{for} \quad \frac{\varepsilon_d}{\xi\varepsilon_0}> 1 \tag{25}$$

$$\xi = \frac{5.8}{\sqrt{f_c'}}\frac{1}{\sqrt{1+400\varepsilon_r}} \le \frac{0.9}{\sqrt{1+400\varepsilon_r}} \tag{26}$$

where ε_0 is a concrete cylinder strain corresponding to the cylinder strength f_c', which can be defined approximately by Foster and Gilbert (1996) as

$$\varepsilon_0 = 0.002+0.001\left(\frac{f_c'-20}{80}\right) \quad \text{for} \quad 20\le f_c'\le 100 \;\; \text{MPa} \tag{27}$$

The steel bar is assumed to be elastic-perfectly-plastic material.

3.3 Compatibility relation

The strain compatibility relationship used in this study is that the sum of the normal strain in two perpendicular directions is an invariant

$$\varepsilon_h+\varepsilon_v = \varepsilon_r+\varepsilon_d \tag{28}$$

where ε_h and ε_v are tensile strains of the horizontal and vertical web steel ties, ε_d is the compressive strain of concrete strut, and ε_r is the tensile strain in the direction perpendicular to the concrete strut. Eq. (28) can be found from the strain compatibility condition using the Mohr's circle of strain as shown in Fig. 5.

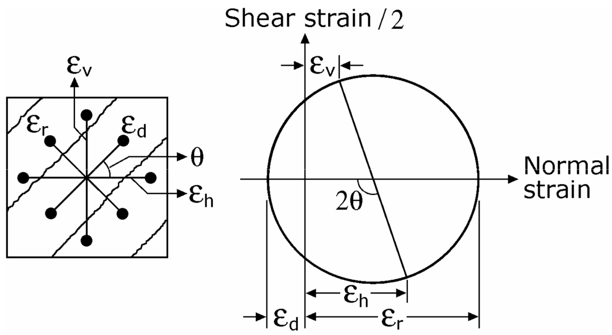


Fig. 5 Mohr circle of strain

3.4 Design procedure

The design procedure for deep beams is summarized as follows.

1) With the given design load V_u, the member forces can be calculated from Eq. (10) to Eq. (14), and the strain of the concrete strut is calculated by Eq. (24) to Eq. (26). When the web reinforcement is yielded or not defined, the limit of the transverse strain of web reinforcement was assumed to be 0.002.
2) Assume the web reinforcement ratio, and compute the amount of horizontal and vertical web reinforcements, that is, A_h and A_v, respectively.
3) Using ε_r as calculated by Eq. (28), the softening coefficient ξ of the diagonal concrete strut is calculated from Eq. (26).
4) Using the state of strain in each member, the stresses were determined from the stress-strain relations of Eq. (24) to Eq. (26) for concrete. The ξ reflects the softening effect for cracked concrete in compression.
5) The nominal strength due to the failure by crushing or splitting of the diagonal concrete strut is given as

$$V_c = \frac{5.8\sqrt{f_c'}}{\sqrt{1+400(\varepsilon_h+\varepsilon_v+\xi\varepsilon_0)}}\left(\frac{a}{2}\sin\theta + kd\cos\theta\right)b\sin\theta \tag{29}$$

The available design strength is determined by the sum of the strength of concrete contribution and the strength of web reinforcement contribution. The iterative process will be required until the convergence is achieved.

6) The design procedure is completed with the examination of the ultimate strength due to the nodal crushing of the diagonal concrete strut at the top and compression block

$$V_n = 0.85f_c'\left(w_p\sin\theta + \frac{h}{4}\cos\theta\right)b\sin\theta \tag{30}$$

$$V_n = 0.85f_c'\left(\frac{hb}{4}\right)\tan\theta \tag{31}$$

The nominal strength by yielding of longitudinal steel tie is given as

$$V_n = \rho_s bdf_y\tan\theta \tag{32}$$

where ρ_s and f_y represent the reinforcement ratio and yield strength of main longitudinal reinforcement. The predicted strength is the minimum value of the nominal strengths computed from the different failure modes, which include crushing or splitting of the diagonal concrete strut, nodal crushing of diagonal concrete strut at the top and compression block, and the yielding of longitudinal reinforcement.

4. Previous test programs

As shown in Table 2, the reinforced concrete deep beams tested in the literature in the following 8 investigations were considered: Clarke 1951, Kong, Robins, and Cole 1970, Smith and Vantsiotis 1982, Anderson and Ramirez 1989, Tan, Kong, Teng, and Guan 1995, Oh and Shin 2001, Aguilar, Matamoros, Parra-Montesinos, Ramirez, and Wight 2002, Quintero-Febres, Parra-Montesinos, and Wight 2006. The references are the same literature used in validating the previously proposed strut-and-tie model approach based on the secant stiffness formulation (Park and Kuchma 2007). The references provided sufficiently complete information on the test setup and material properties for this study. The deep beams include *a*/*d* ratios ranging from 0.27 to 2.7, concrete strengths that range from 13.6 MPa to 73.6 MPa, and various combinations of longitudinal and web reinforcements. A brief description of each testing program is presented below.

Table 2 214 deep beams specimens selected in this study

Ref. No.	Deep beam ID
Smith	0A0-44, 0A0-48, 1A1-10, 1A3-11, 1A4-12, 1A4-51, 1A6-37, 2A1-38, 2A3-39, 2A4-40, 2A6-41, 3A1-42, 3A3-43, 3A4-45, 3A6-46, 0B0-49, 1B1-01, 1B3-29, 1B4-30, 1B6-31, 2B1-05, 2B3-06, 2B4-07, 2B4-52, 2B6-32, 3B1-08, 3B1-36, 3B3-33, 3B4-34, 3B6-35, 4B1-09, 0C0-50, 1C1-14, 1C3-02, 1C4-15, 1C6-16, 2C1-17, 2C3-03, 2C3-27, 2C4-18, 2C6-19, 3C1-20, 3C3-21, 3C4-22, 3C6-23, 4C1-24, 4C3-04, 4C3-28, 4C4-25, 4C6-26, 0D0-47, 4D1-13
Kong	1-30, 1-25, 1-20, 1-15, 1-10, 2-30, 2-25, 2-20, 2-15, 2-10, 3-30, 3-25, 3-20, 3-15, 3-10, 4-30, 4-25, 4-20, 4-15, 4-10, 5-30, 5-25, 5-20, 5-15, 5-10
Clarke	A1-1, A1-2, A1-3, A1-4, B1-1, B1-2, B1-3, B1-4, B1-5, B2-1, B2-2, B2-3, B6-1, C1-1, C1-2, C1-3, C1-4, C2-1, C2-2, C2-3, C2-4, C3-1, C3-2, C3-3, C4-1, C6-2, C6-3, C6-4, D1-1, D1-2, D1-3, D2-1, D2-2, D2-3, D2-4, D3-1, D4-1
Oh	N4200, N42A2, N42B2, N42C2, H4100, H41A2(1), H41B2, H41C2, H4200, H42A2(1), H42B2(1), H42C2(1), H4300, H43A2(1), H43B2, H43C2, H4500, H45A2, H45B2, H45C2, H41A0, H41A1, H41A2(2), H41A3, H42A2(2), H42B2(2), H42C2(2), H43A0, H43A1, H43A2(2), H43A3, H45A2(2), U41A0, U41A1, U41A2, U41A3, U42A2, U42B2, U42C2, U43A0, U43A1, U43A2, U43A3, U45A2, N33A2, N43A2, N53A2, H31A2, H32A2, H33A2, H51A2, H52A2, H53A2
Aguilar	ACI-I, STM-I, STM-H, STM-M
Quintero	A1, A2, A3, A4, B1, B2, B3, B4, HA1, HA3, HB1, HB3
Tan	A-0.27-2.15, A-0.27-3.23, A-0.27-4.30, A-0.27-5.38, B-0.54-2.15, B-0.54-3.23, B-0.54-4.30, B-0.54-5.38, C-0.81-2.15, C-0.81-3.23, D-1.08-2.15, D-1.08-3.23, D-1.08-4.30, D-1.08-5.38, E-1.62-3.23, E-1.62-4.30, E-1.62-5.38, F-2.16-4.30, G-2.70-5.38
Anderson	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12

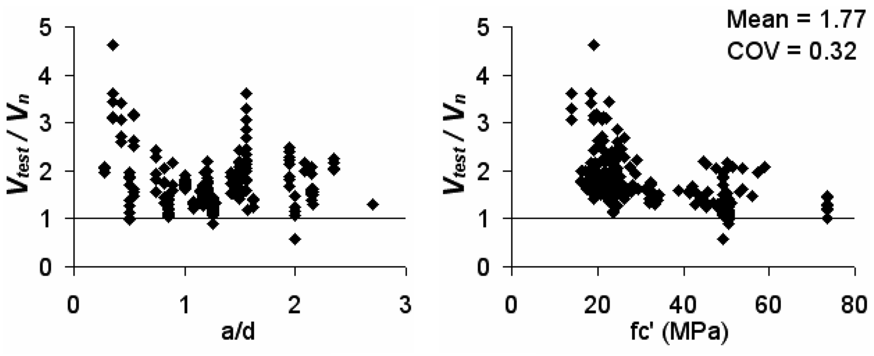
Smith and Vantsiotis (1982) tested 52 simply supported deep beams loaded at two points with shear span-to-depth ratios a/d of 1.0, 1.21, 1.5, and 2.08. The beams capacity was controlled by concrete in either the reduced compression zone at the head of the inclined crack and the region adjacent to the loading block, or fracture of the concrete along the inclined crack. Kong, Robins, and Cole (1970) tested 35 simply supported deep beams with shear span-to-depth ratios a/d ranging from 0.35 to 1.18 to study the influence of web reinforcement on their behavior and strength. Main failure mode is the crushing of concrete near the loading point or the support. The typical behavior of crack patterns and failure mode was similar in most specimens. Clarke (1951) investigated diagonal tension failures in a series of deep beam tests which included beams with no web reinforcement and beams with varying ratios of web reinforcements. All beams were designed for failure under diagonal tension, but in some cases yielding of the main reinforcing bars or compressive failure of the concrete occurring at the time of diagonal tension failure made the primary cause of failure questionable. Oh and Shin (2001) tested 53 simply supported deep beams with concrete compressive strengths of 23.72, 49.10, 50.67, and 73.60 MPa. Despite web reinforcement, in beams with web reinforcement and a lower a/d of 0.5 and 0.85, most showed abrupt failures. At a higher a/d of 1.25 and 2.0, web reinforcement restrained the sudden shear failure. Also, beams cast with higher strength concretes and beams cast with lower-strength concrete failed with some kind of warning and often after more diagonal cracking. Anderson and Ramirez (1989) tested 12 narrow beams with a constant a/d ratio, and longitudinal and stirrup reinforcement ratio. This study showed that the truss model approach can be used to improve the detailing of reinforced concrete members. Tan, Kong, Teng, and Guan (1995) tested 19 simply supported deep beams using high strength concrete with constant longitudinal and vertical web reinforcement ratios. With increasing a/d, the ultimate shear stresses decreased as tied-arch action became less effective. Aguilar, Matamoros, Parra-Montesinos, Ramirez, and Wight (2002) tested 4 deep beams to evaluate the adequacy of the strut-and-tie model provisions of ACI 318-02 Code. Failure occurred after yielding both longitudinal and transverse reinforcement, and large strains were locally recorded in vertical web reinforcement as all four specimens approached failure. Quintero-Febres, Parra-Montesinos, and Wight (2006) tested 12 deep beams with various a/d ratios, concrete strengths, and reinforcement layouts. Failure for all test specimens was brittle and the main failure modes were diagonal splitting and strut crushing, while the specimens with shallow strut angle failed by shear compression near the loading point regardless of the amount of web reinforcement.

5. Verification

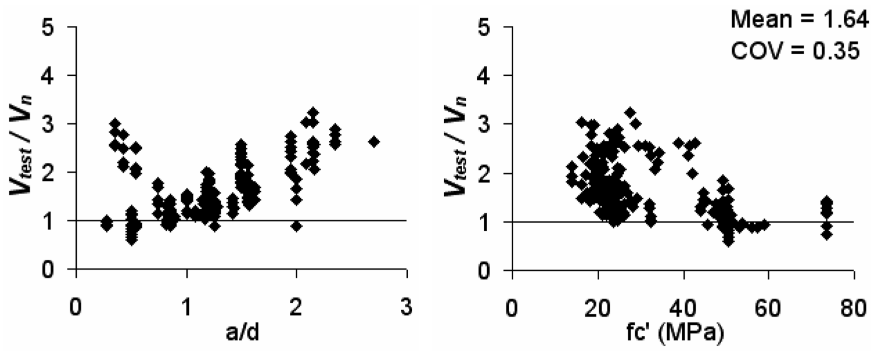
5.1 Strut-and-tie model

The strut-and-tie model is proposed to calculate the amount of web reinforcements of deep beams. The nominal strengths predicted by the proposed method are compared with those obtained by two code procedures of strut-and-tie models (ACI 2005, CSA 1994) as well as the measured capacity of 214 deep beams to investigate the accuracy of the methods. The 214 test specimens are presented for each test group collectively in Table 2, and the strength prediction ratios (V_{test}/V_n) by the three methods are compared in Fig. 6.

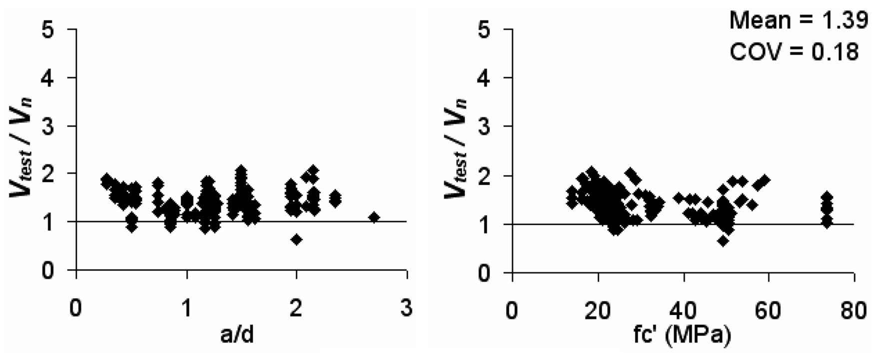
Fig. 6(a) and Fig. 6(b) show that the predictions by the strut-and-tie model approaches of ACI 318-05 and the Canadian Code are very conservative and scattered with mean values of 1.77 and



(a) ACI 318-05



(b) Canadian Code



(c) Proposed approach

Fig. 6 Ratio of measured-to-calculated strength according to (a) ACI 318-05; (b) Canadian Code; (c) Proposed approach

1.64, and COVs of 0.32 and 0.35, respectively. The main cause of this extreme conservatism is that the effective depth of the concrete strut in the code practices is unreasonably small based on the width of loading or bearing plate. The predictions by ACI 318-05 and Canadian Code is scattered with respect to concrete strengths and *a*/*d* ratios as shown in Fig. 6. While deep beams exhibit abrupt failures due to the splitting of the concrete strut when web reinforcement is not placed sufficiently and the concrete strength is higher. This effect cannot be captured by the code procedures.

Fig. 6(c) shows that the predictions of the proposed method are slightly conservative. A good accuracy is obtained with no significant scattering for the deep beams with different concrete strengths, various combinations of web reinforcements and *a*/*d* ratio ranging from 0.27 to 2.7. The proposed method gives a mean value of 1.39, the lowest COV of 0.18, indicating that the method is the both accurate and applicable for the range of available test data.

The softening coefficient of concrete strut is the strut strength divided by the compressive concrete strength. The softening coefficient is to account for the reduction in compressive strength due to transverse tensile strain and the difference of stress fields between the discrete model and structural concrete. There has been considerable debate about the value of the softening coefficient. A number of researchers have conducted independent test programs and have proposed the constitutive models to determine the degree of the softening effects and the parameters that influence it. Batchelor and Campbell (1986) reported that the reduced strength of the web concrete is primarily due to the fact that the diagonal struts are in a state of biaxial tension-compression. They proposed the following relationship for the softening coefficient

$$\ln\left(\beta\frac{d}{b}\right) = 3.342 - 0.1991\left(\frac{a}{d}\right) - 7.471\left(\frac{b}{d}\right) \tag{33}$$

Warwick and Foster (1993) investigated the strength of strut using nonlinear finite element analysis and proposed that

$$\beta = 1.25 - \frac{f_c'}{500} - 0.72\left(\frac{a}{d}\right) + 0.18\left(\frac{a}{d}\right)^2 \le 1 \quad \text{for} \quad \frac{a}{d} < 2 \tag{34a}$$

$$\beta = 0.53 - \frac{f_c'}{500} \quad \text{for} \quad \frac{a}{d} \ge 2 \tag{34b}$$

Foster and Malik (2002) derived the following formula using the softening parameter based on the panel tests of Vecchio and Collins (1986)

$$\beta = \frac{1}{1 + 0.66(a/z)^2} \tag{35}$$

Foster and Malik (2002) also developed the following formula using the relationships presented by Vecchio and Collins (1993)

$$\beta = \frac{1}{0.83 + k_c k_f} \tag{36}$$

where

$$k_f = 0.1825\sqrt{f_c'} \ge 1.0 \tag{37}$$

$$k_c = 0.35\left[0.52 + 1.8\left(\frac{a}{z}\right)^2\right]^{0.8} \tag{38}$$

Fig. 7 shows that the softening coefficients which are determined from Batchelor and Campbell (1986), Warwick and Foster (1993), Eq. (35), and Eq. (36) are direct functions of *a*/*d* ratio. The β value of the proposed method is insensitive to *a*/*d* ratio since the softening coefficient is not an explicit function of *a*/*d* ratio unlike other methods.

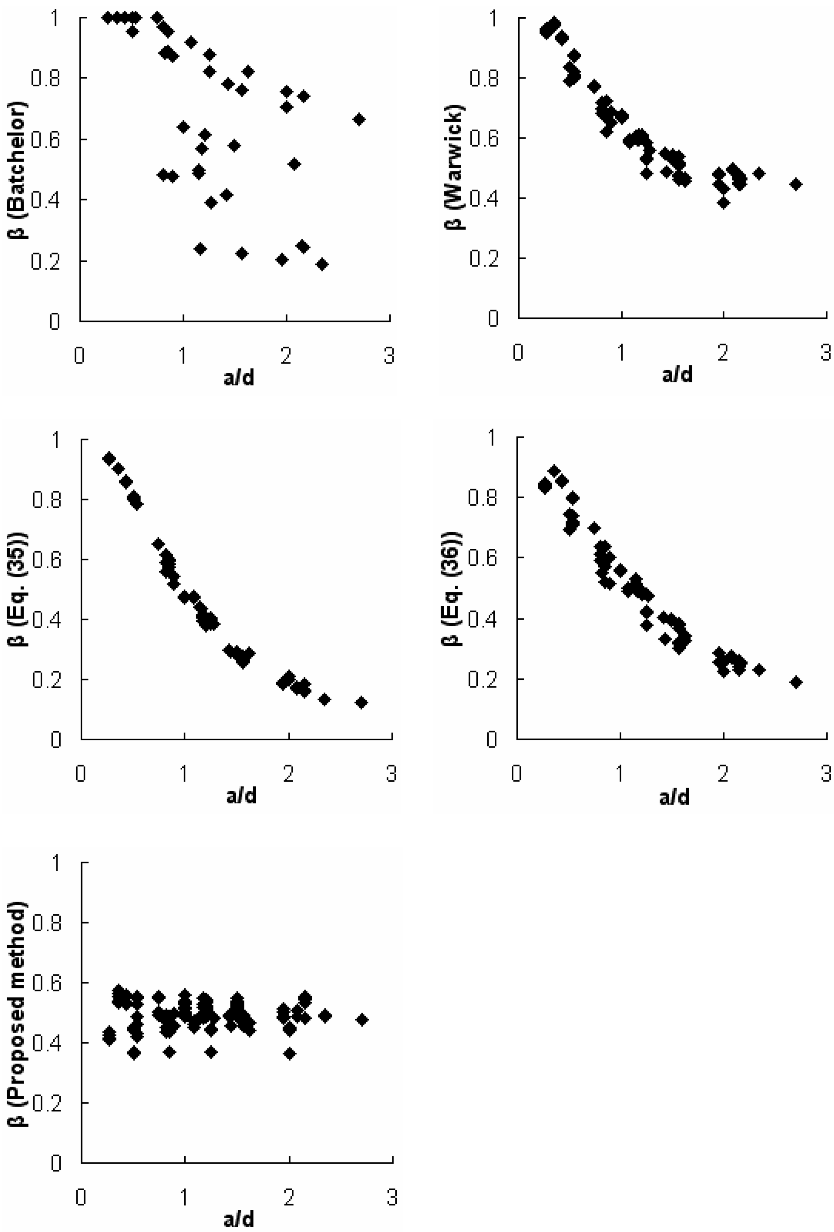


Fig. 7 Softening coefficients determined from Batchelor and Campbell (1986), Warwick and Foster (1993), Eq. (35), Eq. (36), and the proposed method

5.2 Nonlinear finite element analysis

The 28 deep beams were analyzed with VecTor2 commercial software to assess nonlinear finite element analysis (Vecchio 1989, Vecchio and Wong 2003). They were selected to represent the whole range of concrete strengths and *a*/*d* ratios for each set of specimens since they are the main parameters that affect concrete softening.

VecTor2 is a nonlinear finite element program for the analysis of two-dimensional reinforced concrete membrane structures. The theoretical basis of VecTor2 is the Modified Compression Field Theory (Vecchio and Collins 1986) that is the analytical model for predicting the response of reinforced concrete elements subjected to in-plane normal and shear stresses. This model employs the stress-strain relationships to account for a variety of second-order effects including compression softening, tension stiffening, tension softening, and tension splitting. VecTor2 considers cracked concrete as an orthotropic material with smeared and rotating cracks. The solution algorithm is based on a secant stiffness formulation using a total load iterative procedure providing an efficient and robust nonlinear solution. The reinforcements can be modeled with smeared component or discrete element.

In order to illustrate the capability of VecTor2, the compressive stresses, tensile strains, and failure mode for the specimen 'H4100' were compared with the crack patterns observed as shown in Fig. 8. The compressive stresses and the principal tensile strains for the final load step computed from VecTor2 are shown in Fig. 9 and Fig. 10, respectively. In the figures, the numbers represent the ratios of the principal compressive stress to the maximum concrete stress for each continuum element. Fig. 9 and Fig. 10 are useful to determine the cause of failure from the experiment or predicted failure mode. The predicted failure mode is consistent with the observation shown in Fig. 8. The details of the 28 deep beams and strength predictions obtained from the nonlinear finite

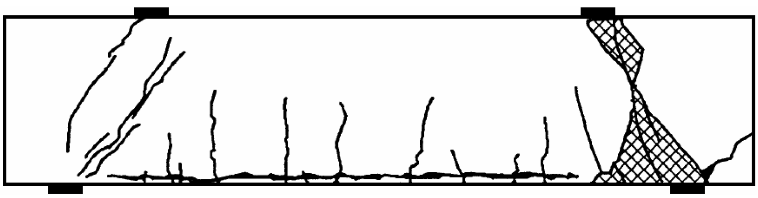


Fig. 8 Crack patterns at failure of specimen 'H4100'

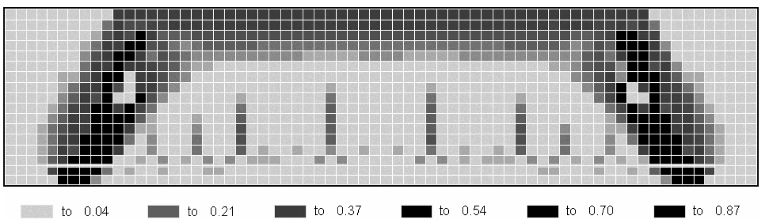


Fig. 9 Ratio of principal compressive stresses to compressive stress capacity at a final load step for the specimen 'H4100'

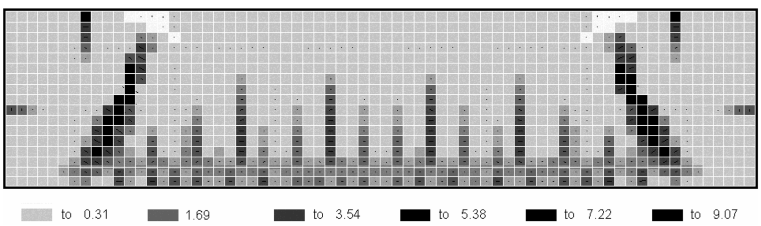


Fig. 10 Principal tensile strains at a final load step for the specimen 'H4100'

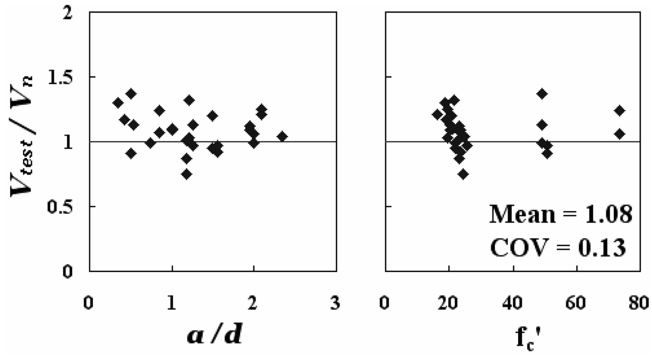


Fig. 11 Ratio of measured-to-calculated strength according to nonlinear finite element analysis

element analysis were compared with the measured failure capacity in Table 1 and Fig. 11. Accuracy was obtained with VecTor2 for the 28 deep beams with different concrete strengths, various combinations of web reinforcements and *a*/*d* ratios giving a mean value of 1.08 and COV of 0.13. The load-carrying capacities and failure modes were also well predicted with VecTor2. Nonlinear FEA tools can also be used to examine the behavior of the structure at service load levels to explore the overall behavior of the structure and to evaluate the performance of the structure in case of an overload. Computer simulation has now joined theory and experimentation as a third path for engineering design and performance evaluation (Chen 2008).

6. Conclusions

In this paper, a strut-and-tie model approach was proposed to calculate the amount of web reinforcements in deep beams. The force transfer mechanisms for this approach were also investigated using linear finite element analysis for 28 deep beams. The load-carrying capacities calculated from the proposed method were compared with those obtained by two code procedures of strut-and-tie models and the measured capacity of 214 deep beams. Also, the 28 deep beams were analyzed with VecTor2 commercial software to assess nonlinear finite element analysis.

According to the results of the comparisons, the proposed strut-and-tie model provides very similar force transfer mechanisms to linear finite element analysis. The capacities calculated from the proposed strut-and-tie method are both accurate and conservative with little scatter or trends for the 214 deep beams. Good accuracy was also obtained with the VecTor2, nonlinear finite element analysis tool based on the Modified Compression Field Theory, which is the analytical model for predicting the response of reinforced concrete elements subjected to in-plane normal and shear stresses. Since the proposed method provides a safe and reliable means for design of deep beams, the results of this study should be considered in future adjustments to code provisions and in the development of design guidelines.

Acknowledgements

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References

AASHTO (2004), AASHTO LRFD Bridge Design Specifications. American Association of State Highway and Transportation Officials.

ACI Committee 318 (2005), Building Code Requirements for Reinforced Concrete (ACI 318-05) and Commentary (ACI 318R-05). American Concrete Institute.

Aguilar, G., Matamoros, A.B., Parra-Montesinos, G., Ramirez, J.A. and Wight, J.K. (2002), "Experimental evaluation of design procedures for shear strength of deep reinforced concrete beams", *ACI Struct. J.*, **99**(4), 539-548.

Anderson, N.S. and Ramirez, J.A. (1989), "Detailing of stirrup reinforcement", *ACI Struct. J.*, **86**(5), 507-515.

Batchelor, D.V.B. and Campbell, T.I. (1986), "Effectiveness factor for shear in concrete beams", *J. Struct. Eng.*, ASCE, **112**(6), 1464-1477.

CEB-FIP (1993), CEB-FIP Model Code for Concrete Structures. Comite Euro-International du Beton/Federation International de la Precontrainte.

Chen, W.F. (2008), "Structural engineering: Seeing the big picture", *KSCE J. Civil Eng.*, **12**(1), 25-29.

Clarke, A.P. (1951), "Diagonal tension in reinforced concrete beams", *ACI J. Proc.*, **48**(10), 145-156.

CSA Committee A23.3 (1994), Design of Concrete Structures: Structures (Design) - A National Standard of Canada. Canadian Standards Association.

Foster, S.J. (1992), Structural Behavior of Reinforced Concrete Deep Beams. Ph.D. Dissertation, School of Civil Engineering, Univ. of New South Wales.

Foster, S.J. and Gilbert, R.I. (1996), "The design of nonflexural members with normal and high-strength concretes", *ACI Struct. J.*, **93**(1), 3-10.

Foster, S.J. and Malik, A.R. (2002), "Evaluation of efficiency factor models used in strut-and-tie modeling of nonflexural members", *J. Struct. Eng.*, ASCE, **128**(5), 569-577.

Guan, H. (2005), "Strut-and-tie model of deep beams with web openings - An optimization approach", *Struct. Eng. Mech.*, **19**(4), 361-379.

Hwang, S.J., Fang, W.H., Lee, H.J. and Yu, H.W. (2001), "Analytical model for predicting shear strength of squat walls", *J. Struct. Eng.*, ASCE, **127**(1), 43-50.

Hwang, S.J. and Lee, H.J. (1999), "Analytical model for predicting shear strengths of exterior reinforced concrete beam-column joints for seismic resistance", *ACI Struct. J.*, **96**(5), 846-857.

Hwang, S.J. and Lee, H.J. (2000), "Analytical model for predicting shear strengths of interior reinforced concrete beam-column joints for seismic resistance", *ACI Struct. J.*, **97**(1), 34-44.

Hwang, S.J., Lu, W.Y. and Lee, H.J. (2000a), "Shear strength prediction for deep beams", *ACI Struct. J.*, **97**(3), 367-376.

Hwang, S.J., Lu, W.Y. and Lee, H.J. (2000b), "Shear strength prediction for reinforced concrete corbels", *ACI Struct. J.*, **97**(4), 543-552.

Hsu, T.T.C. and Zhang, L.X.B. (1997), "Nonlinear analysis of membrane elements by fixed-angle softened-truss model", *ACI Struct. J.*, **94**(5), 483-492.

Kong, F.K., Robins, P.J. and Cole, D.F. (1970), "Web reinforcement effects on deep beams", *ACI J. Proc.*, **67**(12), 1010-1017.

Marti, P. (1985), "Basic tools of reinforced concrete beam design", *ACI Struct. J.*, **82**(4), 46-56.

Oh, J.K. and Shin, S.W. (2001), "Shear strength of reinforced high-strength concrete deep beams", *ACI Struct. J.*, **98**(2), 164-173.

Park, J.W. and Kuchma, D. (2007), "Strut-and-tie model analysis for strength prediction of deep beams", *ACI Struct. J.*, **104**(6), 657-666.

Paulay, T. and Priestley, M.J.N. (1992), Seismic Design of Reinforced Concrete and Masonry Buildings. John Wiley and Sons.

Quintero-Febres, C.G., Parra-Montesinos, G. and Wight, J.K. (2006), "Strength of struts in deep concrete members designed using strut-and-tie method", *ACI Struct. J.*, **103**(4), 577-586.

Rogowsky, D.M., MacGregor, J.G. and Ong, S.Y. (1986), "Tests of reinforced concrete deep beams", *ACI J. Proc.*, **83**(4), 614-623.

Schäfer, K. (1996), "Strut-and-tie models for the design of structural concrete", Notes of workshop, Department of Civil Engineering, National Cheng Kung University, Tainan, Taiwan.

Schlaich, J. and Schäfer, K. (1991), "Design and detailing of structural concrete using strut-and-tie models", *The Struct. Eng.*, **69**(6), 113-125.

Schlaich, J., Schäfer, K. and Jennewein, M. (1987), "Toward a consistent design of structural concrete", *PCI J.*, **32**(3), 75-149.

Smith, K.N. and Vantsiotis, A.S. (1982), "Shear strength of deep beams", *ACI J. Proc.*, **79**(3), 201-213.

Tan, K.H., Kong, F.K., Teng, S. and Guan, L. (1995), "High-strength concrete deep beams with effective span and shear span variations", *ACI Struct. J.*, **92**(4), 395-405.

Vecchio, F.J. (1989), "Nonlinear finite element analysis of reinforced concrete membranes", *ACI J. Proc.*, **86**(1), 26-35.

Vecchio, F.J. and Collins, M.P. (1982), "Response of reinforced concrete to in-plane shear and normal stresses", Report No. 82-03, Univ. of Toronto, Toronto, Canada.

Vecchio, F.J. and Collins, M.P. (1986), "Modified compression field theory for reinforced concrete elements subjected to shear", *ACI J. Proc.*, **83**(2), 219-231.

Vecchio, F.J. and Collins, M.P. (1993), "Compression response of cracked reinforced concrete", *J. Struct. Eng.*, ASCE, **119**(12), 3590-3610.

Vecchio, F.J. and Wong, P.S. (2003), VecTor2 and FormWorks User's Manual, Univ. of Toronto, Toronto, Canada.

Warwick, W.B. and Foster, S.J. (1993), Investigation into the efficiency factor used in nonflexural reinforced concrete member design. UNICIV Rep. No. R-320, School of Civil Engineering, Univ. of New South Wales, Kensington, Sydney, Australia.

Notation

A_c, A_d	: effective area of horizontal and diagonal concrete struts
A_s, A_{sh}, A_{sv}	: cross-sectional areas of longitudinal, horizontal, and vertical steel ties
a	: shear span measured between concentrated load and support
b	: member thickness
d	: effective depth
C, D	: compressive forces of horizontal and diagonal strut
F_h, F_v	: tensile forces of horizontal and vertical ties in the web
f_c'	: compressive strength of concrete cylinder
f_c, f_d	: compressive stress of horizontal and diagonal strut
f_s, f_{sh}, f_{sv}	: tensile stress of longitudinal, horizontal and vertical ties
T	: tensile force of longitudinal steel tie
V_c, V_s	: nominal shear strength provided by concrete and shear reinforcement
V_{test}, V_n	: experimental failure load and nominal failure strength
w_c, w_d	: effective widths of horizontal and diagonal concrete struts
w_{dn}	: width of the top node in the face of the diagonal concrete strut
w_p	: width of loading plate
β	: softening coefficient of the diagonal concrete strut
ε_c, ε_d	: compressive strains of horizontal and diagonal struts
ε_s, ε_h, ε_v	: tensile strains of longitudinal, horizontal, and vertical ties
ε_1, ε_2	: principal tensile and compressive strains
ε_r	: tensile strain of direction perpendicular to diagonal strut
ε_0	: strain at peak stress of standard cylinder
θ	: angle of inclination of diagonal strut with respect to the horizontal axis
ρ_h, ρ_v	: ratios of horizontal and vertical web reinforcements
σ_d	: compressive stress of concrete strut
ϕ_{c1}, ϕ_{c2}	: area reduction factors for diagonal concrete strut

Appendix – Design example of a deep beam

Determine the required web reinforcement of a simply supported deep beam described in the Fig. A1 using the strut-and-tie model approach proposed in the study. Use $f_c' = 21$ MPa, $f_y = 400$ MPa, and $V_u = 400$ kN.

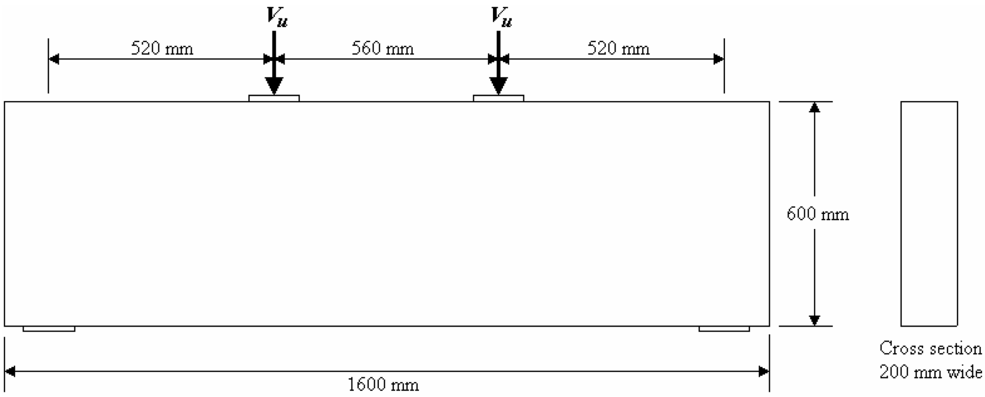

Fig. A1 Design example of deep beam

1. The required amount of main reinforcement is determined from the strut-and-tie model approaches presented in the common design codes such as ACI 318-05 and AASHTO. In this example, AASHTO code is applied.

$$\rho = \frac{A_s}{bd} = 0.01877$$

2. The angle of inclination of diagonal strut is obtained from Eq. (18).

$$\tan\theta = \frac{d - kd/2}{a} = 34.56^{\circ}$$

3. Compute the member forces using Eq. (10)-(14).

$$D = \frac{V}{\sin\theta} = \frac{400kN}{\sin 34.56^{\circ}} = 705.2kN$$

$$C = \frac{V}{\tan\theta} = \frac{400kN}{\tan 34.56^{\circ}} = 580.7kN$$

$$F_h = \frac{D\sin\theta}{4} = \frac{705.2kN \times \sin 34.56^{\circ}}{4} = 100.0kN$$

$$F_v = \frac{D\sin\theta}{4} = \frac{705.2kN \times \sin 34.56^{\circ}}{4} = 145.2kN$$

4. The strain of the diagonal strut is determined from Eq. (24)-(27).

$$\varepsilon_d = 0.001076$$

5. Assume the web reinforcement ratio as 0.1%, and compute A_h and A_v.

$$A_h = \rho_h bz = 0.001 \times 200\ mm \times 358.2\ mm = 71.6\ mm^2$$

$$A_v = \rho_v ba = 0.001 \times 200\ mm \times\ 520mm = 104.0\ mm^2$$

6. ε_r and ξ are determined from Eq. (28) and Eq. (26), respectively.

$$\varepsilon_r = \varepsilon_h + \varepsilon_v - \varepsilon_d = 0.002 + 0.002 - (-0.001076) = 0.005076$$

$$\xi = \frac{0.9}{\sqrt{1 + 400\varepsilon_r}} = \frac{0.9}{\sqrt{1 + 400 \times 0.005076}} = 0.517$$

7. The stress of the diagonal strut is computed from Eqs. (24)-(26).

$$\sigma_d = \xi f'_c \left[2\left(\frac{\varepsilon_d}{\xi\varepsilon_0}\right) - \left(\frac{\varepsilon_d}{\xi\varepsilon_0}\right)^2\right] = 10.84MPa$$

8. Eq. (29) gives the design strength due to the failure by crushing or splitting of the diagonal concrete strut.

$$\phi V_c = 0.7 \times 386.8kN = 270.8kN$$

The available design strength by web reinforcements is given as

$$\phi V_s = 0.9 \times \left(2A_h f_{yh} + \frac{2A_v f_{yv}}{\tan\theta}\right) = 0.9\left(2 \times 71.6 \times 400 + \frac{2 \times 104.0 \times 400}{\tan 34.56^{\circ}}\right) = 160.3kN$$

The total available strength is now given as

$$\phi V_n = 270.8 + 160.3 = 431.1kN$$

Since $V_u = 400kN < \phi V_n = 431.1kN$, the next iteration step will be required until the applied load becomes equal to the calculated available strength.

9. Proceed to the next iteration with new web reinforcement ratio of 0.102%, and it will provide the converged solution as follows

$$A_h = 73.1\ mm^2$$

$$A_v = 106.0\ mm^2$$

$$V_u = \phi V_n = 400.0\ kN$$

The nominal strength can be determined as the minimum value of the shear strengths computed from Eqs. (29)-(32), which represent the failure modes by crushing or splitting of the diagonal concrete strut, nodal crushing of diagonal concrete strut at the top and compression block, and the yielding of longitudinal reinforcement.