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Postbuckling response and failure of symmetric laminated plates with rectangular cutouts under uniaxial compression

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Abstract. This paper deals with the buckling and postbuckling responses and the progressive failure of square symmetric laminates with rectangular cutouts under uniaxial compression. A detailed investigation is made to show the effects of cutout size and cutout aspect ratio on prebuckling and postbuckling responses, failure loads and failure characteristics of $(+45/-45/0/90)_{2s}$, $(+45/-45)_{4s}$ and $(0/90)_{4s}$ laminates. The 3-D Tsai-Hill criterion is used to predict the failure of a lamina while the onset of delamination is predicted by the interlaminar failure criterion. In addition, the effects of boundary conditions on buckling load, failure loads, failure modes and maximum transverse deflection for a $(+45/-45/0/90)_{2s}$ laminate with and without cutout have also been presented. It is concluded that square laminates with small square cutouts have more postbuckling strength than without cutout, irrespective of boundary conditions.

Keywords: buckling; composite laminates; cutouts; failure; postbuckling; strength.

1. Introduction

The ultimate failure of a laminated composite panel does not always occur at the load corresponding to the first-ply failure; because the failure characteristics of heterogeneous and anisotropic composite laminates differ from that of the isotropic ones. Composite laminates can sustain a much higher load after the occurrence of localized damage such as matrix cracking, fiber breaks or delamination (Aggarwal and Broutman 1990). Due to practical requirements cutouts are often required in composite structural panels, such as in wing spars and cover panels of aircraft structures to provide access for hydraulic lines, electrical lines, fuel lines, damage inspection, and to reduce the overall weight of the aircraft. The presence of these cutouts forms free edges in the composite laminates, which in turn cause high interlaminar stresses (Robert 1975) leading to loss of stiffness and premature failure of laminates due to onset of delamination Therefore, stability, overall strength, and failure characteristics of composite panels with cutouts are some of the important parameters for an improved design of structures fabricated with laminated panels. Of the early investigations related to failure of laminated plates without cutouts, are the works by Turvey (1980a, b, c, 1981, 1982, 1987), Turvey and Osman (1989) and Singh *et al.* (1996, 1997, 1998a, b,

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c). Buckling behaviour of laminated plates with circular cutouts has been studied by Nemeth (1988), Lin (1989), Srivatsa and Krishna Murthy (1992), Britt (1993) and Eiblmeier and Loughlan (1997). In addition to this, Akbu*lut et al*. (2001), Yazici *et al*. (2003) and Akbulut *et al*. (2007) investigated the buckling behaviour of composite laminates with central square hole, U-shaped cutouts and corner circular notches, respectively. Very recently, Baba (2007) conducted numerical and experimental study to investigate the effect of boundary conditions, cutout shape, length/thickness ratio, and ply orientation on buckling behaviour of E-glass/epoxy laminated composite plates under in-plane compression load and observed that plate with $[0]_8$ lay-up has the highest buckling load and the plate with $[90]_8$ lay-up has the lowest buckling load. He also showed that clamped boundary condition has the highest buckling load because of its high rigidity. Laminated composite circular plates with circular holes and subjected to radial load have also been analyzed for buckling load by Baltaci *et al*. (2006). Moreover, in one of the early investigations on postbuckling analysis of composite laminates with cutouts, Martin (1972) investigated the buckling and postbuckling behaviour of square uniaxial compression-loaded simply supported composite plate with a circular cutout using Rayleigh-Ritz method. Marshall *et al*. (1987) conducted analytical and experimental work for displacement-loaded square plates with circular cutout and showed the effects of cutout size, plate orthotropy and load level on prebuckling and postbuckling stress distribution. Larsson (1987) and Rouse (1990) studied the buckling and postbuckling behaviour of square-orthotropic compression-loaded and square-anisotropic shear-loaded plates with circular cutouts, respectively. Engelstad *et al*. (1992) analyzed postbuckling response and failure characteristics of graphite epoxy panels with and without a circular hole in axial compression using a progressive damage mechanism in conjunction with 3-D degenerated shell element and obtained good correlation between the experimentally observed and analytically predicted postbuckling responses. Lee and Hyer (1993) investigated the postbuckling failure characteristics of square symmetrically laminated plates with circular hole under uniaxial compression using the maximum stress criterion and established the failure mechanisms of various laminates. Bailey *et al*. (1996) presented the effect of different sized and different shaped cutouts on the buckling and postbuckling response of graphite/epoxy panels with a cutout and found circular cutouts to be more efficient than square cutouts based on volume of material removal. A review of early (before 1996) research investigations on the buckling and postbuckling of laminated composite plates with cutouts has been documented by Nemeth (1996). Kong *et al*. (2001) analyzed, numerically and experimentally buckling and postbuckling behaviors of composite plates with a circular hole and showed that buckling and postbuckling behaviors largely depend on the size of the hole and the stacking sequence of the composite plate. Jain and Kumar (2004) studied the postbuckling response of symmetrical square laminates with central circular and elliptical cutout under uniaxial compression and observed that the cutout shape, size and its alignment have a substantial influence on the reserved strength of the laminate beyond buckling. Very lately, Guo (2007) conducted numerical and experimental studies to investigate the effect of reinforcements around cutouts on the bucking behaviour of composite panels under shear load and determined the best cutout reinforcement.

Thus, it is observed that the postbuckling response and failure of laminated plates with non-circular cutouts have not been studied in detail. The objective of this paper is to investigate the effects of rectangular cutout size and cutout aspect ratio on prebuckling and postbuckling responses, failure loads and failure characteristics of $(+45/-45/0/90)_{2s}$, $(+45/-45)_{4s}$ and $(0/90)_{4s}$ laminates with square/rectangular cutouts under uniaxial compression. In addition, the effects of edge boundary conditions on buckling and failure loads and maximum transverse deflection associated with failure loads for a $(+45/-45/0/90)_{2s}$ quasi-isotropic laminate with and without cutout have also been presented.

2. Methodology

A special purpose program was developed to carry out the study which is based on the finite element formulation (Singh 1996) using the first order shear deformation theory with nine noded Lagrangian element having five degrees of freedom per node. Geometric nonlinearities based on von Karman's assumptions and in-plane shear stress and strain material nonlinearities have been incorporated. The non-linear algebraic finite element equations are solved by Newton-Raphson technique. The calculation of stresses is done on the nodal points as well as on the Gauss points. All the six components of stress are calculated at each nodal and Gauss point. Nodal point stresses refer to the average value of stresses obtained from various elements associated with a particular node. However, to predict the failure of a lamina, only five stress components (three in-plane stresses and two transverse stresses) at mid thickness of each layer are used in the tensor polynomial form of the 3-D Tsai-Hill criterion. The onset of delamination is predicted by the interlaminar failure criterion (Singh 1996, see the Appendix) while the progressive failure procedure applied is same as used by Singh *et al.* (1997, 1998a, b, c). Total failure is said to have occurred when the onset of delamination occurs or when the plate is no longer able to carry any further increase in load due to very large transverse deflection at a constant load.

A comparison of buckling load for quasi-isotropic laminate with square cutout is done with the experimental results reported by Bailey and Wood (1996) for the clamped boundary condition. The ratio of buckling load of the plate with square cutout to that of plate without square cutout (i.e., $P_{with}/P_{without}$) as reported by Bailey and Wood (1996) is 0.90 which is of the same order of magnitude as obtained in the present investigation (Table 5) for the same boundary condition. Further, a comparison of the first-ply failure is made with the experimental results for C4 specimen (Engelstad *et al.* 1992) using the Tsai-Wu criterion. The failure load obtained in the present investigation is 1.89 P_c, where P_c is the buckling load of the laminate, while that reported by Engelstad *et al.* is 2.0 P_c. [Non-dimensionlized buckling load i.e., $P_c b^2/E_2 h^3$ is 17.79]. This difference of about 5% in failure loads is because of the fact that Engelstad *et al.* (1992) considered the uniform end shortening in the application of uniaxial compression as against the uniform stress resultant in this investigation.

Properties of the material (Reddy and Reddy 1992) of the laminate are presented in Table 1. In this study, a full square plate of size 279 mm × 279 mm × 2.16 mm with ply thickness 0.135 mm is

Table 1 Material properties of T300/5208 (pre-peg) graphite-epoxy

Mechanical properties	Values	Strength properties	Values
E_1	132.58 GPa	X_t	1.52 GPa
E_2	10.80 GPa	X_c	1.70 GPa
E_3	10.80 GPa	$Y_t = Z_t$	43.80 MPa
$G_{12} = G_{13}$	5.70 GPa	$Y_c = Z_c$	43.80 MPa
$\nu_{12} = \nu_{13}$	0.24	R	67.60 MPa
ν_{23}	0.49	$S = T$	86.90 MPa

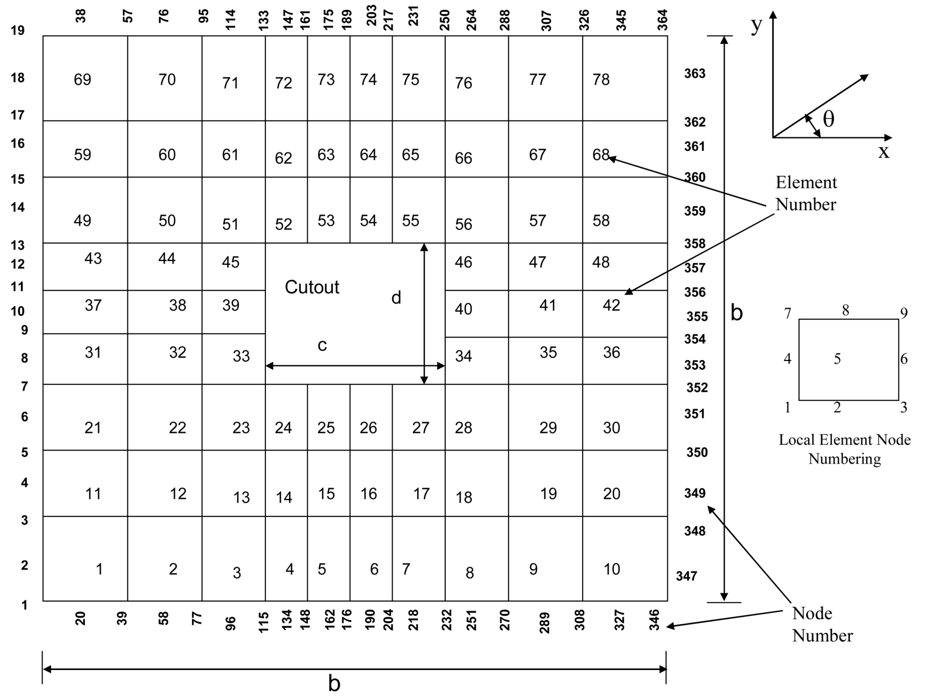

Fig. 1 Finite element mesh pattern for laminate

Table 2 Details of cutout dimensions and aspect ratios

Width of square laminate, b (mm)	Cutout type	Cutout size* c (mm) × d (mm)	c/b	d/b
279	Square	39 × 39	0.14	0.14
279	Square	78 × 78	0.28	0.28
279	Square	117 × 117	0.42	0.42
279	Rectangular	78 × 39	0.28	0.14
279	Rectangular	117 × 39	0.42	0.14

*c and d are dimensions of cutout along x- and y-axis, respectively, (Fig. 1).

used with a central cutout of size $c \times d$; where, c and d refers to the dimensions of the cutout along x- and y-direction, respectively (Fig. 1 and Table 2). Schematics of finite element mesh and element nodes numbers are shown in Fig. 1. Three types of flexural edge boundary conditions, namely BC1, BC2 and BC3 are considered; BC1 refers to a plate with all edges simply supported, BC2 refers to a plate with two longitudinal edges ($y = 0$ and $y = b$) simply supported and the other two edges clamped and BC3 refers to a plate with all edges clamped. In all three cases, the in-plane boundary conditions are identical and compression load is applied on the edges $x = b$; where, b refers to the width of the square plate. Most of the results of this investigation refer to the boundary condition BC1 (all edges simply supported) except those in the section dealing with the effects of boundary conditions. Results for failure loads and corresponding deflections are presented in the following non-dimensionalized forms:

Uniaxial compression in the x-direction $N_x b^2/E_2 h^3$
Uniaxial compression in the y-direction $N_y b^2/E_2 h^3$
Maximum transverse deflection w_{max}/h
Where, E_2 is the transverse elastic modulus of a lamina; h is the thickness of the laminate; b is the width of the square plate; N_x and N_y are the uniaxial compression loads per unit width of the plate along x- and y-direction, respectively; and, w_{max} is the maximum transverse deflection.

3. Results and discussion

3.1 Response and failure under uniaxial compression

The postbuckling and the progressive failure responses of $(+45/-45/0/90)_{2s}$, $(+45/-45)_{4s}$ and $(0/90)_{4s}$ laminates with square cutout of sizes, c/b = 0.14, 0.28 and 0.42 in terms of the load versus the maximum transverse deflection were investigated. A typical plot for c/b = 0.14 is shown in Fig. 2. It can be seen from the plot (Fig. 2) that for maximum transverse deflection $w_{max}/h \leq 1.0$, the strength is maximum for $(+45/-45)_{4s}$ laminate while for $w_{max}/h > 1.0$, $(+45/-45/0/90)_{2s}$ laminate has the largest strength. For $w_{max}/h \geq 2.5$, $(0/90)_{4s}$ laminate shows greater strength than $(+45/-45)_{4s}$ laminate. The reason for this is the larger in-plane stiffness of $(0/90)_{4s}$ laminate than that of $(+45/-45)_{4s}$ laminate. Similar response of laminates with larger square cutouts of sizes c/b = 0.28 and 0.42 was also observed, and no appreciable change in the response was noted due to increase in the cutout size. However, failure characteristics of laminates were different for two cutout sizes. It was observed that there was no variation in the first-ply failure mode of $(+45/-45/0/90)_{2s}$ laminate with increase in the cutout size, but the location of failure was shifted from loaded edge corner, for cutouts $c/b \leq 0.28$, to the corner of the cutout, for cutouts $c/b > 0.28$. In the case of $(+45/-45)_{4s}$ laminate, observations regarding the location of the first-ply failure were similar, but the mode of failure was found to be different for smaller and larger cutouts. It can be predicted that the in-plane normal stresses transverse to the fiber directions are the failure causing stresses; for $(+45/-45/0/90)_{2s}$

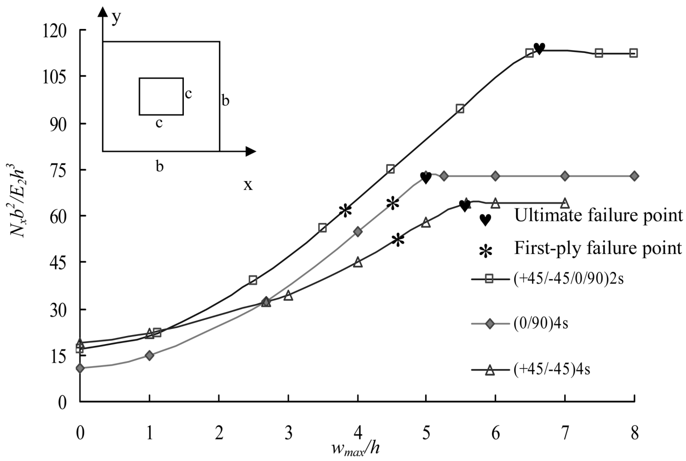

Fig. 2 Load versus maximum transverse deflection response of laminates with square cutouts (c/b = 0.14) under uni-axial compression

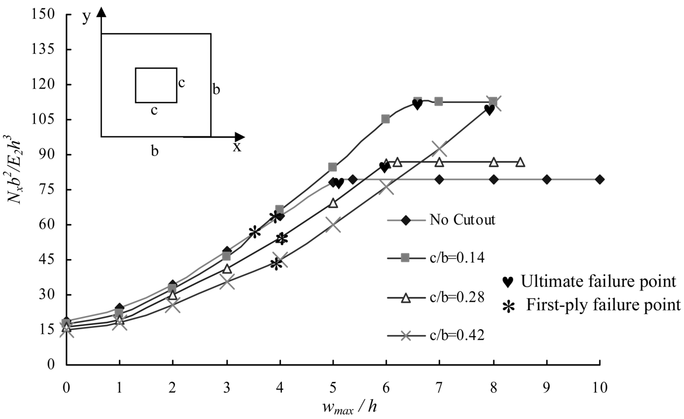

Fig. 3 Load versus maximum transverse displacement of $(+45/-45/0/90)_{2s}$ laminate without and with square cutouts of three different sizes under uni-axial compression

laminate (irrespective of the size of the cutout), and $(+45/-45)_{4s}$ laminate with cutouts of size $c/b < 0.42$, and for the $(0/90)_{4s}$ laminate with cutout size $c/b > 0.42$. However, the mode of the first-ply failure is the in-plane shear mode of failure in $(0/90)_{4s}$ laminate for cutouts $c/b < 0.42$.

Fig. 3 shows the load versus maximum transverse deflection for $(+45/-45/0/90)_{2s}$ laminate without cutout and with square cutouts ($c = d$) of three different sizes. It is observed that there is a monotonic decrease in the strength of laminate with increase in the size of the cutout for a fixed value of the maximum transverse deflection. However, contrary to normal expectation, laminates with cutouts of size $c/b \leq 0.14$ have higher failure loads in comparison to that without cutout due to redistribution of stresses. It can be stated that the first-ply failure load of laminate depends to a great extent on the size of the square cutout and the ultimate failure depends on the mode of the first-ply failure, the location of the failure and the subsequent progressive failure modes. For example, although the first-ply failure load of the laminate with square cutout with $c/b = 0.42$ is smaller than that with cutout $c/b = 0.14$, however, the ultimate failure loads are almost the same.

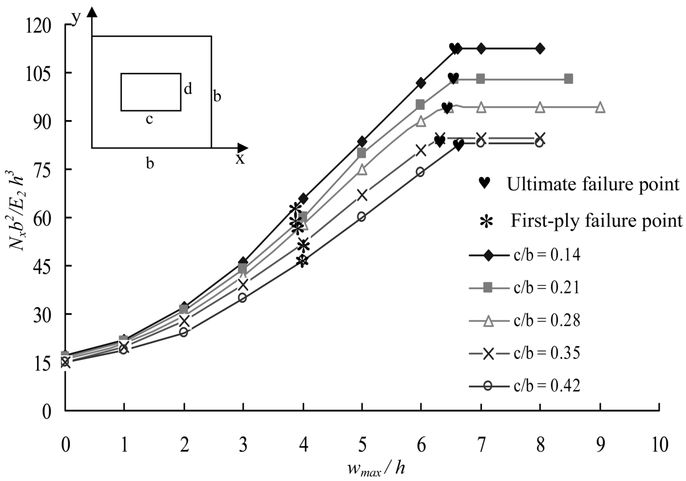

Fig. 4 Load versus maximum transverse displacement of $(+45/-45/0/90)_{2s}$ laminate with rectangular cutouts of different aspect ratios ($d/b = 0.14$, constant) under uni-axial compression

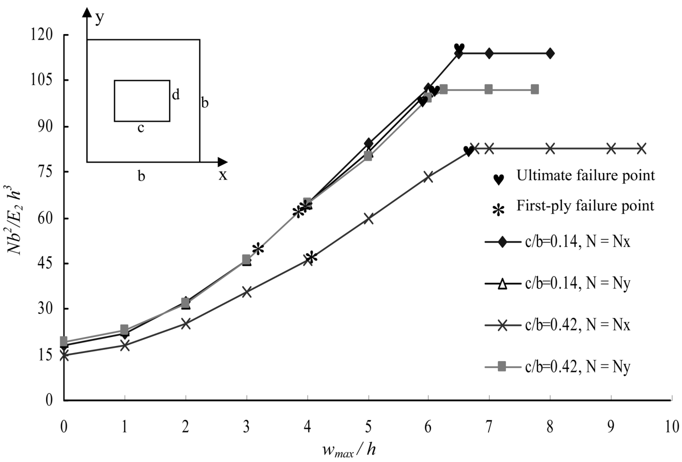

Fig. 5 Load versus maximum transverse deflection of $(+45/-45/0/90)_{2s}$ laminates with cutouts (d/b = 0.14, constant) under uni-axial compression loads in x- and y-directions

The non-dimensionalized values of the first-ply failure loads of the laminate with square cutouts of size c/b = 0.42 and c/b = 0.14 are 44.32 and 63.26, respectively, while the corresponding ultimate failure loads are 111.74 and 113.03, respectively.

The load versus the maximum transverse deflection graph for $(+45/-45/0/90)_{2s}$ laminate with rectangular cutouts of different aspect ratios is shown in Fig. 4. The aspect ratio (c/b) of the cutouts is expressed as the ratio of its longer side (c) to the length of the plate, keeping the ratio of width (d) of the cutout to the width (b) of the plate as constant (d/b = 0.14). It can be observed that for a particular maximum transverse deflection, the strength of laminate decreases with increase in the aspect ratio of the cutout. Moreover, for aspect ratios c/b < 0.42, the first-ply failure is observed to occur at the loaded edge ($x = b$) and for c/b = 0.42, the failure location is found to occur on the support at $x = 0$.

Fig. 5 shows the progressive failure response of square $(+45/-45/0/90)_{2s}$ laminate under uniaxial compression taken independently in the x- and y-directions for cutouts of aspect ratios c/b = 0.14 & 0.42 with d/b = 0.14 (constant). It is shown that with cutout of aspect ratio 0.14, there is no appreciable difference in responses for the two directions of compression loading. However, for the aspect ratio 0.42, significantly higher prebuckling, postbuckling stiffness and strength are observed when the loading is applied along the shorter direction (y-direction) of the cutout in comparison to that for the loading in longer direction (x-direction). It is attributed to the presence of tensile stresses in the x-direction of the laminate, which are of the same order of magnitude as the compression stresses in the y-direction. It is noted that with the cutout of aspect ratio c/b = 0.14, there is no change in the failure mode and the location of the first-ply failure due to change in the loading directions. However, for the aspect ratio c/b = 0.42, the location of the first-ply failure and the cause of ultimate failure are different for the two directions of loading; for the x-direction of compression, the first-ply failure occurs at the corner of the cutout while for y-direction of loading, it occurs at the corner (x = 0 and $y = b$) on the loaded edge of the laminate. It is worth noting that the ultimate failure of laminate is caused by the onset of delamination when loaded in y-direction, while it is caused by large transverse deflection for the x-direction of loading.

Non-dimensionlized values of buckling, first-ply and ultimate failure loads, and maximum

Table 3 Effect of the cutout aspect ratio on buckling and failure loads of square $(+45/-45/0/90)_{2s}$ laminate, [for d/b = 0.14 (constant)]

Cutout aspect ratio (c/b)	Buckling load	First-ply failure load	Ultimate failure load	$(w_{max}/h)^*$
No Cutout	18.93	58.09	79.60	3.70
0.14	17.36	63.26	113.03	3.93
0.21	16.64	58.95	103.56	3.94
0.28	15.78	56.23	94.53	4.01
0.35	15.20	53.52	85.92	4.15
0.42	14.63	45.90	83.20	3.97

*Non-dimensionalized maximum transverse deflection associated with the first-ply failure.

transverse deflections (associated with the first-ply failure) for $(+45/-45/0/90)_{2s}$ laminate without and with rectangular cutouts of different aspect ratios (c/b) with constant d/b = 0.14 are presented in Table 3. It is observed that buckling and failure loads of the laminate decrease with increase in the aspect ratio of the cutout while there is no appreciable change in the maximum transverse deflection associated with the first-ply failure. As expected, the buckling load of the laminate is smaller in the presence of a cutout than that of laminate without a cutout. It is also shown that the first-ply failure loads for cutouts of sizes c/b = 0.14 & 0.21, are higher than that without a cutout; however, the same is not true for all other cutout sizes. In fact this is the general observation for central cutouts of sizes c/b = 0.06 to 0.21, because of the combined effect of increase in postbuckling stiffness and redistribution of stresses in the highly stressed region of the laminate without a cutout. It is worth noting that the ultimate load of laminate with cutouts are larger than those without cutout; this is attributed to the fact that the laminate without cutout loses its stiffness drastically after the first-ply failure which brings in an early ultimate failure, while, in the presence of cutouts, a drastic stress redistribution is observed to take place which results in stress relieving in the advanced stage of the postbuckling deformation and hence the higher ultimate strength. From the detailed investigation on the effect of size of square cutouts of size c/b = 0.06 to 0.21, it is noted that these small size cutouts are beneficial for increasing the postbuckling strength, especially the increase in the first-ply failure load is most appreciable. Here, the lower limit of cutout size is based on failure due to the onset of delamination while the upper limit is based on the fact that the first-ply failure load of laminate for this cutout size is reduced to that of laminate without a cutout.

Table 4 shows the variation of buckling, first-ply and ultimate failure loads, and maximum

Table 4 Effect of uniaxial compression loading direction for square $(+45/-45/0/90)_{2s}$ laminate with rectangular cutouts on buckling load, first-ply failure loads, maximum transverse deflection and ultimate failure loads, [for d/b = 0.14 (constant)]

Cutout aspect ratio (c/b)	Buckling load		First-ply failure loads				Ultimate failure load	
	N_xb^2/E_2h^3	N_yb^2/E_2h^3	N_xb^2/E_2h^3	$(w_{max}/h)^*$	N_yb^2/E_2h^3	$(w_{max}/h)^*$	N_xb^2/E_2h^3	N_yb^2/E_2h^3
0.14	17.36	17.36	63.26	3.93	62.11	3.89	113.03	100.00
0.42	14.63	18.79	45.90	3.97	50.70	3.80	83.19	99.55

*Non-dimensionalized maximum transverse deflection associated with the first-ply failure.

deflection associated with first-ply failure of $(+45/-45/0/90)_{2s}$ laminate for cutout of two aspect ratios, c/b = 0.14 & 0.42 with d/b = 0.14, for the independent loading in x- and y- directions. There is no change in the buckling load of the laminate with square cutout with change in the direction of loading from x- to y-direction. However, a higher buckling load is observed when load is applied along the shorter direction of the cutout. This is attributed to higher bending stiffness of the plate in the direction parallel to the shorter direction of the cutout. It is observed that for c/b = 0.14 & d/b = 0.14 (square cutout), there is no appreciable variation in the first-ply failure load and associated maximum transverse deflection with the change in direction of uniaxial compression. However, for the cutout of aspect ratio c/b = 0.42 & d/b = 0.14, the first-ply failure load is larger while the associated maximum transverse deflection is smaller for the y-direction of uniaxial compression than the corresponding values for the x-direction of loading. The ultimate failure load of the laminate with square cutout is smaller for the loading in y-direction than that for the x-direction of loading, while for the larger cutout size (c/b = 0.42), higher ultimate strength is observed for the y-direction of loading. The above appreciable difference in the ultimate failure loads of laminate with square cutout for two directions of loading is due to the difference in bending stiffness of the laminate in two principal directions.

3.2 Effect of edge boundary conditions

The buckling, first-ply and ultimate failure loads, and the maximum transverse deflections (associated with failure loads) of $(+45/-45/0/90)_{2s}$ quasi-isotropic laminate with the square cutout (c/b = 0.14 & d/b = 0.14) and without a cutout for different edge boundary conditions (BC1, BC2 and BC3) are shown in Table 5. As expected, the buckling loads (Table 5) with cutouts are smaller than those without a cutout irrespective of types of boundary conditions at the edges. However, it is important to note that the first-ply and ultimate failure loads of the laminate with cutout (c/b = 0.14) are larger than the corresponding values for laminate without a cutout for all edge boundary conditions except the first-ply failure load for BC2 boundary condition (which is lower in the case of laminate with cutout). This is attributed to the fact that the net section of cutout is the most critical section of failure for BC2 boundary condition while the loaded edge is the critical failure

Table 5 Buckling and failure loads of $(+45/-45/0/90)_{2s}$ quasi-isotropic laminate with and without square cutout for different edge boundary conditions[#]

Load	Without cutout			With cutout		
	BC1	BC2	BC3	BC1	BC2	BC3
Buckling load	18.93	30.55	43.89	17.26	27.61	39.62
First-ply failure load	58.09	90.80	81.80	63.26	74.59[*]	86.24
Ultimate failure load	79.60	95.53	89.50	113.00	112.00[**]	100.70
$(w_{max}/h)^{f\,\$}$	3.70	3.50	1.56	3.93	3.25	1.67
$(w_{max}/h)^{u\$}$	5.36	3.72	1.83	6.56	6.32	2.12

[#]BC1 refers to a plate with all edges simply supported; BC2 refers to a plate with two longitudinal edges (y = 0 and $y = b$) simply supported and the other two edges clamped; and BC3 refers to a plate with all edges clamped.
[*]First-ply failure occurs at the cutout section; [**]A very large transverse deflection occurs prior to this load.
[$]$(w_{max}/h)^f$ and $(w_{max}/h)^u$ are the non-dimensionalized maximum transverse deflections corresponding to first-ply failure and ultimate failure, respectively.

location for BC1 and BC3 boundary conditions. The plausible reason for a higher ultimate failure load of laminate with cutout, in comparison to that without cutout is the preclusion of early onset of delamination in the laminate with cutout. The onset of delamination is the primary cause of ultimate failure in laminates without cutout, especially in the case of BC2 and BC3 boundary conditions, while the large transverse deflection is cause of ultimate failure of laminates with cutouts for all boundary conditions.

4. Conclusions

Based on the results and discussion, following observations are made:

- The postbuckling strength of square laminates is increased due to introduction of central square cutouts of size c/b = 0.06 to 0.21.
- In the case of simply supported $(+45/-45/0/90)_{2s}$ with a rectangular cutout, the prebuckling and postbuckling stiffness is larger in the shorter direction of the cutout.
- The most critical first-ply failure location for simply supported $(+45/-45/0/90)_{2s}$, $(+45/-45)_{4s}$ and $(0/90)_{4s}$ laminates is the loaded edge for cutouts of size $c/b \leq 0.28$ and the corner of the cutout for $c/b > 0.28$.
- The first-ply failure mode of laminates is dependent on the size of the cutout; it is caused by in-plane normal stresses transverse to the fiber direction (irrespective of the size of the cutout) for $(+45/-45/0/90)_{2s}$ laminate, $(+45/-45)_{4s}$ laminate (with cutout size $c/b < 0.42$) and $(0/90)_{4s}$ laminate (with cutout size $c/b > 0.42$). However, for $(0/90)_{4s}$ laminate with cutout size $c/b <$ 0.42, it is the in-plane shear mode of failure.
- In case of simply supported $(+45/-45/0/90)_{2s}$ laminate with cutout $c/b < 0.06$, the onset of delamination occurs even before the first-ply failure load.
- There is no appreciable change in the mode and the location of the first-ply failure in $(+45/-45/0/90)_{2s}$ laminate with a square cutout due to change in the loading direction, i.e. from x- to y-direction. However, for a rectangular cutout (c/b = 0.42 & d/b = 0.14), the first-ply failure occurs at the corner of the cutout for x-direction of uniaxial compression and at the corner (x = 0 and $y = b$) on the loaded edge of the laminate for y-direction loading.
- For simply supported $(+45/-45/0/90)_{2s}$ laminate with a rectangular cutout (c/b = 0.42 & d/b = 0.14), the onset of delamination is the mode of the ultimate failure for y-direction of uniaxial compression while the large transverse deflection is the cause of the ultimate failure for x-direction of the load.
- There is a monotonic decrease in the buckling and postbuckling strength of simply supported $(+45/-45/0/90)_{2s}$ laminate with an increase in the aspect ratio, c/b of the cutout for a constant d/b ratio of 0.14. However, the variation in the maximum transverse deflection associated with the first-ply failure is not appreciable.
- For $(+45/-45/0/90)_{2s}$ laminate with square cutout (c/b = 0.14), there is no change in the mode of the first-ply failure due to change in the boundary condition at edges (from the simply supported to the clamped). However, the location of the first-ply failure is the loaded edge of the laminate with all edges simply supported/clamped and the corner of the cutout with two parallel edges simply supported & the other two clamped.
- First-ply and ultimate failure loads of $(+45/-45/0/90)_{2s}$ laminate with a square cutout (c/b = 0.14) are larger in comparison to those with no cutout for all boundary conditions except in the

case with two parallel edges simply supported & the other two clamped.

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Notation

The following symbols are used in this paper:

b	: in-plane dimensions of the square plate in x- and y-direction
c and d	: dimensions of cutout in x- and y-direction, respectively
E_1, E_2 and E_3	: principal Young's moduli in fiber direction and other two transverse directions, respectively
G_{12}, G_{13} and G_{23}	: shear moduli associated with planes 1-2, 1-3 and 2-3, respectively
h	: thickness of the square plate
N_x and N_y	: applied uniaxial compression loads per unit width in x- and y-direction, respectively
R, S and T	: shear strengths of lamina in planes 2-3, 1-3 and 1-2, respectively
w_{max}	: maximum transverse deflection
X_t and X_c	: tensile and compressive strengths of lamina in fiber direction, respectively
Y_t and Y_c	: tensile and compressive strengths of lamina in direction transverse to fiber, respectively

Z_t and Z_c : tensile and compressive strengths of lamina in principal material direction-3, i.e. perpendicular to plane of lamina, respectively
ν_{12}, ν_{13} and ν_{23} : Poisson's ratios associated with planes 1-2, 1-3 and 2-3, respectively
σ_3 : transverse normal stress component in principal material direction 3
σ_{DN} : peel strength equal to the tensile normal transverse strength of lamina
σ_{DS} : interlaminar shear strength equal to transverse shear strength corresponding to the plane 1-3 of lamina
θ : fiber orientation with respect to x-direction; and
τ_{13}, τ_{23} : transverse shear stress components in principal material planes 1-3 & 2-3, respectively.

Appendix

Interlaminar failure criterion (Singh 1996)

As per the interlaminar failure criterion, the onset of delamination takes place when the interlaminar transverse stress (calculated by integration of equilibrium equations) components satisfy the following expression:

$$\left(\frac{\sigma_3}{\sigma_{DN}}\right)^2 + \frac{\tau_{13}^2 + \tau_{23}^2}{\sigma_{DS}^2} \geq 1$$

Where, σ_3 is the transverse normal stress component; τ_{13}, τ_{23} are the transverse shear stress components in principal material planes 1-3 and 2-3, respectively; σ_{DN} is the peel strength and σ_{DS} is the interlaminar shear strength; these are taken equal to the tensile normal transverse strength and transverse shear strength (corresponding to the plane 1-3) of lamina, respectively.