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Frequency analysis of moderately thick uniform isotropic annular plates by discrete singular convolution method

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Abstract. In the present study, free vibration analysis of thick annular plates is analyzed by discrete singular convolution method. The Mindlin plate theory is employed. The material is isotropic, homogeneous and obeys Hook's law. In this paper, discrete singular convolution method is used for discretization of equations of motion. Axisymmetric frequency values are presented illustrating the effect of radius ratio and thickness to radius ratio of the annular plate. The influence of boundary conditions on the frequency characteristics is also discussed. Comparing results with those in the literature validates the present analysis. It is shown that the obtained results are very accurate by this approach.

Keywords: free vibration; discrete singular convolution; frequency; annular plate; thick plate.

1. Introduction

Thick circular, sectorial and annular plates have been widely used in many engineering applications, for example, civil and mechanical engineering, nuclear, petroleum and aerospace structures (Han and Liew 1998, Irie *et al.* 1982, Lime and Wang 2000, Liu *et al.* 2001). The bending and vibration analysis of such plates is, therefore, of great importance in practical design. In the literature, various methods have been used for vibration analysis of annular or sectorial plates. Han and Liew (1998, 1999) used differential quadrature method to obtain static and free vibration solutions of plates. Wang and Wang (2004) and Wang *et al.* (1995) proposed a new kind differential quadrature for free vibration analysis of circular and annular plates with uniform and non-uniform thickness. Annular sector and stepped circular and rectangular Mindlin plates have been studied by Xiang *et al.* (1993, 2002) and Xiang and Zhang (2005). Liew and Liu (2000) and Liew and Yang (2000) presented a numerical solution for free vibration of shear deformable annular

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plates. Irie *et al*. (1982) give some benchmark results for annular plates. Zhou *et al.* (2003, 2006, 2006a) have investigated three-dimensional vibration of circular and annular plates. Transverse vibration analysis of circular, annular and sector plates have been studied by Liew *et al.* (1995), Liew and Lam (1993), Wu *et al.* (2002). By using finite element method, axisymmetric vibration analysis of circular and annular plates was presented by Liu and Chen (2001). Some results related to bending analysis of circular and annular plates have also been in literature (Wu and Liu 2001, Civalek 2007a). Detailed reviews have been made by Leissa (1987). The present study deals with the free vibration analysis of thick annular plates by the method of discrete singular convolution (DSC). This is the first time that the DSC method is used for vibration analysis of thick annular plates.

2. Basic formulations

Following references (Han and Liew 1999) the annular plate is shown in Fig. 1. The governing equations for axisymmetric free vibration are given (Han and Liew 1999)

$$D\left(r^2\frac{\partial^2\psi}{\partial r^2}+r\frac{\partial\psi}{\partial r}-\psi\right)-6\kappa(1-v)r^2\left(\psi+\frac{\partial w}{\partial r}\right)-\frac{h^2}{a^4}r^2\frac{\partial^2\psi}{\partial t^2} = 0 \qquad (1a)$$

$$\left(r\frac{\partial^2 w}{\partial r^2}+\frac{\partial w}{\partial r}+r\frac{\partial\psi}{\partial r}+\psi-\frac{2r}{(1-v)\kappa}\frac{\partial^2 w}{\partial t^2}\right) = 0 \qquad (1b)$$

where D is the flexural rigidity, v is the Poisson's ratio, κ is the shear correction factor, t is the time, W and Ψ are the transverse deflection and angular rotation, h is the thickness.

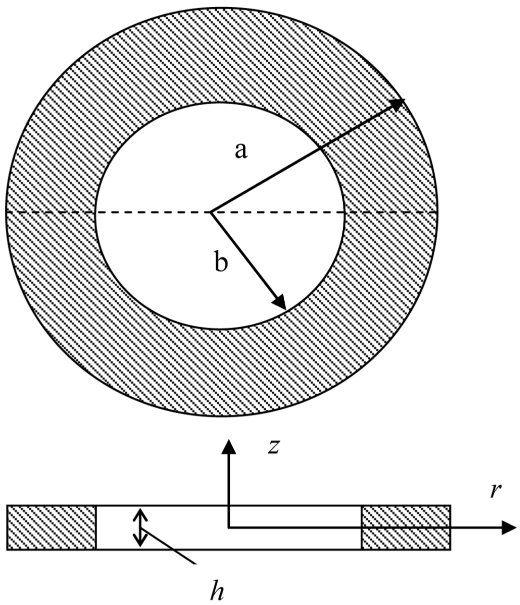

Fig. 1 Geometry of annular plate

For free vibration we can write

$$w(R,t) = W(R)\cos(\omega t) \tag{2a}$$

$$\psi(R,t) = \Psi(R)\cos(\omega t) \tag{2b}$$

Using Eq. (2) the governing equations of the motion given in non-dimensional form as

$$\frac{h^2}{a^2}\left(R^2\frac{d^2\Psi}{dR^2} + R\frac{d\Psi}{dR} - \Psi\right) - 6\kappa(1-v)R^2\left(\Psi + \frac{dW}{dR}\right) - \frac{h^2}{a^2}R^2\Omega^2\Psi = 0 \tag{3}$$

$$\left(R\frac{d^2W}{dR^2} + \frac{dW}{dR} + R\frac{d\Psi}{dR} + \Psi - \frac{2R}{(1-v)}\frac{\Psi^2 W}{\kappa}\right) = 0 \tag{4}$$

In these two equations following non-dimensional parameters are used

$$R = r/a,\ W = w/a,\ \Omega^2 = \omega a^2\sqrt{\rho h/D},\ \ T = t\sqrt{E/\rho a^2(1-\upsilon)^2} \tag{5}$$

3. Discrete singular convolution (DSC)

In recent years, a new kind of numerical approach based on distributions and wavelet analysis called discrete singular convolution is being developed in the field of mathematical physics and computational mechanics by Wei (1999, 2000, 2000a, 2000b). Discrete singular convolution (DSC) is an efficient technique for the numerical solutions of differential equations (Wei and Gu 2002, Wei *et al.* 2002, Zhou and Wei 2006). The method of discrete singular convolution has been used many of problems related to solid and fluid mechanics (Wei 2001b, Wei *et al.* 2002a, Wan *et al.* 2002, Zhao *et al.* 2002,2002a, Zhao and Wei 2002, Ng *et al.* 2004, Zhao *et al.* 2005, Lim *et al.* 2005, Civalek 2006, 2006a, 2007, 2008) by this time. Following the related reference (Wei 2001) and using same notation, consider a distribution, T and $\eta(t)$ as an element of the space of the test function. A singular convolution can be defined by Wei (2001)

$$F(t) = (T*\eta)(t) = \int_{-\infty}^{\infty} T(t-x)\eta(x)dx \tag{6}$$

where $T(t-x)$ is a singular kernel. The DSC algorithm can be realized by using many approximation kernels. However, it was shown (Wei *et al.* 2001, 2002, Zhao *et al.* 2002, Lim *et al.* 2005a, Civalek, 2007c, 2007d, 2007e) that for many problems, the use of the regularized Shannon kernel (RSK) is very efficient. The RSK is given by Wei (2001a)

$$\delta_{\Delta,\sigma}(x-x_k) = \frac{\sin[(\pi/\Delta)(x-x_k)]}{(\pi/\Delta)(x-x_k)}\exp\left[-\frac{(x-x_k)^2}{2\sigma^2}\right];\quad \sigma > 0 \tag{7}$$

where $\Delta = \pi/(N-1)$ is the grid spacing and N is the number of grid points. The parameter σ determines the width of the Gaussian envelope and often varies in association with the grid spacing, i.e., $\sigma = rh$. In the DSC method, the function $f(x)$ and its derivatives with respect to the x coordinate at a grid point x_i are approximated by a linear sum of discrete values $f(x_k)$ in a narrow bandwidth

$[x - x_M, x + x_M]$. This can be expressed as (Xiang *et al.* 2002)

$$\left.\frac{d^n f(x)}{dx^n}\right|_{x = x_i} = f^{(n)}(x_i) \approx \sum_{k=-M}^{M} \delta_{\Delta,\sigma}^{(n)}(x_i - x_k) f(x_k)\,; \quad (n = 0, 1, 2, \ldots) \tag{8}$$

where superscript n denotes the nth-order derivative with respect to x. After employing the DSC method, the governing equations for vibration (Eq. (3)) become

$$\frac{h^2}{a^2}\left[R^2 \sum_{k=-M}^{M} \delta_{\Delta,\sigma}^{(2)}(k\Delta R)\Psi_k + R\sum_{k=-M}^{M} \delta_{\Delta,\sigma}^{(1)}(k\Delta R)\Psi_k - \Psi_i\right]$$
$$-6\kappa(1-\nu)R^2\left[\Psi_i + \sum_{k=-M}^{M} \delta_{\Delta,\sigma}^{(1)}(k\Delta R)W_k\right] - \frac{h^2}{a^2}R^2\Omega^2\Psi_i = 0 \tag{9}$$

$$R\sum_{k=-M}^{M} \delta_{\Delta,\sigma}^{(2)}(k\Delta R)W_k + \sum_{k=-M}^{M} \delta_{\Delta,\sigma}^{(1)}(k\Delta R)W_k + R\sum_{k=-M}^{M} \delta_{\Delta,\sigma}^{(1)}(k\Delta R)\Psi_k$$
$$+\Psi_i + \frac{2R}{(1-\nu)\kappa}\Omega^2 W_i = 0 \tag{10}$$

The higher order derivative terms $\delta_{\Delta,\sigma}^{(n)}(x - x_k)$ in these equations are given as below (Zhao *et al.* 2002)

$$\delta_{\Delta,\sigma}^{(n)}(x - x_k) = \left(\frac{d}{dx}\right)^n [\delta_{\Delta,\sigma}(x - x_k)]$$

where, the differentiation can be carried out analytically. Simply supported, clamped and free boundary conditions are considered. The discretized forms of these boundary conditions are as follows for inner edge:

For clamped boundary

$$W = 0 \quad \text{and} \quad \Psi = 0 \tag{11}$$

For simply supported boundary

$$W = 0 \tag{12}$$

$$\sum_{k=-M}^{M} \delta_{\Delta,\sigma}^{(1)}(k\Delta R)\Psi_k + \frac{\nu}{R}\Psi_1 = 0 \tag{13}$$

For free edge

$$\Psi_1 + \sum_{k=-M}^{M} \delta_{\Delta,\sigma}^{(1)}(k\Delta R)\Psi_k = 0 \tag{14}$$

$$\sum_{k=-M}^{M} \delta_{\Delta,\sigma}^{(1)}(k\Delta R)\Psi_k + a\frac{\nu}{b}\Psi_1 = 0 \tag{15}$$

For the outer edge the DSC form of these boundary conditions can be written by substituting the subscripts 1 with N, and $a/b = 1$.

4. Numerical results

For numerical results, the Poisson's ratio is taken to be 0.3 and the shear correction factor is taken as $k = 0.823(\pi^2/12)$. In order to simplify the results, the boundary conditions for plates are denoted by letters S (simply supported), F (free) and C (clamped). For example, SC denotes that the annular plate is simply supported at inner edge and clamped at outer edge. Some comparison study of the present results for thick annular plates with different thickness to radius ratios and CC, CS and SS boundary conditions is made for verification of the present DSC formulation. The obtained results are compared with the analytical results given by Irie *et al.* (1982) and with the DQ results given by Han and Liew (1999). These results are summarized in Tables 1, 2 and Fig. 2.

It can be seen that the values calculated by the present method shows good agreement. It is also concluded that, the accuracy of the results is increased by increasing N. The reasonable accurate results are obtained for $N = 15$. Natural frequencies of annular plate are calculated for different grid numbers to illustrate the convergence of the proposed method ratio. The results are given in Fig. 2 for three different thickness to radius ratios of CS annular plates ($b/a = 0.3$). It is also seen from this figure that when the grid point numbers reaches $N = 11$ the present method gives accurate predictions for $h/a = 0.2$. For other plate with different thickness, however, the accurate results are obtained for $N = 15$. A different comparison study of the present DSC results with the finite

Table 1 Comparison of frequency parameters of CC annular plates ($h/a = 0.1$)

b/a	Han and Liew (1999)	Irie *et al.* (1982)	Present
0.1	24.629	24.63	24.632
0.2	30.841	30.84	30.843
0.3	39.398	39.40	39.405
0.5	70.277	70.28	70.286

Table 2 Convergence of natural frequency of SS annular plate with different h/a ratio ($b/a = 0.2$)

Sources	h/a			
	0.001	0.1	0.2	0.3
Irie *et al.* (1982)	-	16.16	14.69	12.97
Han and Liew (1999)	16.780	16.164	14.688	-
Present DSC results $N = 9$	17.236	17.883	15.227	13.914
Present DSC results $N = 11$	17.011	16.174	14.703	13.018
Present DSC results $N = 13$	16.804	16.169	14.698	12.981
Present DSC results $N = 15$	16.785	16.166	14.695	12.975
Present DSC results $N = 18$	16.784	16.163	14.694	12.973
Present DSC results $N = 21$	16.784	16.163	14.694	12.973

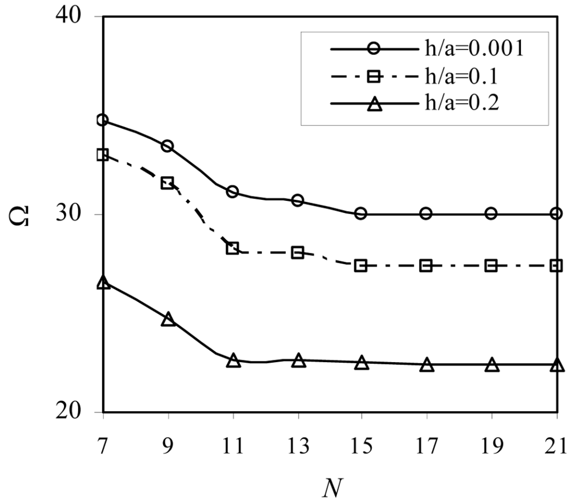

Fig. 2 Convergence of the first frequency with grid numbers of CS annular plates (b/a = 0.3)

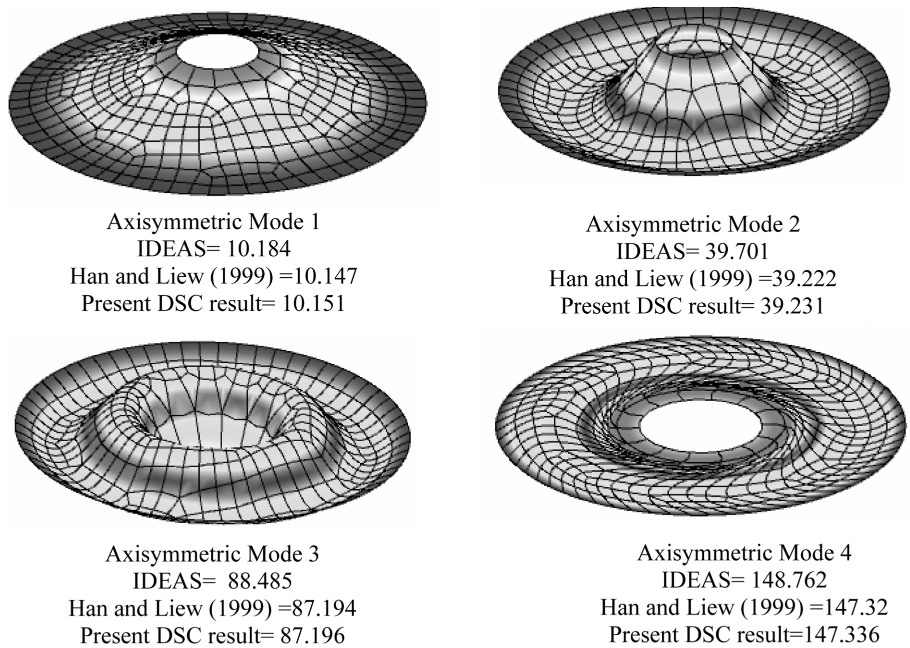

Fig. 3 First four axisymmetric non-dimensional frequencies of C-F annular Mindlin plates (h/a = 0.1, b/a = 0.2)

element solutions obtained using the IDEAS software package has been made and presented in Fig. 3. First four axisymmetric frequencies of C-F annular Mindlin plates are presented in this figure with the results given by Han and Liew (1999) using differential quadrature method. A good agreement is achieved amongst the present results, results given by Han and Liew (1999) and the results produced by IDEAS using Mindlin shell element. The results given by IDEAS are included both the symmetric and axisymmetric modes. We choose only axisymmetric results for comparison.

Variation of frequency parameter with radius ratio *a/b* for annular plate with different boundary conditions has been shown in Figs. 4-6. Nine different boundary conditions are taken into

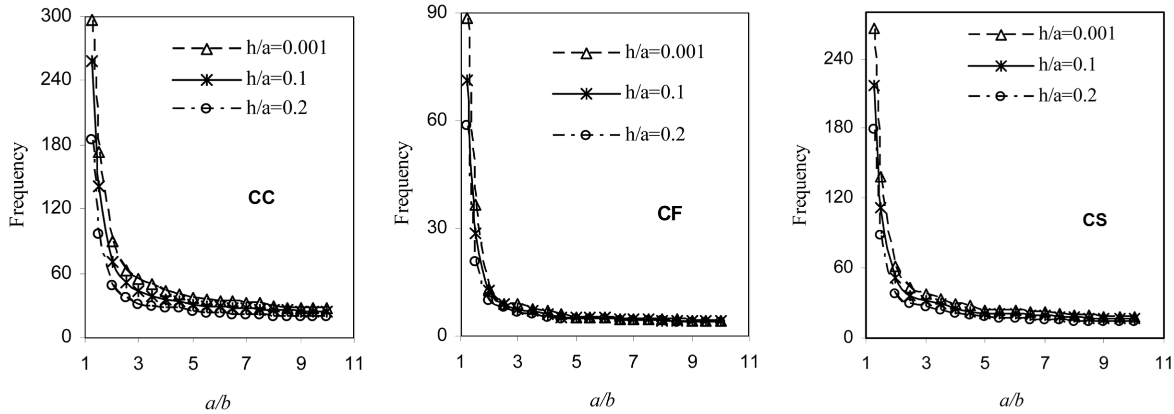

Fig. 4 Variation of frequency parameter with radius ratio *a/b* for annular plate with clamped inner edge

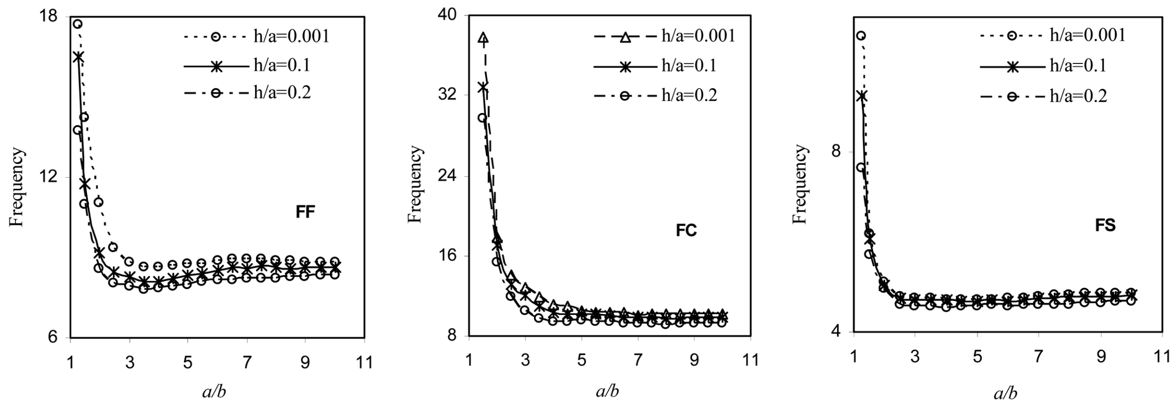

Fig. 5 Variation of frequency parameter with radius ratio *a/b* for annular plate with free inner edge

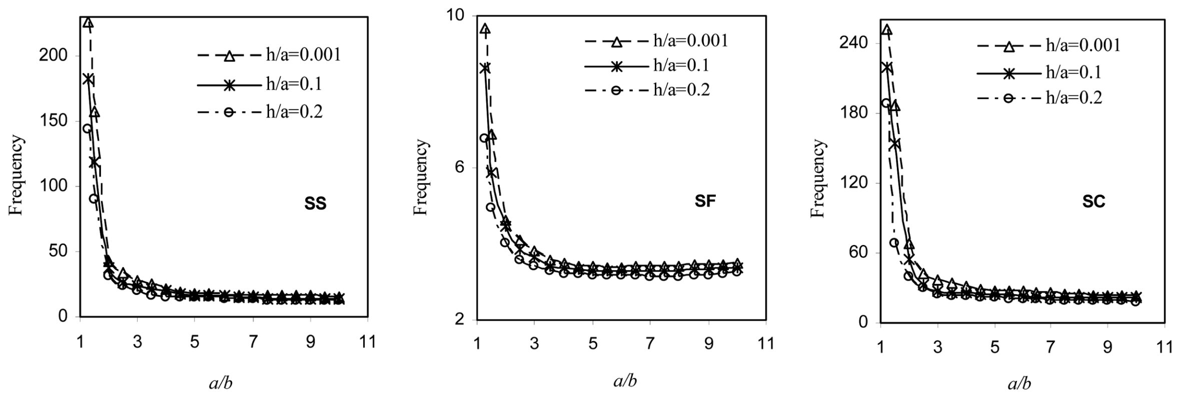

Fig. 6 Variation of frequency parameter with radius ratio *a/b* for annular plate with simply supported inner edge

consideration in these figures. From Figs. 4-6, it is seen that, the frequency parameter decreases rapidly for small radius ratio a/b ($a/b \leq 3$). With the increase of radius ratio ($a/b > 3$) the effect of the a/b ratio on the frequency parameter is insignificant. It is shown that the increasing value of h/a ratio always decreases the frequency parameter. It is also shown in these figures, the plate with clamped inner edges have the highest frequency parameter, followed by the simply supported and free.

The relationships between the frequency parameter with thickness-to-radius ratio h/R for annular plate are depicted in Figs. 7-9 for different a/b ratios. Generally, it can be seen that, as h/R increases, with a/b fixed, the frequencies decrease. Furthermore, the effect of the h/a ratio on the frequency parameter is more significant for small a/b ratios. In Figs. 10 and 11, the variations of the first four frequency parameters for SS and CC plates with radius ratio and thickness to radius ratio are demonstrated respectively. The frequency parameter decreases rapidly for small radius ratio a/b. With the increase of this ratio the effect of the a/b ratio on the frequency parameter is insignificant.

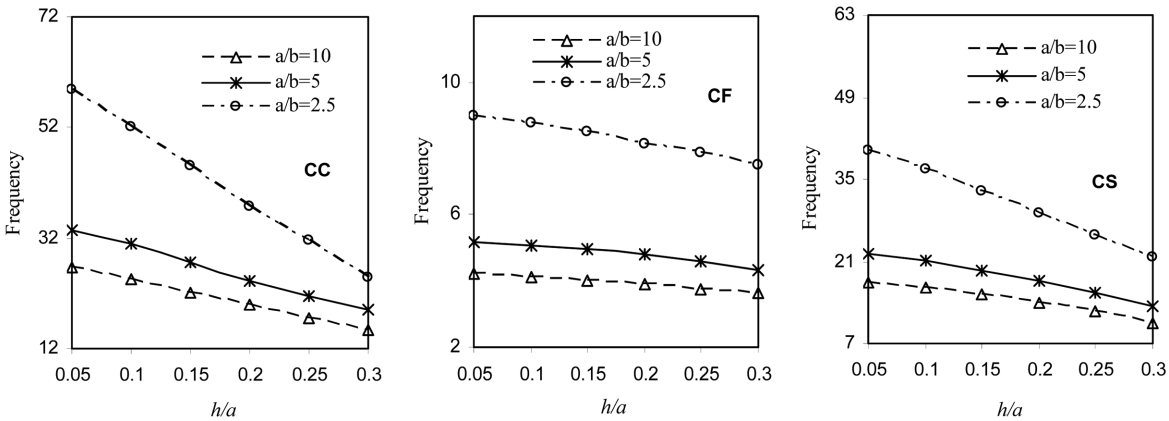

Fig. 7 Variation of frequency parameter with thickness to radius ratio h/a for annular plate with clamped inner edge

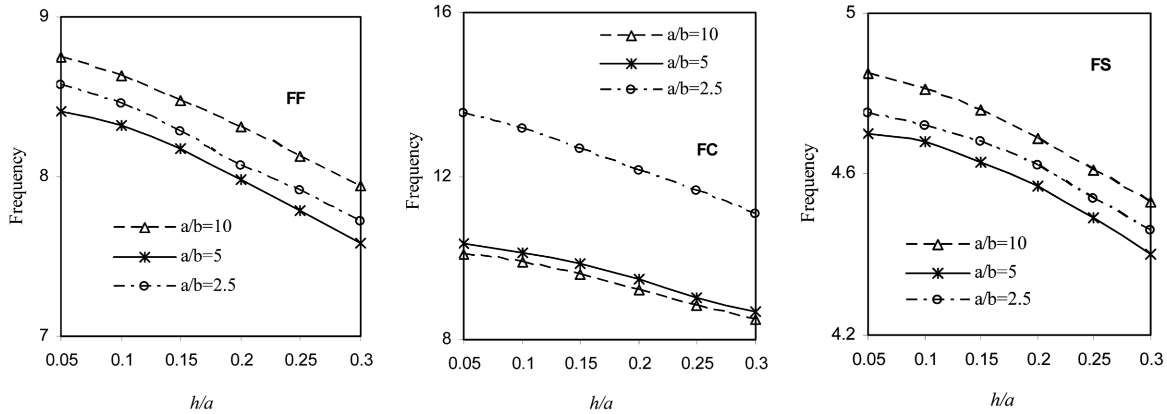

Fig. 8 Variation of frequency parameter with thickness to radius ratio h/a for annular plate with free inner edge

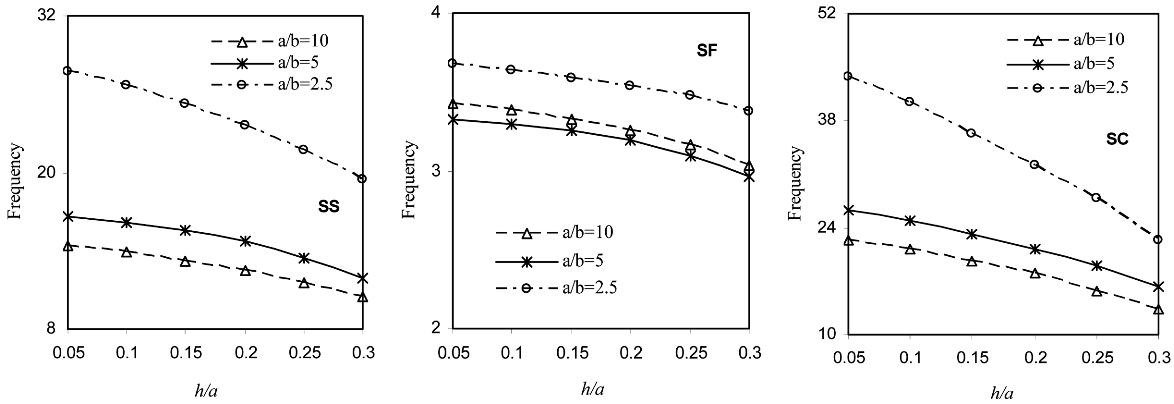

Fig. 9 Variation of frequency parameter with thickness to radius ratio *h/a* for annular plate with simply supported inner edge

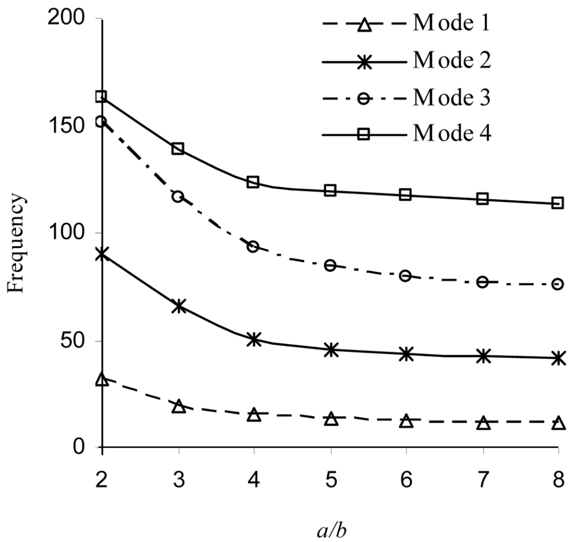

Fig. 10 Variation of frequency parameter of SS plate with radius ratio *a/b* for different mode numbers ($h/a = 0.2$)

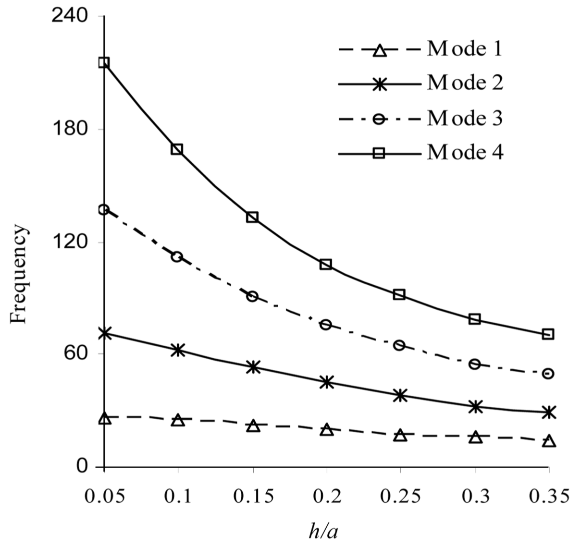

Fig. 11 Variation of frequency parameter of CC plate with thickness to radius ratio *h/a* for different mode numbers ($b/a = 0.1$)

Furthermore, with the increase of mode number, the effect of the *a*/*b* on the frequency parameter is more significant. Similarly, the frequency parameter decreases rapidly for thickness to radius ratio *h/a*. The effect of the mode numbers on the frequency is more significant for the ratio of *h*/*a* than the *a/b* ratio.

The relationship between the frequency parameter and mode numbers at different radius ratio is shown in Fig. 12. It is shown that the increasing value of mode numbers always increases the frequency parameter.

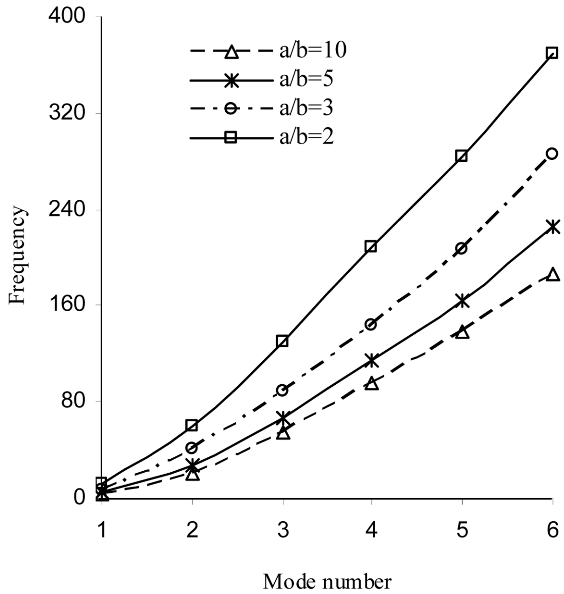

Fig. 12 Variation of frequency parameter of CF plate with mode number for different radius ratio (h/a = 0.15)

5. Conclusions

In the present paper, the free vibration behavior of annular late is studied by the method of discrete singular convolution. Mindlin plate theory is adopted. The obtained results are verified with available analytical and numerical solutions and also the finite element solutions obtained by IDEAS package programs. The effect of some geometric parameters on frequency parameters is investigated. Different combinations of inner and outer boundary conditions were also investigated. It is also important to note that the present method provides a controllable numerical accuracy by using the suitable bandwidth. The results show that the thicknesses to radius ratio, boundary conditions and radius ratio have more important effect on frequencies.

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