Structural Engineering and Mechanics, Vol. 3, No. 3 (1995) 215-228
DOI: http://dx.doi.org/10.12989/sem.1995.3.3.215

A high precision direct integration scheme for non-stationary random seismic responses of non-classically damped structures

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Abstract. For non-classically damped structures subjected to evolutionary random seismic excitations, the non-stationary random responses are computed by means of a high precision direct (HPD) integration scheme combined with the pseudo excitation method. Only real modes are used, so that the reduced equations of motion remain coupled for such non-classically damped structures. In the given examples, the efficiency of this method is compared with that of the Newmark method.

Key words: non-stationary random direct integration damping.

1. Introduction

Seismic loadings are typically non-stationary random ones. However, because of the complexity and considerable computational demands of the analysis of structural random response, a simple but somewhat conservative and rough response spectrum method is instead widely used in practical engineering, except for some very important projects, e.g. for nuclear power stations (Lee and Penzien 1983) or surface-mounted pipelines (Lin, Y. K., *et al.* 1990), which require more careful analysis. However, even for such important projects, stationary random excitations are usually assumed, to avoid over-complicating the computation of their structural non-stationary random response.

Since the 1980's, the theory and methods associated with non-stationary random response

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have received great attention in the literature. It has been possible to analyse some comparatively simple engineering models by using such methods (To 1986, Gasparini and Chaudhury 1980, Langley 1986). However, efforts are still being made to find more efficient methods.

During the past few years, an efficient pseudo excitation method for the analysis of structural stationary random responses has been suggested by Lin, Williams and Zhang (Lin 1985, 1992, Lin, *et al.* 1992). This method has recently been extended to the analysis of non-stationary responses of classically damped structures subjected to evolutionary random seismic excitations (Lin, *et al.*). However, for general structures equipped with seismic isolation devices, which usually impose heavy local damping on the structure, the characteristics of non-classical damping must be taken into account. Obviously, it is of great engineering significance to search for an efficient and reliable method for dealing with such problems. In this paper, only real modes are used to reduce the order of the equations of motion of the non-classically damped structure, so that the resulting damping matrix remains non-diagonal. This procedure is obviously efficient for the present problem (Clough and Penzien 1975).

Recently a high precision direct (HPD) integration scheme was proposed (Zhong and Williams) for structures subjected to deterministic dynamic loadings. This is an unconditionally stable explicit integration method with high precision and efficiency. The present paper shows that by extending the pseudo excitation method to non-classically damped structures, and combining it with this high precision step-by-step scheme, various non-stationary responses due to evolutionary random seismic excitations can be easily solved. Three numerical examples are given, for which Newmark's method is also used to compare with the efficiency and precision of the HPD method. It is shown that, for the same level of precision, the time-step size for the HPD scheme can be about four times that for the Newmark scheme, whilst the computational time is approximately halved.

2. Basic principle of the pseudo excitation method

Consider a linear time-invariant system subjected to a non-stationary random excitation which takes the following widely used evolutionary form (To 1986, Gasparini and Chaudhury 1980, Priestley 1965)

$$f(t)=g(t)x(t) \tag{1}$$

in which t is time, $g(t)$ is a specified envelope function and $x(t)$ is a zero-mean-valued stationary random process with given PSD (power spectral density) $S_{xx}(\omega)$. An arbitrary response $y(t)$ caused by the excitation $f(t)$ can be expressed by means of Duhamel integration as (Lin 1967)

$$y(t)=\int_0^t h(t-\tau)f(\tau)d\tau \tag{2}$$

in which $h(t)$ is the pulse response function of the system. The auto-correlative function of $y(t)$ is (To 1986)

$$\begin{aligned} R_{yy}(t_1, t_2)&=E[y(t_1)y(t_2)] \\ &=E\left[\int_0^{t_1}\int_0^{t_2} h(t_1-\tau_1)h(t_2-\tau_2)f(\tau_1)f(\tau_2)d\tau_1 d\tau_2\right] \\ &=\int_0^{t_1}\int_0^{t_2} h(t_1-\tau_1)h(t_2-\tau_2)E[g(\tau_1)x(\tau_1)g(\tau_2)x(\tau_2)]\,d\tau_1 d\tau_2 \end{aligned}$$

$$=\int_0^{t_1}\int_0^{t_2} h(t_1-\tau_1)h(t_2-\tau_2)\,g(\tau_1)g(\tau_2)R_{xx}(\tau_1,\ \tau_2)d\tau_1 d\tau_2 \tag{3}$$

where $E[\cdots]$ means taking the mean value, and

$$R_{xx}(\tau_1,\ \tau_2)=E[x(\tau_1)x(\tau_2)] \tag{4}$$

Because $x(t)$ is a stationary random process

$$R_{xx}(\tau_1,\ \tau_2)=R_{xx}(\tau)=\int_{-\infty}^{\infty} S_{xx}(\omega)e^{i\omega\tau}d\omega \tag{5}$$

in which $\tau=\tau_2-\tau_1$, and the Wiener-Khinchene relation has been used. Substituting Eq. (5) into Eq. (3) gives

$$R_{yy}(t_1,\ t_2)=\int_{-\infty}^{\infty} I^*(\omega,\ t_1)I(\omega,\ t_2)S_{xx}(\omega)d\omega \tag{6}$$

where the superscript * denotes complex conjugate and

$$I(\omega,\ t)=\int_0^t h(t-\tau)g(\tau)e^{i\omega\tau}d\tau \tag{7}$$

By letting $t_1=t_2=t$ in Eq. (6), the variance of $y(t)$ is obtained as

$$\sigma_y^2(t)=R_{yy}(t,\ t)=\int_{-\infty}^{\infty} I^*(\omega,\ t)I(\omega,\ t)S_{xx}(\omega)d\omega \tag{8}$$

The integral of Eq. (8) is simply the time-variant PSD of the response $y(t)$, i.e.

$$S_{yy}(\omega,\ t)=I^*(\omega,\ t)I(\omega,\ t)S_{xx}(\omega) \tag{9}$$

It is seen from Eq. (7) that $I(\omega,\ t)$ is the response caused by the excitation $g(t)e^{i\omega t}$ with both the initial displacement and initial velocity of the response assumed to be zero. Therefore, provided a pseudo excitation

$$f(t)=\sqrt{S_{xx}(\omega)}g(t)e^{i\omega t} \tag{10}$$

is exerted on the system, then the response caused must be

$$y(\omega,\ t)=\sqrt{S_{xx}(\omega)}I(\omega,\ t) \tag{11}$$

Obviously

$$y^*(\omega,\ t)y(\omega,\ t)=I^*(\omega,\ t)I(\omega,\ t)S_{xx}(\omega) \tag{12}$$

From Eqs. (9) and (12),

$$S_{yy}(\omega,\ t)=y^*(\omega,\ t)y(\omega,\ t)=|y(\omega,\ t)|^2 \tag{13}$$

Thus, the time-variant PSD $S_{yy}(\omega,\ t)$ of an arbitrary non-stationary random response can be

$$f(t)=\sqrt{S_{xx}(\omega)}\,g(t)e^{i\omega t} \rightarrow \boxed{\begin{array}{l}\text{time invariant}\\ \text{linear system}\end{array}} \rightarrow y(t)=\sqrt{S_{xx}(\omega)}\,I(\omega,t)$$

Fig. 1 Non-stationary pseudo-excitation and response

calculated in terms of a deterministic dynamic analysis, see Fig. 1. In general, pseudo dynamic responses $y(\omega, t)$ can be calculated in the time-domain by using time domain integration, as follows.

3. Non-stationary random seismic responses of non-classically damped structures

The motion equation of an n DOF structure subjected to the action of an earthquake is

$$[M]\{\ddot{y}\}+[C]\{\dot{y}\}+[K]\{y\}=-[M]\{E\}\ddot{x}_g \tag{14}$$

in which $[M]$ and $[K]$ are the mass and stiffness matrices, $[C]$ is a non-classically damped matrix, $\{E\}$ is the inertia index vector, and $\ddot{x}_g(t)$ is the ground acceleration, which takes the form of the RHS of Eq. (1). Using the pseudo excitation method, Eq. (10) gives the pseudo ground acceleration as

$$\ddot{x}_g(t)=\sqrt{S_{xx}(\omega)}g(t)e^{i\omega t} \tag{15}$$

Substituting Eq. (15) into the RHS of Eq. (14) produces the following deterministic equations of motion

$$[M]\{\ddot{y}\}+[C]\{\dot{y}\}+[K]\{y\}=-[M]\{E\}\sqrt{S_{xx}(\omega)}g(t)e^{i\omega t} \tag{16}$$

Its initial conditions are

$$\{y(0)\}=\{\dot{y}(0)\}=\{0\} \tag{17}$$

If the order n of Eq. (16) is low, it can be solved directly by a direct integration method. Otherwise it needs to be reduced by means of prior application of the mode-superposition scheme. For non-classically damped problems, it is possible to use the complex modes to decouple the equation system (16) into uncoupled complex SDOF equations. However, in this paper, only real modes are instead used for the reduction of the problem. Accordingly, the following eigenproblem should first be solved

$$[K][\Phi]=[M][\Phi][\Omega^2] \tag{18}$$

$$[\Phi]^T[M][\Phi]=[I] \tag{19}$$

in which $[I]$ is the unit matrix and $[\Phi]$ and $[\Omega^2]$ are the matrices of the first q modes and eigenvalues, respectively. By using the first $q(q\ll n)$ eigenmodes $\{\phi\}_j$, $\{y\}$ can be expressed as

$$\{y(\omega, t)\}=\sum_{j=1}^{q}u_j(\omega, t)\{\phi\}_j=[\Phi]\{u\} \tag{20}$$

and then Eq. (16) can be reduced into the q dimensional equation of motion

$$[I]\{\ddot{u}\}+[C]^\circ\{\dot{u}\}+[\Omega^2]\{u\}=-\{\gamma\}\sqrt{S_{xx}(\omega)}g(t)e^{i\omega t} \tag{21}$$

in which

$$[C]^\circ=[\Phi]^T[C][\Phi] \tag{22}$$

$$\{\gamma\}=[\Phi]^T[M]\{E\} \tag{23}$$

where $[C]^\circ$ will not be a diagonal matrix if $[C]$ is not classically damped. However, because

the order of this reduced equation is rather low, it would be convenient and efficient to solve it by a proper step-by-step scheme. With the structure initially at rest, i.e.

$$\{y(0)\}=\{0\},\ \{\dot{y}(0)\}=\{0\} \tag{24}$$

or, using Eqs. (19) and (20),

$$\{u(0)\}=[\Phi]^T[M]\{y(0)\}=\{0\},\ \{\dot{u}(0)\}=[\Phi]^T[M]\{\dot{y}(0)\}=\{0\} \tag{25}$$

the transient solution of Eq. (21), i.e. $\{y(\omega, t)\}$, can be computed at discrete times. Here Eq. (13) gives the PSD of the i-th element of $\{y(\omega, t)\}$ as

$$S_{y_iy_i}(\omega, t)=y_i^*(\omega, t)y_i(\omega, t) \tag{26}$$

and the corresponding time-dependent variance is

$$\sigma_{y_i}^2(t)=2\int_0^\infty S_{y_iy_i}(\omega, t)d\omega \tag{27}$$

The PSD's and variances of other quantities which can be found through linear transformations of $\{y(\omega, t)\}$, e.g. an arbitrary internal force $n(\omega, t)$ or strain $\varepsilon(\omega, t)$, can be obtained from equations similar to Eqs. (26) and (27).

4. High precision direct integration scheme

The high precision direct (HPD) integration scheme presented by Zhong and Williams applies to both damped and gyroscopic systems. This section summarizes its key relevant points, with the gyroscopic case omitted because it is not relevant to the present paper. Hence, the equations of motion (16) (or (21)) are first written in the form

$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[K]\{x\}=\{f(t)\} \tag{28}$$

then letting

$$\{p\}=[M]\{\dot{x}\}+[C]\{x\}/2 \tag{29}$$

or

$$\{\dot{x}\}=[M]^{-1}\{p\}-[M]^{-1}[C]\{x\}/2 \tag{30}$$

Eq. (28) becomes

$$\{\dot{p}\}=-([K]-[C][M]^{-1}[C]/4)\{x\}-[C][M]^{-1}\{p\}/2+\{f\} \tag{31}$$

Eqs. (30) and (31) can be further rearranged into the linear dual form

$$\{\dot{v}\}=[H]\{v\}+\{r\} \tag{32}$$

in which

$$[H]=\begin{bmatrix} A & D \\ B & G \end{bmatrix},\ \{v\}=\begin{Bmatrix} q \\ p \end{Bmatrix},\ \{r\}=\begin{Bmatrix} 0 \\ f \end{Bmatrix} \tag{33}$$

$$\{q\}=\{x\},\ [A]=-[M]^{-1}[C]/2,\ [B]=-([K]-[C][M]^{-1}[C]/4)$$

$$[G]=-[C][M]^{-1}/2,\ [D]=[M]^{-1}$$

The homogeneous solution to Eq. (32) should satisfy the equation

$$\{\dot{v}_h\}=[H]\{v_h\} \tag{34}$$

and is simply

$$\{v_h(t)\}=\exp([H]\times\tau)\{c\} \tag{35}$$

where $\{c\}$ will be determined according to the initial conditions, and it has been assumed that within the k-th time-step, $t\in(t_k,\ t_{k+1})$, $\tau=t-t_k$. Furthermore, let

$$[T(\tau)]=\exp([H]\times\tau) \tag{36}$$

For the present method, it is very important to compute $[T(\tau)]$ very accurately (Zhong and Williams). Accordingly, let

$$\Delta t=\tau/m \tag{37}$$

where $m=2^N$. The use of $N=20$ was suggested, so that $\Delta t\cong 10^{-6}\tau$. Eq. (36) can then be expressed as

$$[T(\tau)]=[\exp([H]\times\Delta t)]^m \tag{38}$$

or

$$[T(\tau)]\cong[I+H\times\Delta t+(H\times\Delta t)^2/2!+(H\times\Delta t)^3/3!+(H\times\Delta t)^4/4!]^{2^N}\equiv[I+T_{a0}]^{2^N} \tag{39}$$

Because $O(\Delta t^5)=O(10^{-30}\tau^5)$ is of the order of the round-off errors of ordinary computers, taking up to the fourth order term of the Taylor series, as done in Eq. (39), would generally be extremely accurate. Thus it would usually give practically the exact matrix $[T(\tau)]$, i.e., its accuracy approximately matches computer accuracy.

Note that

$$\begin{aligned}
&[I+T_{a0}]^2=[I+2\times T_{a0}+T_{a0}\times T_{a0}]\equiv[I+T_{a1}]\\
&[I+T_{a1}]^2=[I+2\times T_{a1}+T_{a1}\times T_{a1}]\equiv[I+T_{a2}]\\
&\cdots\cdots\cdots\cdots\\
&[I+T_{a,N-1}]^2=[I+2\times T_{a,N-1}+T_{a,N-1}\times T_{a,N-1}]\equiv[I+T_{aN}]
\end{aligned} \tag{40}$$

Clearly

$$[I+T_{aN}]=[I+T_{a,N-1}]^2=[I+T_{a,N-2}]^4=\cdots\cdots=[I+T_{a0}]^{2^N}=[T(\tau)] \tag{41}$$

Eqs. (40) and (41) suggest the computing strategy. The main point is that the unit matrix $[I]$ must not take part in the intermediate computations, so as to avoid loss of significant digits of $[T_{a0}]$, $[T_{a1}],\cdots,[T_{a,N-1}]$. Cleary, Eqs. (40) enable this to be achieved very simply, by obtaining $[T_{a1}]$ from $[T_{a0}]$, $[T_{a2}]$ from $[T_{a1}]$, etc. when coding a computer program, by using the simple recursive statement obtained by cancelling the $[I]$'s on each side of the identity symbols of Eqs. (40).

If the given loading varies linearly within the time-step $(t_k,\ t_{k+1})$, i.e.

$$\{r\}=\{r_0\}+\{r_1\}\times(t-t_k) \tag{42}$$

then the particular solution of Eq. (32), denoted by $\{v_p\}$, should satisfy

$$\{\dot{v}_p\}=[H]\{v_p\}+\{r_0\}+\{r_1\}\times(t-t_k) \tag{43}$$

It is easy to verify that

$$\{v_p(t)\}=([H]^{-1}+[I]\times t)(-[H]^{-1}\{r_1\})-[H]^{-1}(\{r_0\}-\{r_1\}\times t_k) \tag{44}$$

meets this requirement. Thus, using Eqs. (35) and (36), the general solution to Eq. (32) is

$$\{v(t)\}=\{v_h(t)\}+\{v_p(t)\}=[T(\tau)]\{c\}+\{v_p(t)\} \tag{45}$$

For every time-step $[t_k, t_{k+1}]$, the initial state vector $\{v(t_k)\}$ is known. By letting $t=t_k$ in Eq. (45), $\{c\}$ can be determined as

$$\{c\}=\{v(t_k)\}-\{v_p(t_k)\} \tag{46}$$

because $[T(0)]$ is a unit matrix. Finally, by substituting Eqs. (44) and (46) into Eq. (45), the state vector at time t_{k+1} can be obtained as

$$\begin{aligned}\{v(t_{k+1})\}=&[T(\tau)]\times[\{v(t_k)+[H]^{-1}(\{r_0\}+[H]^{-1}\{r_1\})]\\&-[H]^{-1}(\{r_0\}+[H]^{-1}\{r_1\}+\{r_1\}\times\tau)\end{aligned} \tag{47}$$

in which $\tau=t_{k+1}-t_k$.

This scheme is unconditionally stable. In fact, it can be readily proved for an SDOF system that the eigenvalues of matrix $[H]$, see Eqs. (33), must have non-positive real parts, and so because of Eq. (36) the spectral radius of $[T(\tau)]$ cannot be greater than unity (Bathe and Wilson 1976).

5. Examples

5.1. Example 1

Consider an SDOF system initially at rest subjected to an evolutionary random excitation $f(t)$. Its equation of motion is

$$M\ddot{y}+C\dot{y}+Ky=f(t) \tag{48}$$

and the initial conditions are

$$y(0)=0 \text{ and } \dot{y}(0)=0 \tag{49}$$

Suppose that

$$M=10.2,\ C=1.0 \text{ and } K=14000.0 \tag{50}$$

where units are omitted for convenience, and that the excitation takes the form

$$f(t)=g(t)x(t) \tag{51}$$

in which

$$g(t)=\begin{cases}1.0 & \text{when } t\geq 0\\ 0 & \text{when } t<0\end{cases} \tag{52}$$

Finally, suppose that $x(t)$ is a zero-mean-valued stationary random process with PSD given as the Kanai-Tajimi filtered white noise spectrum

Table 1 Variances $\sigma_y^2(t)$ of the displacement response y

Scheme	t \ τ	0.0025	0.005	0.01	0.02	0.03	0.04	0.06	0.08
HPD	1.2	6.443	6.442	6.437	6.418	6.424	6.479	6.575	6.775
	2.4	13.19	13.19	13.18	13.13	13.14	13.23	13.38	13.72
	3.6	19.65	19.65	19.63	19.57	19.58	19.72	19.96	20.46
Newmark	1.2	6.443	6.432	6.389	6.219	5.986	5.728	5.065	4.497
	2.4	13.20	13.19	13.12	12.87	12.54	12.15	10.87	9.330
	3.6	19.65	19.62	19.49	19.00	18.36	17.72	16.27	14.32

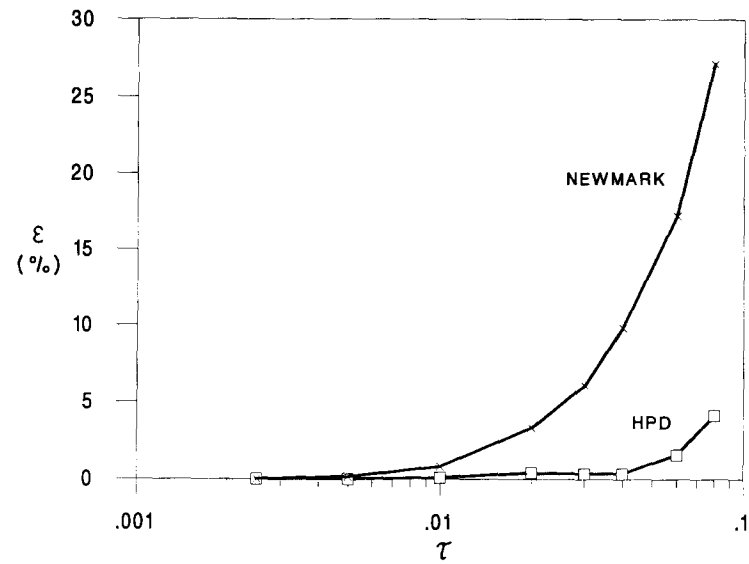

Fig. 2 Error curves for $\sigma_y^2(t)$ with $t=3.6$.

$$S_{xx}(\omega)=\frac{1+4\left(\frac{\zeta_g\omega}{\omega_g}\right)^2}{(1-\omega^2/\omega_g^2)^2+4(\zeta_g\omega/\omega_g)^2}S_0 \tag{53}$$

in which $\zeta_g=0.544$, $\omega_g=19.07$ and $S_0=142.75$.

The response of displacement y was computed by means of the HPD method, and also by the Newmark method with $\alpha=0.5$ and $\delta=0.25$ (Bathe and Wilson 1976). The variances $\sigma_y^2(t)$ at times $t=1.2$, 2.4, and 3.6 with different time-step sizes τ are listed in Table 1. The curves of the variation of relative error ε, at $t=3.6$, with step size for both methods are plotted in Fig. 2. It is seen from both the table and the figure that when the step size is very small, e.g. $\tau=0.0025$, both methods give practically identical results. In order to control the error ε within a certain level, the HPD method can clearly adopt a much higher step size τ than can the Newmark method. For instance, for $\varepsilon=4\%$ the HPD method only needs a step size of $\tau=0.08$, for which the computer time on an IBM/sc386 notebook computer is 26 seconds, whereas $\tau=0.02$ has to be adopted for the Newmark method and the computer time is then 71 seconds.

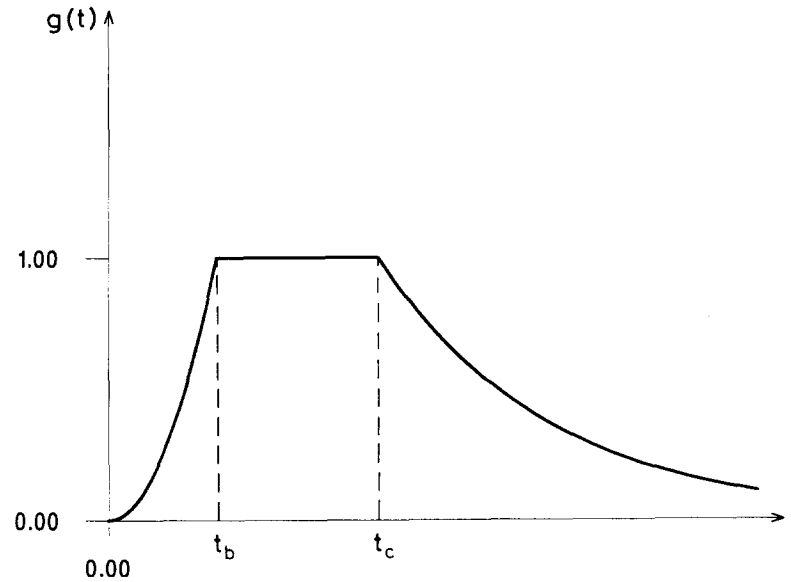

Fig. 3 Modulation function for Eq. (54)

Table 2 Variances $\sigma_y^2(t)$ of the displacement response y with the $g(t)$ of Fig. 3

Scheme	t \ τ	0.0025	0.005	0.01	0.02	0.03	0.04	0.06	0.08
HPD	1.2	3.031	3.030	3.028	3.016	3.016	3.038	3.078	3.141
	2.4	8.819	8.817	8.809	8.778	8.784	8.840	8.942	9.148
	3.6	9.281	9.279	9.271	9.238	9.246	9.301	9.411	9.619
Newmark	1.2	3.032	3.028	3.012	2.946	2.851	2.766	2.485	2.240
	2.4	8.816	8.803	8.754	8.563	8.306	8.028	7.252	6.486
	3.6	9.277	9.264	9.211	9.003	8.719	8.400	7.544	6.758

$g(t)$ was then replaced by the following (Amin and Ang 1968), which is very popular in the earthquake engineering field and is shown in Fig. 3.

$$g(t)=\begin{cases} t^2/t_b^2 & \text{when } t<t_b \\ 1 & \text{when } t_b\leq t\leq t_c \\ \exp\ [-c_1(t-t_c)] & \text{when } t>t_c \end{cases} \tag{54}$$

For $t_b=0.8$ sec, $t_c=2.0$ sec and $c_1=0.1572$ the variance of displacement y was again analysed by the methods used above, and the results are listed in Table 2. They lead to similar conclusions to those drawn from Table 1.

5.2. Example 2

The three DOF system of Fig. 4 is subjected to an evolutionary random ground motion, such that the equations of motion are Eq. (14) with

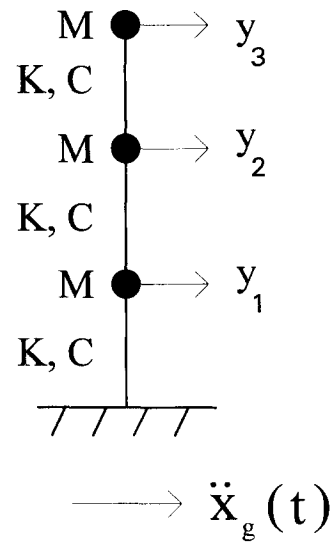

Fig. 4 A three DOF system subjected to evolutionary random seismic excitation.

Table 3 Variances of top displacement y_3

Scheme	t \ τ	0.0025	0.005	0.01	0.02	0.03	0.04	0.06	0.08
HPD	1.2	2.344	2.343	2.339	2.323	2.311	2.307	2.280	2.263
	2.4	2.745	2.744	2.740	2.721	2.708	2.702	2.670	2.650
	3.6	2.797	2.796	2.791	2.773	2.759	2.753	2.719	2.698
Newmark	1.2	2.344	2.340	2.325	2.268	2.193	2.111	1.893	1.600
	2.4	2.746	2.742	2.728	2.673	2.600	2.519	2.298	2.043
	3.6	2.795	2.795	2.782	2.731	2.664	2.589	2.381	2.160

$$[M]=\begin{bmatrix} 10.2 & 0 & 0 \\ 0 & 10.2 & 0 \\ 0 & 0 & 10.2 \end{bmatrix}, \qquad [C]=\begin{bmatrix} 170 & -85 & 0 \\ -85 & 170 & -85 \\ 0 & -85 & 85 \end{bmatrix},$$

$$[K]=\begin{bmatrix} 28000 & -14000 & 0 \\ -14000 & 28000 & -14000 \\ 0 & -14000 & 14000 \end{bmatrix}, \qquad \{E\}=\begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} \tag{55}$$

and $f(t)$ is again given by Eqs. (51)-(53). The time dependent variances of y_3 at times $t=1.2$, 2.4 and 3.6, for different time-steps τ, are listed in Table 3, using the same methods as for Example 1. When the time step size was very small, e.g. $\tau=0.0025$, both methods again gave practically identical results, and so the variance curves for each of the three masses coincided with each other, giving the three curves of Fig. 5. The errors of the variance of y_3 at time $t=3.6$ are shown in Fig. 6 for different time step sizes. Clearly, the error of the HPD method with $\tau=0.08$, for which the computer time was 90 seconds, is approximately equal to that of the Newmark method with either $\tau=0.2$ or $\tau=0.3$, for which the computer times were 222 or 151

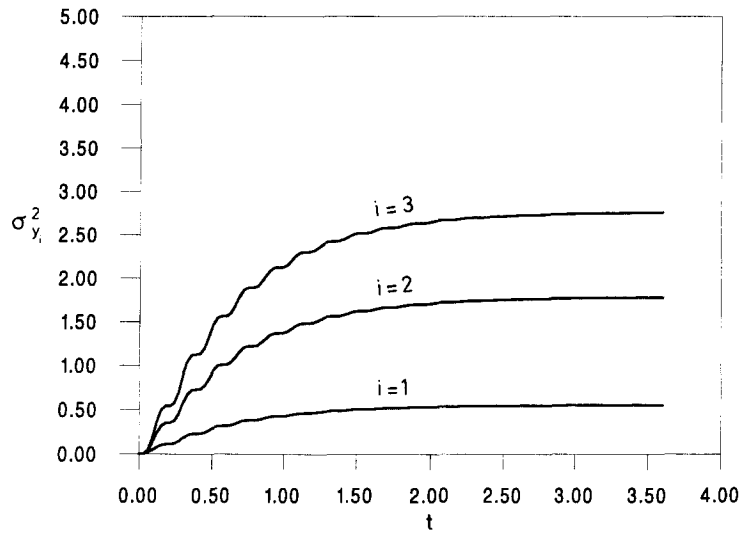

Fig. 5 The time dependent variances of y_1, y_2 and y_3.

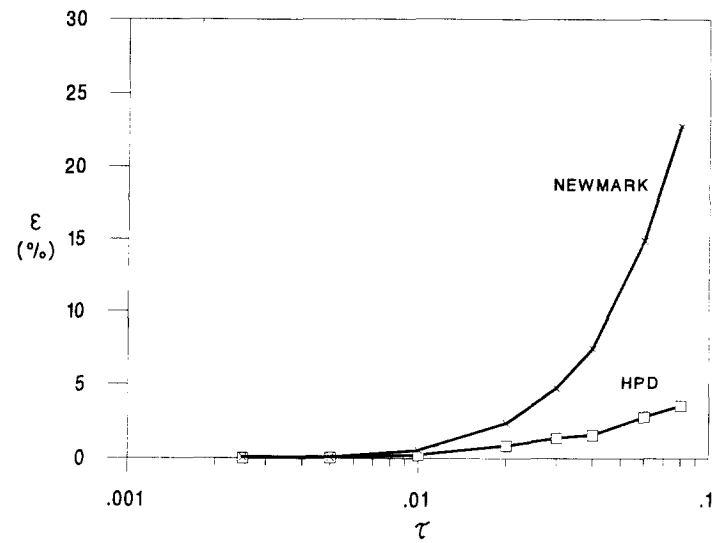

Fig. 6 $t=3.6$ error curves for σ^2_{y3}.

seconds, respectively. Thus the HPD method again needs about half the computer time required by the Newmark method. These times were obtained from an IBM/486 personal computer, with the main frequency of 33 MHz used.

5.3. Example 3

The eight DOF system of Fig. 7 has the mass, damping and stiffness distributions shown, with $M=10^4$ kg, $K=16\times10^6$ N/m and $C=4\times10^4$ (kg N/m)$^{0.5}$. The evolutionary random seismic acceleration is

$$\ddot{x}_g(t)=g(t)\ x(t) \tag{56}$$

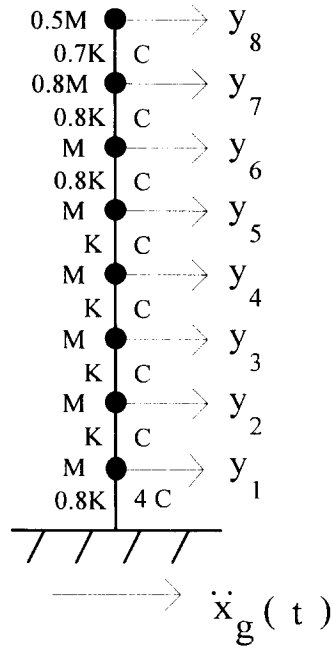

Fig. 7 An eight DOF system subjected to evolutionary random seismic excitation.

Table 4 Variances of top displacement y_8 (cm^2)

Scheme	τ \ t	8.0	16.0	24.0	32.0	40.0	Maximum errors(%)	Compu. time(s)
HPD	0.0025	2.306	6.525	4.881	1.014	0.1547	0.0	68539
	0.0050	2.306	6.525	4.881	1.014	0.1546	0.1	34229
	0.01	2.305	6.522	4.879	1.014	0.1546	0.1	17206
	0.04	2.288	6.475	4.844	1.007	0.1535	0.8	4304
	0.08	2.233	6.326	4.734	0.984	0.1500	3.2	2153
Newmark	0.0010	2.306	6.525	4.881	1.014	0.1547	0.0	106927
	0.0025	2.306	6.525	4.882	1.015	0.1549	0.1	42714
	0.0050	2.305	6.523	4.883	1.017	0.1555	0.5	21748
	0.01	2.300	6.515	4.886	1.023	0.1578	2.0	10728
	0.02	2.282	6.485	4.894	1.046	0.1661	7.4	5360
	0.04	2.210	6.344	3.849	1.069	0.1757	13.6	2672
	0.08	1.956	4.759	3.804	0.6000	0.0528	65.9	1337

in which the modulation function $g(t)$ of Fig. 3, i.e. of Eq. (54), applies with $t_b=8.0$ sec, $t_c=20.0$ sec and $c_1=0.16$. $x(t)$ is a stationary random process with Kanai-Tajimi PSD distribution, see Eq. (53), for which

$$\zeta_g=0.6,\ \omega_g=5\pi \text{ sec}^{-1} \text{ and } S_0=15.74 \text{ cm}^2/\text{sec}^3$$

The time dependent variances of the top displacement are shown in Table 4 and Figs. 8 and 9. Table 4 also shows the maximum percentage error for any of the five values of t shown,

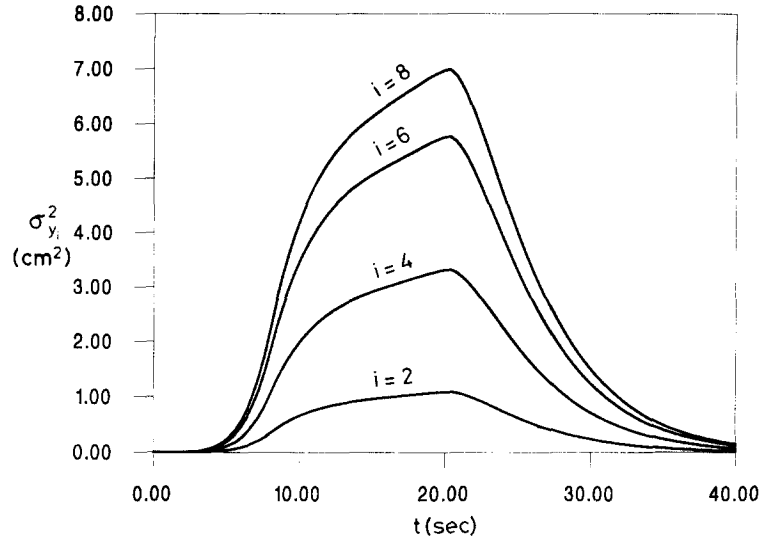

Fig. 8 The time dependent variances of y_2, y_4, y_6 and y_8.

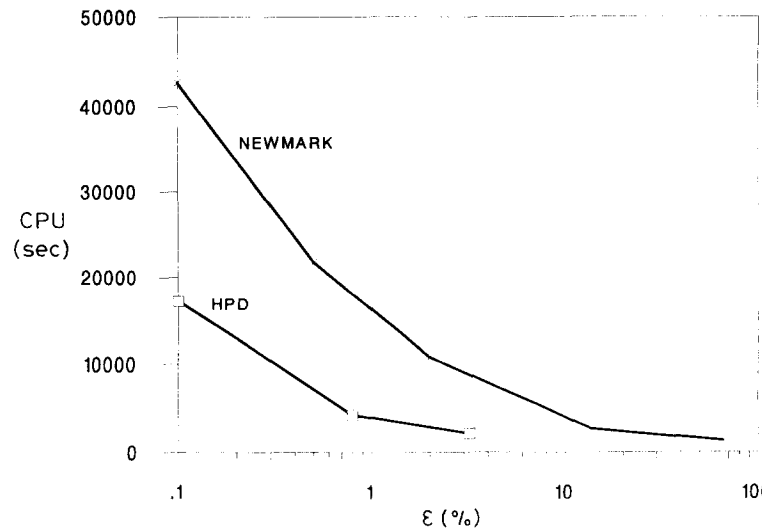

Fig. 9 Error curves of $\sigma^2_{y_8}(t=40.0)$.

and the computer time needed to get to $t=40$. It is seen that if this problem is solved by the HPD scheme with a step size of $\tau=0.08$, only 2153 seconds of computer time are used and the maximum error of the results is 3.2%. However, if the Newmark method with a step size of $\tau=0.02$ is adopted, 5360 seconds, i.e. 2.5 times as much computer time, is required and a larger error, of up to 7.4%, results.

6. Conclusions

Non-stationary random responses of structures subjected to an evolutionary random seismic excitation can be reduced to the solution of deterministic dynamic equations, which can then

be solved very accurately and efficiently in terms of the high precision direct integration scheme. This scheme has been shown by examples to be about twice as fast for such problems than is the Newmark method, which requires about four times as large a time step for given required accuracy.

Acknowledgements

The authors are grateful for the constant support of the National Natural Science Foundation of China and of the British Royal Society. They also owe many thanks to Professor Wanxie Zhong for beneficial discussions.

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