Structural Engineering and Mechanics, Vol. 28, No. 6 (2008) 635-658
DOI: http://dx.doi.org/10.12989/sem.2008.28.6.635

Free vibration analysis of plates resting on elastic foundations using modified Vlasov model

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(Received February 20, 2006, Accepted January 23, 2008)

Abstract. An application is presented of a modified Vlasov model to the free vibration analysis of plates resting on elastic foundations. The effects of the subsoil depth, the ratio of the plate dimensions, the ratio of the subsoil depth to the plate dimension in the longer direction, and the value of the vertical deformation parameter within the subsoil on the frequency parameters of plates on an elastic foundation are investigated. This analysis has been caried out by the aid of a computer program. The first ten frequency parameters are presented in tabular and the graphical forms to evaluate the effects of the parameters considered in this study. Then mode shapes corresponding to the first six of the frequency parameters are given in graphs. It is concluded that the effect of the subsoil depth on the frequency parameters of the plates on an elastic foundation is generally larger than those of the other parameters considered in this study.

Keywords: modified Vlasov model; plates; elastic foundation; free vibration; parametric analysis.

1. Introduction

The concept of plates resting on elastic foundations is extensively used by structural and geotechnical engineers for static and dynamic analyses and for design of many practical soil-structure interaction problems. For this reason, in the technical literature, numerous works have been concerned with such problems. In these kinds of problems, developing a more realistic mathematical model is essential to provide an accurate analysis of the soil-structure system for safe and economical design.

Many researchers use the Winkler model for soil-structure interaction in the static and dynamic analysis of plates resting on elastic foundations, where the vertical surface displacement of the plate are assumed to be proportional at every point to the contact pressure at that point (Hetenyi 1950). In the Winkler model, it is assumed that the foundation soil consists of linear elastic springs which are closely spaced and independent of each other. One of the most important shortcommings of this

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model is that it assumes no interaction between the springs or discontinuous of the foundation.

In order to overcome this problem, several two parameter models have been suggested by many researchers. The model proposed by Filenonko-Borodich acquires continuity between the individual spring elements in the Winkler model by connecting them to a thin elastic membrane under a constant tension. In the model proposed by Hetenyi, interaction between the independent spring elements is accomplished by incorporating an elastic plate in three-dimensional problems. Another model proposed by Pasternak acquires shear interaction between springs by connecting the ends of the springs to a layer consisting of incompressible vertical elements which deform by lateral shear only. Vlasov developed a two-parameter model that accounts for the effect of the neglected shear strain energy in the soil and shear forces that come from surrounding soil by introducing an arbitrary parameter, γ, to characterize the vertical distribution of the deformation in the subsoil (Selvaduari 1979).

All these models are shown to lead to same differential equation. Basically all these models are equivalent and defer only in the definition of the second parameter. The Vlasov model requires the estimation of the value of the vertical deformation parameter, γ. Jones and Xenophontos (1979) established a relationship between the γ parameter and the displacement characteristics, but did not suggest any computational method. Vallabhan and Das (1988) determined γ parameter as a function of the characteristics of the beam resting on an elastic foundation, using an iterative procedure. They named this model as a modified Vlasov model. In this model, they mentioned that the three parameters, k, $2t$, and γ, are affected from loading, from the characteristics of the subsoil and the material properties of the beam, and from the subsoil depth. It is really clear that the deflection of a beam resting on an elastic soil can not be independent of the parameters such as the loading, the characteristics of the subsoil and the material properties of the beam, and the subsoil depth. Straughan (1990) used the modified Vlasov model for the static analysis of rectangular plates by the finite difference method. Turhan (1990) used the same model for the static analysis of plates resting on elastic foundation by the finite element method. Ayvaz *et al.* (1998) used the modified Vlasov model for the earthquake analysis of plates resting on elastic foundation. Daloğlu *et al.* (1999) applied the modified Vlasov model to the forced vibration analysis of rectangular plates on elastic foundations. Omurtag and Kadıoğlu (1998), studied the free vibration analysis of orthotropic plates on Winkler/Pasternak elastic foundation using Gateaux Differential Method. Çelik and Saygun (1999) developed an iterative method to analyze the plates on a two-parameter elastic foundation. Liu (2000) studied the static analysis of isotropic rectangular plates on Winkler foundation. He used the first-order shear deformation theory.

Silva *et al.* (2001) presented a numerical methodology for analysis of plates on tensionless elastic foundation. They used Winkler model and illustrated the methodology by different examples. Huang and Thambiratnam (2001) analyzed the plates resting on elastic supports and elastic foundation by finite strip method. They assumed that the plate is resting on Winkler elastic foundation and discussed the effects of dimension ratio on the static and free vibration responses. Shen *et al.* (2001) examined free and forced vibration analysis of Reissner-Mindlin plates resting on a Pasternak-type elastic foundation. Ayvaz and Özgan (2002) applied the modified Vlasov model to the free vibration analysis of beams on elastic foundations and analyzed the effects of different parameters on the frequency parameters of beams resting on elastic foundations. Xiang (2003) studied the vibration behavior of rectangular Mindlin plates on non-homogenous elastic foundation. He discussed the effects of several parameters on the frequency parameters of square Mindlin plates. Setoodeh and Karami (2004) analyzed the static, free vibration and buckling responses of anisotropic thick laminated composite plates resting on elastic foundation using Winkler and Pasternak models.

Yu *et al.* (2007) presented dynamic response analysis for a Reissner-Mindlin plate free along all four edges resting on a tensionless elastic foundation of Winkler and Pasternak types. Güler and Celep (1995) studied static and dynamic responses of a thin circular plate on a tensionless elastic foundation. Celep and Güler (2004) analyzed static and dynamic responses of a rigid circular plate on a tensionless Winkler foundation. Güler and Celep (2005) also studied response of a rectangular plate-column system on a tensionless Winkler foundation subjected to static lateral load, harmonic ground motion and earthquake motion. Celep and Güler (2007) also investigated axisymmetric forced vibrations of an elastic free circular plate on a tensionless two parameter foundation. Küçükarslan and Banerjee (2004) analyzed inelastic dynamic analysis of pile-soil-structure interaction. Maheshwari *et al.* (2004) investigated three-dimensional nonlinear analysis for seismic soil-pile-structure interaction and presented a three-dimensional method of analysis. Mezaini (2006) investigated effects of soil-structure interaction on the analysis of cylindrical tanks. However, no studies have been found for the free vibration analysis of plates resting on elastic foundations by using the modified Vlasov model.

The aim of this paper is to apply, not to introduce, the modified Vlasov model to the free vibration analysis of plates resting on elastic foundations and to analyze the effects of the subsoil depth, the ratio of the plate dimensions, the ratio of the subsoil depth to the plate dimension in the longer direction, and the value of the vertical deformation parameter, γ, within the subsoil on the frequency parameters of plates on an elastic foundation. For this purpose, a computer program coded by Ayvaz *et al*. (1998) is modified and then used to obtain the stiffness and mass matrices of the plate-soil system. It should be noted that this study is an extension of the study made by Ayvaz and Özgan (2002).

2. Finite element modelling

The governing equation for a plate subjected to the free vibration with no damping is

$$[M]\{\ddot{w}\} + [K]\{w\} = 0 \tag{1}$$

where **[K]** is the stiffness matrix of the plate-soil system, **[M]** is the mass matrix of the plate-soil system, $\{\boldsymbol{w}\}$ and $\{\ddot{w}\}$ are the displacement and acceleration vectors of plate, respectively.

The subsoil considered has a finite depth with a rigid boundary at the bottom (Fig. 1).

The total strain energy in the soil-structure system may be written as

$$\Pi = \frac{D}{2}\int_{-ly/2}^{+ly/2}\int_{-lx/2}^{+lx/2}\left\{(\nabla^2 w)^2 - 2(1-\nu_p)\left[\left(\frac{\partial^2 w}{\partial x^2}\right)\frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2\right]\right\}dxdy +$$
$$\int_0^{H_z}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{1}{2}(\sigma_x\varepsilon_x + \sigma_y\varepsilon_y + \sigma_z\varepsilon_z + \tau_{xy}\gamma_{xy} + \tau_{xz}\gamma_{xz} + \tau_{yz}\gamma_{yz})dxdydz \tag{2}$$

where σ_x, σ_y, σ_z, τ_{xy}, τ_{xz}, τ_{yz}, ε_x, ε_y, ε_z, γ_{xy}, γ_{xz}, and γ_{yz} are the stresses and corresponding strains in the subsoil, $D = E_p h_p^3/12(1-\nu_p^2)$ is the flexural rigidity of plate, w, h_p, E_p, and ν_p are the lateral displacement, the thickness, the modulus of elasticity, and the Poisson's ratio of the plate,

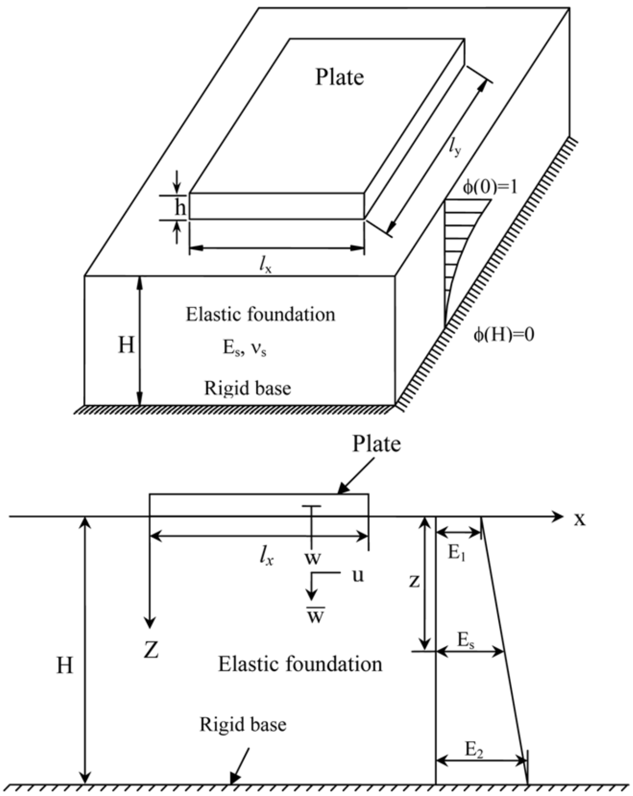


Fig. 1 A simple plate on an elastic foundation

respectively, and H is the height of the subsoil. By using constitutive relations and strain-displacement equations of elasticity, the stresses at any point in the foundation can be expressed as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix} = \frac{E_s(1-\nu_s)}{(1+\nu_s)(1-2\nu_s)} \begin{bmatrix} 1 & \frac{\nu_s}{1-\nu_s} & \frac{\nu_s}{1-\nu_s} & 0 & 0 & 0 \\ \frac{\nu_s}{1-\nu_s} & 1 & \frac{\nu_s}{1-\nu_s} & 0 & 0 & 0 \\ \frac{\nu_s}{1-\nu_s} & \frac{\nu_s}{1-\nu_s} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-2\nu_s}{2(1-\nu_s)} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-2\nu_s}{2(1-\nu_s)} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\nu_s}{2(1-\nu_s)} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} \tag{3}$$

where E_s and ν_s are the modulus of the elasticity and the Poisson's ratio of subsoil, respectively.

If the assumptions of

the vertical displacement $\overline{w}(x,y,z) = w(x,y)\phi(z)$ for $\phi(0) = 1$ ve $\phi(H) = 0$ (4)

the horizontal displacements $\overline{u}(x,y,z) = 0$ and $\overline{v}(x,y,z) = 0$ (5)

are made, and if Eqs. (3), (4), and (5) are substituted into Eq. (2), the following equation can be obtained.

$$\Pi = \frac{D}{2}\int_{-ly/2}^{+ly/2}\int_{-lx/2}^{+lx/2}\left\{(\nabla^2 w)^2 - 2(1-v_p)\left[\frac{\partial^2 w}{\partial x^2}\left(\frac{\partial^2 w}{\partial y^2}\right) - \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2\right]\right\}dxdy$$
$$+\int_0^H\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left[\overline{E}w^2\left(\frac{\partial \Phi}{\partial z}\right)^2 + G_s(\nabla w)^2\Phi^2\right]dxdydz \tag{6}$$

In these equations, $\phi(z)$, $\overline{u}$ and $\overline{v}$ are the mode shapes defining the variation of the deflection $\overline{w}(x,y,z)$ in the z direction, the displacement of the subsoil in the x direction and y direction, respectively, G_s is the shear modulus of the subsoil (Vallabhan and Das 1991), and $\overline{E}$ is equal to $E_s(1-v_s)/(1+v_s)(1-2v_s)$.

By applying variations in Π due to variations in w and ϕ and using variational calculus, the following equations can be obtained.

$$k = \int_0^H \overline{E}\left(\frac{d\Phi}{dz}\right)^2 dz \tag{7}$$

$$2t = \int_0^H G_s \Phi^2 dz \tag{8}$$

where

$$\Phi(z) = \frac{\sinh\gamma\left(1-\frac{z}{H}\right)}{\sinh\gamma} \tag{9}$$

and

$$\left(\frac{\gamma}{H}\right)^2 = \frac{(1-2v_s)\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(\nabla w)^2 dxdy}{2(1-v_s)\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}w^2 dxdy} \tag{10}$$

In these expressions, k, $2t$ and γ are Winkler foundation modulus, shear foundation modulus and vertical deformation parameter within the subsoil, respectively. The other terms are previously defined.

As it can be seen from Eq. (10), the values of γ varies with the displacement of the plate and the depth of the subsoil. Therefore, the variables w, k, $2t$, H and γ are all connected to each other for a plate resting on an elastic foundation.

2.1 Evaluation of the stiffness matrix

The MZC rectangle finite element (Weaver and Johnston 1984) is used in this study. Nodal displacements at each node are

$$w_i, \partial w_i/\partial y, -\partial w_i/\partial x, \quad i = 1, 2, 3, 4 \tag{11}$$

and the displacement function is

$$w = [N]\{w_e\} \tag{12}$$

where $\{w_e\}$ is the nodal displacement vector containing all 12 components of the type shown in Eq. (11). The matrix $[\boldsymbol{N}]$ contains the displacement shape function (Weaver and Johnston 1984, Zienkiewicz 1977).

By using the standard procedure in the finite element methodology for the assemblage of elements, the global stiffness matrix is constructed as a half-banded matrix

$$[K] = \sum_{i=1}^{n}([k_p]+[k_k]+[k_t]) \tag{13}$$

where n is the total number of plate finite elements, and $[k_p]$ is the conventional element stiffness matrix of the plate (Weaver and Johnston 1984, Zienkiewicz 1977). The stiffness matrix for the axial strain effect in the soil, $[k_k]$, is obtained by minimizing the total energy with respect to each component of displacement vector (Turhan 1990), and may be written as

$$[k_k] = kab\int_{-1}^{1}\int_{-1}^{1}[N]^t[N]d\xi d\eta \tag{14}$$

in which a and b are the half of the dimensions of the rectangular element in the x and y directions, and ξ and η are natural co-ordinates. $[k_t]$ is the stiffness matrix which accounts for the shear effect in the soil, expressed as

$$[k_t] = 2tab\int_{-1}^{1}\int_{-1}^{1}\left(\frac{1}{a^2}\left[\frac{\partial N}{\partial \xi}\right]^T\left[\frac{\partial N}{\partial \xi}\right]+\frac{1}{b^2}\left[\frac{\partial N}{\partial \eta}\right]^T\left[\frac{\partial N}{\partial \eta}\right]\right)d\xi d\eta \tag{15}$$

the matrices $[k_k]$ and $[k_t]$ are not presented here since they will take excessive space, so for more information about these matrices, see reference (Turhan 1990).

2.2 Evaluation of the mass matrix

According to Hamilton's variational principle, the total kinetic energy of the plate-soil system may be written as

$$\Pi_k = \frac{1}{2}\int_{\Omega}\dot{w}^T\mu\dot{w}d\Omega \tag{16}$$

where $\dot{w}$ represents the partial derivative of the vector of generalized displacement with respect to the time variable, and μ is the mass density matrix of the form

$$\mu = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \tag{17}$$

where $m_1 = \rho_p h + 1/3(\rho_s H)$, $m_2 = m_3 = 1/12(\rho_p h^3)$, h is the thickness of the plate, and ρ_p and ρ_s are the mass densities of the plate and the soil, respectively (Kolar and Nemec 1989).

Then the consistent mass matrix, M, of the plate on an elastic foundation is obtained by the following equation.

$$M = \int_{\Omega} N_i^T \mu N_i d\Omega \tag{18}$$

In the view of Eq. (12), the following expression can be written for each finite piece

$$[N_i] = \begin{bmatrix} N \\ \dfrac{\partial N}{\partial y} \\ \dfrac{\partial N}{\partial x} \end{bmatrix} \tag{19}$$

The consistent mass matrix of the plate and the soil can be evaluated after substituting Eq. (19) into Eq. (18) and integrating it over the domain. It is symmetric 12×12 matrix, and its upper triangle is given in Ayvaz *et al.* (1998).

As mentioned before, the governing equation for a beam subjected to a free vibration with no damping is represented by Eq. (1). After substituting $w = W\sin\omega t$ into this equation, the following equation can be obtained.

$$([K] - \lambda[M])\{W\} = 0 \tag{20}$$

where $\{W\}$ is a vector of mode shape of vibration and λ (= ω^2, ω is the circular frequency) is the frequency parameter. This is a generalized eigenvalue problem. The eigenvalue solution of this equation yields the frequency parameters and corresponding mode shapes. For the solution of Eq. (20), the program, MATLAB, is used after the matrices [**K**] and [**M**] are included in the program as data.

3. Numerical examples

3.1 Data for numerical examples

In this study, different values of H, l_y/l_x and γ are used for the parametric study of the free vibration analysis of plates resting on elastic foundations. The values of the vertical deformation parameter, γ, are taken to be 1, 2, 3, 4, 5, 6, 7 and 8. The depths, H, of the subsoil are taken to be

5 m, 10 m and 15 m for each γ parameter considered, and the aspect ratio, l_y/l_x, the plate are taken as 1.0, 1.5 and 2.0 for each subsoil depth. Different values of the ratio, H/l_y, are used depending on the subsoil depth, H. In the calculation of the mass matrix, the mass densities of plate and subsoil are taken to be 2500 kg/m^3 and 1700 kg/m^3, respectively. The shorter length, l_x, of the plate is kept constant at 10 m. The properties of the plate-soil system are as follows: The thickness of the plate is 50 cm; the modulus of elasticity of plate is 2.7 10^{10} N/m^2; the Poisson's ratio of the plate is 0.2; the modulus of elasticity of the subsoil is 2.0 10^7 N/m^2 and the Poisson's ratio of the subsoil is 0.25.

For the sake of accuracy in the results, rather than starting with a finite element mesh size, the mesh size required to produce the desired accuracy is determined. To find out the required mesh size, convergence of the frequency parameters is checked for different mesh sizes. It is concluded that the results have acceptable error when equally spaced 10 × 10 elements are used for a 10 m × 10 m square plate. Lengths of the elements in the x and y directions are kept constant for different l_y/l_x ratios.

3.2 Results

The first ten frequency parameters of plate considered for several subsoil depth, plate dimensions, their ratio and the value of the vertical deformation parameter within the subsoil are presented in Tables 1, 2, 3 and 4. In order to see the effects of the changes in these parameters better on the first six frequency parameters, they are planned to be also presented in graphical form, but presentation of all of data obtained in this study in graphical form would take up excessive space. Hence, only the data for $\gamma = 1$, 5 and 8 with different values of l_y/l_x and H/l_y are given in Figs. 2, 3, 4, 5, 6 and 7, respectively.

As seen from Tables and Figs. 2, 3 and 4, the values of the frequency parameters for a constant value of H decrease as the aspect ratio, l_y/l_x, increases. This behavior is understandable in that a plate on an elastic foundation with a larger aspect ratio becomes more flexible and has smaller frequency parameters.

The values of the frequency parameters for a constant value of l_y/l_x ratio decrease as H increases. The decrease in the frequency parameters with increasing aspect ratio, l_y/l_x, for a constant value of H gets larger for larger values of the frequency parameters and smaller for smaller values of the aspect ratio, l_y/l_x.

The changes in the frequency parameters for a constant aspect ratio, l_y/l_x, with increasing subsoil depth, H, are larger than the changes in the frequency parameters for a constant subsoil depth, H, with decreasing the aspect ratio. The changes in the frequency parameters for a constant aspect ratio, l_y/l_x, with increasing subsoil depth, H, are also larger than the changes in the frequency parameters for a constant subsoil depth, H, as the vertical deformation parameter, γ, increases. This shows that the effects of the changes in the subsoil depth, H, on the frequency parameters are larger than those of the changes in the aspect ratio, l_y/l_x, and in the vertical deformation parameter, γ.

As seen from Tables and Figs. 5, 6 and 7, the values of the frequency parameters for a constant value of H increase as H/l_y ratio increases, but the values of frequency parameters for a constant H/l_y ratio decrease as the value of H increases.

It should be noted that the increase in the frequency parameters with increasing H/l_y ratios for a constant value of H gets larger for larger values of the frequency parameters.

Table 1 The first 10 frequency parameters of plates on elastic foundations for different values of H, H/l_y for $\gamma = 1$, and 2

γ	H (m)	l_y (m)	H/l_y	Frequency parameters, λ									
				λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	λ_8	λ_9	λ_{10}
1	5	20	0.250	1571.98	1989.22	2758.50	2918.06	3569.62	5071.84	5507.29	8023.94	9563.05	10099.65
		15	0.333	1640.12	2338.66	2863.57	4073.73	4233.23	7920.53	8250.50	9542.44	11910.23	16971.65
		10	0.500	1803.90	3089.09	3089.09	5918.41	7765.27	9327.86	15838.86	15838.86	34236.77	34236.77
		5	1.000	2285.14	3776.76	5742.41	9434.44	13362.06	34948.61	37186.04	68349.04	92962.04	94392.59
		3	1.667	2860.19	4644.22	10446.42	10753.19	28743.57	36716.85	83515.26	116666.89	192409.67	300566.89
	10	20	0.500	605.09	914.87	1507.94	1645.25	2161.79	3283.42	3645.15	5342.54	6567.80	6918.15
		15	0.667	651.43	1176.17	1598.86	2569.38	2700.27	5476.71	5554.71	6473.31	8264.98	11742.25
		10	1.000	741.04	1742.39	1742.39	3744.54	4899.29	5667.83	9662.21	9662.21	18332.52	18332.52
		6	1.667	971.05	2205.41	3357.43	6597.82	7792.57	20487.14	23307.01	24700.49	37898.67	50676.39
	15	20	0.750	365.33	639.01	1157.27	1297.86	1768.44	2733.01	3102.37	4393.93	5615.87	5781.74
		15	1.000	403.03	867.27	1245.54	2133.15	2270.14	4602.72	4736.06	5435.47	6974.81	9961.25
		10	1.500	485.63	1437.38	1437.38	3552.00	4715.92	5315.61	9733.97	9733.97	17957.96	17957.96
2	5	20	0.250	1743.43	2124.21	2798.29	2955.66	3536.32	4947.94	5342.12	7728.20	9207.23	9675.46
		15	0.333	1806.76	2436.11	2889.37	3992.44	4136.05	7595.97	7918.18	9202.04	11376.28	16327.00
		10	0.500	1959.92	3083.24	3083.24	5663.11	7417.53	8883.01	15111.52	15111.52	33270.31	33270.31
		5	1.000	2401.08	3670.28	5284.45	8843.91	12463.95	33694.43	35617.99	66467.82	90437.79	91737.48
		3	1.667	2926.37	4413.81	9063.01	9898.30	26614.44	35026.84	80475.21	113966.21	188018.35	296628.31
	10	20	0.500	639.67	909.10	1413.94	1535.52	1974.99	2982.77	3282.28	4869.20	5934.33	6267.01
		15	0.667	681.27	1134.57	1488.79	2327.08	2434.85	4901.16	5041.47	5883.00	7476.13	10695.59
		10	1.000	760.92	1606.26	1606.26	3307.47	4315.62	5093.84	8574.87	8574.87	17027.81	17027.81
		6	1.667	970.17	1981.92	2901.73	5790.43	6787.22	18496.26	21580.44	22965.08	35118.89	47263.06
	15	20	0.750	371.32	600.08	1032.86	1148.09	1535.07	2379.41	2663.55	3876.82	4858.81	5078.24
		15	1.000	404.25	791.41	1102.93	1844.28	1947.36	4042.80	4051.86	4756.09	6093.53	8713.99
		10	1.500	477.58	1254.61	1254.61	2999.31	3999.86	4679.12	8381.91	8381.91	16475.23	16475.23

Table 2 The first 10 frequency parameters of plates on elastic foundations for different values of H, H/l_y for $\gamma = 3$, and 4

γ	H (m)	l_y (m)	H/l_y	Frequency parameters, λ									
				λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	λ_8	λ_9	λ_{10}
3	5	20	0.250	2154.17	2513.10	3126.93	3278.24	3811.99	5150.57	5521.02	7806.70	9244.67	9658.59
		15	0.333	2215.08	2801.90	3208.27	4231.37	4366.31	7659.24	7969.31	9247.94	11274.73	16151.64
		10	0.500	2364.04	3380.46	3380.46	5781.04	7467.53	8846.36	14880.75	14880.75	32832.97	32832.97
		5	1.000	2786.78	3899.61	5259.98	8716.16	12119.07	33065.30	34786.15	65352.92	88902.88	90065.71
		3	1.667	3288.45	4559.62	8353.86	9584.76	25365.77	34095.04	78578.59	112278.63	185148.06	294012.03
	10	20	0.500	750.57	992.70	1434.77	1541.62	1925.27	2839.85	3097.64	4592.87	5543.01	5852.18
		15	0.667	789.33	1193.24	1497.61	2232.56	2325.45	4554.77	4732.40	5522.40	6949.93	9992.10
		10	1.000	862.85	1595.48	1595.48	3066.56	3968.60	4722.59	7835.43	7835.43	16086.25	16086.25
		6	1.667	1061.85	1907.00	2649.90	5268.56	6121.55	17099.66	20362.56	21705.68	33081.39	44813.01
	15	20	0.750	416.63	613.46	982.59	1076.08	1400.61	2142.86	2367.62	3510.22	4311.74	4544.21
		15	1.000	446.45	777.94	1038.78	1662.49	1743.20	3563.01	3641.20	4262.41	5434.43	7784.20
		10	1.500	514.37	1159.84	1159.84	2616.02	3499.08	4195.54	7374.52	7374.52	15320.91	15320.91
4	5	20	0.250	2686.49	3033.95	3613.62	3760.43	4267.24	5559.18	5918.71	8138.61	9558.04	9934.28
		15	0.333	2746.30	3310.08	3689.27	4663.69	4794.84	7988.52	8284.63	9563.03	11498.48	16338.52
		10	0.500	2894.01	3848.86	3848.86	6143.63	7788.21	9106.38	15035.83	15035.83	32846.52	32846.52
		5	1.000	3309.10	4328.91	5535.11	8930.99	12208.70	32978.76	34597.62	64953.65	88309.77	89355.16
		3	1.667	3800.54	4940.74	8210.86	9693.54	24917.18	33842.63	77773.24	111582.63	183801.83	292769.71
	10	20	0.500	901.58	1128.40	1532.20	1630.20	1980.45	2835.82	3072.23	4502.58	5395.30	5679.46
		15	0.667	938.87	1314.23	1587.81	2259.30	2344.81	4433.19	4621.23	5389.02	6707.73	9660.74
		10	1.000	1009.08	1674.03	1674.03	3006.75	3846.01	4565.27	7478.80	7478.80	15581.17	15581.17
		6	1.667	1203.74	1947.22	2579.81	5039.01	5811.47	16374.89	19722.97	21010.23	31948.71	43491.06
	15	20	0.750	483.64	662.20	991.58	1072.41	1359.34	2035.66	2229.37	3316.21	4023.37	4249.45
		15	1.000	511.74	810.62	1039.44	1590.25	1659.86	3308.83	3424.59	4000.16	5060.69	7268.97
		10	1.500	577.05	1142.07	1142.07	2424.99	3240.22	3916.75	6813.05	6813.05	14633.80	14633.80

Table 3 The first 10 frequency parameters of plates on elastic foundations for different values of H, H/l_y for $\gamma = 5$, and 6

γ	H (m)	l_y (m)	H/l_y	Frequency parameters, λ									
				λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	λ_8	λ_9	λ_{10}
5	5	20	0.250	3258.05	3598.84	4157.41	4301.49	4791.81	6054.23	6408.32	8584.84	9996.61	10346.86
		15	0.333	3317.26	3867.28	4229.57	5173.82	5303.13	8436.64	8720.65	10000.83	11879.10	16700.54
		10	0.500	3464.53	4381.47	4381.47	6611.94	8230.17	9507.40	15378.48	15378.48	33095.23	33095.23
		5	1.000	3875.58	4837.74	5947.56	9308.45	12511.60	33173.45	34738.97	64945.92	88215.14	89170.56
		3	1.667	4361.49	5420.17	8362.87	10008.45	24893.06	33943.85	77549.27	111420.81	183273.17	292283.48
	10	20	0.500	1066.38	1284.20	1663.76	1756.77	2086.45	2904.75	3129.98	4515.56	5380.54	5643.64
		15	0.667	1102.83	1460.93	1714.93	2347.42	2429.14	4433.48	4621.12	5380.34	6629.20	9537.35
		10	1.000	1171.11	1794.12	1794.12	3043.18	3844.46	4533.50	7338.32	7338.32	15338.20	15338.20
		6	1.667	1363.59	2043.88	2607.10	4972.10	5698.97	16025.86	19413.38	20650.97	31343.77	42801.50
	15	20	0.750	558.72	726.55	1030.54	1104.35	1368.24	2003.17	2180.46	3226.29	3885.85	4099.84
		15	1.000	585.81	865.06	1073.35	1579.50	1643.41	3190.92	3320.29	3875.11	4862.12	6998.18
		10	1.500	649.77	1164.79	1164.79	2344.08	3119.25	3769.77	6514.19	6514.19	14241.48	14241.48
6	5	20	0.250	3840.11	4176.57	4720.95	4863.33	5342.69	6585.32	6936.35	9082.83	10491.44	10823.16
		15	0.333	3898.95	4439.89	4790.78	5715.02	5843.32	8937.59	9211.90	10494.24	12334.41	17145.53
		10	0.500	4046.00	4937.59	4937.59	7125.94	8726.50	9975.15	15810.68	15810.68	33462.53	33462.53
		5	1.000	4454.50	5378.17	6423.49	9762.53	12917.59	33508.63	35043.29	65142.76	88370.55	89258.16
		3	1.667	4936.87	5941.16	8665.34	10422.50	25085.39	34221.51	77619.56	111530.34	183156.75	292185.45
	10	20	0.500	1235.50	1447.55	1810.68	1900.70	2216.79	3010.48	3228.30	4583.07	5433.70	5680.21
		15	0.667	1271.42	1618.19	1858.93	2465.84	2545.41	4495.95	4679.96	5436.03	6637.91	9521.38
		10	1.000	1338.46	1933.54	1933.54	3128.40	3904.94	4570.21	7310.40	7310.40	15237.98	15237.98
		6	1.667	1529.65	2167.93	2684.78	4989.94	5688.83	15869.70	19276.71	20474.45	31017.82	42435.08
	15	20	0.750	636.71	797.72	1084.46	1154.11	1402.87	2010.40	2178.24	3193.87	3827.70	4029.70
		15	1.000	663.17	929.65	1123.97	1601.01	1661.70	3144.77	3278.25	3823.26	4759.97	6856.12
		10	1.500	726.36	1208.12	1208.12	2321.10	3070.64	3698.06	6353.80	6353.80	14012.61	14012.61

Table 4 The first 10 frequency parameters of plates on elastic foundations for different values of H, H/l_y for $\gamma = 7$, and 8

γ	H (m)	l_y (m)	H/l_y	Frequency parameters, λ									
				λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	λ_8	λ_9	λ_{10}
7	5	20	0.250	4425.00	4758.44	5292.61	5433.87	5905.45	7134.00	7483.14	9607.69	11015.11	11333.03
		15	0.333	4483.59	5018.09	5361.81	6270.89	6398.57	9465.45	9732.03	11016.41	12829.58	17634.45
		10	0.500	4630.52	5504.02	5504.02	7662.85	9250.80	10478.54	16290.56	16290.56	33895.24	33895.24
		5	1.000	5037.23	5933.53	6932.91	10257.23	13378.98	33920.51	35435.65	65454.74	88661.61	89497.12
		3	1.667	5517.10	6482.76	9051.73	10889.64	25397.39	34595.03	77850.72	111788.33	183263.92	292300.71
	10	20	0.500	1406.18	1614.25	1965.56	2053.65	2360.15	3136.50	3351.32	4681.52	5524.16	5757.60
		15	0.667	1441.74	1780.55	2011.77	2600.78	2679.01	4592.62	4772.33	5527.44	6695.96	9564.57
		10	1.000	1507.92	2083.19	2083.19	3240.47	3999.75	4646.43	7344.87	7344.87	15219.31	15219.31
		6	1.667	1698.26	2306.80	2790.73	5055.04	5735.37	15818.83	19236.12	20402.26	30847.30	42243.89
	15	20	0.750	715.98	872.33	1146.64	1213.63	1451.36	2040.04	2201.98	3194.74	3813.55	4005.06
		15	1.000	742.00	999.58	1183.86	1640.64	1699.33	3138.54	3272.33	3812.48	4713.11	6785.20
		10	1.500	804.72	1262.92	1262.92	2330.44	3062.31	3670.68	6270.11	6270.11	13877.47	13877.47
8	5	20	0.250	5010.85	5342.05	5868.55	6009.01	6474.79	7692.81	8040.71	10148.57	11555.67	11862.96
		15	0.333	5069.25	5598.95	5935.52	6835.07	6962.36	10009.06	10269.41	11555.57	13348.67	18149.44
		10	0.500	5216.11	6076.06	6076.06	8213.09	9791.57	11003.41	16798.84	16798.84	34367.73	34367.73
		5	1.000	5621.49	6497.34	7462.36	10776.17	13873.54	34378.56	35880.49	65837.30	89033.36	89827.81
		3	1.667	6099.51	7036.35	9489.47	11388.51	25781.81	35025.87	78178.08	112135.12	183503.57	292543.60
	10	20	0.500	1577.62	1782.78	2125.19	2211.95	2511.34	3274.82	3486.99	4799.06	5637.00	5859.99
		15	0.667	1612.92	1945.83	2169.89	2745.71	2823.04	4709.86	4885.40	5640.50	6784.50	9643.19
		10	1.000	1678.47	2238.97	2238.97	3368.76	4115.32	4747.26	7416.88	7416.88	15250.74	15250.74
		6	1.667	1868.20	2454.62	2914.03	5148.81	5816.06	15830.42	19253.35	20394.15	30770.29	42155.91
	15	20	0.750	795.94	948.92	1213.87	1279.04	1509.32	2083.39	2241.37	3216.31	3825.78	4008.39
		15	1.000	821.65	1072.68	1249.40	1691.33	1748.69	3156.34	3288.89	3826.57	4700.22	6756.76
		10	1.500	884.06	1324.73	1324.73	2359.31	3078.30	3671.07	6232.27	6232.27	13799.81	13799.81

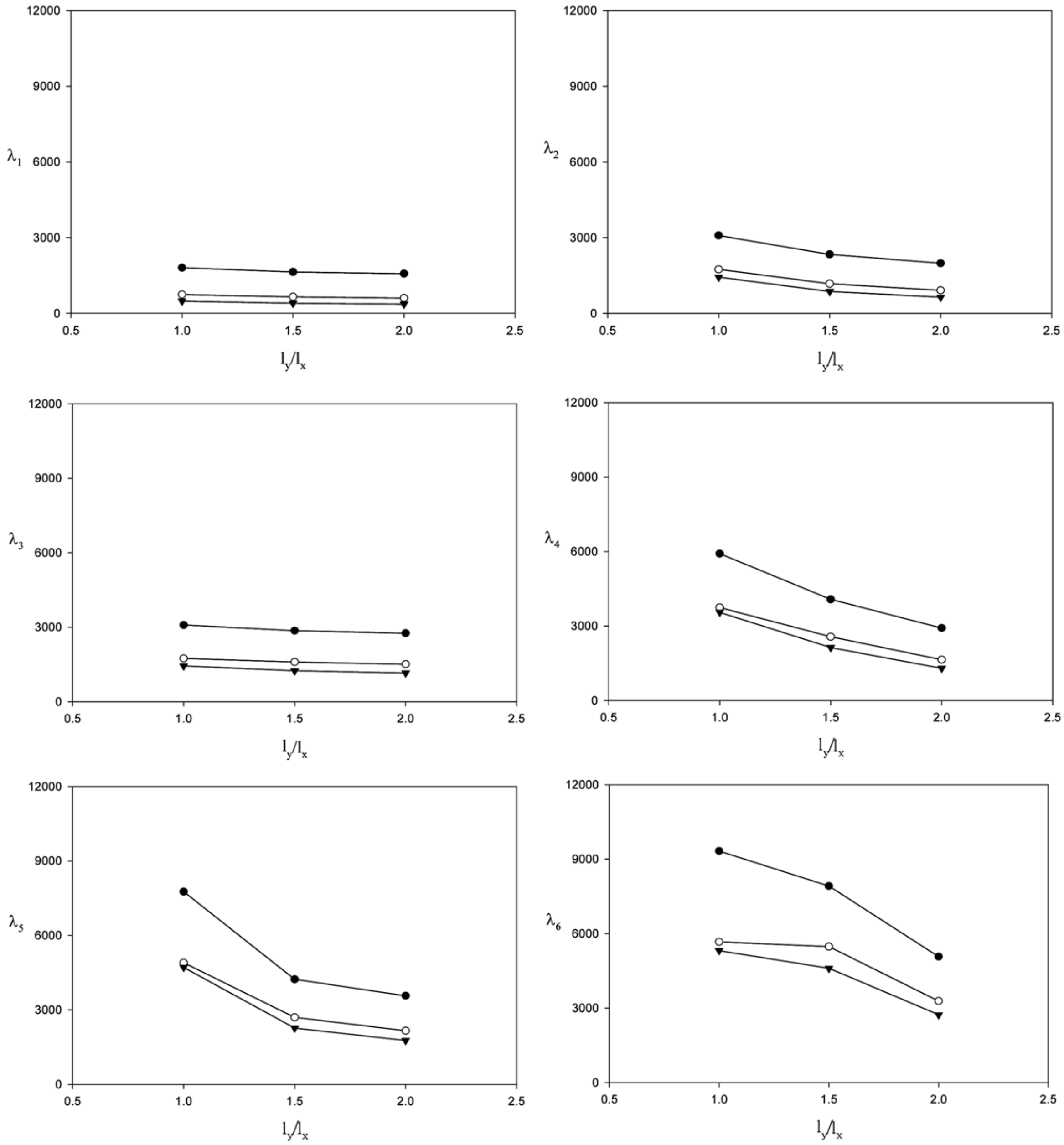


Fig. 2 The effects of different values of H and l_y/l_x on the first six frequency parameters of plates on elastic foundations for $\gamma = 1$. Key for H values: -●-, 5 m; -○-, 10 m; -▼-, 15 m

The decreases in the frequency parameters with increasing value of H for a constant H/l_y ratio gets less for larger values of H. This behavior is also understandable in that a plate on an elastic foundation with a larger subsoil depth becomes more flexible and has smaller frequency parameters.

The changes in the frequency parameters with increasing subsoil depth for a constant value of H/l_y

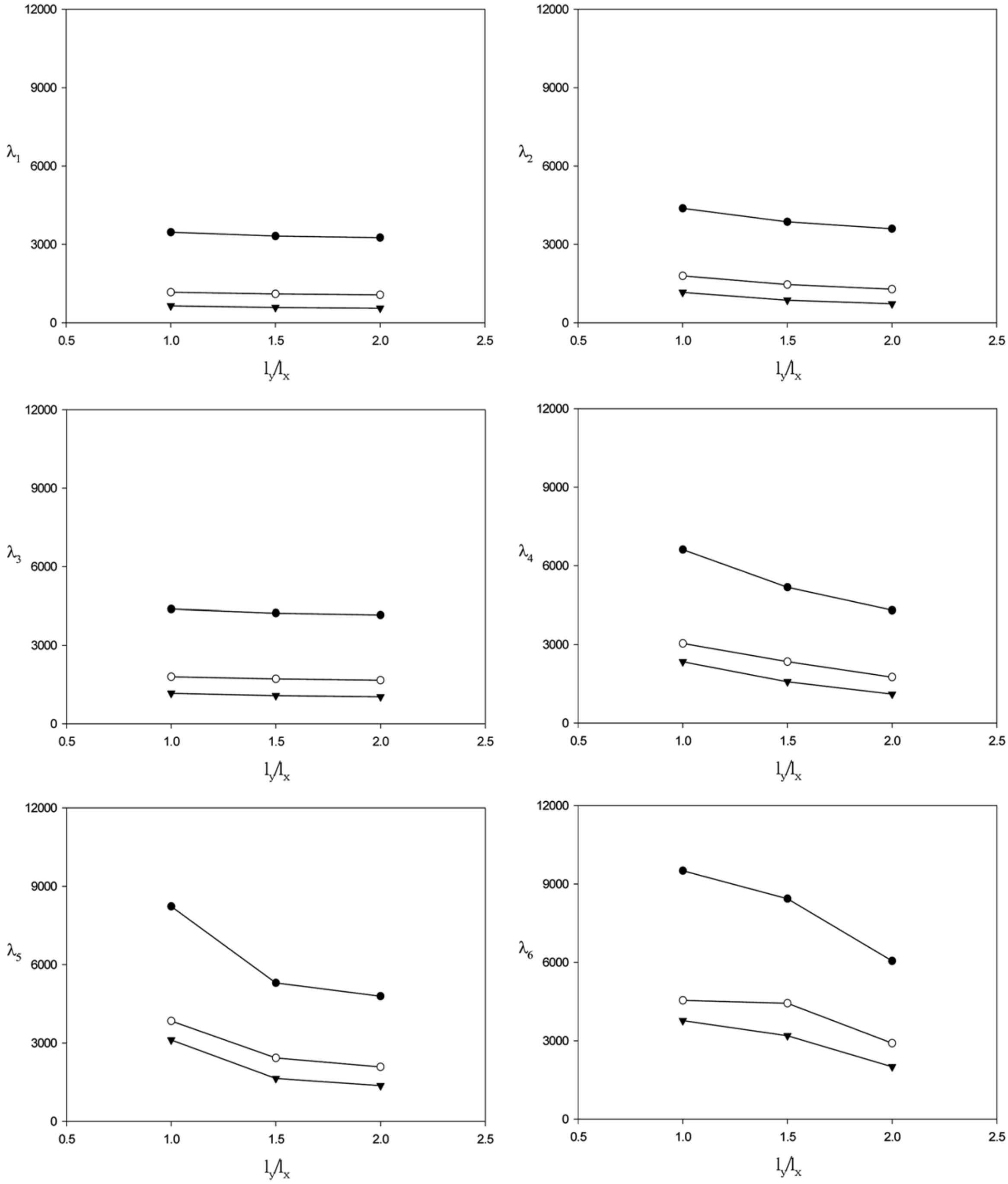


Fig. 3 The effects of different values of H and l_y/l_x on the first six frequency parameters of plates on elastic foundations for $\gamma = 5$. Key for H values: -●-, 5 m; -○-, 10 m; -▼-, 15 m

ratio is larger than that in the frequency parameters with increasing H/l_y ratios for a constant value of H.

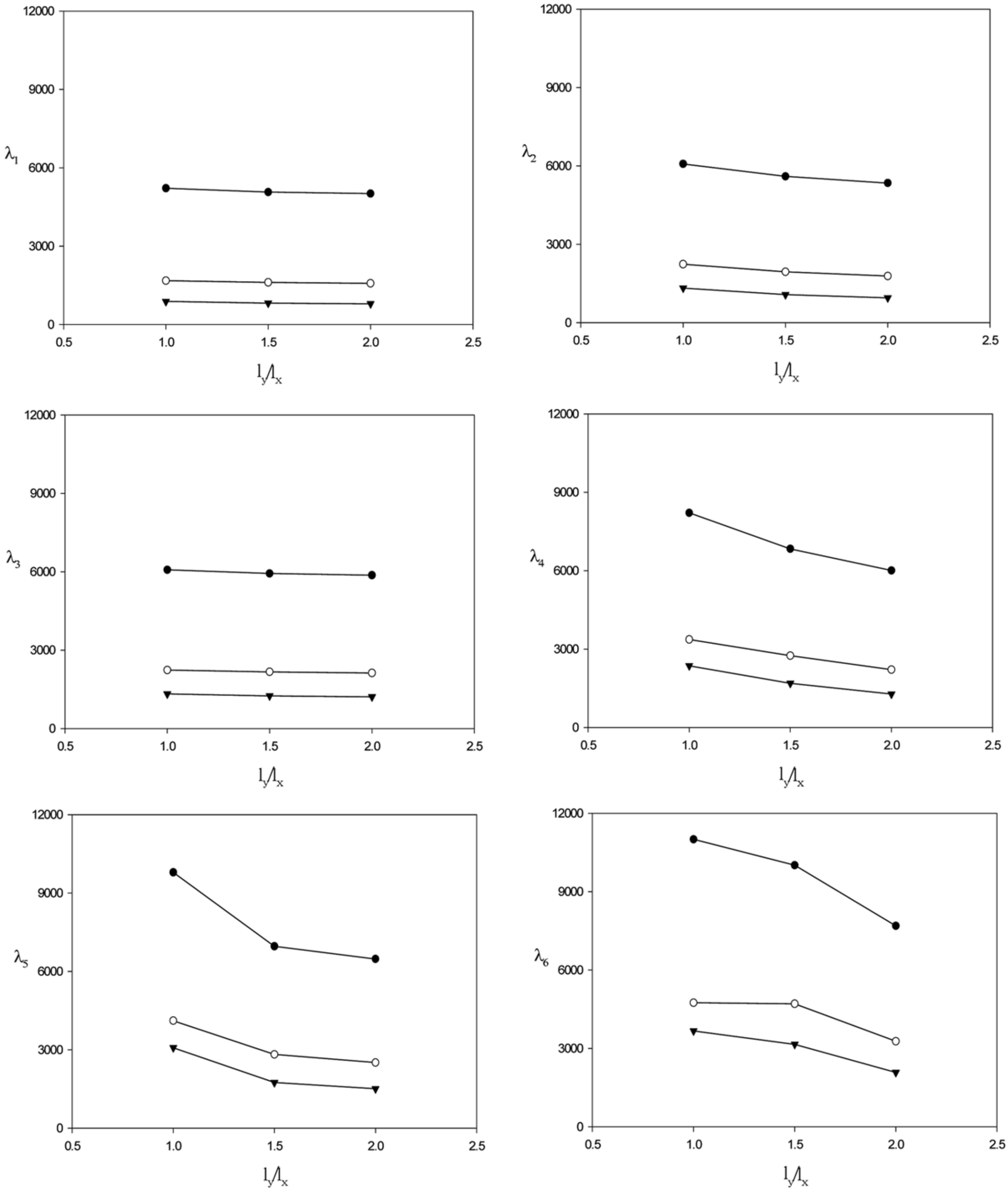


Fig. 4 The effects of different values of H and l_y/l_x on the first six frequency parameters of plates on elastic foundations for $\gamma = 8$. Key for H values: -●-, 5 m; -○-, 10 m; -▼-, 15 m

These observations indicate that the effects of the change in the subsoil depth on the frequency parameter of the plate on an elastic foundation are always larger than those of the change in the

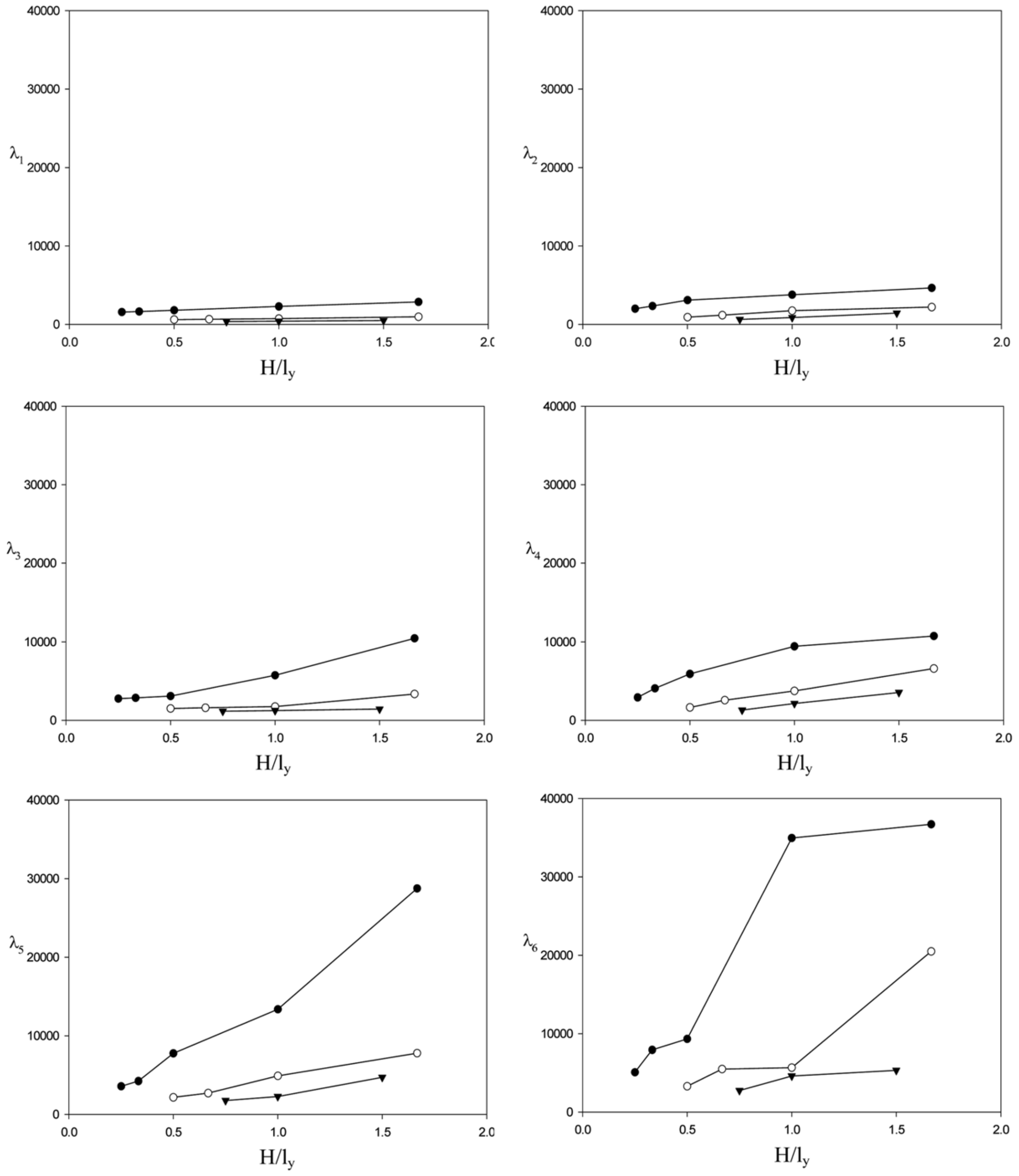


Fig. 5 The effects of different values of H and H/l_y on the first six frequency parameters of plates on elastic foundations for $\gamma = 1$. Key for H values: -●-, 5 m; -○-, 10 m; -▼-, 15 m

other parameters considered in this study.

As also seen from all figures, the curves for a constant value of H/l_y ratio and the aspect ratio, l_y/l_x

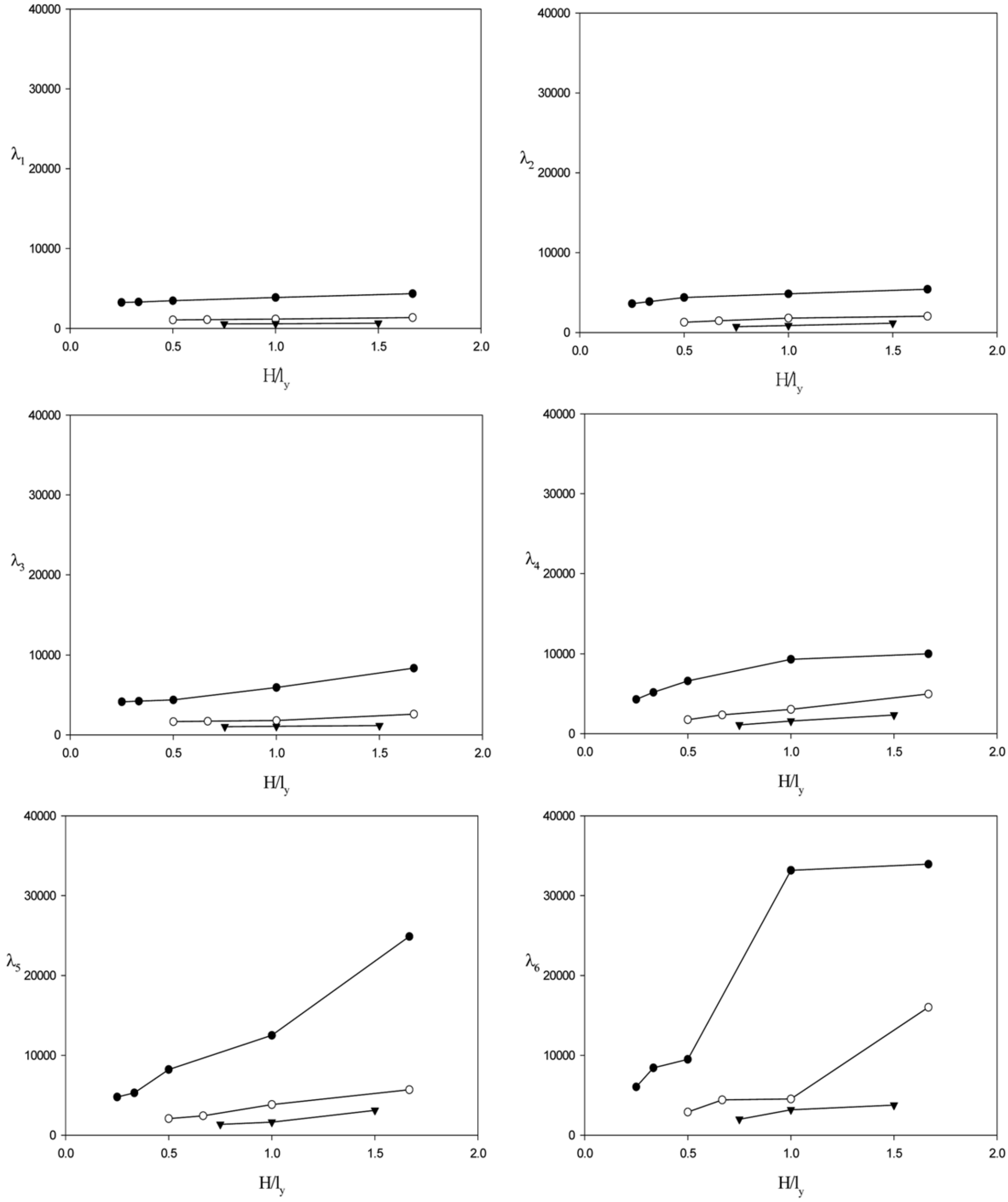


Fig. 6 The effects of different values of H and H/l_y on the first six frequency parameters of plates on elastic foundations for $\gamma = 5$. Key for H values: -●-, 5 m; -○-, 10 m; -▼-, 15 m

are fairly getting closer to each other as the value of H increases. This shows that the curves of the frequency parameters will almost coincide with each other when the value of the subsoil depth, H,

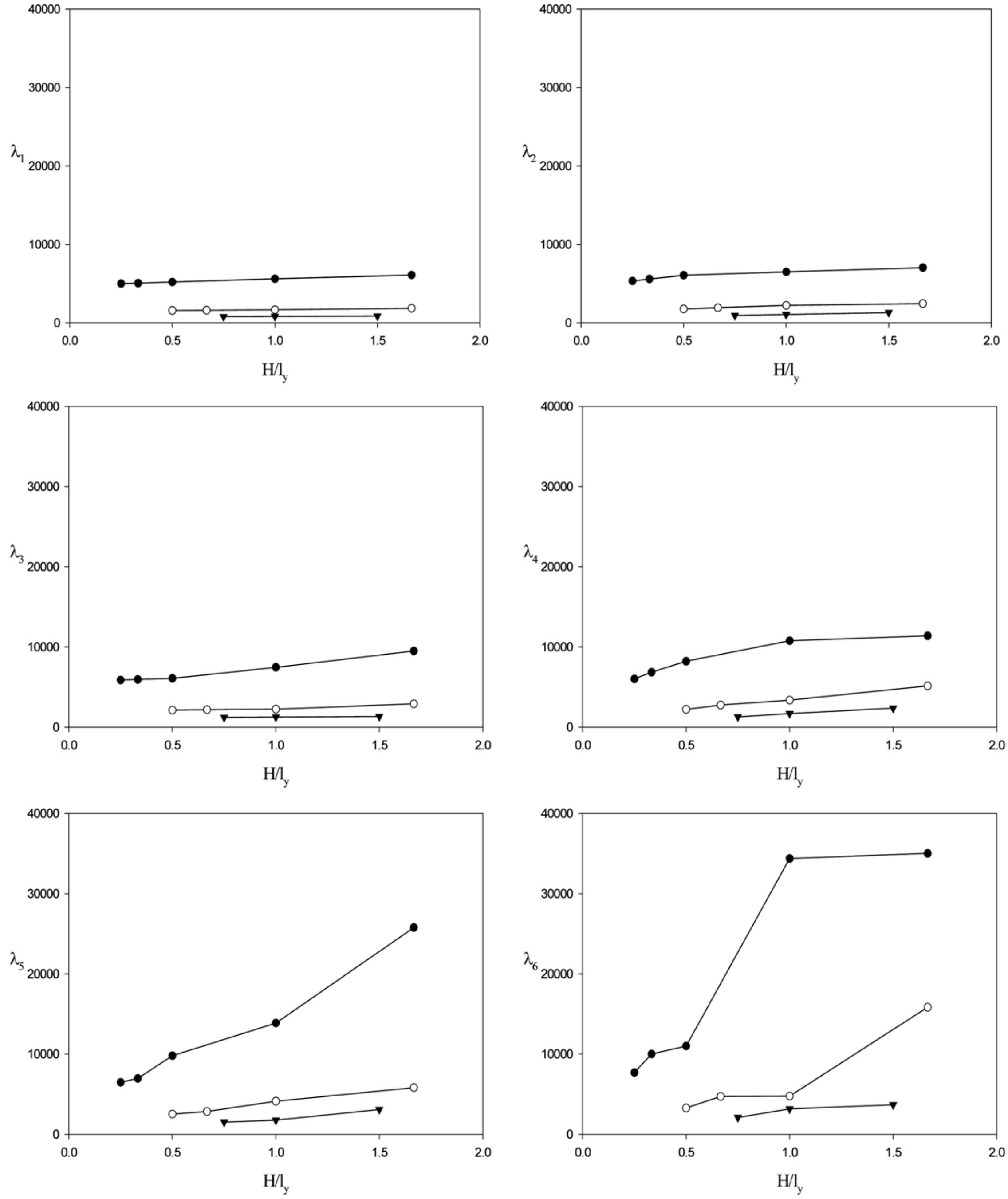


Fig. 7 The effects of different values of H and H/l_y on the first six frequency parameters of plates on elastic foundations for $\gamma = 8$. Key for H values: -●-, 5 m; -○-, 10 m; -▼-, 15 m

increases more. In other words, the increase in the subsoil depth will not affect the frequency parameters after a determined value of H. In addition, variation occurring in the frequency parameters increases as the value of H/l_y ratio increases.

As also seen from these figures, depending on the increase in H/l_y ratio, the increase occuring in the frequency parameters for the larger values of the vertical deformation parameters, γ, gets less as γ increases.

In this study, the mode shapes of the plates on an elastic foundation are also obtained for all parameters considered. Since presentation of all of these mode shapes would take up excessive

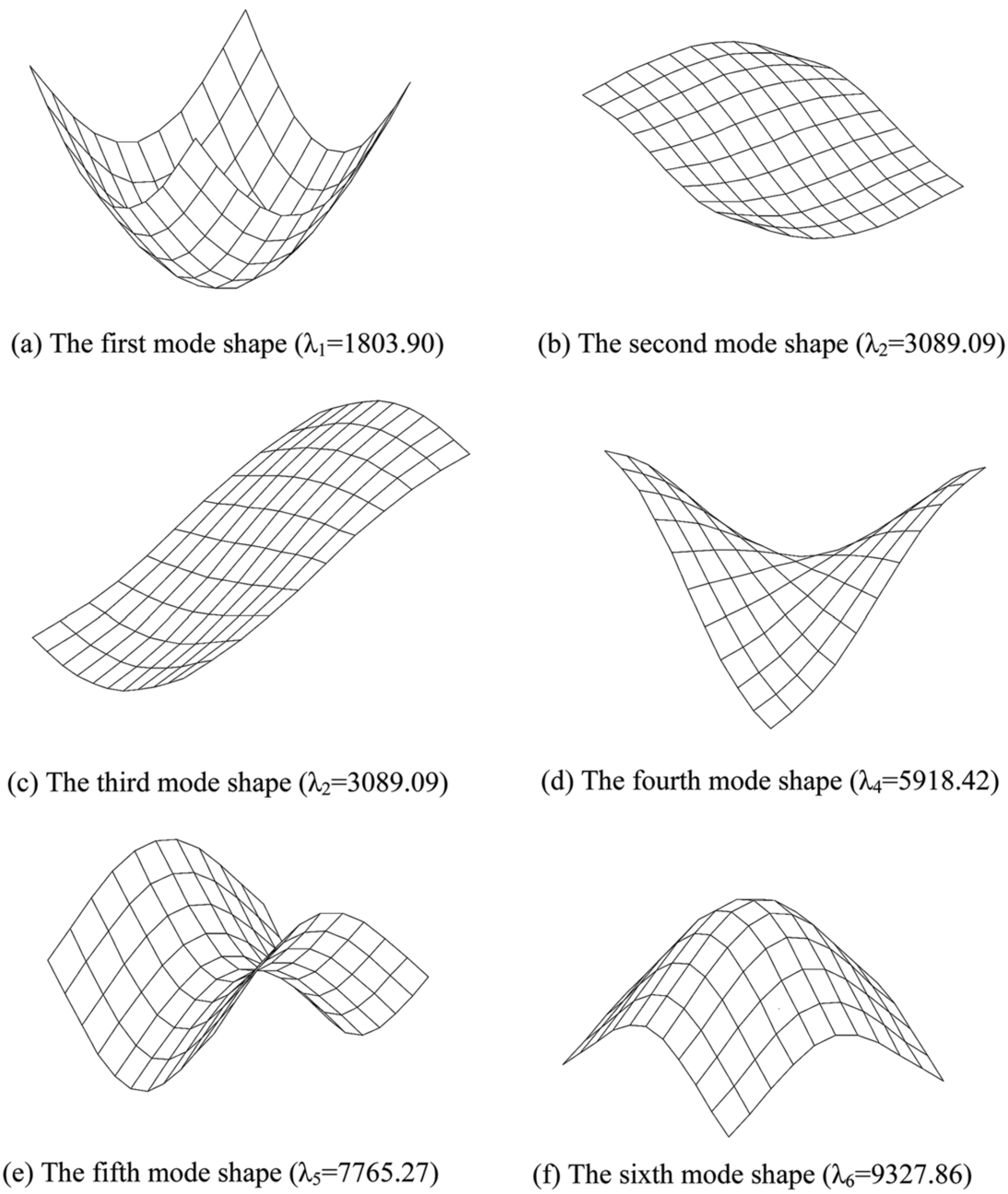
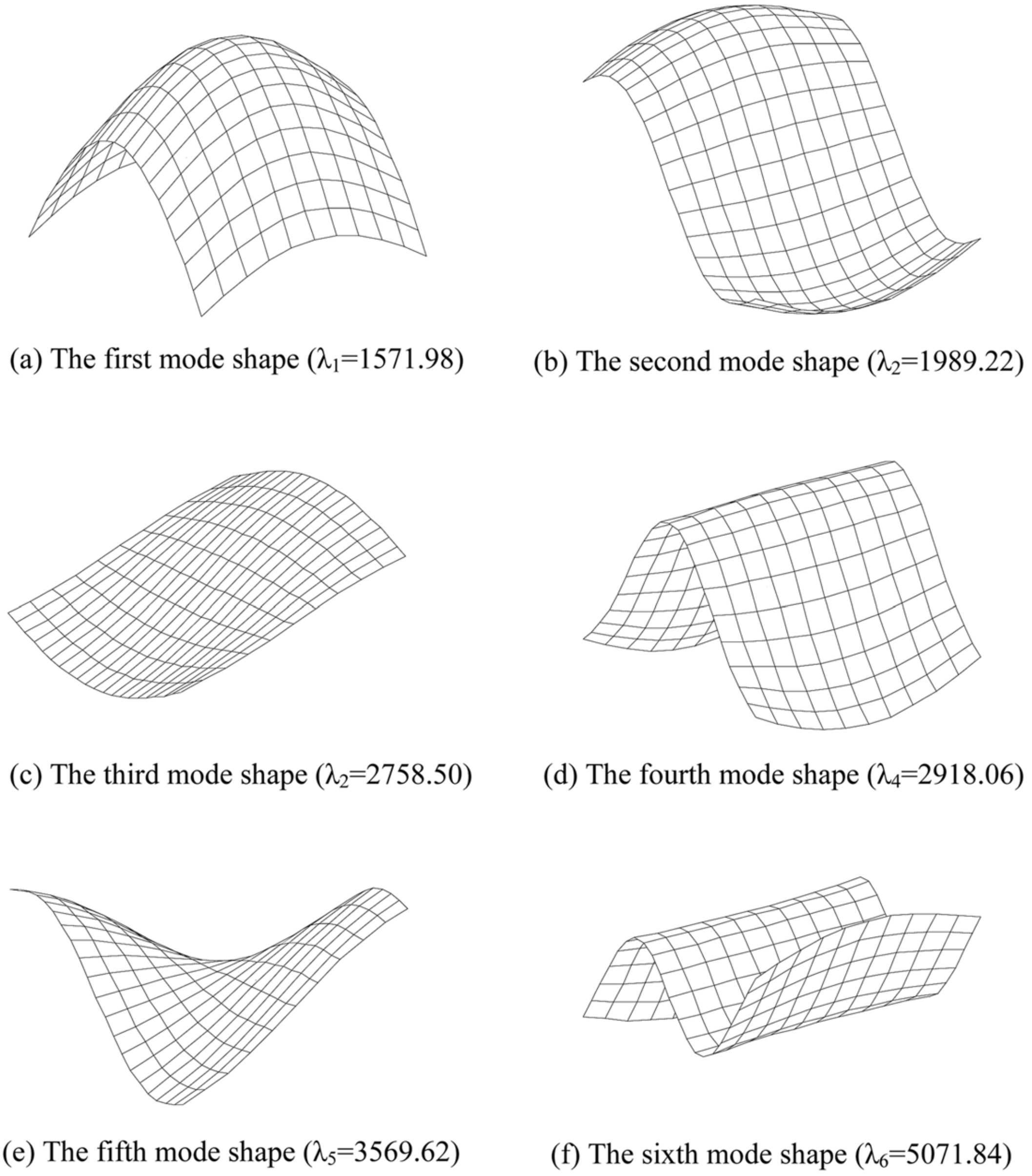


Fig. 8 The first six mode shapes of the plate on elastic foundations for $\gamma = 1$, $H = 5$ m and $l_y/l_x = 1$



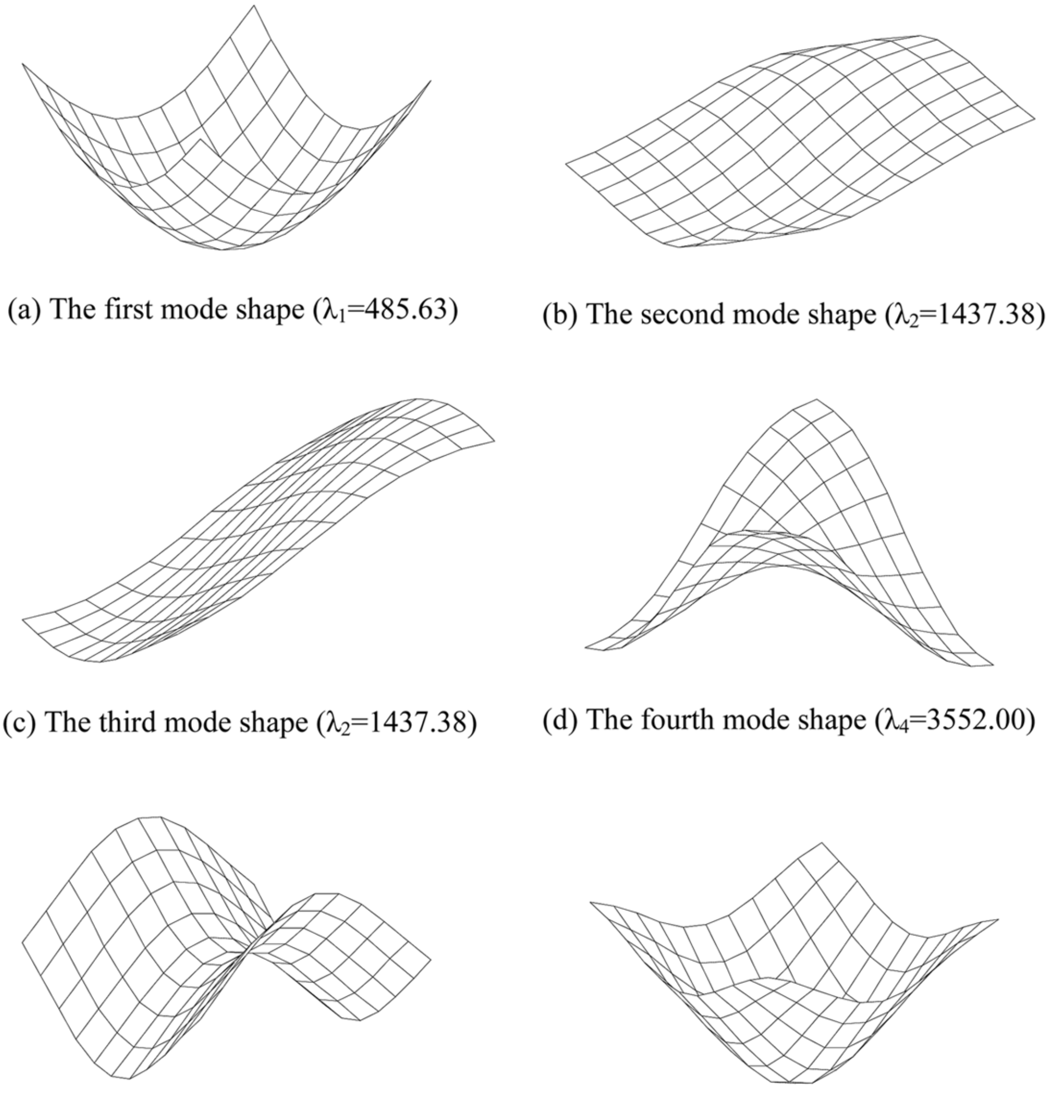
(a) The first mode shape (λ_1=1571.98) (b) The second mode shape (λ_2=1989.22)

(c) The third mode shape (λ_2=2758.50) (d) The fourth mode shape (λ_4=2918.06)

(e) The fifth mode shape (λ_5=3569.62) (f) The sixth mode shape (λ_6=5071.84)

Fig. 9 The first six mode shapes of the plate on elastic foundations for $\gamma = 1$, H = 5 m and $l_y/l_x = 2$

space, only the mode shapes corresponding to the six lowest frequency parameters of the plate for $\gamma = 1$, H = 5 m and l_y/l_x = 1.0, and 2.0 and for γ = 1, H = 15 m and l_y/l_x = 1.0, and 2.0 are presented. These mode shapes are given in Figs. 8, 9, 10, and 11, respectively. In order to make the visibility better, the mode shapes are plotted with exaggerated amplitudes. The scale factors used in these figures are not the same since the important thing in a mode shape is the shape of the mode, not the amplitude. It is well known that when an eigenvector is calculated, one value of the eigenvector is chosen by the user. Therefore, the amplitude of a mode shape can differ depending



(a) The first mode shape (λ_1=485.63)

(b) The second mode shape (λ_2=1437.38)

(c) The third mode shape (λ_2=1437.38)

(d) The fourth mode shape (λ_4=3552.00)

(e) The fifth mode shape (λ_5=4715.92)

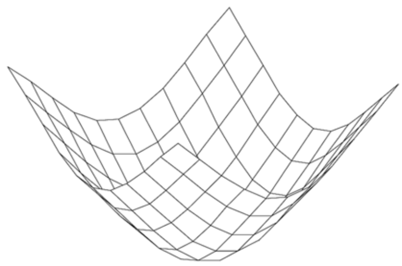
(f) The sixth mode shape (λ_6=5315.61)

Fig. 10 The first six mode shapes of the plate on elastic foundations for $\gamma = 1$, H = 15 m and $l_y/l_x = 1$

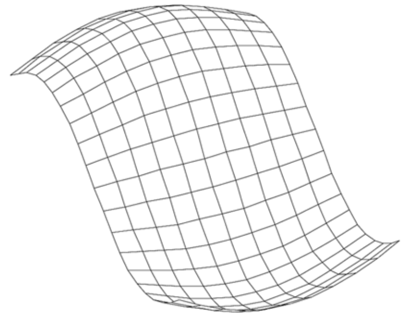
on the first value chosen by the user.

As seen from these figures, the number of half wave increases as the mode number increases. It should be noted that appearences of the mode shapes not given here for the other values of the parameters H, H/L_y, l_y/l_x, and γ are similar to those of the mode shapes presented here.

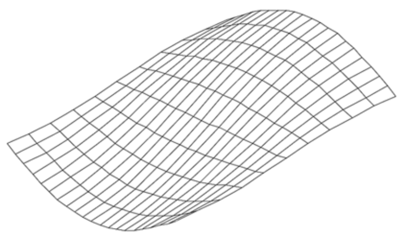
It should be noted that the results obtained by using a Modified Vlasov model are not compared with the results of the Winkler model, which is simpler, because the stiffness parameter, k, is calculated within the program depending on the assumed values of γ, but this parameters in the Winkler model should be given to the program as data.



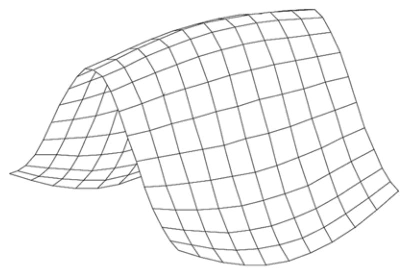
(a) The first mode shape (λ_1=365.33)



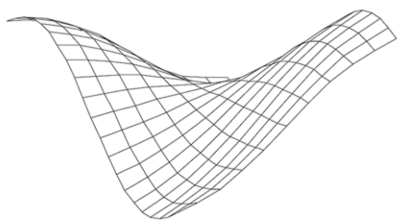
(b) The second mode shape (λ_2=639.01)



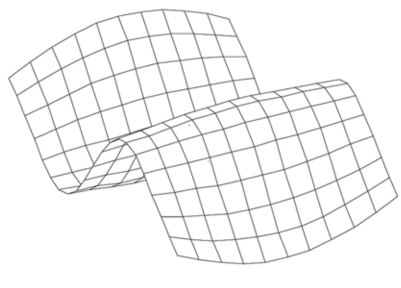
(c) The third mode shape (λ_2=1157.27)



(d) The fourth mode shape (λ_4=1297.86)



(e) The fifth mode shape (λ_5=1768.44)



(f) The sixth mode shape (λ_6=2733.01)

Fig. 11 The first six mode shapes of the plate on elastic foundations for $\gamma = 1$, $H = 15$ m and $l_y/l_x = 2$

4. Conclusions

The purpose of this paper was to apply the modified Vlasov model to the free vibration analysis of plates resting on elastic foundations and to analyze the effects of the subsoil depth, the ratio of the plate dimensions, the ratio of the subsoil depth to the plate dimension in the longer direction, and the value of the vertical deformation parameter, γ, within the subsoil on the frequency parameters of plates on an elastic foundation. As a result, the modified Vlasov model has been applied effectively to the free vibration analysis of plates resting on elastic foundation. In addition, the following conclusions can be drawn from the results obtained in this study.

- The frequency parameter always decreases with increasing aspect ratio, l_y/l_x for any values of H.
- The frequency parameter always increases with increasing H/l_y ratio for any values of subsoil depth.
- The frequency parameter always decreases as the subsoil depth increases for any values of l_y/l_x and H/l_y.
- The frequency parameter generally increases with increasing γ values for any values of H and H/l_y.
- In general, the frequency parameters of the plates on elastic foundations are more sensitive to the changes in the subsoil depth than the changes in the aspect ratio, l_y/l_x, and H/l_y ratio.

It should be noted that the results obtained by using a Modified Vlasov model are not compared with the results of the Winkler model, which is simpler, because the stiffness parameter, k, is calculated within the program depending on the assumed values of γ, but this parameters in the Winkler model should be given to the program as data.

It should also be noted that several similar conclusions were also found in the case of beam resting on elastic foundation (Ayvaz and Ozgan 2002).

References

Ayvaz, Y. and Özgan, K. (2002), "Application of modified Vlasov model to free vibration analysis of beams resting on elastic foundations", *J. Sound Vib.*, **255**, 111-127.

Ayvaz, Y., Daloğlu, A. and Doğangün, A. (1998), "Application of a modified Vlasov model to earthquake analysis of plates resting on elastic foundations", *J. Sound Vib.*, **212**, 499-509.

Celep, Z. and Güler, K. (2004), "Static and dynamic responses of a rigid circular plate on a tensionless Winkler foundation", *J. Sound Vib.*, **276**, 449-458.

Celep, Z. and Güler, K. (2007), "Axisymmetric forced vibrations of an elastic free circular plate on a tensionless two parameter foundation", *J. Sound Vib.*, **301**, 495-509.

Daloğlu, A., Doğangün, A. and Ayvaz, Y. (1999), "Dynamic analysis of foundation plates using a consistent Vlasov model", *J. Sound Vib.*, **224**, 941-951.

Güler, K. and Celep, Z. (1995), "Static and dynamic responses of a circular plate on a tensionless elastic foundation", *J. Sound Vib.*, **183**(2), 185-195.

Güler, K. and Celep, Z. (2005), "Response of a rectangular plate-column system on a tensionless Winkler foundation subjected to static and dynamic loads", *Struct. Eng. Mech.*, **21**(6), 699-712.

Hetenyi, M. (1950), "A general solution for the bending of beams on an elastic foundation of arbitrary continuity", *J. Appl. Phy.*, **21**, 55-58.

Huang, M.-H. and Thambiratnam, D.P. (2001), "Analysis of plate resting on elastic supports and elastic foundation by finite strip method", *Comput. Struct.*, **79**, 2547-2557.

Jones, R. and Xenophontos, J. (1979), "The Vlasov foundation models", *Int. J. Mech. Sci.*, **19**, 317-323.

Kolar, V. and Nemec, I. (1989), Modelling of Soil-Structure Interaction, Amsterdam: Elsevier Scientific Publishing Company.

Küçükarslan, K. and Banerjee, P.K. (2004), "Inelastic dynamic analysis of pile-soil-structure interaction", *Int. J. Comput. Eng. Sci.*, **5**(1), 245-258.

Liu, F.-L. (2000), "Rectangular thick plates on Winkler foundation: Differential quadrature element solution", *Int. J. Solids Struct.*, **37**, 1743-1763.

Maheshwari, B.K., Truman, K.Z., El Naggar, M.H. and Gould, P.L. (2004), "Three-dimensional nonlinear analysis for seismic soil-pile-structure interaction", *Soil Dyn. Earthq. Eng.*, **24**(4), 343-356.

Mezaini, N. (2006), "Effects of soil-structure interaction on the analysis of cylindrical tanks", *Practice Period.*

Struct. Des. Const., **11**(1), 50-57.
Omurtag, M.H. and Kadıoğlu, F. (1998), "Free vibration analysis of orthotropic plates resting on Pasternak foundation be mixed finite element formulation", *Comput. Struct.*, **67**, 253-265.
Selvaduari, A.P.S. (1979), Elastic Analysis of Soil-Foundation Interaction. Amsterdam: Elsevier Scientific Publishing Company.
Setoodeh, A.R. and Karami, G. (2004), "Static, free vibration and buckling analysis of anisotropic thick laminated composite plates on distributed and point elastic supports using a 3-D layer-wise FEM", *Eng. Struct.*, **26**, 211-220.
Shen, H.-S., Yang, J. and Zhang, L. (2001), "Free and forced vibration of Reissner-Mindlin plates with free edges resting on elastic foundations", *J. Sound Vib.*, **244**, 299-320.
Silva, A.R.D., Silveira, R.A.M. and Gonçalves, P.B. (2001), "Numerical methods for analysis of plates on tensionless elastic foundations", *Int. J. Solids Struct.*, **38**, 2083-2100.
Straughan, W.T. (1990), "Analysis of plates on elastic foundation", Ph.D. Thesis, The Graduate School of Texas Tech University, Lubbock, Texas.
Turhan, A. (1990), "A consistent Vlasov model for analysis of plates on elastic foundation using finite element method", Ph.D. Thesis, The Graduate School of Texas Tech University, Lubbock, Texas.
Vallabhan, C.V.G. and Das, Y.C. (1988), "A parametric study of beams on elastic foundations", *J. Eng. Mech. Div.*, **114**, 2072-2082.
Vallabhan, C.V.G. and Das, Y.C. (1991), "A refined model for beams on elastic foundation", *Int. J. Solids Struct.,* **27**, 629-637.
Vallabhan, C.V.G. and Das, Y.C. (1991), "Modified Vlasov model for beams on elastic foundation", *J. Geotech. Eng.*, **117**, 956-966.
Weaver, W. and Johnston, P.R. (1984), *Finite Element for Structural Analysis*. Englewood Cliffs, NJ: Prentice-Hall.
Xiang, Y. (2003), "Vibration of rectangular Mindlin plates resting on non-homogenous elastic foundations", *Int. J. Mech. Sci.*, **45**, 1229-1244.
Yu, L., Shen, H.-S. and Huo, X.-P. (2007), "Dynamic response of Reissner-Mindlin plates with free edges resting on tensionless elastic foundation", *J. Sound Vib.*, **299**, 212-228.
Zienkiewicz, O.C. (1977), *The Finite Element Method*. London: McGraw-Hill Ltd. (The third edition).
Çelik, M. and Saygun, A. (1999), "A method for the analysis of plates on a two-parameter foundation", *Int. J. Solids Struct.*, **36**, 2891-2915.