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Stochastic optimum design of linear tuned mass dampers for seismic protection of high towers

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Abstract. This work deals with the design optimization of tuned mass damper (TMD) devices used for mitigating vibrations in high-rise towers subjected to seismic accelerations. A stochastic approach is developed and the excitation is represented by a stationary filtered stochastic process. The effectiveness of the vibration control strategy is evaluated by expressing the objective function as the reduction factor of the structural response in terms of displacement and absolute acceleration. The mechanical characteristics of the tuned mass damper represent the design variables. Analyses of sensitivities are carried out by varying the input and structural parameters in order to assess the efficiency of the TMD strategy. Variations between two different criteria are also evaluated.

Keywords: TMD; structural optimization; random vibrations.

1. Introduction

The issue of mitigating the response of structures to environmental loads has drawn the interest of many researchers in recent years. The concept of structural control is now widely accepted and has been frequently applied in construction. Among the numerous passive control methods available, the tuned mass damper (TMD) is one of the simplest and most reliable. It is applicable not only in new constructions, but also existing ones. Due to its simplicity and reliability, the device is also used to suppress undesirable high vibration levels in machinery.

The mechanism involved in mitigating the vibration consists in the transfer of the vibration energy

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to the TMD, which dissipates it by damping.

A widespread application of TMDs is found in high-rise buildings in order to reduce the oscillating effect of wind loads. TMDs can also serve to protect integrated plants and equipment, such as air conditioning plants or power supply plants, just to name two.

Originally, building regulations considered the protection of human life as the primary objective of the structural design of new constructions and retrofit interventions. It soon became obvious that this goal was not sufficient to ensure a structure's functionality in the aftermath of an earthquake. Assuring complete structural efficiency is of great importance. Protecting auxiliary systems from damage can also reduce post-calamity reconstruction costs.

Recent earthquakes have shown that damaged auxiliary systems have an enormously negative economic impact. Over 75% of construction costs of high-structures are associated with non-structural components. In fact, localised damage of acceleration sensitive non-structural systems affect the functionality of a large portion of the entire structure.

The social and economic consequences of the loss of structural functionality underline the need for more efficient damage control systems. Anti-seismic design should not only guarantee the safety of lives but also reduce damage levels of vital auxiliary systems within the structure.

The development of the Performance Based Seismic Design (SEAOC Vision 2000, 1995) has proved reliable in achieving targeted performance objectives. For example, preventing or limiting damage to auxiliary systems at fully operational and operational performance levels (for earthquakes in moderately seismic areas) is critically important in Vision 2000, which is designed for earthquakes in moderately seismic areas.

At such levels, the structure remains in the elastic range or may only show a limited excursion beyond this range.

A high-performance design can be efficiently obtained by controlling the vibration level through the adoption of an enhanced TMD strategy. In order to increase the efficiency of a TMD, it is necessary to define its optimum mechanical parameters (tuning frequency, damping ratio and mass ratio). Many researchers have investigated the characteristics of tuned mass damper systems and several design formulae have been proposed to optimize parameters for different kinds of oscillation. The analytical development of an optimal design of a TMD considers several types of procedures and different mathematical models for the primary structural system and the associated external load (Rana and Soong 1998). Abe (1994) derived a formula to estimate the effectiveness of the MTMD subjected to harmonic forces. Takewaki (2000) developed a method for the optimal location of viscous dampers, taking into account the response amplification related to the ground.

An interesting work in this field is the unconstrained optimization of single non-linear (Rundinger, 2006) and multiple linear (Hoang *et al.* 2005) tuned mass dampers. It uses the structural displacement covariance of the protected system as objective function (*O.F*). The input is modelled by a simple stationary white noise stochastic process.

A complete stochastic-based optimization is proposed by Marano (2007) in which a reliable optimization criterion is developed by adopting a covariance approach. The *O.F.* and constraints are defined stochastically. The constraint imposes a limit to the probability of failure associated to the first threshold crossing of structural displacement over a given admissible level.

This paper focuses on high-rise towers which require fully operational performance levels under seismic conditions. The efficiency of the TMD can be improved if the mechanical characteristics are evaluated by means of the following criterion of optimization. The seismic analysis is developed stochastically and ground motion is modelled by a general stationary-filtered stochastic process.

Two different criteria are considered and the O.F. is expressed either by the ratio of the displacement between the protected and unprotected systems or by the absolute acceleration of the top of the tower expressed in stochastic terms by the root mean square value. These quantities are directly related to damage levels in structural elements and auxiliary systems. A comparison between the two criteria is also evaluated. The Design Vector (DV) collects the TMD frequency and damping ratio. Several analyses are carried out in order to assess the sensitivity of the optimum solution under varying ground conditions and structural characteristics.

2. The problem of a continuum tower equipped with a linear elastic TMD

A tuned mass damper (TMD) is a mass-dashpot-spring system (secondary system) attached to a main system designed to reduce the vibration level induced by environmental actions (Fig. 1).

The problem of a tall tower subjected to earthquakes and protected against undesirable vibrations by a TMD is analysed in this paper. An adequate design to cater for acceleration-sensitive non-structural elements requires an adequate estimation of the intensity of horizontal accelerations. This is also true for existing structures. It is obvious that for highly flexible structures, the first vibration mode is so long that it may not lie within the dominant frequency content of earthquakes and the response may be dominated by high vibration modes, especially on stiff soil. Therefore, a system of $n+1$ degrees of freedom is adopted to model a continuum column having height H, an extra mass M_s positioned at the free top end and equipped with a TMD. The column, with an annular cross-section with constant thickness δ (Fig. 2), is excited by a mono-directional horizontal base acceleration $\ddot{x}_g$.

The column is modelled as a visco-elastic discrete system, with n lumped elements having equal heights $\Delta_H = H/n$, whereas H represents the total height of the column.

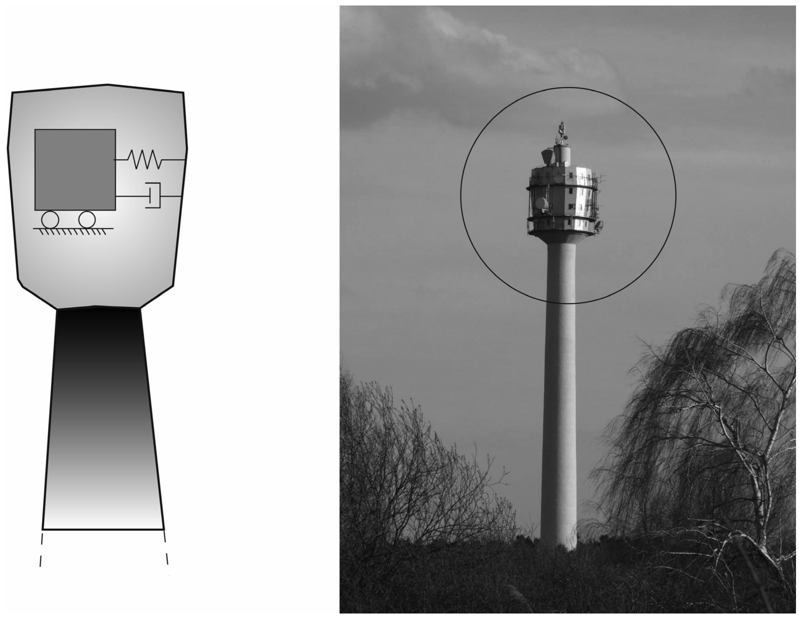

Fig. 1 Possible tuned mass dampers location in a tall tower

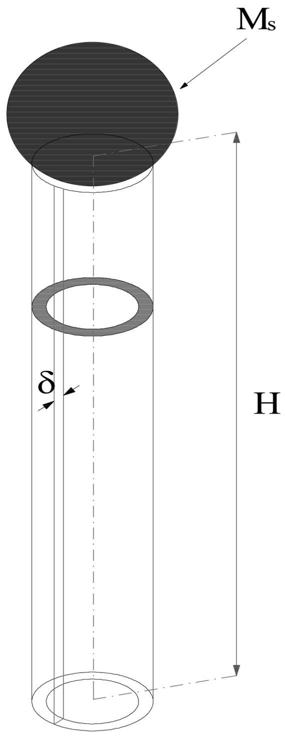

Fig. 2 Schematization a tube-type column with an additional mass at the top free end

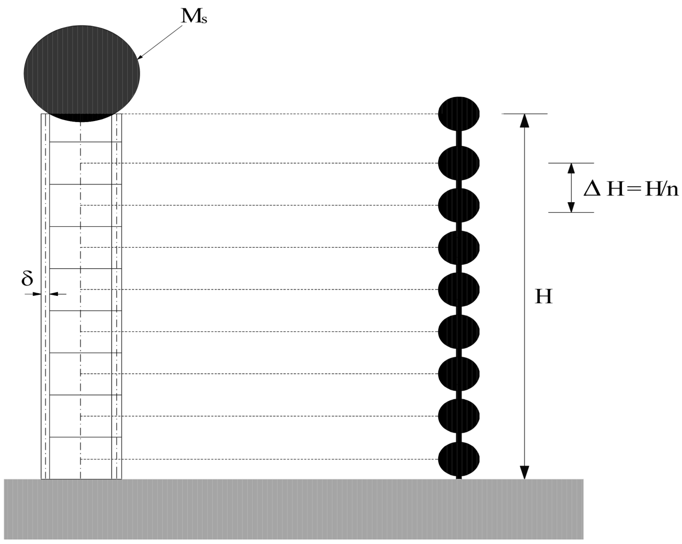

Fig. 3 Discretization of a column into system of lumped masses

The mechanical scheme of this lumped mass system equipped with a TMD is represented in Fig. 3. The mass M_s at the top of the structure is connected to the mass m_{TMD} of the TMD with a linear spring which has a k_{TMD} characteristic and a dashpot with a c_{TMD} characteristic.

Having introduced the filter equation, the structural response of this combined system is determined by solving the dynamic equilibrium Eq. (1)

$$\begin{cases} \overline{\mathbf{M}}\ddot{\mathbf{x}}(t) + \overline{\mathbf{C}}\dot{\mathbf{x}}(t) + \overline{\mathbf{K}}\mathbf{x}(t) = -\overline{\mathbf{M}}\overline{\mathbf{r}}\ddot{x}_g \\ \ddot{x}_g = \ddot{x}_f + w = -(2\xi_g\omega_g\dot{x}_f + \omega_g^2 x_f) \end{cases} \tag{1}$$

In Eq. (1), $\overline{\mathbf{M}}, \overline{\mathbf{K}}, \overline{\mathbf{C}}$ and $\overline{\mathbf{r}}$ stand for the mass, stiffness and damping matrices, the drag vector of the column plus the TMD system. A condensed stiffness matrix has been considered. The dissipation viscous matrix of the column is assumed to be a linear combination of mass and stiffness matrices

$$\overline{\mathbf{M}} = \begin{bmatrix} m_{TMD} & \mathbf{0} \\ \mathbf{0} & \mathbf{M} \end{bmatrix} = \begin{bmatrix} m_{TMD} & 0 & 0 & . & . & 0 \\ 0 & m_1 & 0 & . & . & 0 \\ 0 & 0 & m_2 & . & . & 0 \\ 0 & 0 & 0 & . & . & 0 \\ 0 & 0 & 0 & 0 & . & 0 \\ 0 & 0 & 0 & 0 & 0 & m_n \end{bmatrix} \tag{2}$$

$m_1 = \rho a_1 \Delta H/2 + M_S$; $m_i = \rho(a_i + a_{i-1})\Delta H/2$, $i = 2, \ldots, n$, being ρ the mass density, and a_i denotes the cross-section of the column, which can generally be variable.

$$\overline{\mathbf{K}} = \begin{bmatrix} k_{TMD} & -k_{TMD} & 0 & . & . & 0 \\ -k_{TMD} & k_{TMD} + k_{11}^c & k_{12}^c & . & . & k_{1n}^c \\ 0 & k_{21}^c & k_{22}^c & . & . & k_{21}^c \\ 0 & . & . & . & . & . \\ 0 & . & . & . & k_{n-1n-1}^c & k_{n-1n}^c \\ 0 & . & . & . & k_{nn-1}^c & k_{nn}^c \end{bmatrix} \tag{3}$$

In Eq. (3), k_{ij}^c are the elements of the condensed stiffness matrix $\mathbf{K}^c$, defined by

$$\mathbf{K}^c = \mathbf{K}^{xx} - \mathbf{K}^{x\gamma}(\mathbf{K}^{\gamma\gamma})^{-1}\mathbf{K}^{\gamma x} \tag{4}$$

and the terms $\mathbf{K}^{xx}$, $\mathbf{K}^{x\gamma}$, $\mathbf{K}^{\gamma x}$ and $\mathbf{K}^{\gamma\gamma}$ are given in the appendix.

Moreover:

$$\overline{\mathbf{C}} = \begin{bmatrix} c_{TMD} & -c_{TMD} & 0 & . & . & 0 \\ -c_{TMD} & c_{TMD} + c_{11} & c_{12} & . & . & c_{1n} \\ 0 & c_{21} & c_{22} & . & . & c_{21} \\ 0 & . & . & . & . & . \\ 0 & . & . & . & c_{n-1n-1} & c_{n-1n} \\ 0 & . & . & . & c_{nn-1}^c & c_{nn} \end{bmatrix} \tag{5}$$

In matrix Eq. (5), c_{ij} are the elements of the dissipation viscous matrix **C**, here assumed to be a linear combination of mass and stiffness matrices expressed by a Raleigh assumption

$$\mathbf{C} = a_0\mathbf{M} + a_1\mathbf{K}^c \tag{6}$$

$a_0 = \xi_0\frac{2\omega_1\omega_2}{\omega_1+\omega_2}$ and $a_1 = \xi_0\frac{2}{\omega_1+\omega_2}$, ω_1 and ω_2 are respectively the first and second system natural frequencies.

Detailed formulae of matrices are given in the appendix.

x_f is the response of the Kanai-Tajimi filter which has a time constant frequency ω_g and a damping coefficient ξ_g. w is the white noise process whose Power Spectral Density (PSD) function is S_0.

The displacement vector $\mathbf{x} = (x_{TMD}, x_1, x_2, \ldots)^T$ is defined by the system-ground relative displacements, whereas the velocity and acceleration vectors are respectively $\dot{\mathbf{x}} = (\dot{x}_{TMD}, \dot{x}_1, \dot{x}_2, \ldots)^T$ and $\ddot{\mathbf{x}} = (\ddot{x}_{TMD}, \ddot{x}_1, \ddot{x}_2, \ldots)^T$. Finally, $\mathbf{r} = (1, 1, 1, \ldots)^T$ is the drag vector.

By introducing the state space vector

$$\mathbf{z} = (x_{TMD}, \mathbf{x}, x_f, \dot{x}_{TMD}, \dot{\mathbf{x}}, x_f)^T \tag{7}$$

system Eq. (1) can be replaced by the equation

$$\dot{\mathbf{Z}}(t) = \mathbf{A}\mathbf{Z}(t) + \mathbf{r}^*\ddot{x}_g(t) \tag{8}$$

in which the state matrix **A** is given in the appendix and $\mathbf{r}^* = (0, 0, 0, ..|1, 11, ..)^T$.

The space state covariance matrix $\mathbf{Q}_{ZZ} = E[\mathbf{Z}\mathbf{Z}^T]$ is obtained by solving the *Lyapunov* equation stated for the stationary case

$$\mathbf{A}\mathbf{Q}_{ZZ} + \mathbf{Q}_{ZZ}\mathbf{A}^T + \mathbf{B} = 0 \tag{9}$$

in which the state matrix **A** is

$$\mathbf{A} = \begin{pmatrix} \mathbf{0} & \mathbf{I}_{n+1} \\ -\mathbf{H}_2 & -\mathbf{H}_1 \end{pmatrix} \tag{10}$$

in which

$$\mathbf{H}_1 = \begin{pmatrix} & & & & & & 2\xi_g\omega_g \\ & & & & & & 2\xi_g\omega_g \\ & & & & & & . \\ & & & \overline{\mathbf{M}}^{-1}\overline{\mathbf{C}} & & & . \\ & & & & & & 2\xi_g\omega_g \\ & & & & & & 2\xi_g\omega_g \\ 0 & 0 & 0 & \ldots & 0 & 0 & -2\xi_g\omega_g \end{pmatrix} \tag{11}$$

and

$$\mathbf{H}_2 = \begin{pmatrix} & & 2\xi_g\omega_g \\ & & 2\xi_g\omega_g \\ & & . \\ & \overline{\mathbf{M}}^{-1}\overline{\mathbf{K}} & . \\ & & 2\xi_g\omega_g \\ & & 2\xi_g\omega_g \\ 0\;0\;0 & \dots\;\;0\;0 & -2\xi_g\omega_g \end{pmatrix} \tag{12}$$

The input matrix $\mathbf{B}$ has all elements equal to zero, except $B_{66} = 2\pi S_0$.

By introducing the vector $\ddot{\mathbf{y}} = (\ddot{y}_{TMD}, \ddot{y}_1, \ddot{y}_2, \ddot{y}_3, \dots)^T$, which collects the absolute accelerations, the relative covariance matrix is

$$\mathbf{Q}_{\ddot{y}} = \mathbf{D}\mathbf{Q}_{\mathbf{X}\dot{\mathbf{X}}}\mathbf{D}^T \tag{13}$$

In which

$$\mathbf{D} = [-\overline{\mathbf{M}}^{-1}\overline{\mathbf{K}}\;\; -\overline{\mathbf{M}}^{-1}\overline{\mathbf{C}}] \tag{14}$$

and $\mathbf{Q}_{\mathbf{X}\dot{\mathbf{X}}} = E[X\dot{X}]^T$.

3. Optimum design criterion of TMD

In order to enhance the effectiveness of the vibration control strategy, TMD frequency and damping ratio should be carefully selected.

In dealing with random vibrations, the problem of optimization can be defined as the identification of a suitable set of variables. These variables are the parameters of the design structural configuration and are gathered from the *Design Vector* (DV) $\mathbf{b}$ over an admissible domain $\mathbf{\Omega}$. The optimum DV must be able to minimize a given *objective function O.F.* and satisfy several conditions of constraint in terms of reliability. Both the reliability constraints and the *O.F.* are defined over a given time interval, as the problem deals with a dynamic structural response.

The general optimization problem defined in such terms was first treated by Nigam (1972) and transformed into a standard non-linear approach stated as follows:

find a design vector	$\mathbf{b} \in \mathbf{\Omega}$	
which minimizes	$O.F.(\mathbf{b})$	
subject to	$P_f^i(\mathbf{b}, t) \le \tilde{P}_f^i \quad i = 1 \text{ to } k$	(15)

The *O.F.* may be defined either in a standard deterministic or stochastic manner. In the latter case, response statistics can be used, such as the covariance or spectral moments of variables (displacement, acceleration or structural stress involving important structural elements).

$P_f^i(\mathbf{b}, t)$ is the probability of failure associated to the i^{th} failure mode, k being the total failure mode number and $\tilde{P}_f^i$ its admissible value.

In this study, the optimal design of a TMD concerns the unconstrained optimization of the two-dimensional design vector $\mathbf{b} = (\omega_{TMD}, \xi_{TMD})^T$, in which

$$\omega_{TMD} = \sqrt{\frac{k_{TMD}}{m_{TMD}}} \tag{16}$$

$$\xi_{TMD} = \frac{c_{TMD}}{2\sqrt{m_{TMD}k_{TMD}}} \tag{17}$$

The mass ratio $\gamma_{TMD} = m_{TMD}/M_T$ is assumed to be a given quantity (M_T is the total mass of the system). The process of optimization lies in minimizing the response of the protected structure as compared to the un-protected one. Two different criteria are adopted with two different *O.F.s*.

The first criterion regards the minimization of the absolute acceleration response in which the $O.F._{acc}$ (subscript *acc* denotes the criterion based on acceleration) is defined as a dimensionless ratio between $\sigma_{\ddot{Y}_1}$ and $\sigma^0_{\ddot{Y}_1}$, which are respectively the absolute acceleration at the top of both the protected and unprotected structures.

The objective function is a direct index of the performance of the TMD, whereby detrimental vibrations that can cause damages in acceleration sensitive auxiliary systems are reduced.

Therefore, the following optimization problem regarding the TMD is formulated:

find $$\mathbf{b} = (\omega_{TMD}, \xi_{TMD}) \in \mathbf{\Omega} \tag{18}$$

which minimizes $$O.F._{acc} = \frac{\sigma_{\ddot{Y}_1}(\mathbf{b})}{\sigma^0_{\ddot{Y}_1}} \tag{19}$$

The second optimization criterion concerns the minimization of the displacement response in which the $O.F._{dis}$ (subscript *dis* denotes the criterion based on displacement) is defined as a dimensionless ratio between $\sigma_{\bar{X}_1}$ and $\sigma^0_{\bar{X}_1}$, which are respectively the RMS of the relative displacement $\bar{x}_1 = x_1 - x_2$ at the top of both the protected and unprotected structures.

The second optimization problem of the TMD is formulated as:

find $$\mathbf{b} = (\omega_{TMD}, \xi_{TMD}) \in \mathbf{\Omega} \tag{20}$$

which minimizes $$O.F._{dis} = \frac{\sigma_{\bar{X}_1}(\mathbf{b})}{\sigma^0_{\bar{X}_1}} \tag{21}$$

4. Numerical example

A 35 m-high reinforced concrete tower is taken into consideration. The tower has a constant tube-type cross-section with an internal diameter of 175 cm and a wall thickness of 25 cm. This continuous system is divided into 40 finite elements. The concentrated mass M_s is assumed to be equal to 140.000 kg. The mass density and the Young modulus adopted are respectivel $\rho = 2.5 \times 10^3$ kg/m^3 and $E = 300.000$ kg/cm^2. The first natural period is $T_1 = 1.26$ sec. The tower is equipped with a single TMD whose mechanical characteristics have to be optimized in order to maximize the performance by reducing the vibration level and the structural and non- structural damage.

4.1 Optimization results

The two optimization criteria previously described are applied and the results are plotted in Fig. 4.

The optimization is developed in space $\mathbf{b} = (\omega_{TMD}, \xi_{TMD})^T$, assuming γ_{TMD} to be a given quantity as the possible range of variations can differ from case to case in the adoption of TMDs.

Stiff soil and the soft soil conditions are analysed and their parameters are given in Table 1 (Muscolino 2001).

The optimum solution to the problem corresponds to the global minimum of the O.F. in the design vector space. More precisely, the dimensionless design variable $\rho_\omega = \omega_{TMD}/\omega_1$, in which ω_1 is equal to 4.98 rad/sec, and ξ_{TMD} are represented. Their optimum values are denoted as $\rho_\omega^{opt} = \omega_{TMD}^{opt}/\omega_1$ and ξ_{TMD}^{opt}.

A parametric sensitivity analysis in terms of the O.F. and design variables has been performed to verify the variability of the optimum solution and assess differences between the displacement and acceleration optimization criteria.

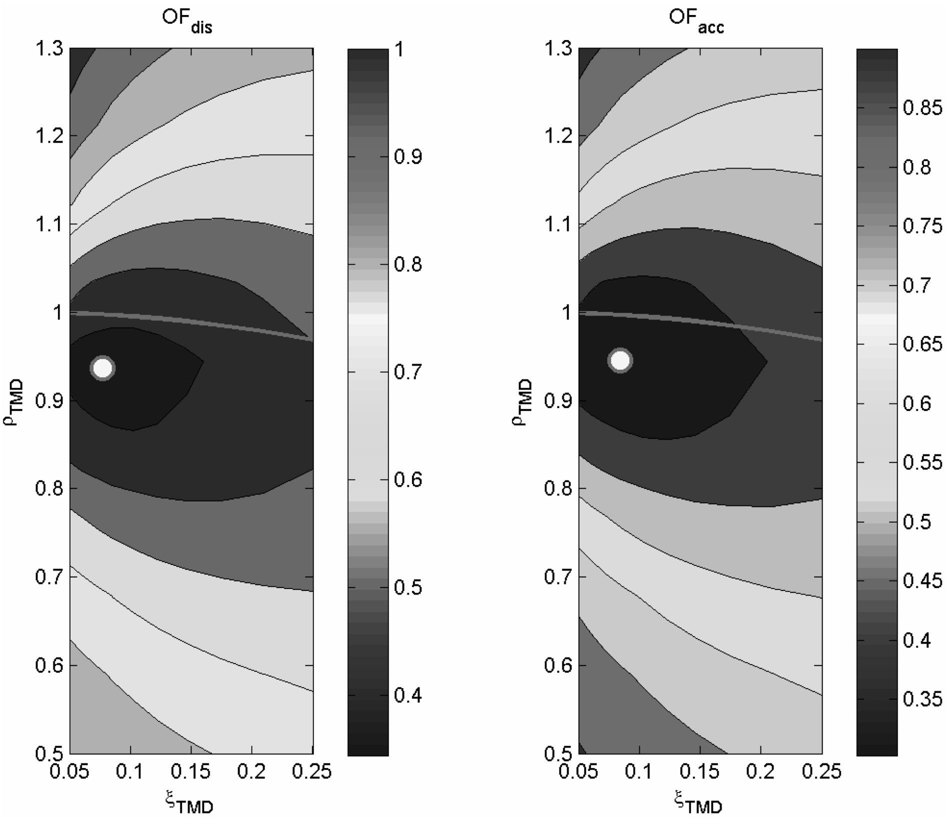

Fig. 4 O.F. surface in the dimensionless design vector space $\rho_{TMD} = \rho_\omega = \omega_{TMD}/\omega_1$ and ξ_{TMD}. Stiff soil is considered. The first natural frequency of the column is $\omega_1 = 4.98$ (*rad*/sec) and $\xi_S = 0.02$. The mass ratio is assumed $\gamma_{TMD} = 0.01$

Table 1 Ground parameters

Filter parameters	Stiff Soil	Soft soil
ω_f	20 rad/sec	4.50 rad/sec
ξ_f	0.65	0.10

The procedure includes both stiff (Fig. 5) and soft soil (Fig. 6) conditions. The mass ratio is assumed to vary from 0.01 to 0.3. A number of practical and economic factors which influence the choice of this parameter must be evaluated. Usually, the additional mass doesn't exceed the 1-2% of the first modal mass. In real civil applications, additional masses specifically used as TMDs can reach a weight of up to 400 t and are often made of steel or concrete allocated in a dedicated location within the main structure (Kwok and Samali 1995). Sometimes, masses serving other functions, such as water tanks, are also used as TMDs. A typical example of this is found in the 305 m-tall Sidney Tower. It is reasonable to assume that auxiliary system masses located on adequate supports can be used as TMDs. Their masses can be potentially greater than those currently adopted. In this paper the mass ratios γ_{TMD} may vary from 1% to 16% in standard civil applications.

Figs. 5 and 6 demonstrate that the performance of a TMD improves when the mass ratio increases. In Fig. 5 (stiff soil), it can be noted that the displacement based optimization criterion offers a better performance compared to that of acceleration. Concerning the optimized design variables, TMD damping ratio attains the same values with both criteria, whereas the optimal TMD frequency ratio $\rho_{\omega}^{opt} = \omega_{TMD}^{opt}/\omega_1$ falls when the displacement criterion is applied. The difference increases as the mass ratio increases, giving proof of the growing efficiency of the TMD strategy. The optimal TMD frequency value tends to decrease faster during the displacement based procedure compared to the acceleration criterion.

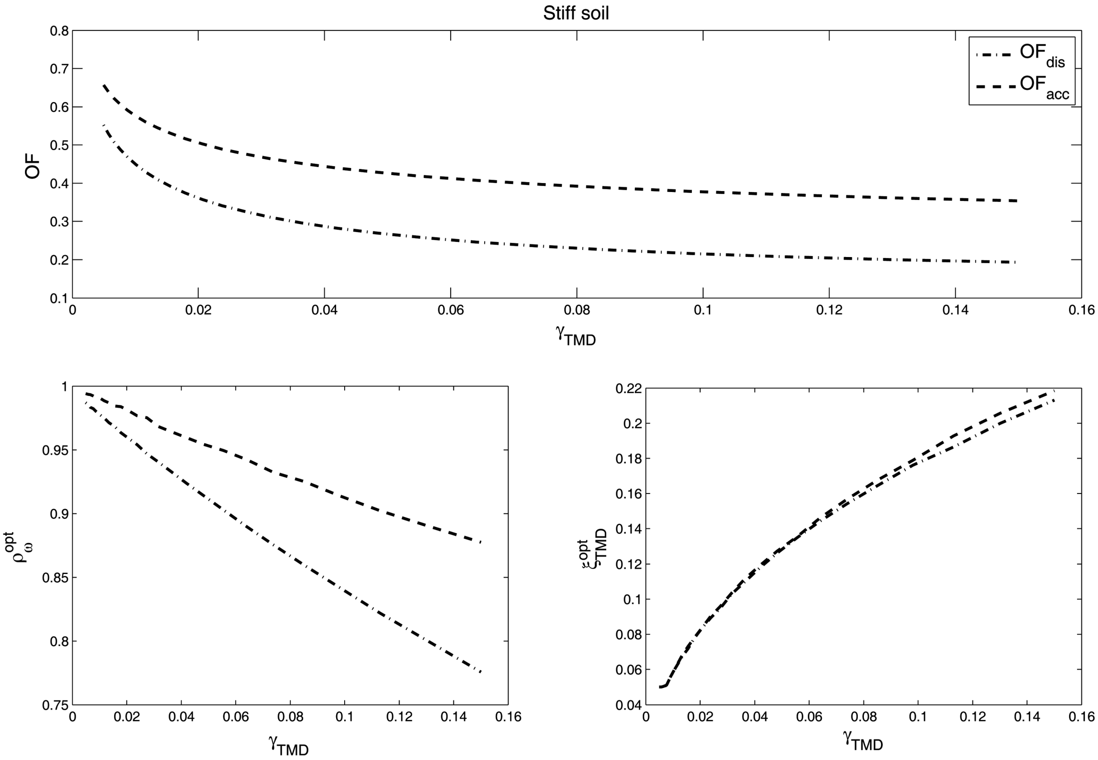

Fig. 5 Differences in OF (a) and DV(b-c) optimal solutions for displacement and acceleration based criteria, for different values of tuned mass ratio. A stiff soil is considered. The first natural frequency of the column is $\omega_1 = 4.98$ (*rad*/sec) and $\xi_S = 0.02$

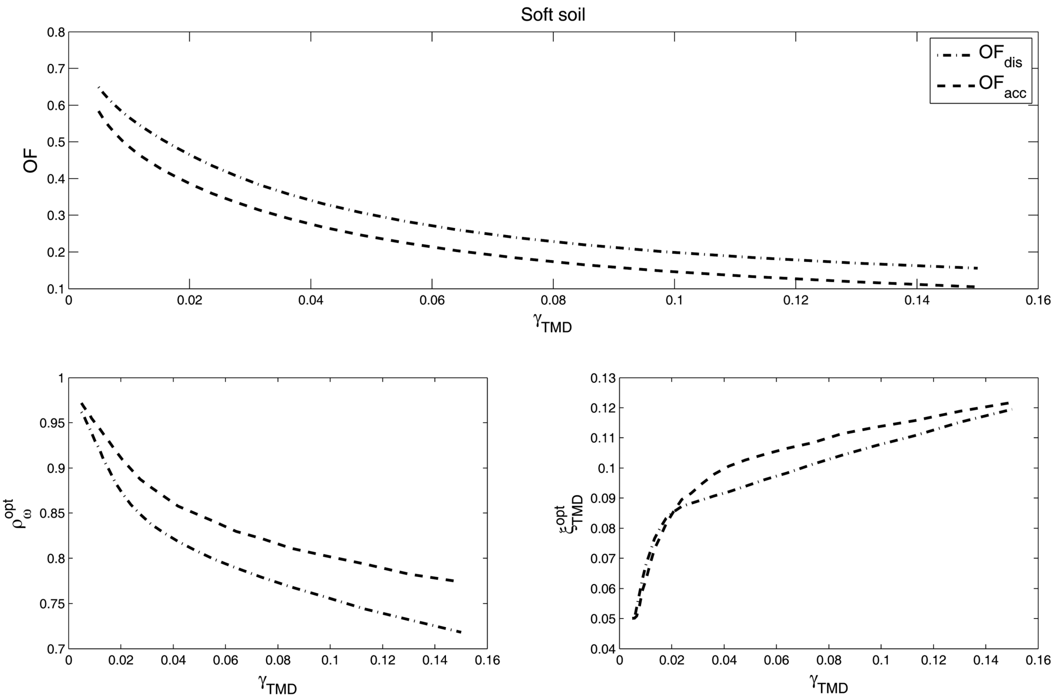

Fig. 6 Differences in O.F. and DV optimal solutions for displacement and acceleration based criteria, for different values of tuned mass ratio. Soft soil is considered. The first natural frequency of the column is $\omega_1 = 4.98$ (*rad*/sec) and $\xi_S = 0.02$

Fig. 6 refers to soft soil condition. In this case, the acceleration based criterion offers a better performance compared to that of displacement. With regards the optimum design variables, some differences arise in the tuned mass damping ratio. ξ_{TMD}^{opt} is larger when applying the acceleration based criterion, whereas is lower in case of the displacement based criterion.

In order to evaluate the different performances between the two criteria under stiff and soft soil conditions, the optimal solution is plotted in Figs. 7 and 8 by varying the frequency content of the seismic acceleration. The ground frequency is on the x-axis. The ground filter damping ratio from 0.65 for $\omega_f = 20$ *rad*/sec to 0.1 for $\omega_f = 4$ *rad*/sec is assumed to be a linear variation. The mass ratio is assumed to be equal to $\gamma_{TMD} = 0.02$. Two fundamental frequencies for two structures are assumed: $\omega_1 = 4.98$ (*rad*/sec) (Fig. 7) and $\omega_1 = 10.58$ (*rad*/sec) (Fig. 8).

A maximum performance can be noted in Fig. 7, which corresponds to the minimum value of the optimized objective function. This occurs when the analysed structure is in resonance with the frequency content of the seismic excitation ($\omega_f \cong 5$ *rad*/sec). With a fundamental frequency of $\omega_1 = 4.98$ (*rad*/sec) and in absence of a TMD, a maximum amplification of the response occurs in resonance with the frequency content of the ground motion.

In case of soft soil, high modes don't have a great influence on the response because the ground excitation is narrow-banded in its energy content. The application of a TMD offers a good performance in reducing structural responses. Both criteria give quite similar optimum *O.F.s*. For soil with $\omega_f \cong 5$ *rad*/sec, the optimum solution gives $\rho_{\omega}^{opt} = \omega_{TMD}^{opt} / \omega_S \cong 1$. Also shown by previous researchers, this solution corresponds to tuning the mass damper to the fundamental frequency of

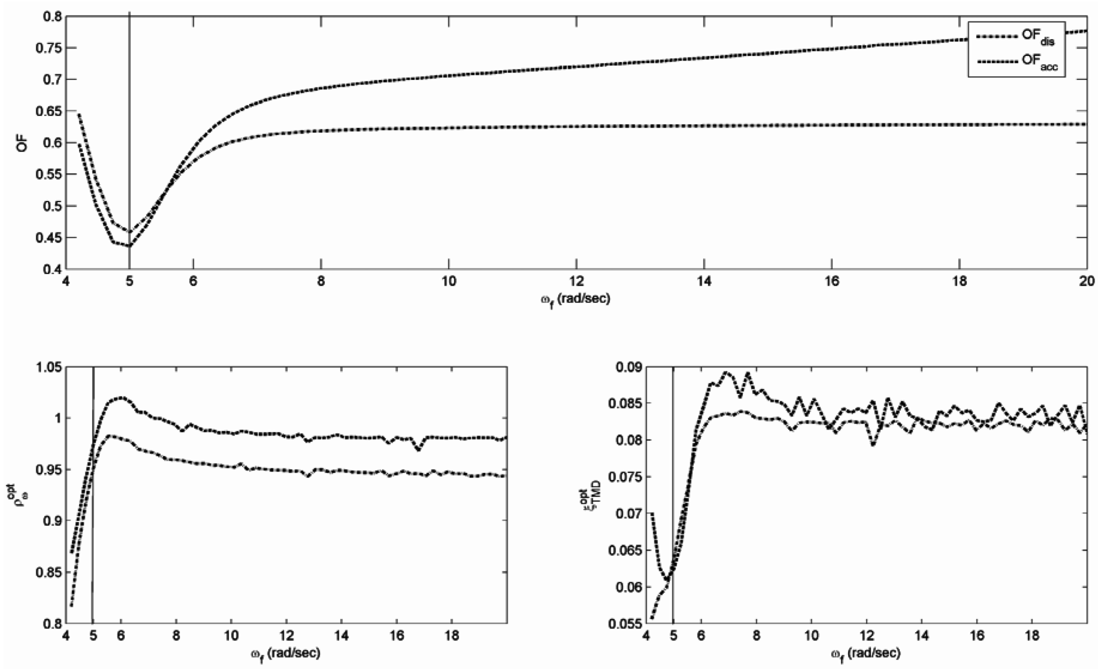

Fig. 7 Differences in OF and DV optimal solutions for displacement and acceleration based criteria, for different values of filter frequency ratio. The first natural frequency of the column is $\omega_1 = 4.98$ (*rad/sec*) and $\xi_S = 0.02$

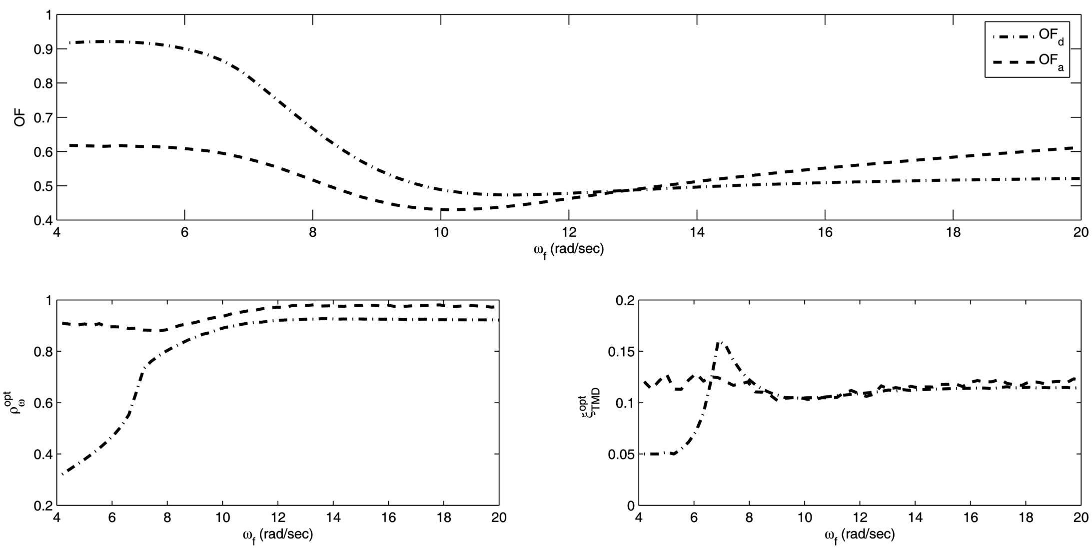

Fig. 8 Differences in OF and DV optimal solutions for displacement and acceleration based criteria, for different values of filter frequency ratio. The first natural frequency of the column is $\omega_1 = 10.58$ (*rad/sec*) and $\xi_S = 0.02$

the structure. The TMD acts only by means of its frequency. This behaviour is demonstrated by a low optimum TMD damping ratio.

With reference to the two response quantities of interest, displacement and absolute acceleration, it can be observed that acceleration on soft soil is better controlled than displacement, although efficiency levels remain similar in both cases.

As the soil becomes stiffer, it can be observed that the TMD efficiency decreases, the optimum *O.F.* values being larger than those obtained on soft soil. This is an expected outcome because the unprotected structure with a natural frequency $\omega_1 = 4.98$ (*rad*/sec) is far from being in resonance with the ground motion and will automatically cause a substantial reduction of the response, while the presence of a TMD will have no significant impact.

The acceleration based optimization is less efficient than that of displacement. It should be noted that on stiff soil, which corresponds to the broadest band excitation, even high vibration modes can influence structural response. In any case, the mass damper is always tuned to the fundamental frequency of the structure and, therefore, cannot control high modes.

In Fig. 8, which corresponds to a more rigid structure, it is possible to note the similarity of efficiency in reducing the acceleration on soft soil and the displacement on stiff soil. The TMD works better from medium to stiff soil because of a maximum amplification resonance effect with the ground motion. In conditions of resonance and near-resonance of the unprotected structure with the ground motion, the optimum performance is obtained by both criteria by means of the same TMD damping ratio and by tuning the TMD frequency almost to the first natural frequency of the structure.

On soft soil, the TMD functions in a different way in reducing acceleration and displacement. An acceleration reduction can be obtained, only by tuning the TMD frequency to the natural frequency of the structure. Any reduction of the displacement can be obtained.

In order to outline the varying efficiency levels obtained with different soil conditions and by adopting either the displacement or acceleration based criteria, the system response transfer

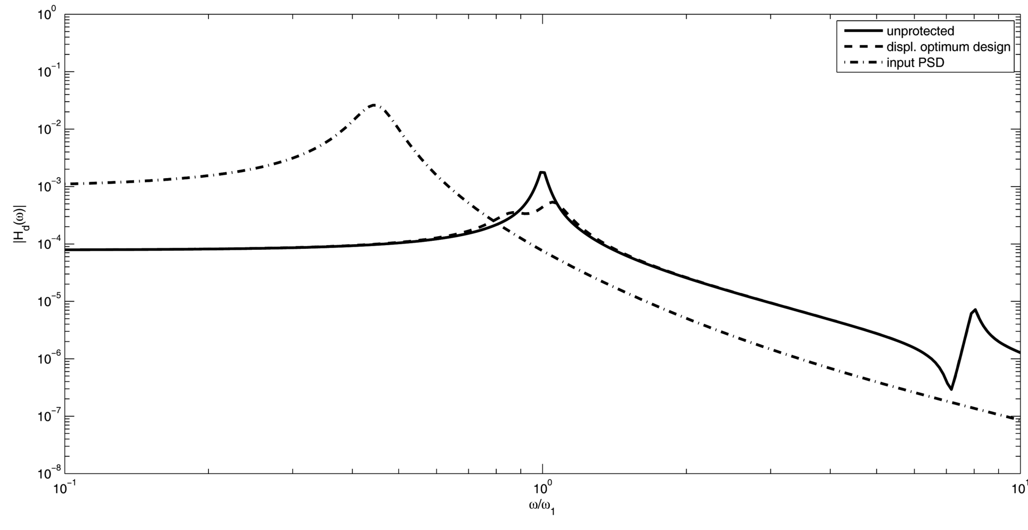

Fig. 9 Top floor displacement transfer function for the unprotected structure as well as the protected. Soft soil condition has been considered; the original structure has $\omega_1 = 4.98$ (*rad*/sec), whereas displacement based optimum TMD parameter have been utilized

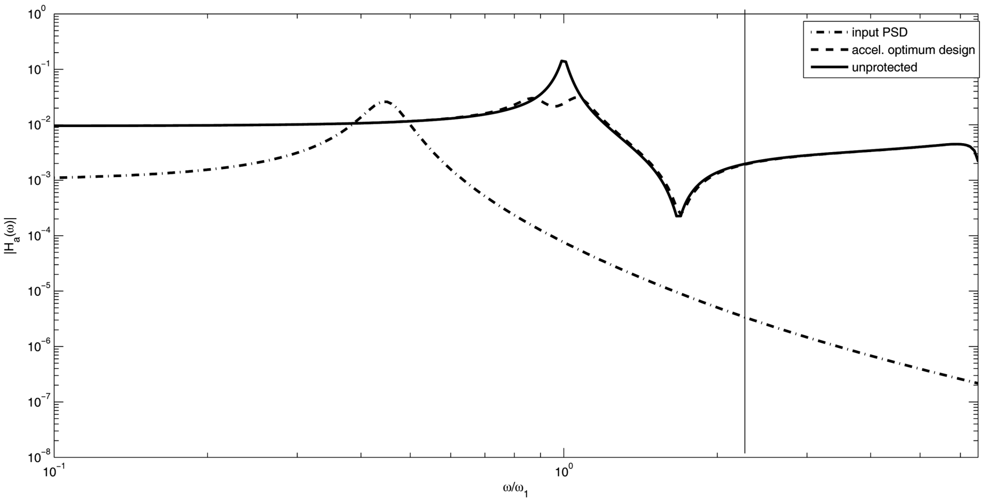

Fig. 10 Top floor acceleration transfer function for the unprotected structure and for the protected. Soft soil condition has been considered; the original structure has $\omega_1 = 4.98$ (*rad*/sec), whereas acceleration based optimum TMD parameter have been utilized

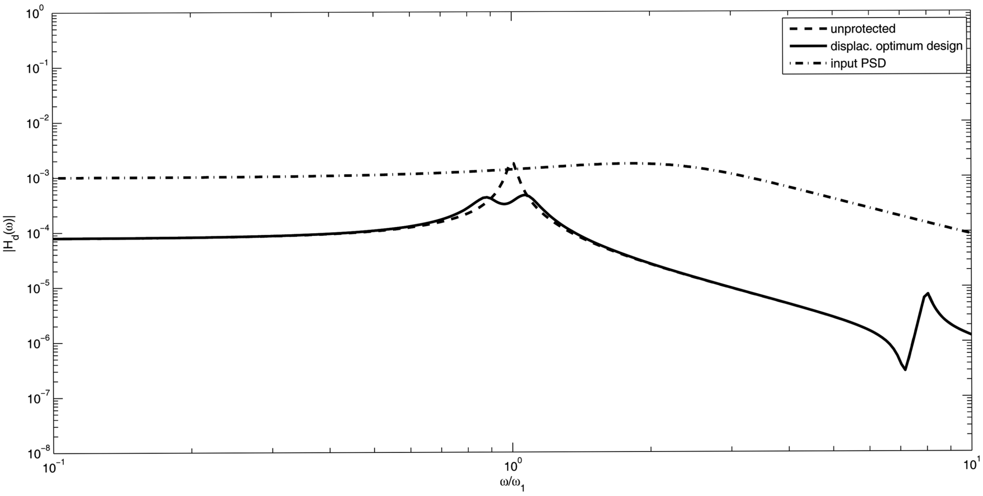

Fig. 11 Top floor displacement transfer function for the unprotected structure as well as the protected. Stiff soil condition has been considered; the original structure has $\omega_1 = 4.98$ (*rad*/sec), whereas displacement based optimum TMD parameter have been utilized

functions are represented in Figs. 9, 10, 11 and 12, together with the Power Spectral Density (PSD) function for soft soil (Figs. 9 and 10) and stiff (Figs. 11 and 12).

Fig. 9 shows the displacement transfer function of the unprotected system (continuum line) and of the system equipped with a TMD whose optimum mechanical characteristics have been obtained on the basis of the displacement criterion. A soft soil condition has been considered, whereas the analysed structure has a fundamental frequency of $\omega_1 = 4.98$ (*rad*/sec).

Whereas the original system shows a single predominant peak which corresponds to its fundamental frequency, the seismically protected structure shows two smaller peaks distant from the

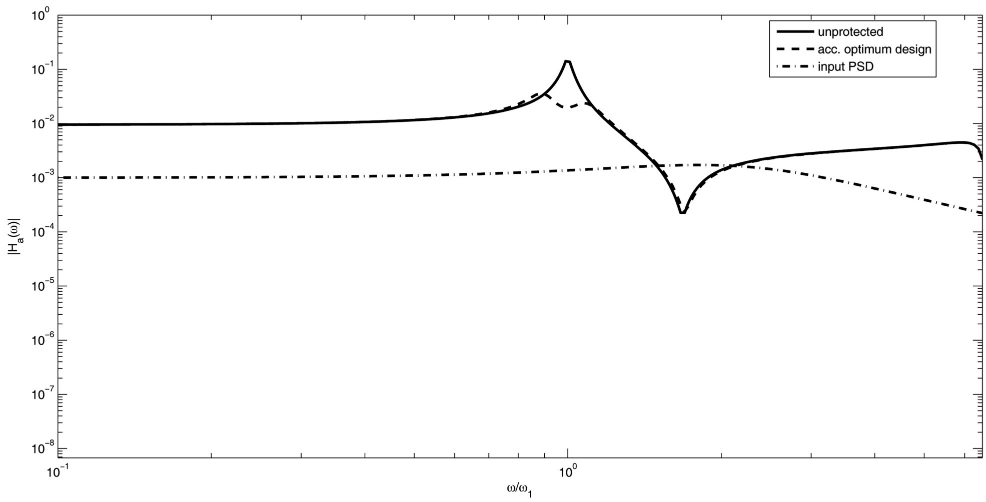

Fig. 12 Top floor acceleration transfer function for the unprotected structure as well as the protected. Stiff soil condition has been considered; the original structure has $\omega_1 = 4.98$ (*rad*/sec), whereas acceleration based optimum TMD parameter have been utilized

original one. These peaks correspond to the two predominant frequencies of the protected structure. The acceleration transfer function also shows two smaller peaks, as shown in Fig. 10, and the TMD characteristics are those obtained with the acceleration criterion.

The main goal of a TMD strategy is to split into two and reduce the single resonant peak.

The results here given are quite interesting because all optimum transfer functions are represented. It can be noted that of the two resonant frequencies, the acceleration peak shows two small ones and the displacement optimization has a predominant peak belonging to the opposite region of the resonant frequency of the ground motion. An opposite tendency can be observed in the case of stiff soil (Figs. 11 and 12). In this condition, it is the acceleration criterion which gives a predominant frequency belonging to the opposite region of the resonant frequency of the ground motion, whereas the displacement criterion furnishes two small peaks.

The efficiency of both criteria under soft and stiff soil conditions can be validated from these observations in the frequency domain.

In Figs. 13, 14, 15 and 16, a sensitivity analysis is carried out by varying the mass ratio for different structural periods. This variation is obtained by adopting several additional masses M_s. The remaining structural characteristics are the same as in the previous examples.

Fig. 13 corresponds to stiff soil and the displacement based criterion. It can be noted that the TMD efficiency in reducing structural displacement depends strongly on the structural period. The best control efficiency occurs when the structure is near to the resonance condition with the ground motion. As explained, this happens when $T_0 = 0.58$ sec ($\omega_1 = 10.8$ *rad*/sec). For a low mass ratio, the optimum response reduction is obtained by tuning the mass damper to the first natural frequency; the optimum required level of dissipation remains low. When the mass ratio increases, the tendency of the optimum design variables changes and the TMD acts by means of its damping characteristic.

The same conclusion can be drawn from Fig. 12, in which the efficiency is evaluated in terms of acceleration reduction.

Significant reduction of the responses to displacement and acceleration can be obtained only by

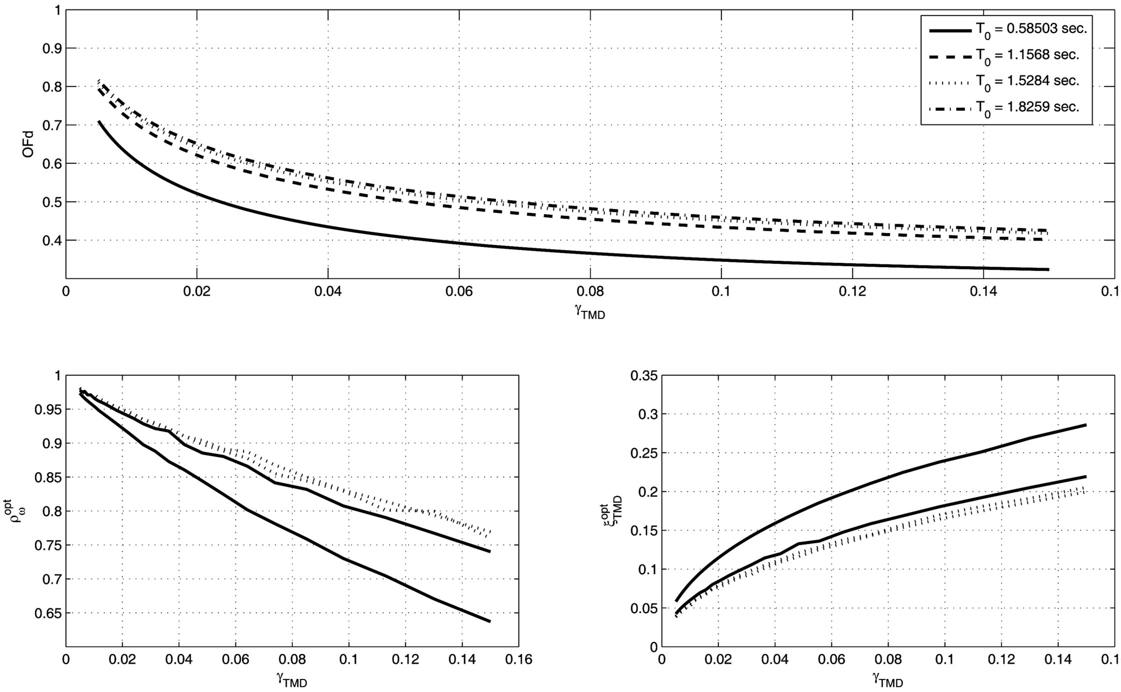

Fig. 13 O.F.$_d$ and DV optimum solutions versus TMD mass ratio. The analysis has been carried out for 4 different main system natural periods T_0. The filter parameters used are referred to a stiff soil type condition

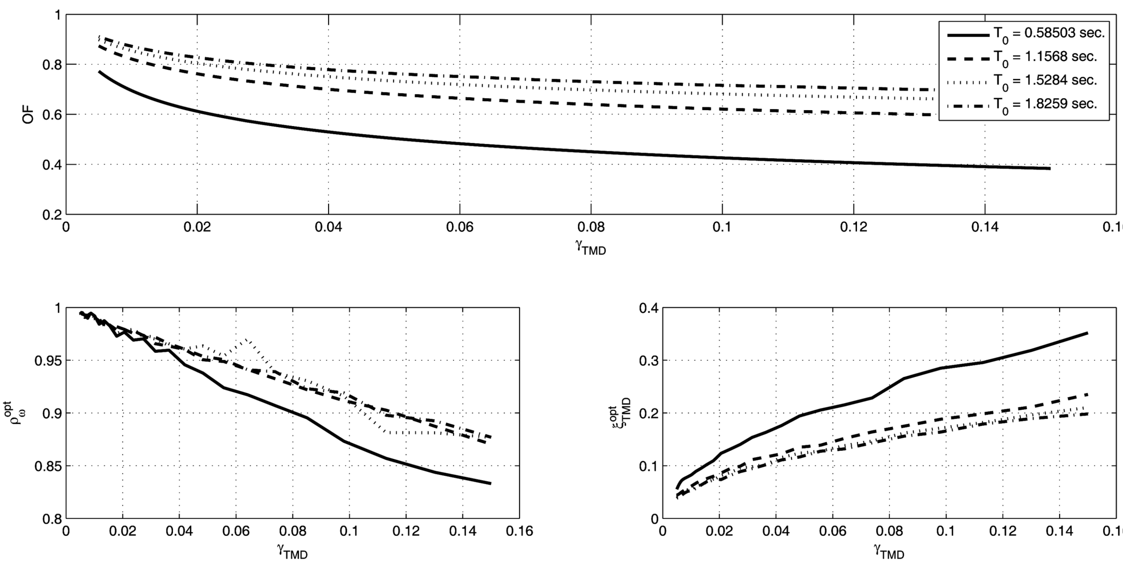

Fig. 14 O.F.$_a$ and DV optimum solutions versus TMD mass ratio. The analysis has been carried out for 4 different main system natural periods T_0. The filter parameters used are referred to a stiff soil type condition

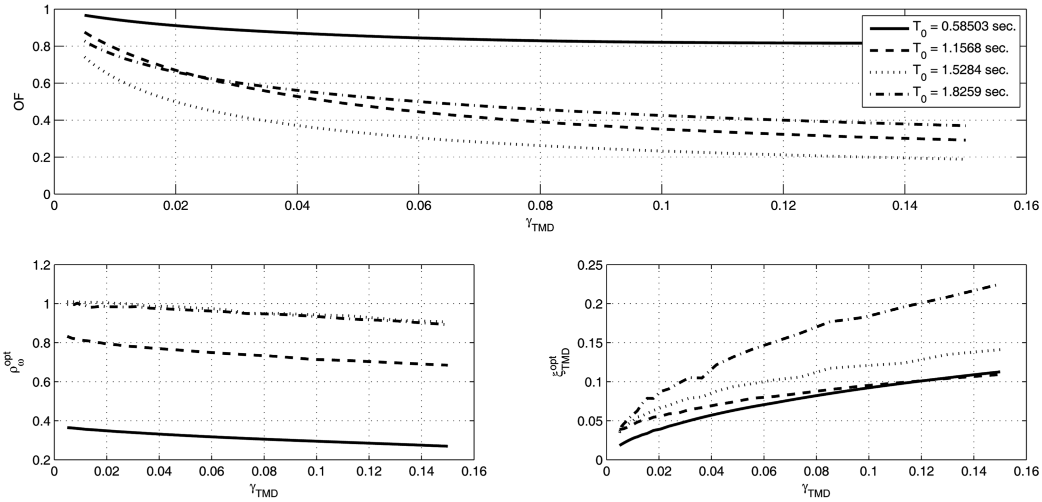

Fig. 15 O.F.$_d$ and DV optimum solutions versus TMD mass ratio. The analysis has been carried out for 4 different main system natural periods T_0. The filter parameters used are referred to a soft soil type condition

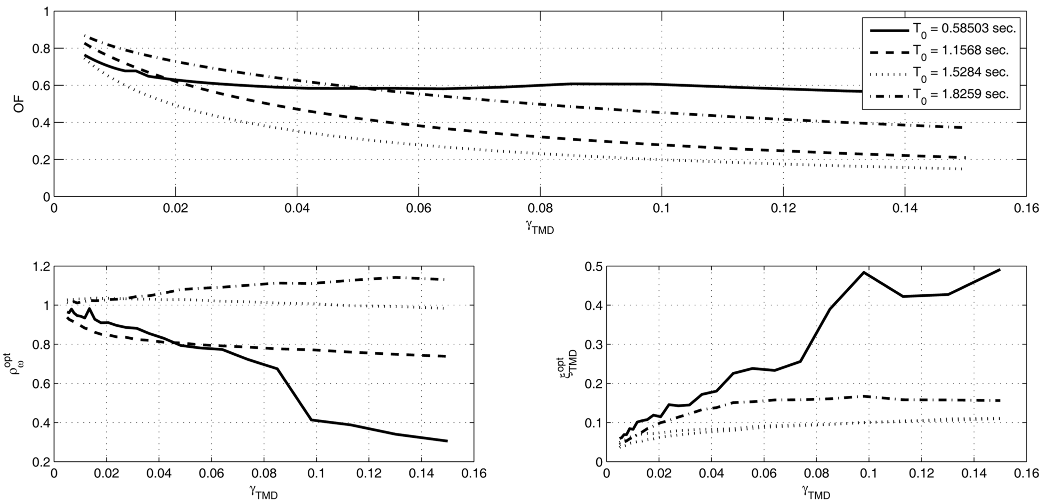

Fig. 16 O.F.$_a$ and DV optimum solutions versus TMD mass ratio. The analysis has been carried out for 4 different main system natural periods T_0. The filter parameters used are referred to a soft soil type condition

adopting a high mass ratio.

Figs. 15 and 16 correspond to soft soil conditions. With the displacement based criterion, the solutions differ both in terms of performance and optimal DV parameters. A low performance solution is related to a structure with a natural period of 0.58 sec. The optimized O.F. tends to decrease, increasing the structural performance, when the main structural period increases.

The DV optimal parameters (i.e., ρ_{ω}^{opt} and ξ_{TMD}^{opt}) also show dissimilar solutions for various system natural periods. The TMD is tuned to the frequency of the main structure only in the case of a structure with a higher period. With more rigid structures, it isn't possible to tune the TMD frequency to the main frequency of the structure, thus causing a TMD strategy failure.

With this ground configuration, the spectral contents are localised almost around the main frequency. This configuration can be considered to be a narrow band if compared to the stiff one. If the structural period is closer to the main ground, optimal O.F. and DV solutions are different from those obtained in the case of greater or smaller structural periods quite distant from the ground.

On the contrary, the results on stiff soil change as the structure varies, but remain close to those obtained from soft soil conditions.

With the acceleration criterion it isn't possible to obtain a uniform tendency of the optimum solution for both structures.

5. Conclusions

This paper presents a full stochastic approach for the optimum design of tuned mass dampers used in tall towers which may be subjected to earthquakes. The optimization concerns the identification of tuned mass damper mechanical parameters which maximize performance in mitigating the vibration level of the protected structure. Two different criteria have been developed which consider the reduction of displacement and acceleration as objective functions. Several numerical analyses have been carried out to assess the sensitivity of the optimal solution confronted with structure and ground motion characterization.

The results obtained show that the optimized TMD can be very effective in reducing the vibration level in the primary structure, but its effectiveness depends on the relation between ground motion characteristics and structural parameters. The optimized TMD works differently in reducing displacement and acceleration with relation to ground motion frequency content and structural parameters. Results show that acceleration is controlled better than displacement on soft soil, whereas an inverse tendency can be noted on stiff soil. These outcomes have been explained by analysing the responses of the optimized system in the frequency space.

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Appendix

$$
[\mathbf{K}^{x\gamma}] = \begin{bmatrix}
-\dfrac{6EJ}{h_1^2} & -\dfrac{6EJ}{h_1^2} & 0 & & & & & & \\
\dfrac{6EJ}{h_1^2} & \dfrac{6EJ}{h_1^2}-\dfrac{6EJ}{h_2^2} & -\dfrac{6EJ}{h_2^2} & 0 & & & & & \\
0 & \dfrac{6EJ}{h_2^2} & \ddots & & 0 & & & & \\
 & 0 & & \ddots & \ddots & 0 & & & \\
 & & 0 & \ddots & \ddots & \ddots & 0 & & \\
 & & & 0 & \ddots & \ddots & \ddots & 0 & \\
 & & & & 0 & \ddots & \ddots & -\dfrac{6EJ}{h_{n-2}^2} & 0 \\
 & & & & & 0 & \dfrac{6EJ}{h_{n-2}^2} & \dfrac{6EJ}{h_{n-2}^2}-\dfrac{6EJ}{h_{n-1}^2} & -\dfrac{6EJ}{h_{n-1}^2} \\
 & & & & & & 0 & \dfrac{6EJ}{h_{n-1}^2} & \dfrac{6EJ}{h_{n-1}^2}-\dfrac{6EJ}{h_n^2}
\end{bmatrix}
$$

$$
[\mathbf{K}^{\gamma\gamma}] = \begin{bmatrix}
\dfrac{4EJ}{h_1} & -\dfrac{2EJ}{h_1} & 0 & & & & & & \\
-\dfrac{2EJ}{h_1} & \dfrac{4EJ}{h_1}+\dfrac{4EJ}{h_2} & -\dfrac{2EJ}{h_2} & 0 & & & & & \\
0 & -\dfrac{2EJ}{h_2} & \ddots & & 0 & & & & \\
 & 0 & & \ddots & \ddots & 0 & & & \\
 & & 0 & \ddots & \ddots & \ddots & 0 & & \\
 & & & 0 & \ddots & \ddots & \ddots & 0 & \\
 & & & & 0 & \ddots & \ddots & -\dfrac{2EJ}{h_{n-2}} & 0 \\
 & & & & & 0 & -\dfrac{2EJ}{h_{n-2}} & \dfrac{4EJ}{h_{n-2}}+\dfrac{4EJ}{h_{n-1}} & -\dfrac{2EJ}{h_{n-1}} \\
 & & & & & & 0 & -\dfrac{2EJ}{h_{n-1}} & \dfrac{4EJ}{h_{n-1}}+\dfrac{4EJ}{h_n}
\end{bmatrix}
$$

$$[\mathbf{K}^{xx}] = \begin{bmatrix} \frac{12EJ}{h_1^3} & -\frac{12EJ}{h_1^3} & 0 & & & & & & \\ -\frac{12EJ}{h_1^3} & \frac{12EJ}{h_1^3}+\frac{12EJ}{h_2^3} & 0 & & & & & & \\ 0 & -\frac{12EJ}{h_2^3} & \ddots & 0 & & & & & \\ & 0 & \ddots & \ddots & 0 & & & & \\ & & 0 & \ddots & \ddots & \ddots & 0 & & \\ & & & 0 & \ddots & \ddots & \ddots & 0 & \\ & & & & 0 & \ddots & \ddots & -\frac{12EJ}{h_{n-2}^3} & 0 \\ & & & & & 0 & -\frac{12EJ}{h_{n-2}^3} & \frac{12EJ}{h_{n-2}^3}+\frac{12EJ}{h_{n-1}^3} & -\frac{12EJ}{h_{n-1}^3} \\ & & & & & & 0 & -\frac{12EJ}{h_{n-1}^3} & \frac{12EJ}{h_{n-1}^3}+\frac{12EJ}{h_n^3} \end{bmatrix}$$

Finally:

$$J = \frac{\pi}{4}(R_e^4 - R_i^4)$$

in which R_e and R_i are respectively the external and internal radius of the section.