

# Shear lag prediction in symmetrical laminated composite box beams using artificial neural network

Rajeev Chandak<sup>†</sup>, Akhil Upadhyay<sup>‡</sup> and Pradeep Bhargava<sup>‡‡</sup>

Department of Civil Engineering, Indian Institute of Technology, Roorkee, 247667, India

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**Abstract.** Presence of high degree of orthotropy enhances shear lag phenomenon in laminated composite box-beams and it persists till failure. In this paper three key parameters governing shear lag behavior of laminated composite box beams are identified and defined by simple expressions. Uniqueness of the identified key parameters is proved with the help of finite element method (FEM) based studies. In addition to this, for the sake of generalization of prediction of shear lag effect in symmetrical laminated composite box beams a feed forward back propagation neural network (BPNN) model is developed. The network is trained and tested using the data base generated by extensive FEM studies carried out for various  $b/D$ ,  $b/t_F$ ,  $t_F/t_W$  and laminate configurations. An optimum network architecture has been established which can effectively learn the pattern. Computational efficiency of the developed ANN makes it suitable for use in optimum design of laminated composite box-beams.

**Keywords:** laminated composites; shear lag; effective width ratio; finite element method; artificial neural networks.

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## 1. Introduction

In the elementary theory of bending, the normal stress in the longitudinal direction is assumed to be proportional to the distance from the neutral axis and therefore uniform across the flange width. However, for a beam with wide flanges such as box beams, this assumption becomes invalid: the normal stress distribution is not uniform in the wide flange. The stress is maximum in general at the web-flange junction, decreasing towards the middle of the flange as shown in Fig. 1. This phenomenon is caused by the lag of shear strain in the flange plate between the web plates and is referred to as the shear lag phenomenon. Neglecting this effect amounts to an overestimate of the strength of the beam (Nakai and Yoo 1988). These effects are far more pronounced for highly anisotropic composites than for isotropic materials.

The shear lag problem has long been recognized in engineering fields. The exact solution of shear lag problem is quite complicated and it is difficult to get close form solutions particularly in case of orthotropic material.

The concept of effective width,  $b_e$ , was first proposed by Von Karman (1924) in order to measure

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<sup>†</sup> Research Scholar

<sup>‡</sup> Associate Professor, Corresponding author, E-mail: [akhilfce@iitr.ernet.in](mailto:akhilfce@iitr.ernet.in)

<sup>‡‡</sup> Professor

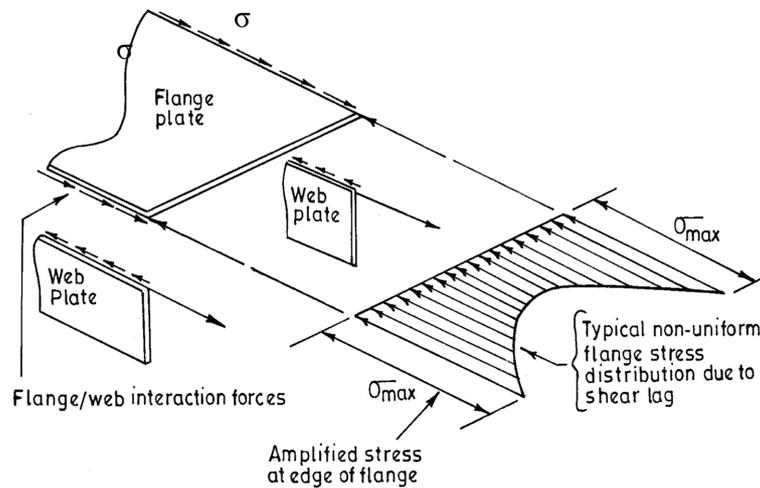


Fig. 1 Stress distribution affected by shear lag

the effect of shear lag in thin-walled structures and has been widely adopted. Several methods are available in the literature for the solution of shear lag problem in box beams. These methods include the energy method (Reissner 1946), the folded-plate method (Kristek 1979), the bar-simulation method (Evans and Taherian 1980), the harmonic analysis method (Kristek *et al.* 1981), the parallel processing method (Fafitis and Rong 1995), the single Fourier series method (Tahan *et al.* 1997) and the finite segment method (Luo *et al.* 2002). Mostly these approaches deal the problem of shear lag with isotropic material.

There are a few studies on shear lag effect in beams made of orthotropic material. Pavlovic *et al.* (1998a, 1998b) studied shear lag effect in rectangular orthotropic plates by deriving closed-form solutions using single Fourier series under in-plane boundary shear. Bending stiffness of flange (being a thin plate) of the girder was neglected and as such the problem is treated as the plane-stress problem. Lopez-Anido and GangaRao (1996) developed a warping solution for shear lag in prismatic thin-walled orthotropic composite beams without considering the ply stress of panels of composite box beam. Wu *et al.* (2002) presented a method for analyzing the shear lag and shear deformation effects on symmetrical laminated thin-walled composite box beams under bending. The method is based on the principle of minimum potential energy. Wu *et al.* (2004) proposed the initial value solutions of static equilibrium differential equations of single-cell thin-walled composite laminated box beams under bending loads. Shear lag effect induced non uniform distribution function of axial displacement along the flange width is expressed by a cubic parabola. Tenchev (1992) conducted two-dimensional finite element analysis using 8-node isoparametric plane stress elements to deal with the shear lag problem. However, orthotropy of material is accounted by considering high E/G ratio. Shear lag effect on glass fiber reinforced plastic (GFRP) pultruded box and wide flange sections was studied by Nagaraj and GangaRao (1997). Pultruded beams, orthotropic in nature, are approximated to be isotropic in arriving at beam rigidity. Upadhyay and Kalyanaraman (2003) presented a semi-empirical method to calculate shear lag in fiber reinforced plastic (FRP) box girders. Expressions are derived for shear-lag parameter using three bar analogy approach.

All these researchers either used analytical approach or FEM to investigate shear lag in box-beams under bending. Analytical methods are based on many simplified assumptions and involve

solution of number of equations whereas FEM is computationally expensive (Seible and Scordelid 1983, Upadhyay and Kalyanaraman 2003). Design of laminated composites is more involved due to large number of design variables, constraints and relative inexperience. Hence necessitates the use of optimization procedure in the design. Optimum design methods require many repeated analysis. The finite element method is prohibitively time consuming and impractical at this stage of design. In view of this a computationally efficient procedure is needed. Recently, ANN has emerged as an efficient tool for such applications.

ANN is a model free estimator and it can be trained provided sufficient database is available. Researchers have started using ANN to study the various problems related with the civil engineering, especially when it is difficult to find an accurate mathematical based solution. Zhang and Friedrich (2003) reviewed the application of ANN to polymer composites and concluded that use of ANN is still in basic stage as far as polymer composites are concerned and recommended the promotion of ANN in this field of research. Mishra and Upadhyay (2004) used ANN for finding out percentage area of steel for rectangular columns subjected to combined axial compression and bending. They used a feed-forward back propagation artificial neural network. It is concluded that developed ANN model have immense potential to predict reasonably accurate values of percentage of steel for columns subjected to combined axial compression and bending. El Kadi (2006) reported the work done in the mechanical modeling of fiber-reinforced composite materials using ANN during the last decade. Use of ANN in medical applications, image and speech recognition, classification and control of dynamic systems is already established but only recently researchers started using this tool in modeling the mechanical behaviour of fiber-reinforced composite materials. Koker *et al.* (2007) investigated the effect of various training algorithms on learning performance of the neural networks on the prediction of bending strength and hardness behaviour of particulate reinforced metal matrix composites. The Levenberg-Marquardt training algorithm was found the fastest converging one and working with high accuracy in prediction. Turias *et al.* (2005) applied ANN to estimate the effective thermal conductivity of a composite in normal direction to the fiber axes. It was shown on the basis of the computing time required that ANN is computationally more efficient than FEM and well-suited for estimation of transverse conductivities.

Lot of work has been done by changing boundary conditions, load type etc. in box-beams made of isotropic material. On the other hand less work is done for beams made of laminated composites where this phenomenon gets enhanced. So, in the present work more emphasis is given on material aspects i.e., on laminated composite beams. ANN also requires large database for training purpose and keeping this fact in mind only one type of boundary condition and load type are considered and large database is developed, in place of dealing with more cases having small databases.

The main objective of the present work is to train an ANN which can serve as a computationally efficient analysis tool. Such tools will be quite useful in the optimum designs of laminated composites. To achieve this objective a large database is generated by FEM studies. Subsequently, key parameters are identified to study the shear lag effect in laminated composite box beam and these are used as input parameters for the ANN.

## 2. Shear lag parameters identification and definitions

An approximate method of dealing with shear lag is to use an effective width concept, in which

the actual width  $b$  of a flange is replaced by a reduced width  $b_e$  given by

$$b_e = b \times (\text{Nominal bending stress/Maximum bending stress}) \quad (1)$$

Nakai and Yoo (1988) reported that effective width of box beam made of isotropic material is affected by two parameters  $\omega$  and  $\kappa$  in addition to  $L/b$  (ratio of span to width of box beam).  $\omega$  and  $\kappa$  represent orthotropy parameter of flange plate and cross-sectional parameter respectively.

On the basis of numerical studies three parameters are identified which govern the shear lag effect in symmetrical laminated composite box beams. These are  $L/b$ ,  $\omega_1$ , and  $\kappa_1$ . The effect of parameter  $L/b$  on shear lag in case of wide flanged box beam is an established fact. As  $L/b$  increases, shear lag effect decreases. The parameter  $\omega_1$  is the orthotropy parameter of the top flange and web and it depends on extensional stiffnesses of these elements. The influence of  $\omega_1$  indicates that the effective width decreases with increase in value of  $\omega_1$ . The parameter  $\kappa_1$  is a cross-sectional shape parameter and the effective width decreases for the larger values of  $\kappa_1$ . Based on work carried out by Upadhyay (1998) on FRP box beam, Rajesh (2005) reported the use of these parameters to study the negative shear lag in laminated composite cantilever box beam. However, the effect of web fiber orientations is not incorporated in parameter  $\omega_1$  which may have significant influence on the phenomenon. In the present work parameter  $\omega_1$  is modified to incorporate the effect of web fiber orientations to study shear lag in symmetrical box beams made up of laminated composites. The modified parameter  $\omega_1$  and parameter  $\kappa_1$  are given

$$\omega_1 = \{(1/2) \times (A_{11}/A_{66})_{TF}\} + \{(1/6) \times (A_{11}/A_{66})_W\} \quad (2)$$

$$\kappa_1 = [(EA)_{TF}/(EA)_G] + [(D_{11})_{TF}/(EI)_G] \quad (3)$$

Where,

$$(EA)_G = [(EA)_{TF} + (EA)_{BF} + (EA)_W] \quad (4)$$

$(EA)_{TF}$ ,  $(EA)_{BF}$  and  $(EA)_W$  are smeared axial stiffness of top flange, bottom flange and web respectively and are given by

$$(EA)_{TF} = (ET)_{TF} \times b \quad (5)$$

$$(EA)_{BF} = (ET)_{BF} \times b \quad (6)$$

$$(EA)_W = (ET)_W \times (h - t_{TF} - t_{BF}) \times 2 \quad (7)$$

Where,

$$(ET)_{TF} = [(A_{11})_{TF} - \{(A_{12})_{TF} \times (A_{12}/A_{22})_{TF}\}] \quad (8)$$

$$(ET)_{BF} = [(A_{11})_{BF} - \{(A_{12})_{BF} \times (A_{12}/A_{22})_{BF}\}] \quad (9)$$

$$(ET)_W = [(A_{11})_W - \{(A_{12})_W \times (A_{12}/A_{22})_W\}] \quad (10)$$

$$(EI)_G = \text{the flexural rigidity of the beam section} = [(EI)_{TF} + (EI)_{BF} + (EI)_W] \quad (11)$$

$(EI)_{TF}$ ,  $(EI)_{BF}$  and  $(EI)_W$  are flexural stiffness of top flange, bottom flange and web respectively and are given by

$$(EI)_{TF} = [\{(D_{11})_{TF} \times b\} + \{(EA)_{TF} \times (n_C - t_{TF})^2\}] \quad (12)$$

$$(EI)_{BF} = [\{(D_{11})_{BF} \times b\} + \{(EA)_{BF} \times (D - n_C - t_{BF})^2\}] \quad (13)$$

$$(EI)_W = \left[ \left\{ (Et)_W \times \frac{(D - t_{TF} - t_{BF})^3}{12} \right\} + \left\{ (EA)_W \times \left\{ \frac{(D - t_{TF} - t_{BF})}{2} - n_C + t_{TF} \right\}^2 \right\} \right] \quad (14)$$

' $D$ ' represents depth of beam,  $t$  represents thickness of element,  $n_C$  represents the depth of neutral axis from top of beam. Suffix  $TF$ ,  $BF$ , and  $W$  correspond to top flange, bottom flange and web respectively. Equivalent axial and flexural stiffness parameters for FRP box beams are adopted as defined by Upadhyay and Kalyanaraman (2003).  $A_{ij}$ ,  $B_{ij}$  and  $D_{ij}$  terms represent extensional, coupling and bending stiffness of the laminate, and the definitions of these terms are readily available in text books.

### 3. Validation of approach

Upadhyay and Kalyanaraman (2003) studied the shear lag phenomenon in a simply supported graphite epoxy composite box beam (Fig. 2(a)) subjected to central point load using MSC-NASTRAN. They reported effective width ratios for two beams having fiber orientations of  $0^\circ$  and

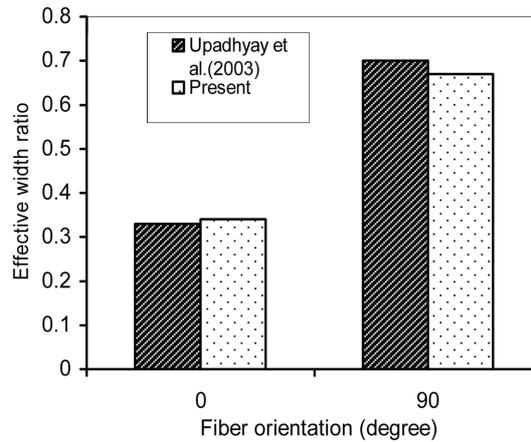
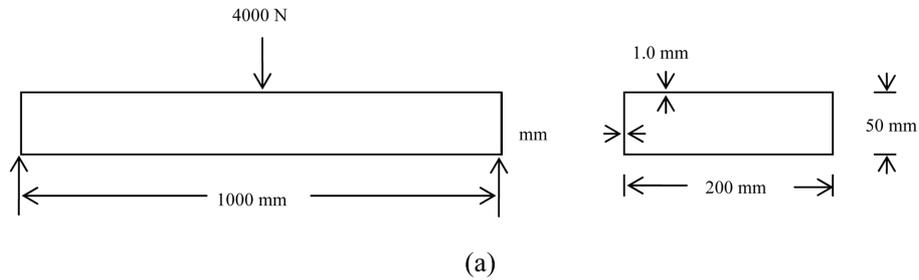


Fig. 2 Shear-lag in composite box beam: (a) beam details, (b) effect of orthotropy on shear-lag

Table 1 Properties of graphite epoxy material

Particulars	Values
Longitudinal elastic modulus, $E_{11}$ (GPa)	145
Transverse elastic modulus, $E_{22}$ (GPa)	16.5
Shear modulus, $G_{12}$ (GPa)	4.48
Poisson's ratio, $\nu_{12}$	0.314
Poisson's ratio, $\nu_{21}$	0.037

Table 2 Comparison of maximum deflections (mm)

Box beam and load type	Tripathy <i>et al.</i> (1994)		Present approach (using ANSYS)
	FEM model	COSMOS/M	
Composite tip load			
Single cell	0.62743	0.67208	0.64824
Twin cell	0.35905	0.37567	0.33218

90° respectively, in all the elements. To validate the approach, identical box beams are analyzed using ANSYS 7.1. The properties of graphite epoxy material are as per Table 1. The comparison of results shown in Fig. 2(b) validates the modeling aspects used in this paper.

Tripathy *et al.* (1994) carried out bending analysis for the study of deflections for box beams of isotropic and laminated composites subjected to different kinds of loading conditions. A finite element model had been developed based on the strain energy principle, and the results were compared with commercial software "COSMOS/M". To validate the approach identical cantilever glass-epoxy composite box beams are analyzed using ANSYS 7.1. The material properties and beam dimensions are longitudinal elastic modulus,  $E_{11} = 39.3$  GPa, transverse elastic modulus,  $E_{22} = 8.30$  GPa, shear modulus,  $G_{12} = 4.14$  GPa, Poisson's ratio,  $\nu_{12} = 0.26$ ,  $\nu_{21} = 0.0549$ . They considered box beams of span 0.4 m, width 0.2 m and depth 0.05 m. Top and bottom flange thickness is 3.8 mm each and web thickness is 3.3 mm. Table 2 shows comparison of deflection with those reported by Tripathy *et al.* (1994) and the good match validates the modeling aspects of present work.

## 4. Numerical studies

### 4.1 Finite element models

Symmetric graphite epoxy box beams are considered with  $b/D = 2$  to 6,  $L/b = 2$  to 7.5,  $t_F/t_W = 0.6$  to 1.5,  $n = 6$ . All elements of the box beam are made of balanced symmetric laminates. Various specially orthotropic symmetric configurations are used in every panel of the beams in order to eliminate the couplings like bending-stretching, bending-twisting etc. The material properties for graphite epoxy material are taken as per Table 1. Box beam model is discretized with 8-node isoparametric laminated shell element (SHELL 99) in ANSYS 7.1 as shown in Fig. 3. In total 243 studies are carried out.

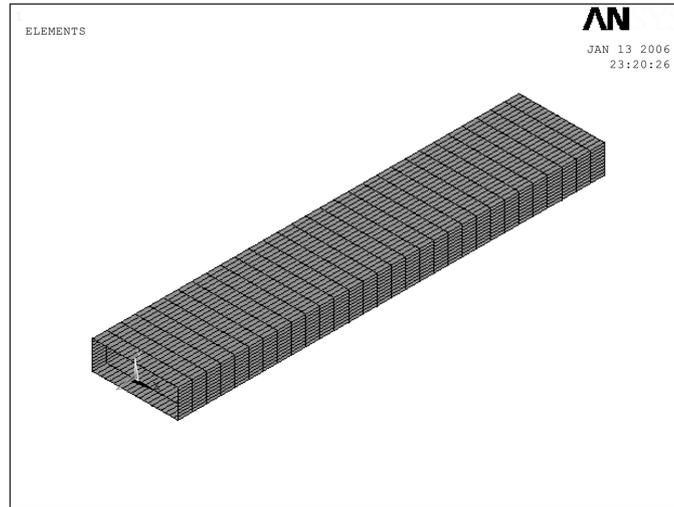


Fig. 3 Discretized model of laminated composite box beam

#### 4.2 Simply supported composite box beam under point load at mid span

A simply supported box beam subjected to an identical pair of concentrated loads at the mid span is considered to get stress distribution in top flange. Using these stresses the force is calculated in the top flange of the box beam. This force is used to evaluate the effective width ratio of top flange in the analysis of this paper.

#### 4.3 Evaluation of $\omega_1$ , and $\kappa_1$

$\omega_1$ , and  $\kappa_1$  values are calculated using expressions (2) and (3) for each case. Few sets of numerical studies are reported in Table 3. Using developed computer program it is possible to keep shear lag parameters  $\omega_1$ , and  $\kappa_1$  almost same for given value of  $L/b$  by varying other parameters of box beam such as  $b/D$ ,  $b/t_F$ ,  $t_F/t_W$ .

Table 3 Comparison of  $(b_e/b)$  for given values of  $L/b$ ,  $\omega_1$ , and  $\kappa_1$ 

$b/D$	$b/t_F$	$t_F/t_W$	Fiber orientation In flanges	Fiber orientation In web	$L/b$	$\omega_1$	$\kappa_1$	$b_e/b$	% Error in corresponding $b_e/b$
2.933	122.222	1	[0/0/0] <sub>s</sub>	[0/0/0] <sub>s</sub>	5	21.823	0.3779	0.44	0
3.611	147.727	0.804	[0/0/0] <sub>s</sub>	[0/0/0] <sub>s</sub>	5	21.823	0.3771	0.44	
2.933	122.222	1	[0/0/0] <sub>s</sub>	[90/90/90] <sub>s</sub>	5	16.987	0.4826	0.59	1.69
3.611	147.727	0.804	[0/0/0] <sub>s</sub>	[90/90/90] <sub>s</sub>	5	16.987	0.4828	0.58	
2.933	122.222	1	[0/0/0] <sub>s</sub>	[0/90/90] <sub>s</sub>	5	18.599	0.4415	0.52	5.77
3.611	147.727	0.804	[0/0/0] <sub>s</sub>	[0/90/90] <sub>s</sub>	5	18.599	0.4416	0.49	
2.933	122.222	1	[0/0/0] <sub>s</sub>	[0/0/0] <sub>s</sub>	5	21.823	0.3779	0.45	2.17
4	111.111	0.703	[0/0/0] <sub>s</sub>	[0/0/0] <sub>s</sub>	5	21.823	0.3771	0.46	

Table 3 Continued

$b/D$	$b/t_F$	$t_F/t_w$	Fiber orientation In flanges	Fiber orientation In web	$L/b$	$\omega_1$	$\kappa_1$	$b_e/b$	% Error in corresponding $b_e/b$
2.933	122.222	1	[0/0/0] <sub>s</sub>	[0/0/0] <sub>s</sub>	5	21.823	0.3779	0.45	0
2	222.222	1.513	[0/0/0] <sub>s</sub>	[0/0/0] <sub>s</sub>	5	21.823	0.3775	0.45	
2	222.222	1.513	[0/0/0] <sub>s</sub>	[90/90/90] <sub>s</sub>	5	16.987	0.4824	0.57	3.39
2.933	122.222	1	[0/0/0] <sub>s</sub>	[90/90/90] <sub>s</sub>	5	16.987	0.4826	0.59	
2.933	122.222	1	[0/45/-45] <sub>s</sub>	[0/0/0] <sub>s</sub>	5	6.955	0.2817	0.56	3.45
2	222.222	1.525	[0/45/-45] <sub>s</sub>	[0/0/0] <sub>s</sub>	5	6.955	0.2820	0.58	
6	166.667	0.652	[0/0/0] <sub>s</sub>	[0/0/0] <sub>s</sub>	3.33	21.823	0.4049	0.32	0
4	166.667	1	[0/0/0] <sub>s</sub>	[0/0/0] <sub>s</sub>	3.33	21.823	0.4042	0.32	
4	166.667	1	[0/45/-45] <sub>s</sub>	[0/0/0] <sub>s</sub>	3.33	6.956	0.3187	0.47	0
6	166.667	0.643	[0/45/-45] <sub>s</sub>	[0/0/0] <sub>s</sub>	3.33	6.956	0.3183	0.47	
3	166.667	1	[0/0/0] <sub>s</sub>	[0/0/0] <sub>s</sub>	7.5	21.823	0.3833	0.55	1.78
2.933	122.222	1	[0/0/0] <sub>s</sub>	[0/0/0] <sub>s</sub>	7.5	21.823	0.3779	0.56	
3	166.667	1	[0/0/0] <sub>s</sub>	[0/0/0] <sub>s</sub>	7	21.823	0.3833	0.53	1.85
2.933	122.222	1	[0/0/0] <sub>s</sub>	[0/0/0] <sub>s</sub>	7	21.823	0.3779	0.54	
3	166.667	1	[0/0/0] <sub>s</sub>	[0/0/0] <sub>s</sub>	6.5	21.823	0.3833	0.51	1.92
2.933	122.222	1	[0/0/0] <sub>s</sub>	[0/0/0] <sub>s</sub>	6.5	21.823	0.3779	0.52	
3	166.667	1	[0/0/0] <sub>s</sub>	[0/0/0] <sub>s</sub>	6	21.823	0.3833	0.49	0
2.933	122.222	1	[0/0/0] <sub>s</sub>	[0/0/0] <sub>s</sub>	6	21.823	0.3779	0.49	

From the sets of numerical studies as given in Table 3 it has been observed that for given  $L/b$ ,  $\omega_1$ , and  $\kappa_1$  the value of effective width ratio ( $b_e/b$ ) is almost constant. Maximum percentage error is 5.77. These studies show that the parameters proposed in present work can be used as key parameters for estimating shear lag effect in symmetrical laminated composite box beam with reasonable accuracy. Also, these parameters form the basis to develop ANN model for further studies.

## 5. ANN based shear lag prediction

ANN has emerged as a powerful tool for function approximation used in the various engineering applications. Neural network architecture consists of a number of interconnected units or layers: the layer where the input patterns are applied is called the input layer, the layer where the output is obtained is the output layer, and the layers between the input and output layers are the hidden layers (Fig. 4).

A supervised training mechanism called Feed forward back-propagation training algorithm (Kartalopoulos 2000) is commonly used in most of the engineering applications. In the back-propagation training mechanism, the input data are presented at the input layer, the information is processed in the forward direction, and the output is calculated at the output layer. The target values are known at the output layer, so that the error can be estimated. The total error at the output layer

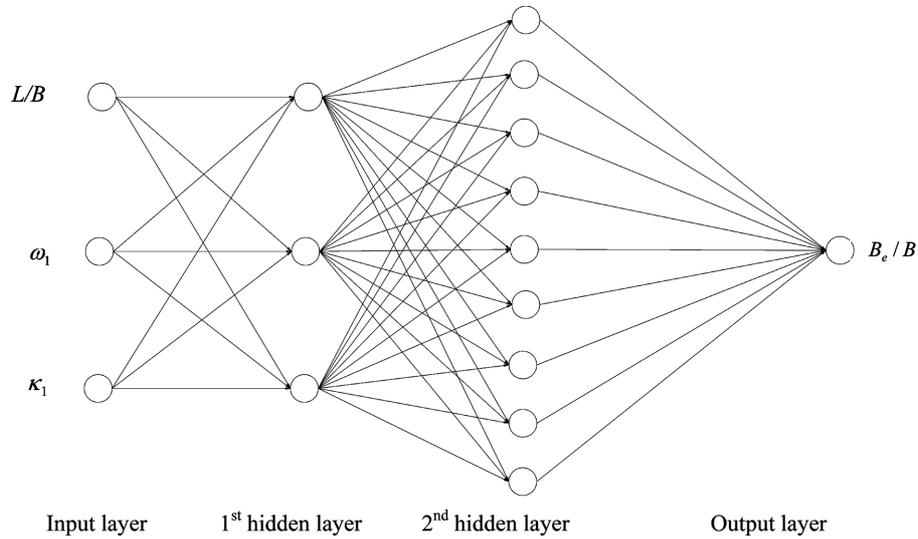


Fig. 4 Structure of a Feed-Forward ANN

is distributed back and the connection weights are adjusted. This process of feed-forward mechanism and back propagation of errors and weight adjustment is repeated iteratively until convergence in terms of an acceptable level of error is achieved. This whole process is called the training of the ANN. The trained ANN is then validated on the testing data set, which it has not seen before. Once an ANN has been trained and tested, it can be used for prediction or modeling the physical system for which it has been designed.

### 5.1 Development and training of the network

The  $L/b$ ,  $\omega_1$ , and  $\kappa_1$  are taken as input parameters and effective width ratio  $b_e/b$  is taken as output parameter for the preparation of network. All the available 243 input numerical data sets are normalized between 0 and 1. Out of these, 198 representative data sets have been chosen for the purpose of training the network and remaining 45 data sets have been kept for testing. Table 4 shows the range of maximum and minimum values of the training and testing data that is used in this study. In the training process of the network, these normalized 198 data sets of input parameters have been fed to the input nodes of the network and the corresponding values of  $b_e/b$  have been fed to the network as desired outputs. After sufficient number of training iterations, the network attains an ability to promptly provide the effective width ratio ( $b_e/b$ ) information for a given set of input parameters.

Table 4 Range of training and testing parameters

Parameters	Training data		Testing data	
	Maximum	Minimum	Maximum	Minimum
$L/b$	7.50	3.33	7.50	3.33
$\omega_1$	21.82	1.99	21.82	1.99
$\kappa_1$	0.4921	0.1620	0.4824	0.2016

### 5.2 Architecture of the network

As stated earlier, BPNN training approach was used in this investigation. The network consists of 3 input nodes (corresponding to  $L/b$ ,  $\omega_1$ , and  $\kappa_1$ ) and one output node (corresponding to  $b_e/b$ ). Connection weights were adjusted during the training process through minimization of the mean square error (MSE) using the gradient descent based algorithm in Neural Network toolbox of *MATLAB 7.0* for error back propagation. As Levenberg-Marquardt algorithm appears to be the fastest method for training moderate-sized feed forward neural networks (up to several hundred weights) it is used in the present study. The Levenberg-Marquardt method is based on approaching second-order training speeds without having the computation of Hessian matrix, in other words, second derivatives (Hagan and Menhaj 1994, Hagan *et al.* 1996). The advantage of this training method is its fast convergence about minimum and giving more accurate results. The transfer function determines the relationship between input and outputs of a node and a network. *Tansig* transfer function was used in the hidden layer and *Purelin* transfer function was used in the output layer. The number of hidden layers and the number of nodes in the hidden layers were decided by trial and error based on comparative performance of large number of network architectures as shown in Table 5. Studies carried out to select appropriate architecture for the ANN show that performance of 3-3-9-1 is better than other architectures. Hence, in present work architecture 3-3-9-1 is finally selected.

### 5.3 Prediction of shear lag by BPNN

The network is trained for sufficiently large number of iterations till the training error and testing error reach minimum and the network is stable. In the present case, training of 3-3-9-1 BPNN has been continued till 5,000 iterations and the training error reaches its minimum value of 7.48e-4. When the performance of the network is checked for the testing data, MSE comes out to be 5.42e-3.

Fig. 5 shows the ANN predicted and target values of effective width ratio, when the network has been assigned to recall the 198 samples used for training the network and the network has predicted

Table 5 Training and testing error for the various BPNN architectures

S. No	No. of layers	No. of Nodes				MSE	
		Input layer	Hidden layer		Output layer	up to 5,000 iterations	
			1 <sup>st</sup>	2 <sup>nd</sup>		Training	Testing
1	4	3	2	2	1	8.20e-3	2.78e-2
2	4	3	3	2	1	7.92e-3	1.97e-2
3	4	3	3	3	1	6.85e-3	1.69e-2
4	4	3	3	5	1	6.10e-3	1.01e-2
5	4	3	3	7	1	4.41e-3	8.65e-3
6	4	3	3	8	1	2.19e-3	7.23e-3
7	4	3	3	9	1	7.48e-4	5.42e-3
8	4	3	3	10	1	7.11e-4	5.73e-3
9	4	3	3	11	1	6.32e-4	7.45e-3

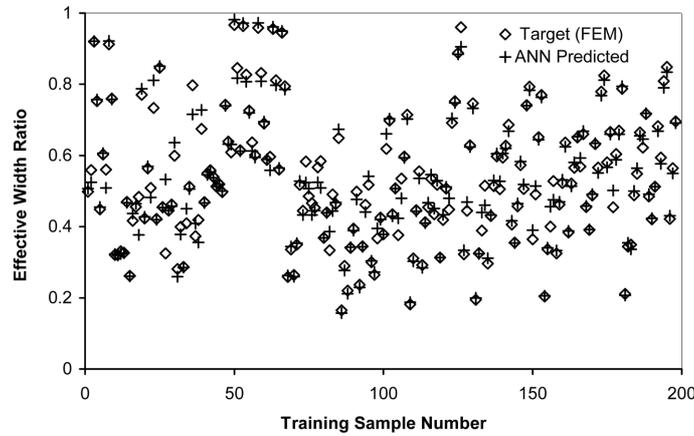


Fig. 5 Results of ANN predictions with training data

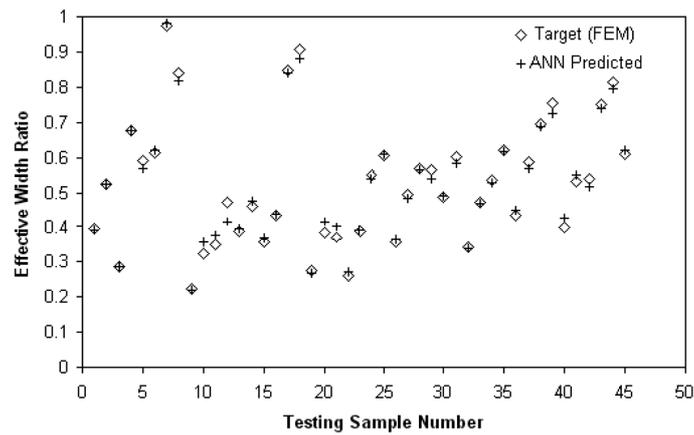


Fig. 6 Results of ANN predictions with testing data

them very well. Fig. 6 shows the ANN predicted and target values of effective width ratio, when the network has been assigned to predict for 45 number of testing data set and even in this case the prediction of the network is very close to those obtained by a rigorous FEM analysis. It has been observed from the results that the network can predict the shear lag (in terms of  $b_e/b$ ) almost accurately with error less than 7%. On the other hand, considering the computing time required by the proposed analysis takes around 3s and the corresponding FEM analysis takes 147s. From this it is evident that this procedure based on ANN is computationally efficient.

## 6. Conclusions

Shear lag is very important phenomenon in case of wide flanged box beams. Especially when material is orthotropic (FRP), this phenomenon enhances and exists till failure of member. Thus it should be accounted in design. In this paper a simple and quick approach for estimating shear lag effect in symmetrical laminated composite box beam is presented. Three unique parameters ( $L/b$ ,

$\omega_1$ , and  $\kappa_1$ ) governing the shear lag behaviour of laminated composite box beams are identified and their uniqueness is validated with numerical studies. A numbers of FEM studies are carried out to generate training and testing data sets for the development of an ANN model (3-3-9-1) for prediction of shear lag in laminated composite box beam. Average and maximum error on testing data are 1.7% and 6.28% respectively. It will be very useful for the designers as this provide a handy way to predict shear lag in simply supported laminated composite box beam subjected to mid-span point load. This neural network will be quite useful in predicting shear lag behaviour during the optimum design of FRP box beams.

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## Notations

$A_{ij}$	: extensional stiffness coefficients of the laminate
$b$	: width of flange of box beam
$b_e$	: effective width of top flange
$B_{ij}$	: coupling stiffness coefficients of the laminate
$D$	: depth of beam
$D_{ij}$	: bending stiffness coefficients of the laminate
$E$	: Young's elastic modulus
$E_{11}, E_{22}$	: longitudinal and transverse elastic modulus
$(EA)_G$	: the smeared axial rigidity of the beam section
$(EA)_{TF}, (EA)_{BF}$ and $(EA)_w$	: the smeared axial stiffness of top flange, bottom flange and web respectively
$(EI)_{TF}, (EI)_{BF}$ and $(EI)_w$	: the flexural stiffness of top flange, bottom flange and web respectively
$(ET)_{TF}, (ET)_{BF}, (ET)_w$	: smeared extensional stiffness of top flange, bottom flange, and web per unit width
$G$	: shear modulus
$G_{12}$	: longitudinal shear modulus
$h$	: total thickness of laminate
$h_k$	: z-coordinate of ply 'k' from mid plane the flexural rigidity of the beam section
$L$	: span of beam
$n$	: number of plies in the laminate
$n_c$	: the depth of neutral axis from top of beam section
$t_F$	: thickness of flange
$t_{TF}, t_{BF}$	: thicknesses of top and bottom flanges respectively
$t_w$	: thickness of web
$\kappa, \kappa_1$	: cross-sectional parameters
$\nu_{12}, \nu_{21}$	: major and minor Poisson's ratio
$\omega, w_1$	: orthotropic parameters