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Estimation of active multiple tuned mass dampers for asymmetric structures

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Abstract. This paper proposes the application of active multiple tuned mass dampers (AMTMD) for translational and torsional response control of a simplified two-degree-of-freedom (2DOF) structure, able to represent the dynamic characteristics of general asymmetric structures, under the ground acceleration. This 2DOF structure is a generalized 2DOF system of an asymmetric structure with predominant translational and torsional responses under earthquake excitations using the mode reduced-order method. Depending on the ratio of the torsional to the translational eigenfrequency, i.e. the torsional to translational frequency ratio (TTFR), of asymmetric structures, the following three cases can be distinguished: (1) torsionally flexible structures (TTFR < 1.0), (2) torsionally intermediate stiff structures (TTFR = 1.0), and (3) torsionally stiff structures (TTFR > 1.0). The even distribution of the AMTMD within the whole width and half width of the asymmetric structure, thus leading to three cases of installing the AMTMD (referred to as the AMTMD of case 1, AMTMD of case 2, AMTMD of case 3, respectively), is taken into account. In the present study, the criterion for searching the optimum parameters of the AMTMD is defined as the minimization of the minimum values of the maximum translational and torsional displacement dynamic magnification factors (DMF) of an asymmetric structure with the AMTMD. The criterion used for assessing the effectiveness of the AMTMD is selected as the ratio of the minimization of the minimum values of the maximum translational and torsional displacement DMF of the asymmetric structure with the AMTMD to the maximum translational and torsional displacement DMF of the asymmetric structure without the AMTMD. By resorting to these two criteria, a careful examination of the effects of the normalized eccentricity ratio (NER) on the effectiveness and robustness of the AMTMD are carried out in the mitigation of both the translational and torsional responses of the asymmetric structure. Likewise, the effectiveness of a single ATMD with the optimum positions is presented and compared with that of the AMTMD.

Keywords: vibration control; damping; active multiple tuned mass dampers (AMTMD); asymmetric structures; ground acceleration; torsional to translational frequency ratio (TTFR); normalized eccentricity ratio (NER).

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1. Introduction

It is well known that the main disadvantage of the TMD is its sensitivity to the fluctuation in the tuning the natural frequency of the TMD to the controlled natural frequency of the structure and/or that in the damping of the TMD. The mistuning or off-optimum damping will reduce the effectiveness of the TMD significantly. Employing more than one TMD, with different dynamic characteristics, has then been proposed to improve the effectiveness. The multiple tuned mass dampers (MTMD), with distributed natural frequencies, were proposed by Xu and Igusa (1992) and also studied by several researchers, such as Yamaguchi and Harnpornchai (1993), Abe and Fujino (1994), Abe and Igusa (1995), Kareem and Kline (1995), Jangid (1999), and Li (2000). It is shown that the MTMD is more effective in the mitigation of the oscillations of structures than a single TMD. On the other hand, the effectiveness of the TMD can also be further enhanced through introducing an active force to act between the structure and the TMD (thus comprising the active tuned mass damper, ATMD) (Chang and Soong 1980). However, the robustness (against the change or the estimation error in the structural natural frequency) of the ATMD is not to be compared with that of the MTMD. Studies in optimizing the feedback gains and damper characteristics of an ATMD in order to minimize the structural displacements and/or accelerations have nowadays been carried out by several researchers, for example, Chang and Yang (1995), Ankireddi and Yang (1996), Yan *et al*. (1999). However, as a building gets taller and more massive, in order to achieve the required level of response reduction during strong earthquake, a heavier additional mass (such as the TMD or MTMD) which is anticipated to suffer from a larger stroke, is required. This heavier additional mass will need the extraordinarily large space and thus, its use becomes economically impractical. If the active control system is taken into account, then a large control force must be created and the power limitation of actuator prevents this system from being implemented in actual buildings. Evidently, it is imperative and of practical interest to search for the control systems, which can relax the requirements for masses and/or control forces. In view of this, the active multiple tuned mass dampers (AMTMD) have been proposed by Li and Liu (2002) to attenuate undesirable oscillations of structures under the ground acceleration. In the studies on the AMTMD, it is assumed that a structure vibrates in only one direction or in multiple directions independently with its fundamental modal properties to design the AMTMD. This assumption simplifies the analysis of a system and the synthesis of a controller. In real structures, however, this assumption is not always appropriate because structures generally possess multidirectional coupled vibration modes and the control performance of controllers will degrade due to parameter variation or spillover induced by the effect of coupling. Furthermore, there exist not only transverse vibrations but also torsional vibrations in real structures and they generally possess coupling; that is, a real structure is actually asymmetric to some degree even with a nominally symmetric plan and will undergo lateral as well as torsional vibrations simultaneously under purely translational excitations. Consequently, the controllers have to be designed by taking into account the effect of transverse–torsional coupled vibration modes in such cases.

Representative studies of designing the TMD, ATMD, and MTMD through taking into account the effect of transverse-torsional coupled vibration modes are the following: Jangid and Datta (1997), Lin *et al*. (1999, 2000), Pansare and Jangid (2003), Arfiadi and Hadi (2000), Singh *et al.* (2002), Ahlawat and Ramaswamy (2003), Wang and Lin (2005), and Li and Qu (2006). With a view to practical applications of the AMTMD, it is imperative and of practical interest to include the effects of torsional coupling into consideration in estimating the performance of the AMTMD. It is well known that structures characterized by non-coincident center of mass and center of stiffness will develop a coupled lateral-torsional response when subjected to earthquake ground motions. For conceptual design purposes, the idealized two-degree-of-freedom (2DOF) system can be considered as a generalized 2DOF system of an asymmetric structure with predominant translational and torsional response under earthquake excitations. In such a case, rather than using the real mass, the generalized translational and torsional masses will be introduced in all derivations below. A careful examination of the governing equations of motion of such a generalized 2DOF system with the AMTMD will shed insight into the effects of the coupled lateral–torsional dynamic behavior of asymmetric structures on the effectiveness and robustness of the AMTMD. Consequently, the present work on the AMTMD has mainly attempted to control only the translational and torsional responses of such a generalized 2DOF system; notwithstanding this, this concept can easily be extended to multistory asymmetric structures using the mode reduced-order method. In the current study, the even distribution of the AMTMD within the whole width and half width of the asymmetric structure, thus leading to three cases of installing the AMTMD (referred to as the AMTMD of case 1, AMTMD of case 2, AMTMD of case 3, respectively), is taken into account. The effectiveness and robustness of the AMTMD will be investigated and demonstrated for three cases of the uncoupled torsional to translational frequency ratio (TTFR), namely TTFR = 0.5, 1, 2. Simultaneously, the effectiveness of the ATMD with the optimum layouts is presented and compared with that of the AMTMD.

2. Damping determination of asymmetric structures (Li and Qu 2006)

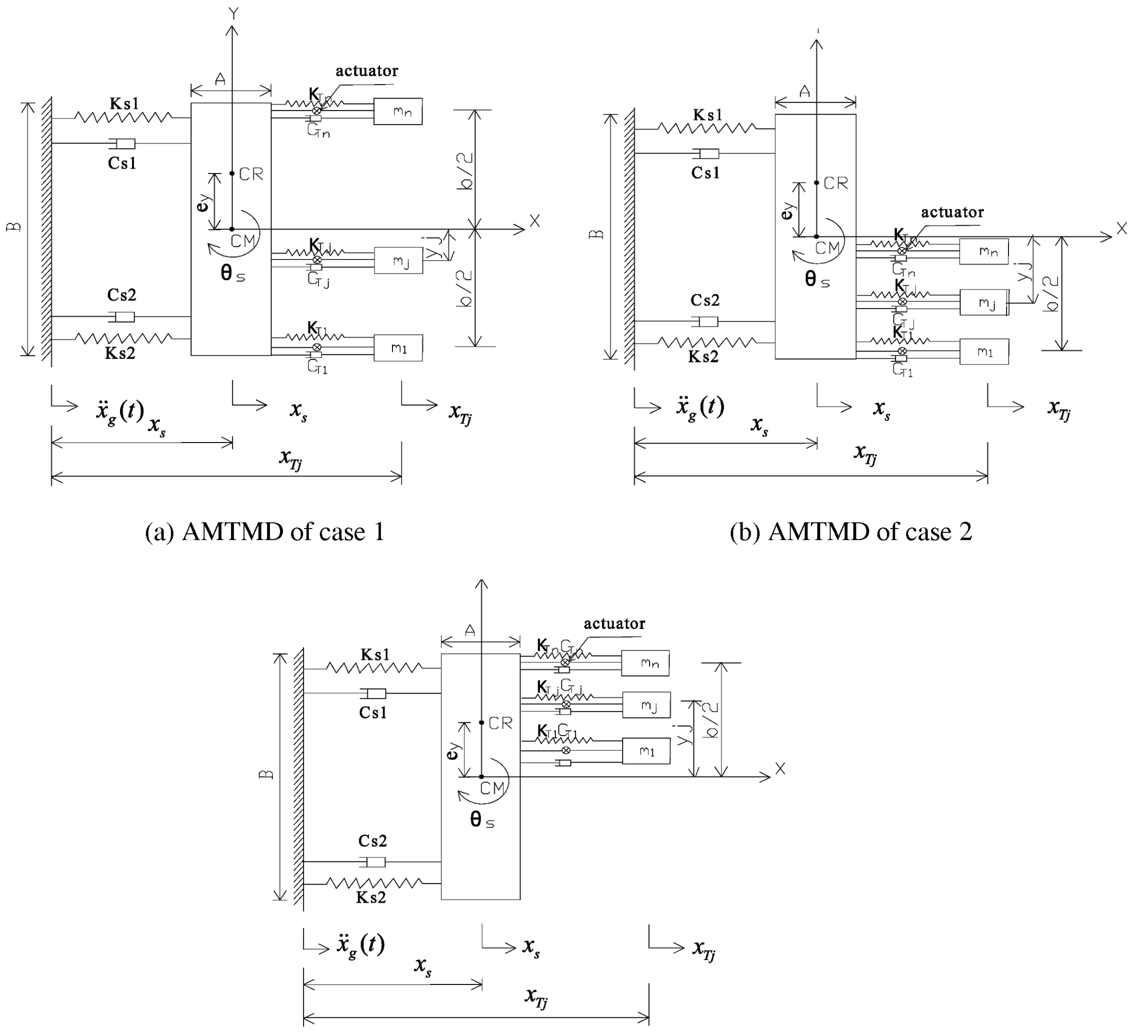
The structure to be controlled with an AMTMD is the asymmetric structure [i.e., the center of resistance (CR) of the structure does not coincide with the center of mass (CM)], as shown in Fig. 1. Fig. 1 provides three distributions of the AMTMD, referred to as the AMTMD of case 1, AMTMD of case 2, and AMTMD of case 3, respectively. The two uncoupled frequencies of the asymmetric structure are defined as below:

$$\omega_s = \sqrt{\frac{k_s}{m_s}} \tag{1}$$

$$\omega_\theta = \sqrt{\frac{k_\theta}{m_s r^2}} \tag{2}$$

in which m_s is the mode-generalized mass of the structure; $k_s = k_{s1} + k_{s2}$ is the mode-generalized lateral stiffness of the structure in the x direction, where k_{s1} and k_{s2} are the stiffness of two resisting elements, respectively; $k_\theta = k_{s1}y_{s1}^2 + k_{s2}y_{s2}^2$ is the mode-generalized torsional stiffness of the structure with respect to the CM, where y_{s1} and y_{s2} are the distances from the CM to the two resisting elements, respectively; r represents the radius of gyration of the deck about the vertical axis through the CM. The equations of motion of the asymmetric structure can be written in the following matrix form

$$\begin{bmatrix} m_s & 0 \\ 0 & m_s r^2 \end{bmatrix}\begin{bmatrix} \ddot{x}_s \\ \ddot{\theta}_s \end{bmatrix} + \begin{bmatrix} c_s & c_{s\theta} \\ c_{s\theta} & c_\theta \end{bmatrix}\begin{bmatrix} \dot{x}_s \\ \dot{\theta}_s \end{bmatrix} + \begin{bmatrix} k_s & k_s e_y \\ k_s e_y & k_\theta \end{bmatrix}\begin{bmatrix} x_s \\ \theta_s \end{bmatrix} = -\begin{bmatrix} m_s \\ 0 \end{bmatrix}\ddot{x}_g(t) \tag{3}$$



(a) AMTMD of case 1

(b) AMTMD of case 2

(c) AMTMD of case 3

Fig. 1 Generalized two-degree-of-freedom (2DOF) system of an asymmetric structure with predominant translational and torsional response set with the active multiple tuned mass dampers (AMTMD), referred in terms of different distributions as to the AMTMD of case 1, AMTMD of case 2, and AMTMD of case 3

in which c_s, $c_{s\theta}$ and c_θ denote the elements of the damping matrix to be determined next; $e_y = (k_{s1}y_{s1} - k_{s2}y_{s2})/(k_{s1} + k_{s2})$ represents the eccentricity between the CR and CM; and $\ddot{x}_g(t)$ is the ground acceleration.

Denoting the fundamental and second natural frequencies with ω_{s1} and $\omega_{s2}(\omega_{s2} > \omega_{s1})$, respectively, they can be derived through solving the eigenvalue problem associated with Eq. (3) as follows

$$\frac{\omega_{s1}}{\omega_s} = \sqrt{\frac{1 + \lambda_\omega^2 - \sqrt{(\lambda_\omega^2 - 1)^2 + 4E_R^2}}{2}} \tag{4}$$

$$\frac{\omega_{s2}}{\omega_s} = \sqrt{\frac{1 + \lambda_\omega^2 + \sqrt{(\lambda_\omega^2 - 1)^2 + 4E_R^2}}{2}} \tag{5}$$

in which $E_R = e_y/r$ represents the ratio of the eccentricity to the radius of gyration of the deck, referred to as the normalized eccentricity ratio, *NER*; $\lambda_\omega = \omega_\theta/\omega_s$ denotes the uncoupled torsional to translational frequency ratio, *TTFR*.

With the hypothesis of the same damping ratio $\xi_{s1} = \xi_{s2} = \xi_s$ (letting $\xi_s = 0.02$ in this study) for the two modes and superposing the modal damping matrices, the damping matrix can be expressed in the following form

$$\begin{bmatrix} c_s & c_{s\theta} \\ c_{s\theta} & c_\theta \end{bmatrix} = a_0 \begin{bmatrix} m_s & 0 \\ 0 & m_s r^2 \end{bmatrix} + b_0 \begin{bmatrix} k_s & k_s e_y \\ k_s e_y & k_\theta \end{bmatrix} \tag{6}$$

$$a_0 = \left[\frac{2(\xi_{s2}\omega_{s1} - \xi_{s1}\omega_{s2})}{\omega_{s1}^2 - \omega_{s2}^2}\right] \times \omega_{s1}\omega_{s2} \tag{7}$$

$$b_0 = \frac{2(\xi_{s1}\omega_{s1} - \xi_{s2}\omega_{s2})}{\omega_{s1}^2 - \omega_{s2}^2} \tag{8}$$

Rearranging the above Eq. (6) yields the each elements of the damping matrix as shown below

$$c_s = 2a_s m_s \xi_s \omega_s \tag{9}$$

$$c_{s\theta} = 2a_{s\theta} m_s r \xi_s \omega_s \tag{10}$$

$$c_\theta = 2a_\theta m_s r^2 \xi_s \omega_s \tag{11}$$

in which

$$a_s = \frac{\left[\frac{\omega_{s1}}{\omega_s} \times \frac{\omega_{s2}}{\omega_s}\right] + 1}{\left[\frac{\omega_{s1}}{\omega_s} + \frac{\omega_{s2}}{\omega_s}\right]} \tag{12}$$

$$a_{s\theta} = \frac{E_R}{\left[\frac{\omega_{s1}}{\omega_s} + \frac{\omega_{s2}}{\omega_s}\right]} \tag{13}$$

$$a_\theta = \frac{\left[\frac{\omega_{s1}}{\omega_s} \times \frac{\omega_{s2}}{\omega_s}\right] + \lambda_\omega^2}{\left[\frac{\omega_{s1}}{\omega_s} + \frac{\omega_{s2}}{\omega_s}\right]} \tag{14}$$

3. State equations of the asymmetric structure with the AMTMD

An AMTMD with different dynamic characteristics, which is evenly placed within the width b

with the center at the CM and within the width $[b/2]$ with the centers, respectively, at $y_{(n+1)/2} = [\mp b/4]$ is utilized here for reducing both the translational and torsional responses of the asymmetric structure, as shown in Figs. 1(a), (b), (c). For the three cases above, the ordinate of each ATMD in the AMTMD can be, respectively, determined by

Each ATMD in the AMTMD of case 1

$$y_j = \left[-\frac{1}{2} + \frac{j-1}{n-1}\right] \times b \qquad (j = 1, 2, \ldots, n) \tag{15}$$

Each ATMD in the AMTMD of case 2

$$y_j = \left[-1 + \frac{j-1}{n-1}\right] \times \frac{b}{2} \qquad (j = 1, 2, \ldots, n) \tag{16}$$

Each ATMD in the AMTMD of case 3

$$y_j = \left[\frac{j-1}{n-1}\right] \times \frac{b}{2} \qquad (j = 1, 2, \ldots, n) \tag{17}$$

When the relative displacements of both the structure (x_s) and each ATMD (x_{Tj}) with reference to the ground are introduced, the equations of motion for the asymmetric structure with the AMTMD under ground acceleration can be formulated as follows

$$m_s\ddot{x}_s + c_s\dot{x}_s + k_s x_s + c_{s\theta}\dot{\theta}_s + k_s e_y \theta_s = -m_s\ddot{x}_g(t) + \sum_{j=1}^{n} F_j(t) \tag{18}$$

$$m_s r^2 \ddot{\theta}_s + c_\theta\dot{\theta}_s + k_\theta\theta_s + c_{s\theta}\dot{x}_s + k_s e_y x_s = \sum_{j=1}^{n} y_j F_j(t) \tag{19}$$

$$m_{Tj}[\ddot{x}_g(t) + \ddot{x}_{Tj}] + c_{Tj}[\dot{x}_{Tj} - (\dot{x}_s + y_j\dot{\theta}_s)] + k_{Tj}[x_{Tj} - (x_s + y_j\theta_s)] = u_j(t) \qquad (j = 1, 2, \ldots, n) \tag{20}$$

$$F_j(t) = c_{Tj}[\dot{x}_{Tj} - (\dot{x}_s + y_j\dot{\theta}_s)] + k_{Tj}[x_{Tj} - (x_s + y_j\theta_s)] - u_j(t) \qquad (j = 1, 2, \ldots, n) \tag{21}$$

$$u_j(t) = -m_{tj}\ddot{x}_s - c_{tj}[\dot{x}_{Tj} - (\dot{x}_s + y_j\dot{\theta}_s)] + k_{tj}[x_{Tj} - (x_s + y_j\theta_s)] \qquad (j = 1, 2, \ldots, n) \tag{22}$$

in which m_{Tj}, c_{Tj}, and k_{Tj} are the mass, damping, and stiffness of the jth ATMD in the AMTMD, respectively; m_{tj}, c_{tj}, and k_{tj} gains of the acceleration, velocity, and displacement feedback for the jth ATMD in the AMTMD.

In the present paper, the MTMD in the AMTMD is manufactured by keeping the identical stiffness and damping (i.e., $k_{T1} = k_{T2} = \ldots = k_{Tn} = k_T$ and $c_{T1} = c_{T2} = \ldots = c_{Tn} = c_T$) and varying the mass (i.e., $m_{T1} \neq m_{T2} \neq \ldots \neq m_{Tn}$). Likewise, the control forces of the AMTMD are generated through keeping the identical displacement and velocity feedback gains (i.e., $k_{t1} = k_{t2} = \ldots = k_{tn} = k_t$ and $c_{t1} = c_{t2} = \ldots = c_{tn} = c_t$) and varying the acceleration feedback gain (i.e., $m_{t1} \neq m_{t2} \neq \ldots \neq m_{tn}$).

Let us now introduce the following notations

$$\omega_j^2 = \frac{k_{Tj} + k_{tj}}{m_{Tj}} = \frac{k_T + k_t}{m_{Tj}}; \quad \mu_T = \frac{m_T}{m_s} = \sum_{j=1}^{n} \frac{m_{Tj}}{m_s} = \sum_{j=1}^{n} \mu_{Tj}$$

$$\xi_j = \frac{c_{Tj} + c_{tj}}{2m_{Tj}\omega_j} = \frac{c_T + c_t}{2m_{Tj}\omega_j}; \quad \alpha = \frac{m_{tj}}{m_{Tj}} \text{ (referred to as the NAFGF)}$$

Defining the average natural frequency of the AMTMD $\omega_T = \sum_{j=1}^{n} \frac{\omega_j}{n}$, the natural frequency of each ATMD in the AMTMD can then be calculated from

$$\omega_j = \omega_T\left[1 + \left[j - \frac{n+1}{2}\right]\frac{\beta}{n-1}\right] = f\omega_s\left[1 + \left[j - \frac{n+1}{2}\right]\frac{\beta}{n-1}\right] \quad (j = 1, 2, \ldots, n) \tag{23}$$

where the nondimensional parameter $\beta = (\omega_n - \omega_1)/\omega_T$ is defined to be the frequency spacing, used for measuring the robustness, of the AMTMD.

Making use of Eq. (23), the ratio of the natural frequency of each ATMD in the AMTMD to the controlled natural frequency of the structure can then be written explicitly as follows

$$r_j = \frac{\omega_j}{\omega_s} = f\left[1 + \left[j - \frac{n+1}{2}\right]\frac{\beta}{n-1}\right] \quad (j = 1, 2, \ldots, n) \tag{24}$$

in which $f = \omega_T/\omega_s$ is defined to be the tuning frequency ratio of the AMTMD.

Taking into consideration the fact that the damping ratio of each ATMD in the AMTMD is unequal, it is needed to introduce the average damping ratio $\xi_T = \sum_{j=1}^{n} \frac{\xi_j}{n}$. Employing the above definitions and deduced expressions, the total mass ratio and the damping ratio of each ATMD in the AMTMD can be, respectively, calculated by (Chunxiang 2000, Li and Qu 2006)

$$\mu_T = \sum_{j=1}^{n} \mu_{Tj} = \left[\sum_{j=1}^{n} 1/r_j^2\right]\mu_{T1}r_1^2 = \left[\sum_{j=1}^{n} 1/r_j^2\right]\mu_{T2}r_2^2 = \ldots = \left[\sum_{j=1}^{n} 1/r_j^2\right]\mu_{Tn}r_n^2 \tag{25}$$

$$\xi_T = \sum_{j=1}^{n} \frac{\xi_j}{n} = \xi_1 r_1^{-1} f = \xi_2 r_2^{-1} f = \ldots = \xi_n r_n^{-1} f \tag{26}$$

For the purpose of deducing the state equations of the asymmetric structure with the AMTMD, Eqs. (18)-(22) are rewritten in the following matrix form:

$$M\ddot{x} + C\dot{x} + Kx = \Gamma \ddot{x}_g(t) \tag{27}$$

in which

$$x = [x_s \;\; r\theta_s \;\; x_{T1} \;\; \ldots \;\; x_{Tj} \;\; \ldots \;\; x_{Tn}]^T \tag{28}$$

$$\dot{x} = [\dot{x}_s \;\; r\dot{\theta}_s \;\; \dot{x}_{T1} \;\; \ldots \;\; \dot{x}_{Tj} \;\; \ldots \;\; \dot{x}_{Tn}]^T \tag{29}$$

$$\ddot{x} = [\ddot{x}_s \;\; r\ddot{\theta}_s \;\; \ddot{x}_{T1} \;\; \ldots \;\; \ddot{x}_{Tj} \;\; \ldots \;\; \ddot{x}_{Tn}]^T \tag{30}$$

$$\Gamma = [-1 \;\; 0 \;\; -\mu_{T1} \;\; \ldots \;\; -\mu_{Tj} \;\; \ldots \;\; -\mu_{Tn}]^T \tag{31}$$

$$M = \begin{bmatrix} A_1 & 0 & 0_{1\times n} \\ B_1 & B_2 & 0_{1\times n} \\ (C_1)_{n\times 1} & (C_2)_{n\times n} & 0_{n\times 1} \end{bmatrix} \tag{32}$$

$$C = \begin{bmatrix} A_2 & A_3 & (A_4)_{1\times n} \\ B_3 & B_4 & (B_5)_{1\times n} \\ (C_3)_{n\times 1} & (C_4)_{n\times 1} & (C_5)_{n\times n} \end{bmatrix} \tag{33}$$

$$K = \begin{bmatrix} A_5 & A_6 & (A_7)_{1\times n} \\ B_6 & B_7 & (B_8)_{1\times n} \\ (C_6)_{1\times n} & (C_7)_{n\times 1} & (C_8)_{n\times n} \end{bmatrix} \tag{34}$$

where

$$A_1 = 1 - \sum_{j=1}^{n} \alpha\mu_{Tj}$$

$$A_2 = 2\omega_s\left[\alpha_s\xi_s + \sum_{j=1}^{n}\mu_{Tj}\xi_j f\left[1 + \left[j - \frac{n+1}{2}\right]\frac{\beta}{n-1}\right]\right]$$

$$A_3 = 2\omega_s\left[a_{s\theta}\xi_s + \sum_{j=1}^{n}\mu_{Tj}\xi_j f\left[\frac{y_j}{r}\right]\left[1 + \left[j - \frac{n+1}{2}\right]\frac{\beta}{n-1}\right]\right]$$

$$A_4 = -2\omega_s\left\{\mu_{T1}\xi_1 f\left[1 - \frac{\beta}{2}\right] \ \ldots\ \mu_{Tj}\xi_j f\left[1 + \left[j - \frac{n+1}{2}\right]\frac{\beta}{n-1}\right] \ \ldots\ \mu_{Tn}\xi_n f\left[1 + \frac{\beta}{2}\right]\right\}$$

$$B_1 = -\sum_{j=1}^{n}\alpha\mu_{Tj}\left[\frac{y_j}{r}\right]$$

$$B_2 = 1$$

$$B_3 = A_3$$

$$B_4 = 2\omega_s\left[\alpha_\theta\xi_s + \sum_{j=1}^{n}\mu_{Tj}\xi_j f\left[\frac{y_j}{r}\right]^2\left[1 + \left[j - \frac{n+1}{2}\right]\frac{\beta}{n-1}\right]\right]$$

$$B_5 = -2\omega_s\left\{\mu_{T1}\xi_1 f\left[\frac{y_j}{r}\right]\left[1 - \frac{\beta}{2}\right] \ \ldots\ \mu_{Tj}\xi_j f\left[\frac{y_j}{r}\right]\left[1 + \left[j - \frac{n+1}{2}\right]\frac{\beta}{n-1}\right] \ \ldots\ \mu_{Tn}\xi_n f\left[\frac{y_j}{r}\right]\left[1 + \frac{\beta}{2}\right]\right\}$$

$$C_1 = [\alpha\mu_{T1} \ \ \ldots\ \ \alpha\mu_{Tj} \ \ \ldots\ \ \alpha\mu_{Tn}]^T$$

$$C_2 = \begin{bmatrix} \mu_{T1} & & & & \\ & \ddots & & & \\ & & \mu_{Tj} & & \\ & & & \ddots & \\ & & & & \mu_{Tn} \end{bmatrix}$$

$$C_3 = A_4^T$$

$$C_4 = B_5^T$$

$$C_5 = 2\omega_s \begin{bmatrix} \mu_{T1}\xi_1 f\left[1-\frac{\beta}{2}\right] & & & & \\ & \ddots & & & \\ & & \mu_{Tj}\xi_j f\left[1+\left[j-\frac{n+1}{2}\right]\frac{\beta}{n-1}\right] & & \\ & & & \ddots & \\ & & & & \mu_{Tn}\xi_n f\left[1+\frac{\beta}{2}\right] \end{bmatrix}$$

$$A_5 = \omega_s^2\left[1+\sum_{j=1}^{n}\mu_{Tj}f^2\left[1+\left[j-\frac{n+1}{2}\right]\frac{\beta}{n-1}\right]^2\right]$$

$$A_6 = \omega_s^2\left[E_R+\sum_{j=1}^{n}\mu_{Tj}f^2\left[\frac{y_j}{r}\right]\left[1+\left[j-\frac{n+1}{2}\right]\frac{\beta}{n-1}\right]^2\right]$$

$$A_7 = -\omega_s^2\left\{\mu_{T1}f^2\left[1-\frac{\beta}{2}\right]^2 \ \dots \ \mu_{Tj}f^2\left[1+\left[j-\frac{n+1}{2}\right]\frac{\beta}{n-1}\right]^2 \ \dots \ \mu_{Tn}f^2\left[1+\frac{\beta}{2}\right]^2\right\}$$

$$B_6 = A_6$$

$$B_7 = \omega_s^2\left[\lambda_\omega^2+\sum_{j=1}^{n}\mu_{Tj}f^2\left[\frac{y_j}{r}\right]^2\left[1+\left[j-\frac{n+1}{2}\right]\frac{\beta}{n-1}\right]^2\right]$$

$$B_8 = -\omega_s^2\left\{\mu_{T1}f^2\left[\frac{y_j}{r}\right]\left[1-\frac{\beta}{2}\right]^2 \ \dots \ \mu_{Tj}f^2\left[\frac{y_j}{r}\right]\left[1+\left[j-\frac{n+1}{2}\right]\frac{\beta}{n-1}\right]^2 \ \dots \ \mu_{Tn}f^2\left[\frac{y_j}{r}\right]\left[1+\frac{\beta}{2}\right]^2\right\}$$

$$C_6 = A_7^T$$

$$C_7 = B_8^T$$

$$C_8 = \omega_s^2 \begin{bmatrix} \mu_{T1}f^2\left[1-\frac{\beta}{2}\right]^2 & & & & \\ & \ddots & & & \\ & & \mu_{Tj}f^2\left[1+\left[j-\frac{n+1}{2}\right]\frac{\beta}{n-1}\right]^2 & & \\ & & & \ddots & \\ & & & & \mu_{Tn}f^2\left[1+\frac{\beta}{2}\right]^2 \end{bmatrix}$$

The aforementioned formulation Eq. (27) can be transformed into the following state equations

$$\dot{X} = AX + Bw \tag{35}$$

$$Y = CX + Dw \tag{36}$$

in which the $[2(n+2) \times 1]$ state vector X is defined as

$$X = \begin{bmatrix} x \\ \dot{x} \end{bmatrix} \tag{37}$$

The state matrix A, input matrix B, output matrix C, output vector Y, and input w are, respectively, given by

$$A = \begin{bmatrix} 0_{(n+2)\times(n+2)} & E_{(n+2)\times(n+2)} \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \tag{38}$$

$$B = \begin{bmatrix} 0_{(n+2)\times 1} \\ M^{-1}\Gamma \end{bmatrix} \tag{39}$$

$$C = \begin{bmatrix} 0_{(n+2)\times(n+2)} & 0_{(n+2)\times(n+2)} \\ 0_{(n+2)\times(n+2)} & E_{(n+2)\times(n+2)} \end{bmatrix} \tag{40}$$

$$D = \begin{bmatrix} 0_{(n+2)\times 1} \\ 0_{(n+2)\times 1} \end{bmatrix} \tag{41}$$

$$Y = \begin{bmatrix} 0_{(n+2)\times 1} \\ x_{(n+2)\times 1} \end{bmatrix} \tag{42}$$

$$w = \ddot{x}_g(t) \tag{43}$$

4. Evaluation criteria of the AMTMD

With the hypothesis of $\ddot{x}_g(t) = X_g e^{-i\omega t}$, the translational and torsional displacements of the asymmetric structure-AMTMD can then be written as

$$x_s = [H_{x_s}(-i\omega)]e^{-i\omega t} \tag{44}$$

$$x_{Tj} = [H_{Tj}(-i\omega)]e^{-i\omega t} \qquad (j = 1, 2, \ldots, n) \tag{45}$$

$$r\theta_s = [H_{\theta_s}(-i\omega)]e^{-i\omega t} \tag{46}$$

Now, the dynamic magnification factors (DMF) of both translational and torsional displacements of the asymmetric structure with the AMTMD can then be calculated from

$$DMF_s = \left|\omega_s^2[H_{x_s}(-i\omega)]\right| \tag{47}$$

$$DMF_\theta = \left|\omega_s^2[H_{\theta_s}(-i\omega)]\right| \tag{48}$$

Finally, the assessment can be performed on the optimum parameters and effectiveness of the AMTMD for asymmetric structures through the implementation of the following particular criteria

$$R_I = min.min.max.DMF_s \tag{49}$$

$$R_{II} = min.min.max.DMF_\theta \tag{50}$$

$$R_{III} = \frac{min.min.max.DMF_s}{max.DMF_s^*} \tag{51}$$

$$R_{IV} = \frac{min.min.max.DMF_\theta}{max.DMF_\theta^*} \tag{52}$$

in which DMF_s^* and DMF_θ^* denote the dynamic magnification factors (DMF) of the asymmetric structure without the AMTMD, corresponding respectively to both the translational and torsional responses.

Eqs. (49) and (50), termed as the optimum criteria or objective functions, mean that the examination of the optimum parameters is conducted through the minimization of the minimum values of the maximum translational and torsional displacement dynamic magnification factors of asymmetric structures with the AMTMD. They can be explicitly explained in the following steps. First of all, for a fixed value of λ (set within the range from 0.4 to 3.4) and a fixed tuning frequency ratio, the maximum amplitudes for different average damping ratios and frequency spacings are found, and the minimum amplitudes are selected from the maximum amplitudes, which is the minimax amplitude for that tuning frequency ratio. Then the foregoing procedure is repeated for different tuning frequency ratios to find the minimax of each tuning frequency ratio. Finally, the smallest minimaxes are selected and the corresponding tuning frequency ratio, average damping ratio, and frequency spacing are optimum values. The aforementioned whole process is repeated for various total mass ratios and normalized acceleration feedback gain factors (NAFGF).

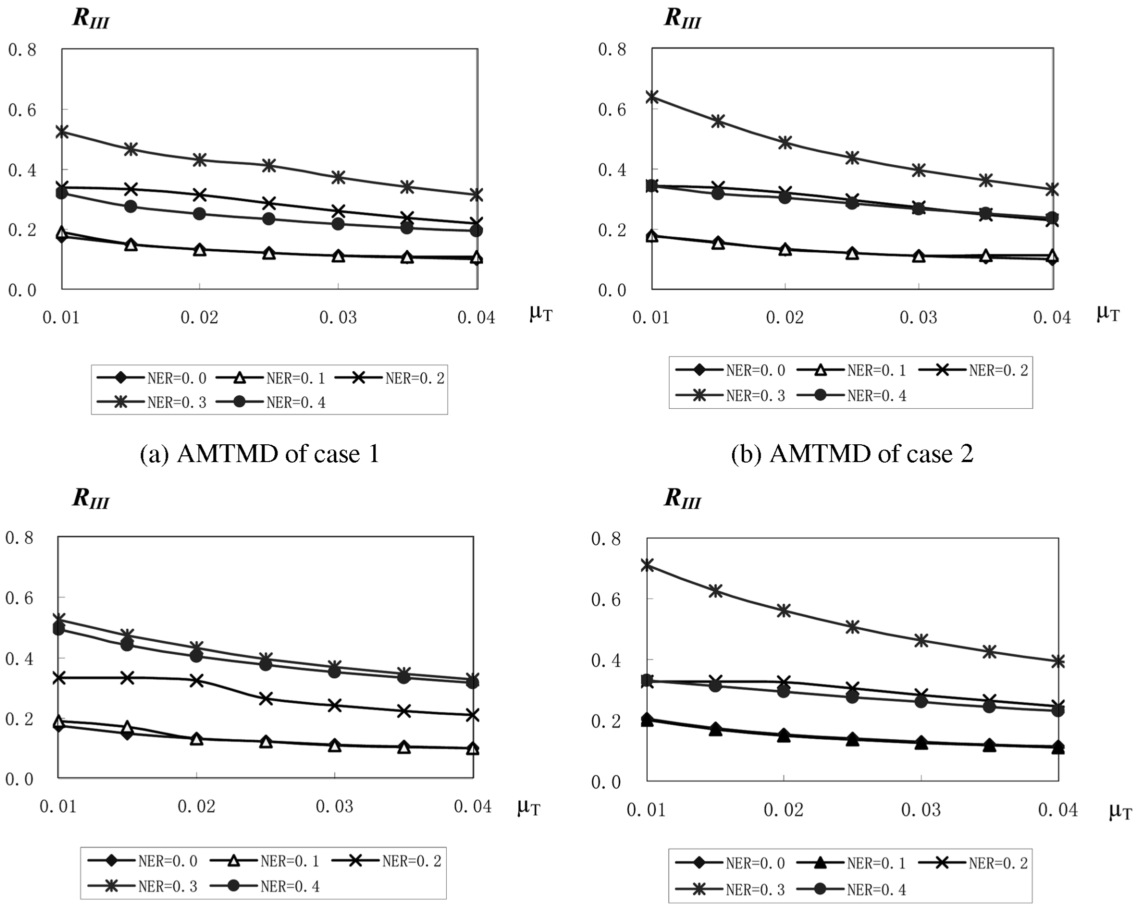
It is worth pointing here out that for the ATMD the obtained optimum parameters based on Eqs. (49) and (50) are, respectively, the optimum tuning frequency ratio, optimum damping ratio, and optimum position.

Eqs. (51) and (52), i.e., the minimization of the minimum values of the maximum translational and torsional displacement dynamic magnification factors, nondimensionalised respectively by the maximum translational and torsional displacement dynamic magnification factors of asymmetric structures without the AMTMD, called the effectiveness criteria, are used to quantitatively measure the effectiveness of the AMTMD in controlling the translational and torsional displacements of asymmetric structures, respectively.

5. Estimation of the AMTMD

Displayed in Figs. 2-15 are the numerical results of the present research, in which the normalized width (i.e., *b*/*r*) is set equal to 1.0. Simultaneously, the structural damping ratio and normalized acceleration feedback gain factor (NAFGF) are, respectively, set equal to 0.02 and 4. The degree of asymmetry (i.e., torsional coupling) of the structure depends upon the normalized eccentricity ratio (NER) and the torsional to translational frequency ratio (TTFR). In terms of the TTFR, asymmetric structures may be classified as the torsionally flexible structures (TTFR < 1.0), torsionally intermediate stiff structures (TTFR = 1.0), and torsionally stiff structures (TTFR > 1.0). The NER and TTFR thus are the key parameters of assessing the effectiveness and robustness of the AMTMD for asymmetric structures. The superscript opt denotes the optimum values of the frequency spacing of the AMTMD and the position of the ATMD.

Fig. 2 show the variation of the effectiveness of the AMTMD with $n = 5$ and ATMD with respect to the total mass ratio for reduction of translational response of asymmetric structures with



(a) AMTMD of case 1

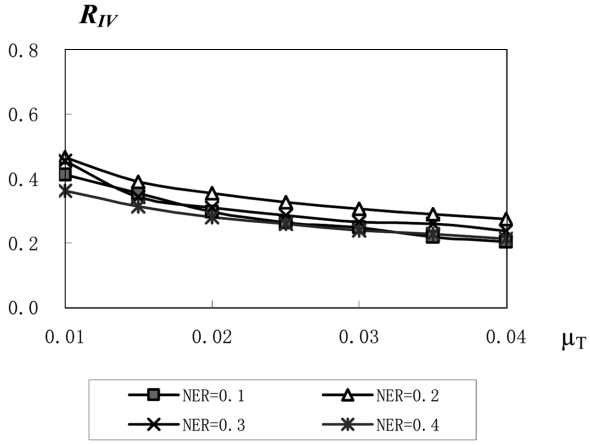
(b) AMTMD of case 2

(c) AMTMD of case 3

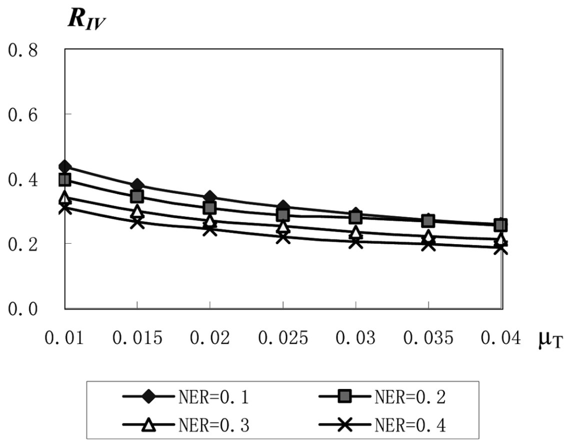
(d) ATMD

Fig. 2 Variation of the effectiveness of the AMTMD with total number equal to 5 and ATMD for reduction of the translational response with respect to total mass ratio in the case of TTFR = 0.5

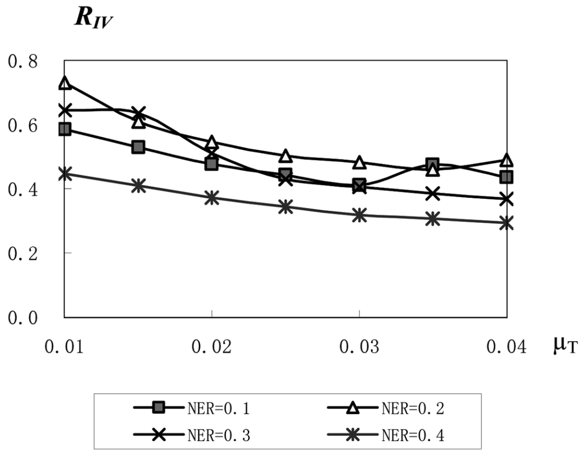
TTFR = 0.5, namely the torsional flexible structures. It is seen from Fig. 2 that with the exception of NER = 0.4, the effectiveness of both the AMTMD and ATMD reduce with the increase of the NER. However, the effectiveness of both the AMTMD and ATMD for NER = 0.4 is slightly better than that for NER = 0.2. This is because the torsional response has greater contribution to the translational response in asymmetric structures with excessively great NER. For smaller NER = 0.1, the effectiveness of both the AMTMD and ATMD for asymmetric structures is practically equal to that for symmetric structures (i.e., NER = 0). Likewise, in this case, the effectiveness of both the AMTMD and ATMD practically maintains constant for larger total mass ratio above 0.03, implying that increasing the total mass ratio can not further enhance the effectiveness of both the AMTMD and ATMD. More importantly, the AMTMD of case 1 is superior to the AMTMD of case 2, AMTMD of the case 3, and ATMD. Hence, the AMTMD of case 1 is preferably suitable for attenuating undesirable translational response of asymmetric structures with TTFR = 0.5.



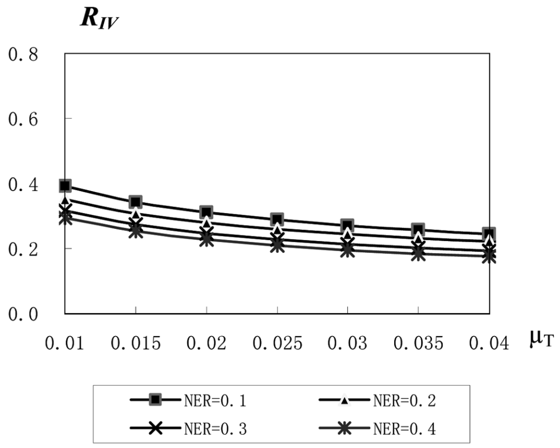
(a) AMTMD of case 1



(b) AMTMD of case 2



(c) AMTMD of case 3

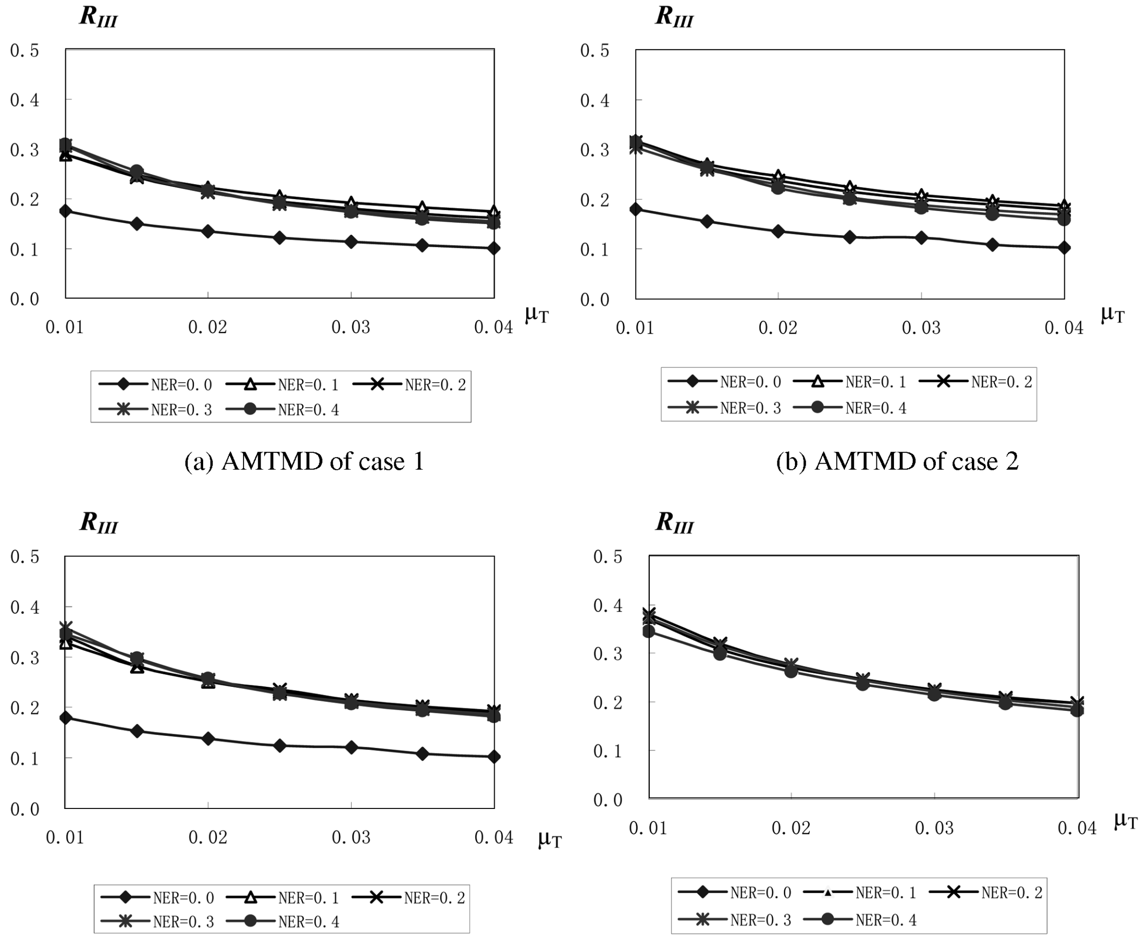


(d) ATMD

Fig. 3 Variation of the effectiveness of the AMTMD with total number equal to 5 and ATMD for reduction of the torsional response with respect to total mass ratio in the case of TTFR = 0.5

Fig. 3 presents the variation of the effectiveness of the AMTMD with $n = 5$ and ATMD relative to the total mass ratio for reduction of torsional response of asymmetric structures with TTFR = 0.5, namely the torsional flexible structures. In term of Fig. 3, the AMTMD of case 1, AMTMD of case 2, and ATMD should be preferably selected for reducing the torsional response of the torsionally flexible structures (TTFR = 0.5). Generally, the effectiveness of both the AMTMD and ATMD increase with an increase in the NER. A possible explanation of such as a changing trend for the effectiveness is that the torsotional response of asymmetric structures increases with the increase of the NER. In regard to the effectiveness, the ATMD with the optimum position can render better effectiveness than the AMTMD of case 1 and AMTMD of case 2.

Fig. 4 demonstrates the variation of the effectiveness of the AMTMD with $n = 5$ and ATMD with regard to the total mass ratio for suppression of translational response of asymmetric structures with TTFR = 1, namely the torsionally intermediate stiff structures. It is seen from Fig. 4 that in this case, the effectiveness of the AMTMD is greater than that of the ATMD. Evidently, the effectiveness of



(a) AMTMD of case 1

(b) AMTMD of case 2

(c) AMTMD of case 3

(d) ATMD

Fig. 4 Variation of the effectiveness of the AMTMD with total number equal to 5 and ATMD for reduction of the translational response with respect to total mass ratio in the case of TTFR = 1 both the AMTMD and ATMD for torsionally intermediate stiff structures (TTFR = 1) is significantly lower than that for symmetric structures (NER = 0). Additionally, it is worth noting that the influence of the NER is rather negligible on the effectiveness of both the AMTMD and ATMD. Hence, the AMTMD (ATMD) almost attains the same level of reducing the translational response of the torsionally intermediate stiff structures with various NER values.

Fig. 5 displays the variation of the effectiveness of the AMTMD with $n = 5$ and ATMD with reference to the total mass ratio for attenuating the torsional response of asymmetric structures with TTFR = 1, namely the torsionally intermediate stiff structures. Comparing Fig. 5 with Fig. 4 indicates that the change trends of the AMTMD (ATMD) for reducing the torsional response are generally similar to those for attenuating the translational response.

Fig. 6 gives the variation of the effectiveness of the AMTMD with $n = 5$ and ATMD with respect to the total mass ratio in the mitigation of the translational response of asymmetric structures with

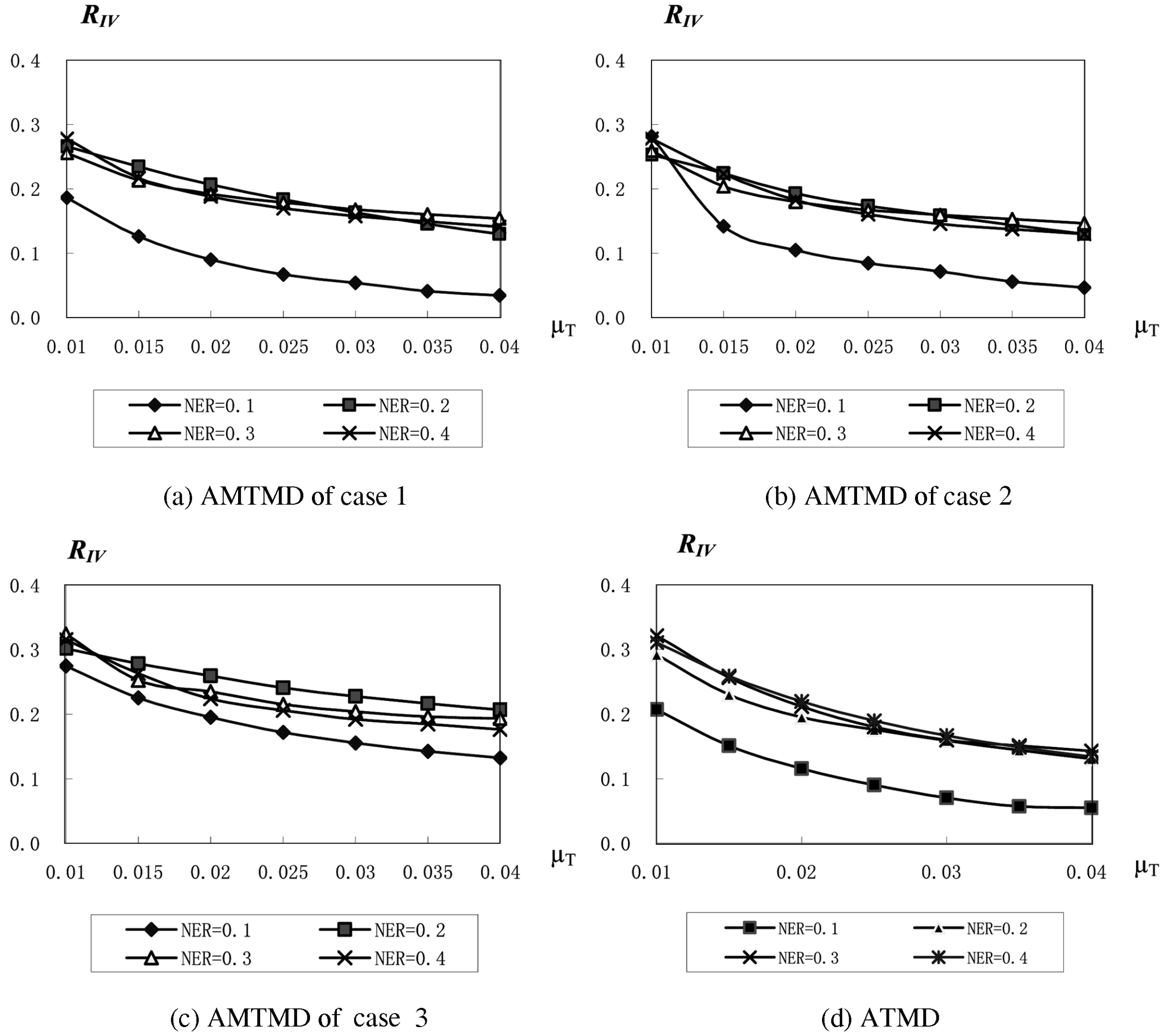
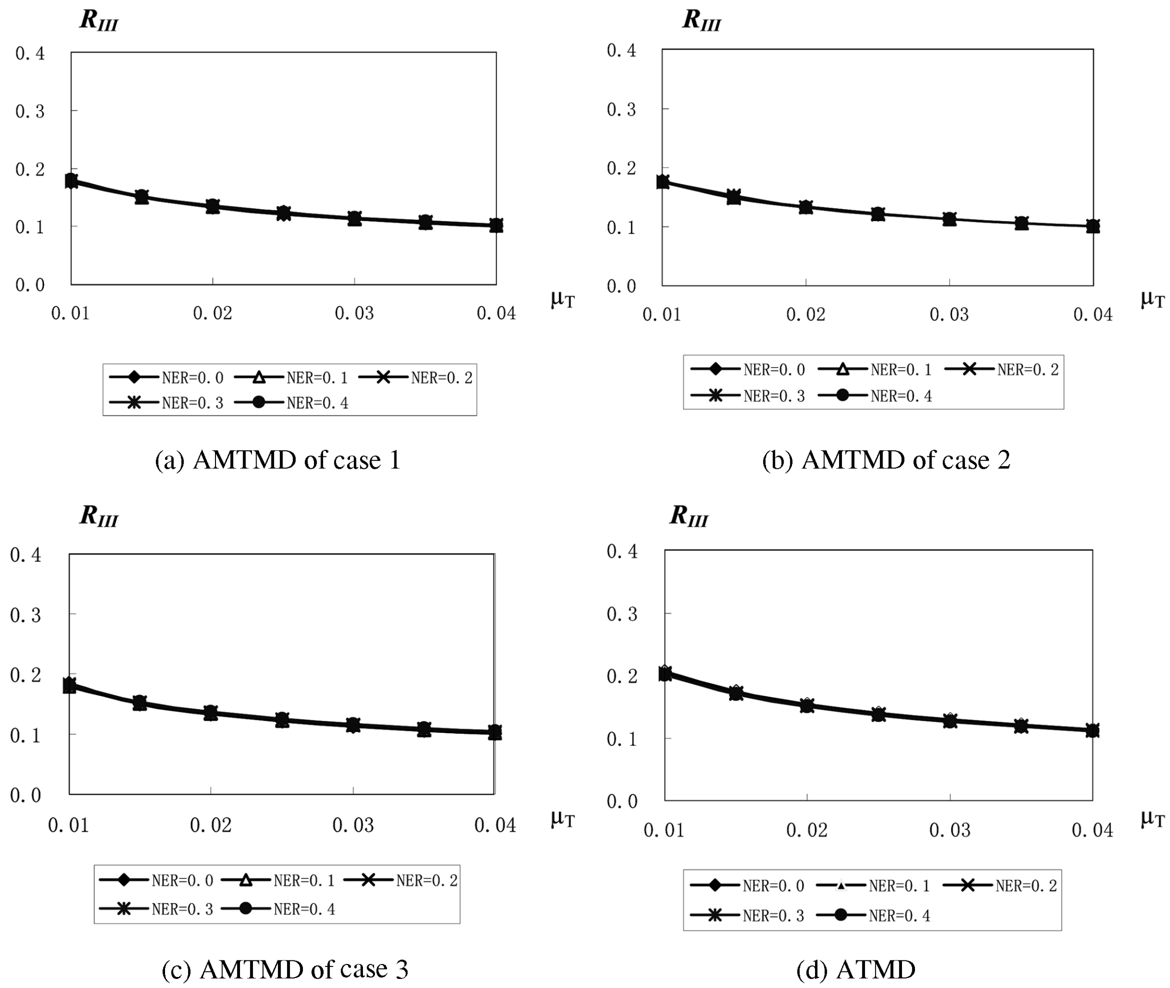


Fig. 5 Variation of the effectiveness of the AMTMD with total number equal to 5 and ATMD for reduction of the torsional response with respect to total mass ratio in the case of TTFR = 1



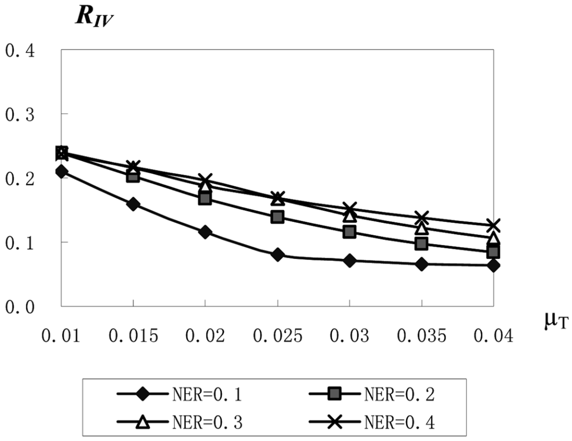
(a) AMTMD of case 1 (b) AMTMD of case 2

(c) AMTMD of case 3 (d) ATMD

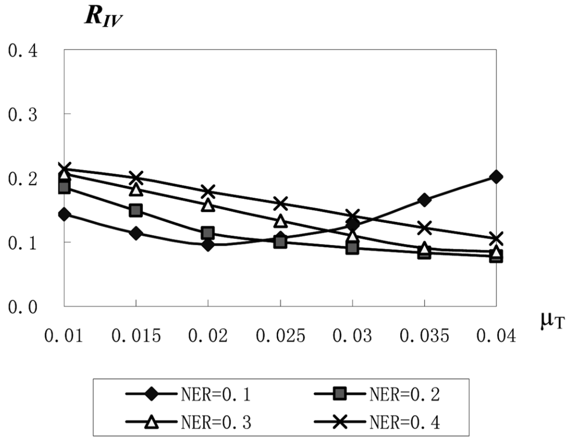
Fig. 6 Variation of the effectiveness of the AMTMD with total number equal to 5 and ATMD for reduction of the translational response with respect to total mass ratio in the case of TTFR = 2

TTFR = 2, namely the torsionally still structures. Fig. 6 clearly indicate that the AMTMD of case 1, of AMTMD of case 2, AMTMD of case 3, and ATMD have identical effectiveness in the mitigation of the translational response of asymmetric structures with TTFR = 2, though various NER values. A possible explanation for such a phenomenon is that the translational response is mainly attributed to the torsional response.

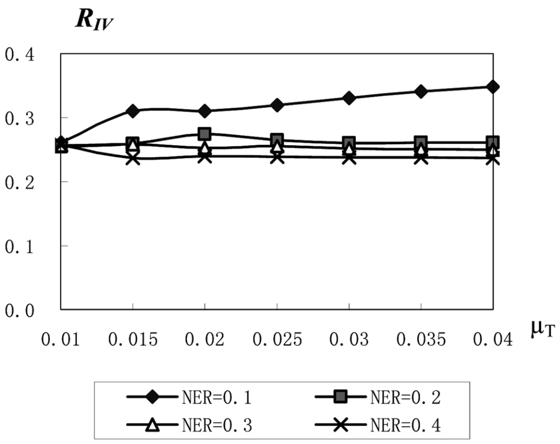
Fig. 7 provides the variation of the effectiveness of the AMTMD with $n = 5$ and ATMD with reference to the total mass ratio for attenuating the torsional response of asymmetric structures with TTFR = 2, namely the torsionally still structures. With reference to the schematic representation of Fig. 7, with the exception of the ATMD, the NER has influence on the effectiveness of the AMTMD in reducing the torsional response of the torsionally stiff structures. Note that the AMTMD of case 1 and ATMD should preferably be selected for this case. It is worth noting that the effectiveness of the AMTMD of case 1 and ATMD decreases with the increase of the NER.



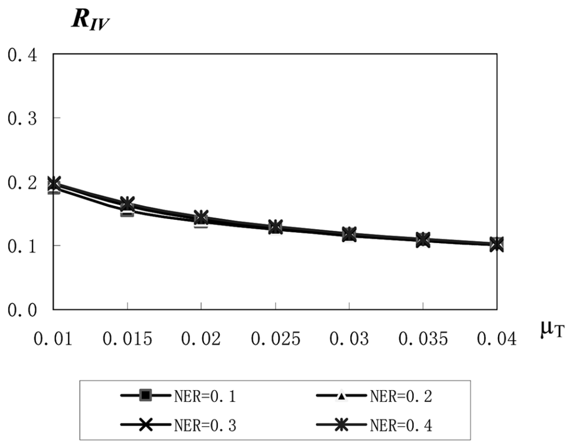
(a) AMTMD of case 1



(b) AMTMD of case 2



(c) AMTMD of case 3

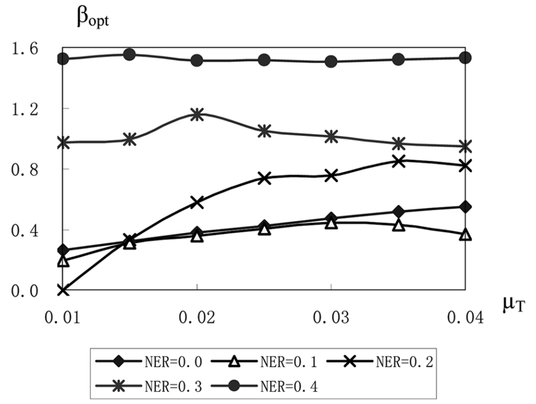


(d) ATMD

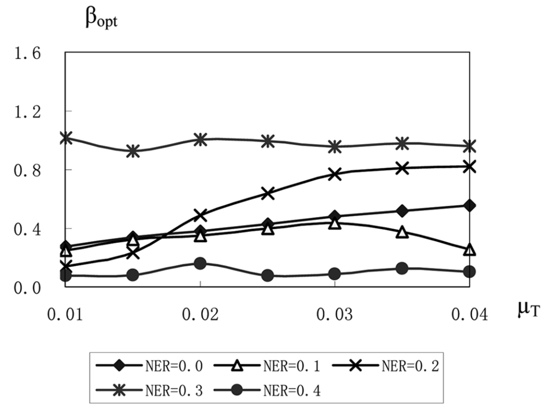
Fig. 7 Variation of the effectiveness of the AMTMD with total number equal to 5 and ATMD for reduction of the torsional response with respect to total mass ratio in the case of TTFR = 2

Fig. 8 shows the variation of the robustness of the AMTMD with $n = 5$ for reduction of translational response with reference to the total mass ratio in the case of TTFR = 0.5. It can be seen from Fig. 8 that the NER affects significantly the robustness of the AMTMD for suppressing the translational response of torsionally flexible structures. Note that the AMTMD of case 1 relatively takes on better robustness. Likewise, the AMTMD of case 1 possesses a stable change trend, namely the robustness increases with the increase of the NER.

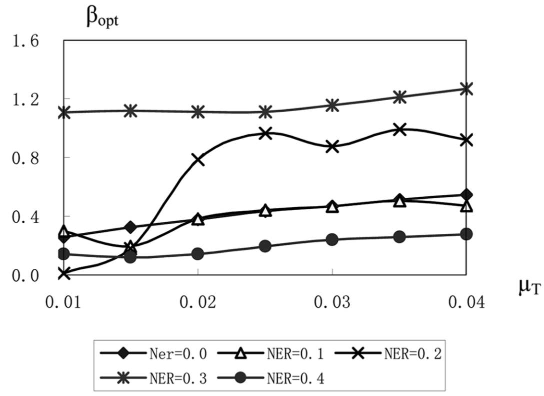
Fig. 9 renders the variation of the AMTMD with $n = 5$ for suppression of the torsional response with regard to the total mass ratio in the case of TTFR = 0.5. It is seen from Fig. 9 that the NER affects remarkably the robustness of the AMTMD of case 3. However, the NER slightly affects the robustness of both the AMTMD of case 1 and the AMTMD of case 2. With reference to the schematic representation of Fig. 8, the robustness of the AMTMD of case 1 and AMTMD of case 2



(a) AMTMD of case 1

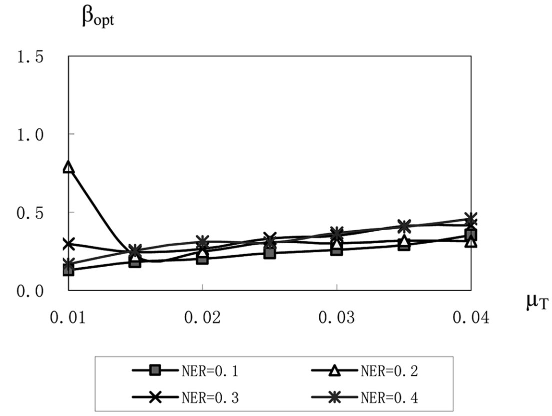


(b) AMTMD of case 2

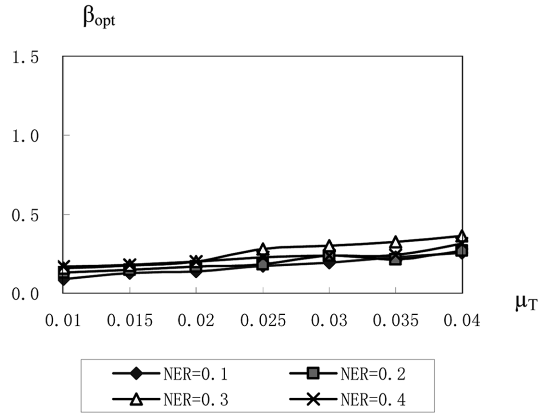


(c) AMTMD of case 3

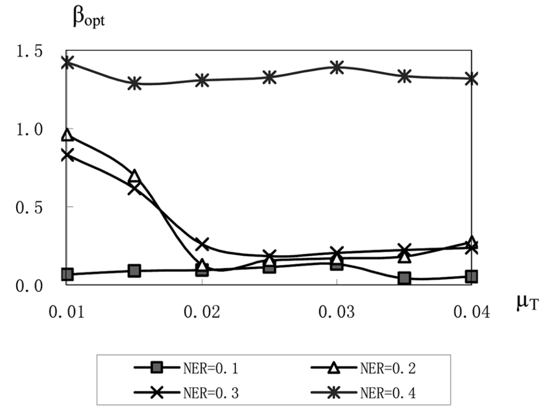
Fig. 8 Variation of the robustness of the AMTMD with total number equal to 5 for reduction of the translational response with respect to total mass ratio in the case of TTFR = 0.5



(a) AMTMD of case 1

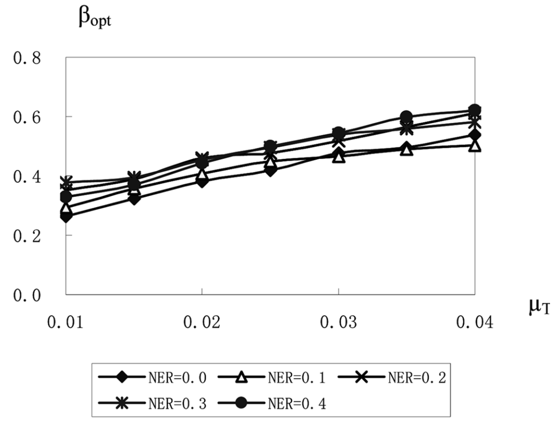


(b) AMTMD of case 2

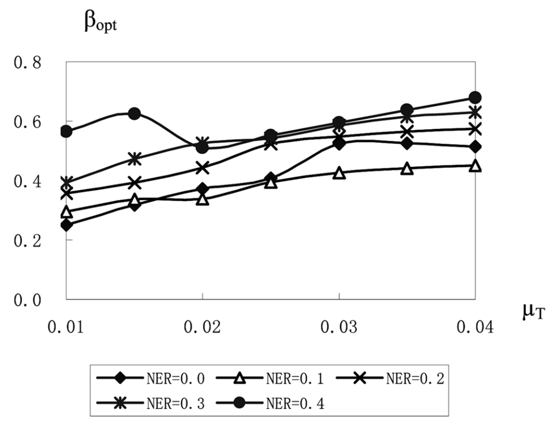


(c) AMTMD of case 3

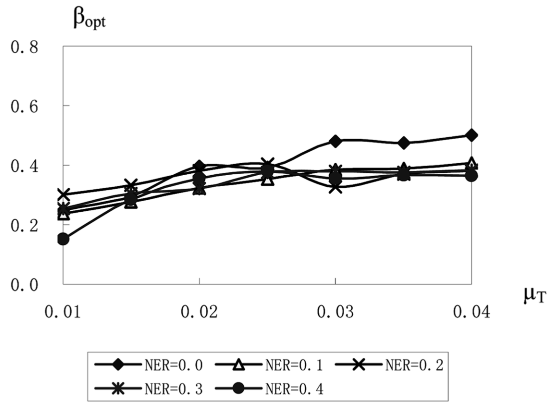
Fig. 9 Variation of the robustness of the AMTMD with total number equal to 5 for reduction of the torsional response with respect to total mass ratio in the case of TTFR = 0.5



(a) AMTMD of case 1

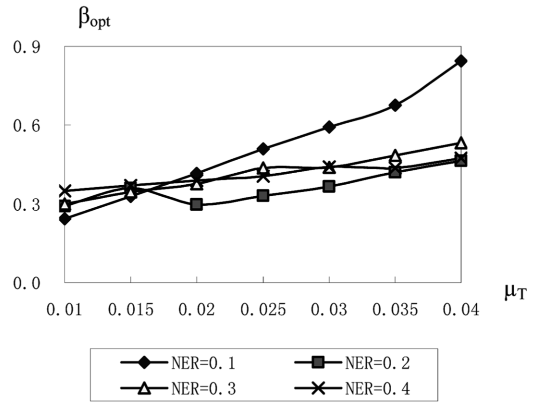


(b) AMTMD of case 2

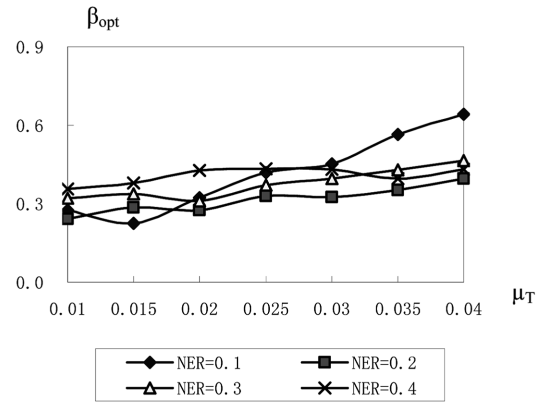


(c) AMTMD of case 3

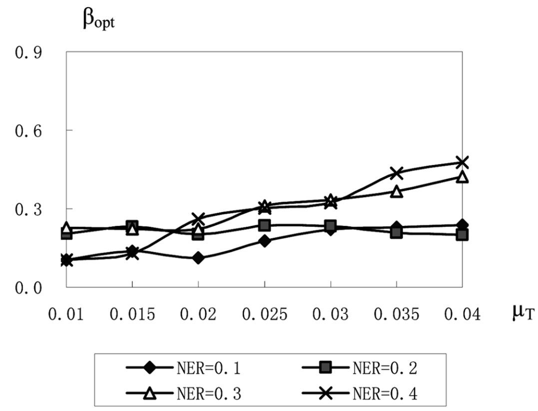
Fig. 10 Variation of the robustness of the AMTMD with total number equal to 5 for reduction of the translational response with respect to total mass ratio in the case of TTFR = 1



(a) AMTMD of case 1

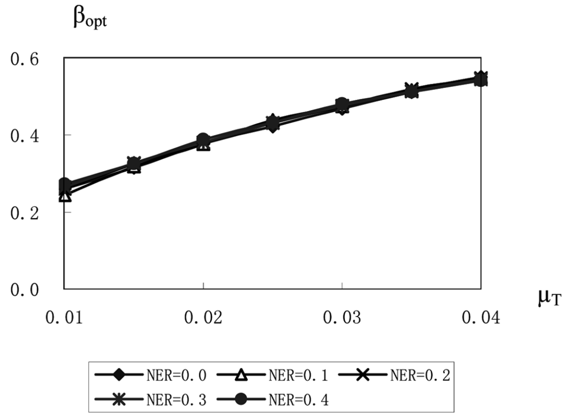


(b) AMTMD of case 2

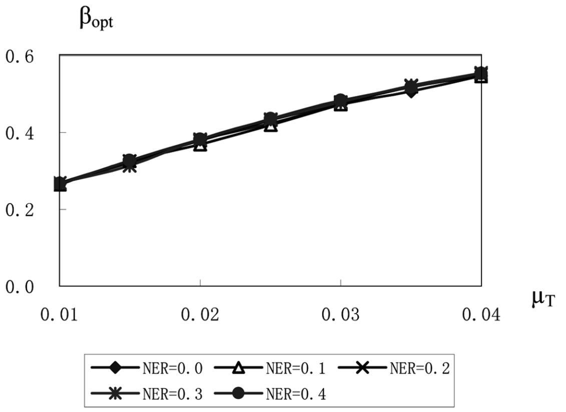


(c) AMTMD of case 3

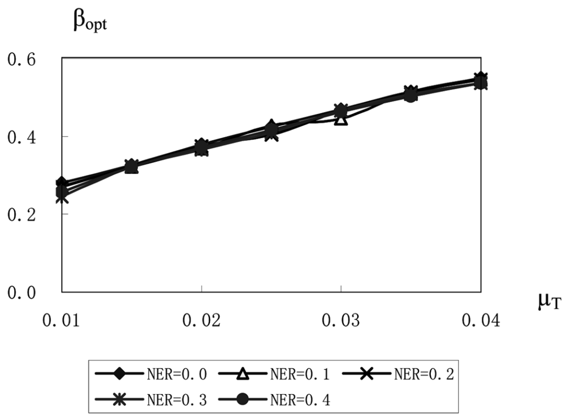
Fig. 11 Variation of the robustness of the AMTMD with total number equal to 5 for reduction of the torsional response with respect to total mass ratio in the case of TTFR = 1



(a) AMTMD of case 1

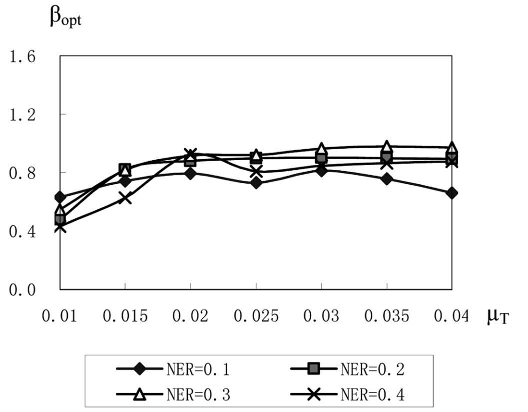


(b) AMTMD of case 2

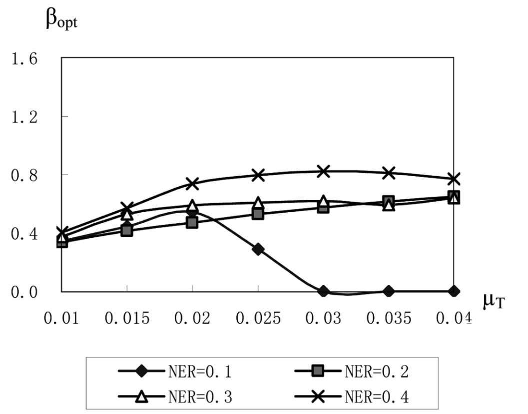


(c) AMTMD of case 3

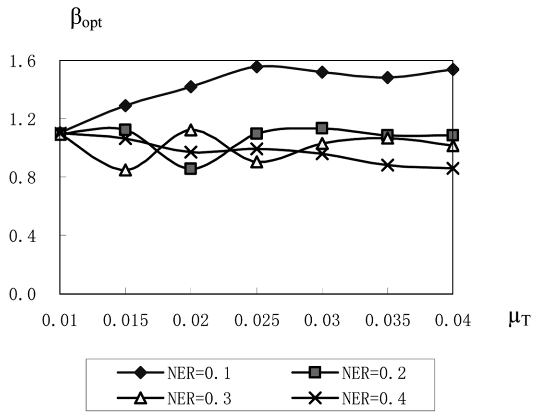
Fig. 12 Variation of the robustness of the AMTMD with total number equal to 5 for reduction of the translational response with respect to total mass ratio in the case of TTFR = 2



(a) AMTMD of case 1

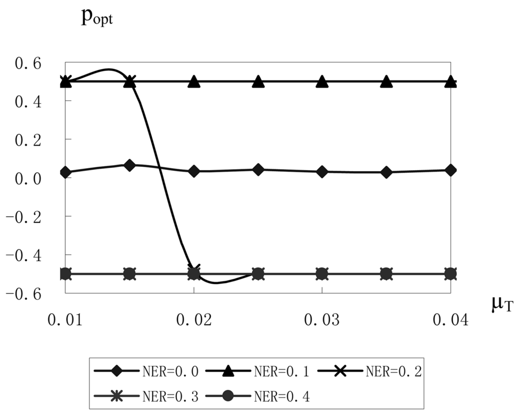


(b) AMTMD of case 2

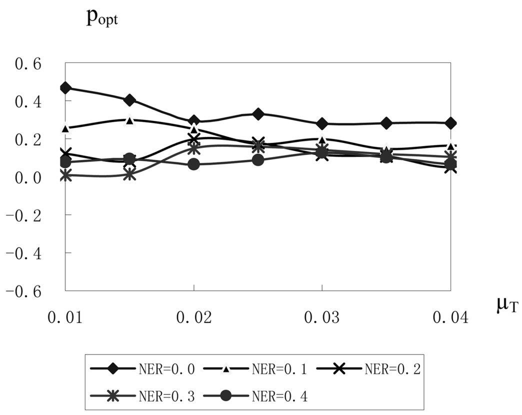


(c) AMTMD of case 3

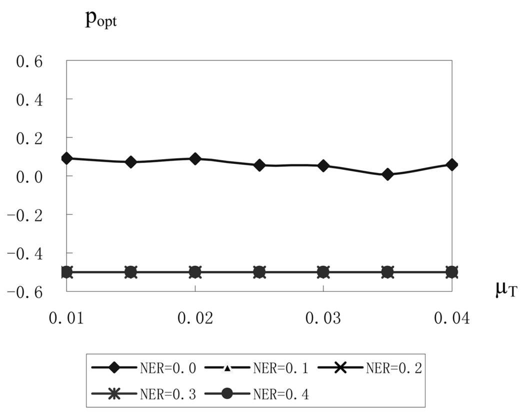
Fig. 13 Variation of the robustness of the AMTMD with total number equal to 5 for reduction of the torsional response with respect to total mass ratio in the case of TTFR = 2



(a) TTFR = 0.5

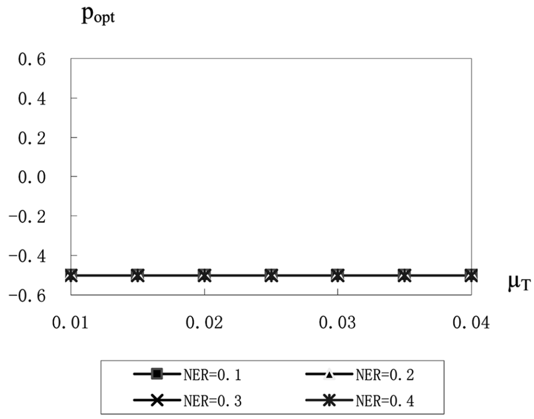


(b) TTFR = 1.0

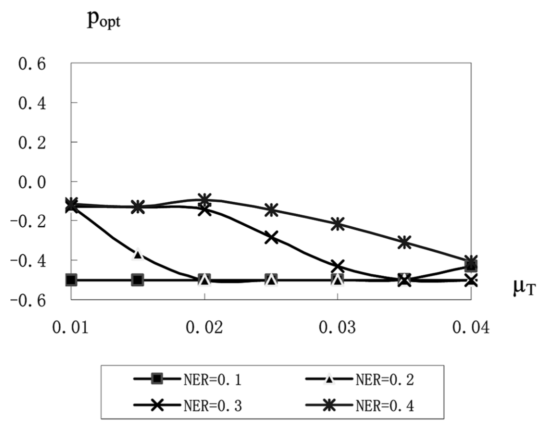


(c) TTFR = 2.0

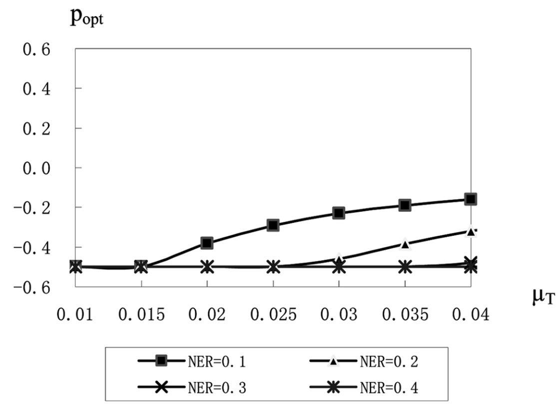
Fig. 14 Variation of the optimum position of the ATMD for reduction of the translational response of asymmetric structures



(a) TTFR = 0.5



(b) TTFR = 1.0



(c) TTFR = 2.0

Fig. 15 Variation of the optimum position of the ATMD for reduction of the torsional response of asymmetric structures decrease significantly in reducing the torsional response of the torsionally flexible structures.

Fig. 10 depicts the variation of the robustness of the AMTMD with $n = 5$ for reduction of the translational response with regard to the total mass ratio in the case of TTFR = 1. It is seen from Fig. 10 that with reference to the AMTMD of case 2 and AMTMD of case 3, the AMTMD of case 1 provide better robustness. Likewise, the robustness of the AMTMD of case 1 increases with the increase of the NER.

Fig. 11 presents the variation of the robustness of the AMTMD with $n = 5$ in the mitigation of the torsional response with respect to the total mass ratio in the case of TTFR = 1. Fig. 11 clearly indicates that the AMTMD of case 1 possesses relatively larger robustness than the AMTMD of case 2 and AMTMD of case 3. It is also shown that the NER affects significantly the robustness of the AMTMD of case 1.

Fig. 12 offers the variation of the robustness of the AMTMD with $n = 5$ in mitigating the translational response with regard to the total mass ratio in the case of TTFR = 2. It is shown that the AMTMD of case 1, AMTMD of case 2, and AMTMD of case 3 achieve the same robustness. It is also shown that the NER has little influence on the robustness of the AMTMD.

Fig. 13 presents the variation of the robustness of the AMTMD with $n = 5$ in mitigating the torsional response with regard to the total mass ratio in the case of TTFR = 2. Note that the distribution of the AMTMD makes large difference in the robustness. As far as the effectiveness of the AMTMD is concerned, the AMTMD of case 1 can provide better effectiveness. Note also that the influence of the NER is relatively lesser on the robustness of the AMTMD of case 1.

Figs. 14 and 15 demonstrate how the variation of the optimum position of the ATMD varies with the total mass ratio for reduction of the translational and torsional responses of asymmetric structures, respectively. It can be observed that there is a large difference in the optimum position between the torsionally flexible, intermediate stiff, and stiff structures. Note also that a large discrepancy exists between reducing the translational and torsional responses.

Further numerical analysis indicates that for the effectiveness and robustness of the AMTMD there exists generally a similar changing pattern between different normalized acceleration feedback gain factors (NAFGF). As far as the magnitude is concerned, the effectiveness and robustness of the AMTMD increases with an increase in the NAFGF.

6. Conclusions

From the preceding elucidation, the following main conclusions can be drawn:

(1) In attenuating the translational response of the torsionally flexible structures (TTFR = 0.5), the AMTMD of case 1 is preferably suitable. In attenuating the torsional response of the torsionally flexible structures (TTFR = 0.5), the AMTMD of case 1, AMTMD of case 2, and ATMD should be preferably selected. In regard to the effectiveness, the ATMD with the optimum position can render better effectiveness than the AMTMD of case 1 and AMTMD of case 2.

(2) In reducing the translational response of the torsionally flexible structures (TTFR = 0.5), the AMTMD of case 1 relatively takes on better robustness. Likewise, the AMTMD of case 1 possesses a stable change trend, namely the robustness increases with the increase of the NER. However, in reducing the torsional response of the torsionally flexible structures (TTFR = 0.5), the robustness of the AMTMD of case 1 and AMTMD of case 2 decrease significantly.

(3) The effectiveness of both the AMTMD and ATMD for torsionally intermediate stiff structures

(TTFR = 1) is significantly lower than that for symmetric structures (NER = 0). The AMTMD (ATMD) almost attains the same level of reducing the translational response of the torsionally intermediate stiff structures with various NER values. Likewise, in the mitigation of the torsional response, the change trends of the AMTMD (ATMD) are generally similar to those for attenuating the translational response.

(4) In suppressing the translational response of torsionally intermediate stiff structures (TTFR = 1), the AMTMD of case 1 provide better robustness. Likewise, the robustness of the AMTMD of case 1 increases with the increase of the NER. In suppressing the torsional response of torsionally intermediate stiff structures (TTFR = 1), the AMTMD of case 1 possesses relatively larger robustness than the AMTMD of case 2 and AMTMD of case 3. Also, the NER affects significantly the robustness of the AMTMD of case 1.

(5) In the mitigation of the translational response of torsionally stiff structures (TTFR = 2), the AMTMD of case 1, of AMTMD of case 2, AMTMD of case 3, and ATMD have identical effectiveness, though various NER values. In the mitigation of the torsional response of the torsionally stiff structures, the AMTMD of case 1 and ATMD should preferably be selected. Likewise, the effectiveness of the AMTMD of case 1 and ATMD decreases with the increase of the NER.

(6) In the mitigation of the translational response of torsionally stiff structures (TTFR = 2), the AMTMD of case 1, AMTMD of case 2, and AMTMD of case 3 achieve the same robustness. Also, the NER has little influence on the robustness of the AMTMD. In the mitigation of the torsional response of torsionally stiff structures (TTFR = 2), the distribution of the AMTMD makes large difference in the robustness. Also the influence of the NER is relatively lesser on the robustness of the AMTMD of case 1.

Eventually, it is worth pointing out that with resorting to the present approach, Li *et al.* (2007) has numerically investigated the earthquake resistant performance of the ATMD and AMTMD for asymmetric buildings, so as to further validate the effectiveness and robustness of the ATMD and AMTMD in reducing both the translational and torsional responses of asymmetric buildings in the time-domain. *SIMULINK* analysis has been implemented to a three-storey asymmetric steel structure building under various earthquakes, taking into account both the certainty and uncertainty in the structural stiffness. Numerical simulations indicate that the ATMD and AMTMD can effectively control both the translational and torsional responses of asymmetric buildings subjected to earthquakes. Likewise, the AMTMD generally has better performance than the ATMD for seismically excited asymmetric buildings.

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Nonation

AMTMD	: active multiple tuned mass dampers
b/r	: normalized width of asymmetric structures, set in the present paper equal to 1.0
CM	: center of mass of asymmetric structures
CR	: center of resistance of asymmetric structures
c_s	: mode-generalized damping coefficient of asymmetric structures
c_T	: constant damping coefficient of the AMTMD
c_{Tj}	: damping coefficient of the jth ATMD in the AMTMD
c_{tj}	: velocity feedback for the jth ATMD in the AMTMD
c_t	: constant velocity feedback for the jth ATMD in the AMTMD
DMF	: dynamic magnification factors of structures with the AMTMD
DMF_s, DMF_s^*	: translational displacement dynamic magnification factors of asymmetric structures with and without the AMTMD

DMF_θ, DMF_θ^*	: torsional displacement dynamic magnification factors of asymmetric structures with and without the AMTMD
DOF	: degree-of-freedom
E_R	: ratio of the eccentricity to the radius of gyration of the deck, referred in this paper to as the normalized eccentricity ratio (*NER*)
e_y	: eccentricity between the CR and CM of asymmetric structures
f	: tuning frequency ratio of the AMTMD
j	: ATMD number in the AMTMD
k_s	: mode-generalized lateral stiffness of asymmetric structures in the translational (x) direction
k_T	: constant spring stiffness of the AMTMD
k_{Tj}	: spring stiffness of the jth ATMD in the AMTMD
k_t	: constant displacement feedback for the jth ATMD in the AMTMD
k_{tj}	: displacement feedback for the jth ATMD in the AMTMD
k_θ	: mode-generalized torsional stiffness of asymmetric structures with respect to the CM
MTMD	: multiple tuned mass dampers
m_s	: mode-generalized mass of asymmetric structures
m_{Tj}	: mass of the jth ATMD in the AMTMD
m_{tj}	: acceleration feedback for the jth ATMD in the AMTMD
n	: ATMD total number in the AMTMD
R_I	: minimization of the minimum values of the maximum translational displacement dynamic magnification factors of asymmetric structures with the AMTMD
R_{II}	: minimization of the minimum values of the maximum torsional displacement dynamic magnification factors of asymmetric structures with the AMTMD
R_{III}	: minimization of the minimum values of the maximum translational displacement dynamic magnification factors, nondimensionalized by the maximum translational displacement dynamic magnification factors of asymmetric structures without the AMTMD
R_{IV}	: minimization of the minimum values of the maximum torsional displacement dynamic magnification factors, nondimensionalised by the maximum torsional displacement dynamic magnification factors of asymmetric structures without the AMTMD
r	: radius of gyration of the deck about the vertical axis through the CM
r_j	: ratio of the natural frequency of the jth ATMD to the uncoupled translational natural frequency of asymmetric structures
$H_{x_s}(-i\omega)$	: transfer function for translational displacement of asymmetric structures with the AMTMD
$H_{\theta_s}(-i\omega)$	: transfer function for torsional displacement of asymmetric structures with the AMTMD
$\ddot{x}_g(t)$	: ground acceleration
x_s	: translational displacement of asymmetric structures with respect to the ground
x_{Tj}	: translational displacement of each ATMD with reference to the ground
y_j	: translational displacement of each ATMD with reference to the ground
$y_{(n+1)/2}$	: center of the AMTMD, i.e., placement of the $(n+1)/2$th ATMD in the AMTMD
θ_s	: torsional displacement of asymmetric structures
β	: frequency spacing of the AMTMD
λ	: ratio of the external excitation frequency to the uncoupled translational natural frequency, which is set within the range from 0.4 to 3.4
λ_ω	: uncoupled torsional to translational frequency ratio (*TTFR*)
ξ_j	: damping ratio of the jth ATMD in the AMTMD
ξ_s	: structural damping ratio, which is set in this study equal to 0.02
ξ_T	: average damping ratio of the AMTMD
μ_{Tj}	: mass ratio of the jth ATMD in the AMTMD
μ_T	: total mass ratio of the AMTMD
ω	: external excitation frequency
ω_j	: natural frequency of the jth ATMD in the AMTMD
ω_s	: uncoupled translational natural frequency of asymmetric structures

ω_{s1}	: coupled fundamental natural frequency of asymmetric structures
ω_{s2}	: coupled second natural frequency of asymmetric structures
ω_{θ}	: uncoupled torsional natural frequency of asymmetric structures
ω_{T}	: average natural frequency of the AMTMD