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Brittle fracture analysis of the offset-crack DCDC specimen

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Abstract. Applications of fracture mechanics in the strength analysis of ceramic materials have been lately studied by many researchers. Various test specimens have been proposed in order to investigate the fracture resistance of cracked bodies under mixed mode conditions. Double Cleavage Drilled Compression (DCDC) specimen, with a hole offset from the centerline is a configuration that is frequently used in sub-critical crack growth studies of ceramics and glasses. This specimen exhibits a strong crack path stability that is due to the strongly negative *T*-stress term. In this paper the maximum tensile stress (MTS) criterion is employed for investigating theoretically the initiation of brittle fracture in the DCDC specimen under mixed mode conditions. It is shown that the *T*-stress has a significant influence on the predicted fracture load and the crack initiation angle. The theoretical results suggest that brittle fracture in the DCDC specimen is controlled by a combination of the singular stresses (characterized by K_I and K_{II}) and the non-singular stress term, *T*-stress.

Keywords: brittle fracture; mixed mode loading; ceramic materials; fracture analysis.

1. Introduction

It is widely recognized that stress concentration around the crack tip has a great influence on the strength of cracked bodies. Accordingly it seems necessary to have a careful study of the factors which contribute to this concentration. There have been many studies concerned with the determination of fracture parameters and various criteria to predict brittle fracture in ceramic materials. While studies of failure resistance in ceramics have traditionally been concerned with the determination of tensile (or opening) fracture toughness K_{Ic}, flaws in real structures are rarely subjected to pure tensile loading. Catastrophic fracture can be initiated in structural ceramic components at cracks or pre-existing flaws which are oriented at arbitrary angles to the far-field loading directions or subjected to multi-axial stresses. Thus the efficient and safe design of advanced ceramics for potential structural applications very often requires the development of fracture data and failure criteria for combinations of mode-I and mode-II loading (Suresh *et al.* 1990, Fett *et al.* 1995, Ayatollahi and Bagherifard 2005).

In this paper, the Double Cleavage Drilled Compression (DCDC) specimen with a hole offset from the centerline (Ritter 2000) is studied as one of the favorite practical configurations used for

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the determination of crack growth condition in ceramic materials under mixed mode I/II loading. After compilation of literature results on fracture parameters including the mixed mode stress intensity factors, K_I and K_{II} and the T-stress term, brittle fracture of the specimen is predicted using a generalized maximum tangential stress.

2. Elastic stresses around the crack tip

Crack growth in ceramic materials is usually abrupt and unexpected, and hence its mechanism is categorized as brittle fracture. Brittle fracture contains little macro or micro plasticity and involves low-energy absorption and very high crack growth velocity during the fracture process (Meguid 1989). Therefore linear elastic fracture mechanics (LEFM) which is based on the assumption that material condition is predominantly linear elastic during the fracture process can be applicable to ceramics. The loading modes of cracked bodies, the stress distribution in the vicinity of crack tip and the essential fracture parameters are briefly discussed here. There are different modes of crack deformation in fracture mechanics. The basic modes are defined as:

- Mode I: opening mode
- Mode II: in-plane shear or sliding mode
- Mode III: tearing or anti-plane shearing mode
- Mixed mode: a combination of two or three of the above basic modes

Mode I loading condition has received the greatest amount of attention. However, in real ceramic structures due to arbitrary distribution of flaws, the cracks are very often subjected to multi-axial stresses. Therefore, a criterion taking the mixed mode conditions into account should be employed to predict brittle fracture of ceramic materials.

Consider a through-thickness sharp crack in a linear elastic isotropic body (as shown schematically in Fig. 1). Elastic stress field under mixed mode loading at any point near the crack tip can be described as below.

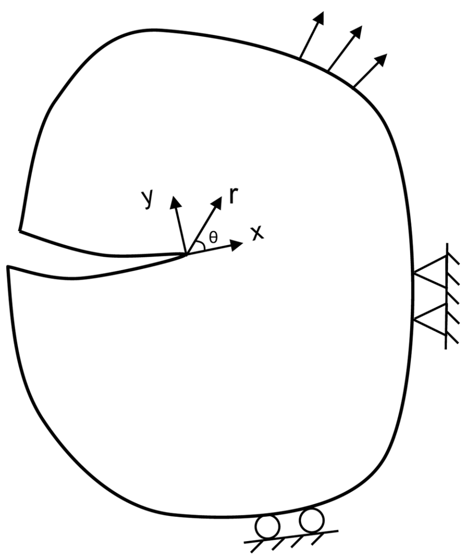


Fig. 1 Cartesian (x, y) and polar (r, θ) co-ordinate systems at the crack tip

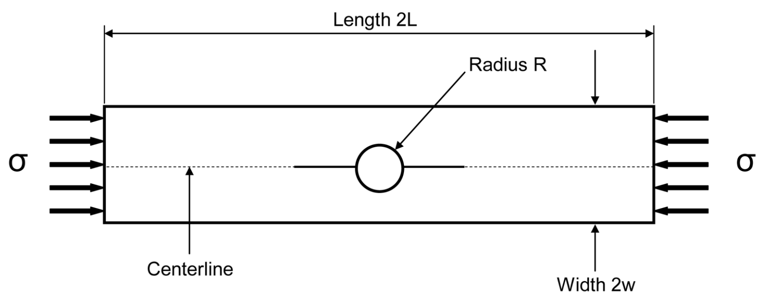
$$\sigma_{ij} = \frac{K_I}{\sqrt{2\pi r}} f_{ij}^I(\theta) + \frac{K_{II}}{\sqrt{2\pi r}} f_{ij}^{II}(\theta) + T g_{ij}(\theta) + O(r^{1/2}) \quad (1)$$

where σ_{ij} denotes the stress tensor, K_I and K_{II} are the mode I and mode II stress intensity factors, respectively, T is the non-singular elastic T-stress, and $f_{ij}(\theta)$ and $g_{ij}(\theta)$ are angular functions. In Eq. (1) the higher order terms shown here by $O(r^{1/2})$ exist, but they are negligible in the vicinity of the crack tip.

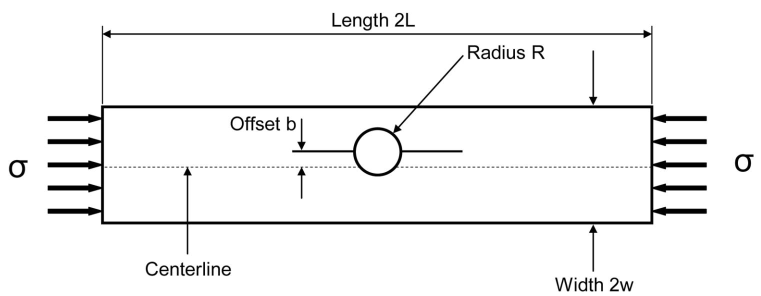
K_I and K_{II} are important fracture parameters which contribute to the fracture process. The values of stress intensity factors depend on the applied load, crack shape, mode of crack deformation, and finally the configuration of specimen or structure. For simple geometries, the numerical values of stress intensity factors can be calculated using analytical methods but for complicated configurations, the use of computational techniques or experimental methods is unavoidable. The T-stress term which is a constant and non-singular stress component acting parallel to the crack plane, can have considerable effect on crack growth process as well (Smith *et al.* 2001).

3. DCDC specimen

The Double Cleavage Drilled Compression (DCDC) specimen is one of the favorite configurations used for the determination of crack growth conditions. The specimen in its standard form, as illustrated in Fig. 2(a), consists of a rectangular bar with a circular hole along its centerline and loaded by a uniform compressive pressure. Due to the presence of the central hole, tensile stresses are generated in the vicinity of the crack tip and cause a positive mode-I stress intensity factor. This specimen was developed by Janssen (1974, 1980) in an effort to obtain a specimen with



(a) Mode I specimen.



(b) Mixed mode specimen.

Fig. 2 Schematic representations of DCDC hole/crack configurations crack length independent fracture toughness. While partial success in developing a constant K_I fracture specimen was achieved, later studies (Smith 1987) showed that this specimen also has many advantages for investigating fracture in brittle materials. The DCDC configuration, offers distinct experimental advantages for measuring crack growth conditions, including experimental simplicity of compressive loading, ease of pre-cracking, mid-plane crack stability and stable crack growth (Michalske *et al.* 1993, He *et al.* 1995). The compressive loading facilitates fixturing for high temperature controlled environment while the auto pre-cracking and crack plane stability eliminate the need for complex machining steps which can introduce an additional source of experimental error in fracture measurements.

In order to obtain results for mixed mode stress intensity factors, the energy release rate, the crack opening displacement and other fracture parameters, many modified versions of this geometry have been developed and studied. One of the mostly used configurations of the DCDC specimen generating mixed mode I/II loading condition is shown in Fig. 2(b). In this configuration, the axis of the hole and the crack plane are offset from the centerline.

Brittle fracture has been studied in various experimental and analytical research studies using these configurations. The fracture parameters are available in the literature for both geometries (Turnet *et al.* 1995, Lardner *et al.* 2001, Fett and Munz 2003, Fett and Rizzi 2005).

The DCDC specimen under mode I has been analyzed for interface energy and crack opening displacement measurements by several researchers (Michalske *et al.* 1993, Turner *et al.* 1995). The mixed mode specimen has also been utilized in studies of sub-critical crack growth in both monolithic glass and glass/epoxy/glass specimens (Ritter *et al.* 1998, 2000). The results of the studies indicate that no significant deviation from the initial fracture plane is expected in crack extension tests carried out with these specimens (Fett and Munz 2003, Fett and Rizzi 2005, Fett *et al.* 2005).

This fact is recognized as a consequence of strongly negative *T*-stress in these special configurations.

As mentioned before, the purpose of this paper is to study the initiation of crack growth in the offset-crack DCDC specimen under mixed mode loading. A key requirement for fracture predictions is the availability of fracture parameters (K_I, K_{II} and the *T*-stress). Therefore, the experimental calibrations and analytical solutions reported in previous studies were reviewed and finally the results obtained in the recent finite element analyses (Lardner *et al.* 2001, Fett and Rizzi 2005) were selected on account of the wider geometrical scope and also fair agreement with the experimental results. Further details will be given in the next section.

4. Brittle fracture prediction in mixed mode DCDC specimen

In order to predict the onset of brittle fracture, several criteria have been suggested for specimens subjected to different combinations of mode I and mode II. Most of the criteria employed in fracture predictions ignore the effect of *T*-stress in their estimations and just account for the dominant singular stresses existing in the vicinity of crack tip e.g. (Erdogan and Sih 1963, Sih 1973, Hussain *et al.* 1974).

The fracture behavior of the offset-crack DCDC specimen can be studied for example by using the well-known maximum tangential stress (MTS) criterion (Erdogan and Sih 1963). This criterion postulates that brittle fracture in mixed mode loading initiates from the crack tip along the direction

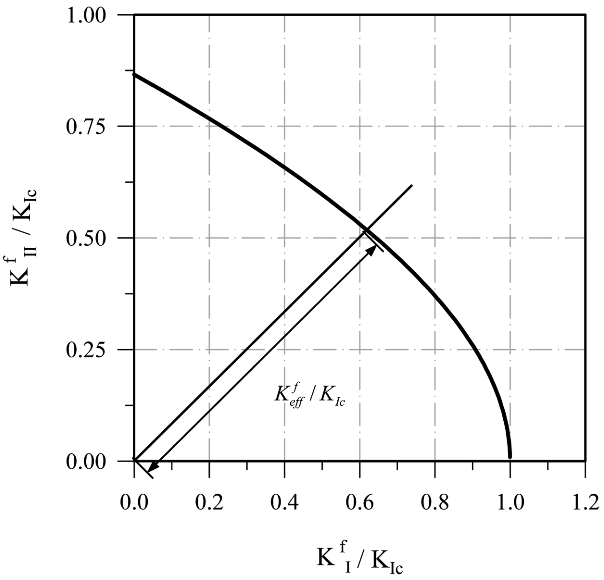


Fig. 3 Onset of crack growth in mixed mode brittle fracture based on the MTS criterion (Erdogan and Sih 1963)

of maximum tangential stress. The onset of brittle fracture also takes place when the tangential stress along this direction attains a critical value. Erdogan and Sih (1963) formulated this criterion in terms of mode I and mode II stress intensity factors and mode I fracture toughness, K_{Ic}. The fracture envelope resulting from the MTS criterion is shown in Fig. 3. For a mixed mode crack problem with given values of K_I and K_{II}, a line with the slope of K_{II}/K_I is plotted from the origin (see Fig. 3). The crack growth initiates when the effective stress intensity factor $K_{eff} = (K_I^2 + K_{II}^2)^{1/2}$, reaches its corresponding critical value, K^f_{eff}. According to the maximum tangential stress criterion, brittle fracture occurs in the case of pure mode I (horizontal axis) when $K_I = K_{Ic}$ and for pure mode II (vertical axis) when $K_{II} = 0.87K_{Ic}$, and for general mixed mode I/II when $(K_I^2 + K_{II}^2)^{1/2} = \alpha K_{Ic}$, in which $0.87 < \alpha < 1$.

Meanwhile, recent studies have revealed that the *T*-stress is increasingly being recognized as an important additional characterizing parameter in the analyses of cracked bodies (Smith *et al.* 2001, Ayatollahi and Aliha 2006, 2007).

Some researchers have attempted to achieve a general criterion which considers the *T*-stress effects on fracture process. A generalized maximum tangential stress criterion in which the tangential stress is calculated at a critical distance r_c from the crack tip has been recently suggested by Smith *et al.* (2001). The critical distance r_c is considered to be a material property. The generalized criterion demonstrates that the *T*-stress also plays a significant role in mixed mode brittle fracture. Smith *et al.* developed a series of curves similar to that shown in Fig. 3 for different levels of *T*-stress. According to those curves mixed mode fracture toughness of cracked components increases in the presence of negative *T*-stress and decreases due to a positive *T*-stress. The effects of *T*-stress on mixed mode fracture toughness are more significant for materials having larger critical distance r_c. The theoretical predictions obtained from the generalized maximum tangential stress criterion have been validated by a series of experimental results (Smith *et al.* 2001, Ayatollahi and Aliha 2005, 2006, 2007, Smith *et al.* 2006).

Here we use both the MTS and the generalized MTS criteria for predicting the onset of mixed mode brittle fracture in the offset-crack DCDC specimen. Therefore two cases with different assumptions are considered for this specimen:

Case1- mixed mode I/II with *T*-stress

Case2- mixed mode I/II without *T*-stress

The results obtained for different geometries are compared and discussed. The effects of *T*-stress on the onset of fracture and also on the direction of fracture initiation are presented here by using the parameters, K_{Norm} and θ_{Norm} defined as

$$K_{Norm} = \frac{K_{eff,T,I,II}}{K_{eff,I,II}} \tag{6}$$

$$\theta_{Norm} = \frac{\theta_{0,T,I,II}}{\theta_{0,I,II}} \tag{7}$$

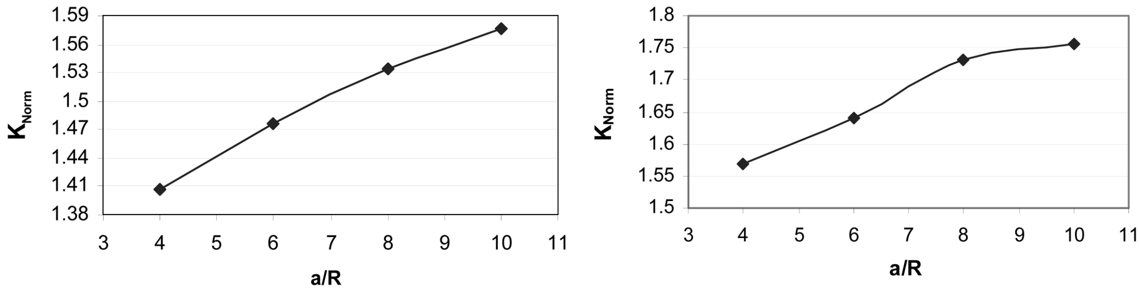
where $K_{eff,I,II}$ and $\theta_{0,I,II}$ are the values of fracture toughness and the crack initiation angle predicted by the conventional MTS criterion, respectively. $K_{eff,T,I,II}$ and $\theta_{0,T,I,II}$ are the similar values but estimated using the generalized MTS criterion which considers the effects of K_I, K_{II} and the *T*-stress. When the parameters K_{Norm} and θ_{Norm} are equal to one, the solution corresponds to the conventional MTS criterion in which the *T*-stress contribution to the failure process is entirely ignored.

5. Results and discussion

In this section the initiation of fracture in the mixed mode DCDC specimen (Fig. 2b) is studied for a variety of material properties and geometry parameters in order to compare the results of the MTS and the generalized MTS criteria. Brittle fracture in the DCDC specimen has been well studied in the past but only for pure mode I. The aim in this paper is to investigate particular conditions in which mode II is dominant. Therefore, our study concentrates only on the specimens having $K_{II}/K_I > 1$. For this purpose, the crack parameters K_I, K_{II} and T should first be known for the specimens. The mode I stress intensity factor and the *T*-stress are calculated using the solutions available in the literature (He *et al.* 1995, Lardner *et al.* 2001, Fett and Munz 2003, Fett and Rizzi 2005, Fett *et al.* 2005) as described earlier. But for the case of mode II stress intensity factor, the available formulae are limited only to very limited number of specimen geometries (He *et al.* 1995, Fett and Munz 2003, Fett and Rizzi 2005). Thus the mode II stress intensity factor is extracted by interpolating the results available in (Fett and Rizzi 2005). The values of fracture parameters related to an arbitrary compressive load P = 100 MPa, b/w = 1/2 and w/R = 4 are listed in Table 1 for

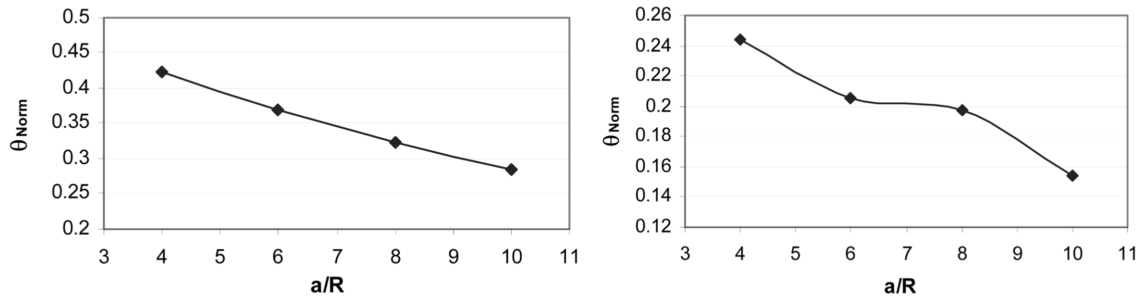
Table 1 Fracture parameters for P = 100 MPa and different values of a/R

a/R	K_I (Mpa$\sqrt{\text{mm}}$)	K_{II} (Mpa$\sqrt{\text{mm}}$)	*T*-stress (MPa)
4	59.29	63.03	−71.94
6	49.98	55.45	−76.25
8	43.12	49.09	−74.90
10	37.94	43.54	−81.82



(a) Alumina (b) Graphite

Fig. 4 K_{Norm} versus crack length ratio for two types of ceramics



(a) Alumina (b) Graphite

Fig. 5 θ_{Norm} versus crack length ratio for two types of ceramics

different values of *a/R*.

These fracture parameters were used in a computer program developed based on the stress field around the crack tip and the equations concerning the MTS and generalized MTS criteria. The results were obtained for two types of ceramics: alumina and graphite. Fracture toughness for alumina and graphite was considered as 3.72 MPa.mm$^{1/2}$ and 0.94 MPa.mm$^{1/2}$, and the critical distance r_c was taken as 0.25 mm and 1 mm, respectively (Aliha 2006).

Fig. 4 shows the variations of K_{Norm} with the crack length ratio *a/R* for both materials. It is seen that K_{Norm} is always larger than 1. Therefore, the negative sign of *T*-stress in the DCDC specimen gives rise to larger values of toughness compared with the conventional MTS criterion. This implies that the generalized MTS criterion, predicts a higher fracture resistance in the mixed mode DCDC specimen. Fig. 4 also indicates that K_{Norm} increases for both materials as the crack length ratio *a/R* becomes larger.

Regarding the influence of *T*-stress on the predicted fracture initiation angle θ_o, Fig 5 shows that the angle between the direction of fracture initiation and the crack line decreases when the effect of *T*-stress is taken into account. In other words, the magnitude of the fracture angle estimated by the generalized MTS criterion is less than that predicted by the conventional MTS criterion. It is also shown that θ_{Norm} becomes less as the crack length ratio *a/R* increases.

The results shown in Fig. 4 and Fig. 5 for the DCDC specimen reveal that there is a significant difference between the fracture loads calculated by the MTS and the generalized MTS criteria. While the sign and magnitude of *T*-stress are the main parameters that affect the generalized MTS criterion, the critical distance r_c also influences the results predicted by this criterion (Smith *et al.*

Table 2 Crack tip parameters for the selected geometries with $w/R = 4$

Geometry number	a/R	b/R	K_I (Mpa$\sqrt{\text{mm}}$)	K_{II} (Mpa$\sqrt{\text{mm}}$)	T-stress (MPa)
1	6	1/2	43.33	20.21	−84.69
2	8	1/4	43.33	8.16	−87.18
3	8	3/4	37.99	29.15	−86.59

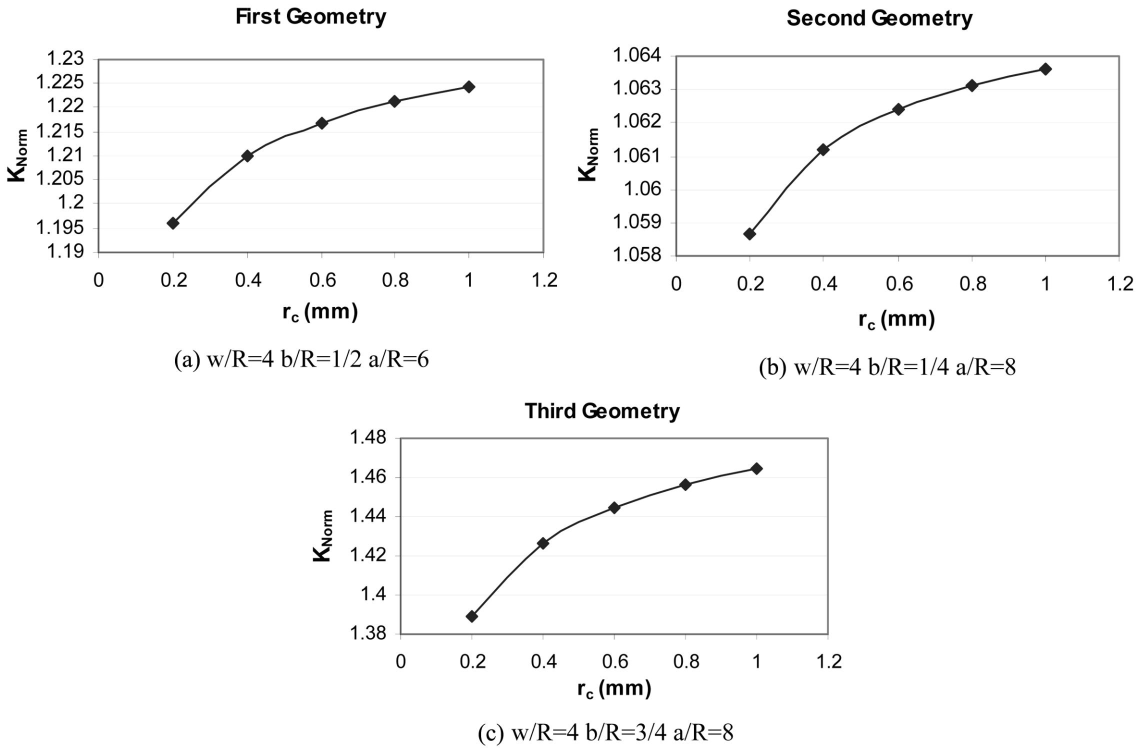


Fig. 6 K_{Norm} for DCDC specimens made of graphite and with different values of r_c

2001). The values of K_{Norm} and θ_{Norm} presented in Fig. 4 and Fig. 5 for alumina and graphite are based on a constant-r_c assumption. Here we examine how sensitive K_{Norm} is to the size of r_c. For this purpose, three different geometries are considered and their fracture parameters K_I, K_{II} and T are calculated from (Lardner *et al.* 2001, Fett and Rizzi 2005). The details of selected geometries are given in Table 2. The material properties related to graphite are used but the size of r_c is assumed to vary from 0.2 mm to 1 mm.

Fig. 6 presents the parameter K_{Norm} versus r_c calculated for the three different specimens. It is seen that K_{Norm} increases for larger values of r_c, that is the effect of T-stress on the results becomes more significant as r_c increases. However, according to Fig. 6(a) to Fig. 6(c), K_{Norm} is not generally much sensitive to r_c. This is because when r_c increases from 0.2 to 1 mm, the change in K_{Norm} is only a few percents.

Recent studies by Ayatollahi and his co-workers have shown that the generalized MTS criterion provides significantly more accurate predictions for mode II and mixed mode fracture toughness of brittle and quasi-brittle materials (Ayatollahi and Aliha 2006, 2007, Smith 2006). For example, when the effect of *T*-stress is ignored (i.e., when the conventional MTS criterion is used), there is an error of about 50 percent between the theoretical estimates and the experimental findings for a rock material under pure mode II loading (Lim *et al.* 1994). This error vanishes when the generalized MTS criterion is employed (Smith *et al.* 2001). Therefore, the results obtained in this research from the generalized MTS criterion for mixed mode fracture analysis of the DCDC specimen are expected to be more accurate than those of the conventional MTS criterion. Fig. 4 demonstrates that the fracture resistance of the DCDC specimens made of alumina and graphite is predicted to enhance even more than 50 percent when the generalized MTS criterion is employed. The enhanced fracture resistance is due to the highly negative *T*-stress existing in the offset-crack DCDC specimen. As reported by researchers in the past (Fett and Munz 2003, Fett and Rizzi 2005), the negative *T*-stress is an advantage for the DCDC specimen since it provides a stable and self similar crack growth in laboratory experiments. However, it is important to note that due to this negative *T*-stress the mixed mode fracture toughness can be significantly overestimated for a ceramic material if measured by using the offset-crack DCDC specimen. This is because the cracked ceramic components in real applications rarely have a *T*-stress as negative as those existing in the DCDC specimen.

6. Conclusions

The generalized MTS criterion which includes the effects of both the singular term and the T-term in the tangential stress around the crack tip was employed to predict brittle fracture in the offset-crack DCDC specimen. Using the generalized criterion, the direction and the onset of mixed mode brittle fracture were predicted for this specimen. A comparison between the results showed that the predictions of the conventional MTS criterion for mixed mode fracture resistance of the DCDC specimen are much lower than those of the generalized MTS criterion. This can be attributed to the highly negative *T*-stress in the DCDC specimen. By increasing the crack length, the *T*-stress becomes more negative and a higher fracture resistance is predicted by the generalized MTS criterion for the specimen. The results also indicate that an improvement is expected in the fracture resistance of the offset-crack DCDC specimen, when the critical distance r_c increases.

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