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Technical Note

Equivalent transform from the force-densities of cable nets to the stresses of membrane elements

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1. Introduction

The form-finding analysis of membrane structures is to find the geometry shape and the corresponding stress distribution on membrane surface. For cable nets the force-density method is an efficient form-finding method (Zhang 2002, Li and Guan 2004).

A simple method for form-finding analysis of membrane structures is to simulate the membrane surface as a cable net and to adopt the force-density method to find its shape and corresponding cable forces. But it will be encountered how to transform the cable forces to the membrane stresses that is necessary in the further loading case computation by finite element method. To avoid such transform and improve the form-finding efficiency, some approximate models and methods have been proposed. But all these approximation will lead to the losing of accuracy as the improving of the efficiency.

In this paper the membrane surface is described by the triangular membrane elements. The mesh lines can be also taken as the cable segments of a cable net with uniform force in all directions. The boundary cable segments belong to one triangle element and the others joint two triangle elements. The force-density method is adopted to carry out the form-finding analysis for the cable net. The obtained shape of the cable-net is taken as the membrane surface, and the transform relationship is derived from the forces of the cable net to the stresses of the triangular membrane elements. Such stresses fulfill the equilibrium conditions on the membrane surface.

This method makes it possible to use the force-density method to perform the form-finding analysis, then to transfrom the cable forces to the stresses of triangular elements, and finally directly to use the finite element method to carry out the loading case computation. With this transform the form-finding of membrane structures will be very efficient.

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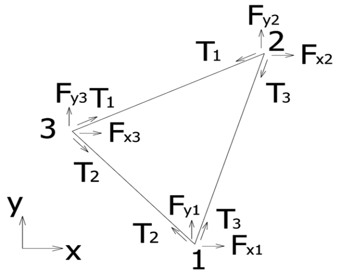


Fig. 1 Forces of cables and its equivalent nodal force in a single triangle element

2. Equilibrium conditions for cables and triangle elements

A single triangle element j is shown in Fig. 1. Taking the side lines of the element as the cable segments, we can denote $\{T\}^j = [T_1 T_2 T_3]^T$ as the force vector and $\{q\}^j = \{q_1^j q_2^j q_3^j\}^T$ as the corresponding force density vector of cables, where $q_i^j = T_i/l_i$ and l_i is the length of the cables. In Fig. 1 $\{F\}^j = [F_{x1} F_{y1} F_{x2} F_{y2} F_{x3} F_{y3}]^T$ is the equivalent nodal force vector.

Eq. (1) exists according to the equilibrium conditions at nodes show at Fig. 1. The relationship between $\{F\}$ and $\{q\}$ can be expressed by the coefficient matrix $[A]$.

$$\{F\}^j = [A]\{q\}^j \tag{1}$$

According to the theory of finite element method (Zhang 2002), the nodal force vector of the single triangle element can be also expressed as

$$\{F\}^j = \int_v [B]^T \{\sigma\}^j dv = [B]^T \{\sigma\}^j \Delta t \tag{2}$$

where, v, Δ and t are the volume, area and thickness of element j, respectively.

From Eqs. (1) and (2) we have

$$\{q\}^j = [c]^j \{\sigma\}^j \tag{3}$$

The relationship between the stress vector and the force-density vector of membrane structures can be obtained and expressed as in Eq. (4) by assembling Eq. (3) for all elements ($j = 1, ..., m$), where m is the amount of elements.

$$[C]\{\sigma\} = \{q\} \tag{4}$$

Here, $[C]$ is a maxtrix with $n \times 3m$ order, where n is the amount of the mesh lines on the membrane surface. $[C]$ relates only to membrane thickness and the local coordinates of the nodes at the state from form-finding analysis. $\{q\} = \{q_1 \ldots q_n\}^T$ is the force density vector of the cable nets and $\{\sigma\} = \{[\sigma]^1 \ldots [\sigma]^j \ldots [\sigma]^m\}^T$. Assuming the k'th cable segment is composed of the k_1'th line of the j_1'th triangle element and the k_2'th line of the j_2'th triangle elements, we can have

$$q_k = q_{k_1}^{j_1} + q_{k_2}^{j_2} \tag{5}$$

It should be noticed that the cable forces of the cable net from the force-density method fulfill the equilibrium conditions at all nodes. The stresses obtained from Eq. (4) will lead to the same equivalent nodal forces as the cable forces. Hence, the stresses of all triangle elements corresponding the cable net can also fulfill the equilibrium conditions at nodes.

3. Transfrom from cable forces to element stresses

At the common side of the two conjointed elements the strains along this side obtained from the two elements should be compatible.

According to the phycical equtaion (Xu 1990), the strain in the direction of α with x can be expressed by σ_x, σ_y and τ_{xy} in the element.

Let $\varepsilon_{\alpha^1} = \varepsilon_{\alpha^2}$, the strain equation along the common side can be expressed by σ_x, σ_y, τ_{xy} of every two conjointed elements. There are $3m-n$ common sides in membrane surfaces described by m triangle elements. For all the common sides we can establish and assemble Eq. (6). Finally we can obtain

$$[B]\{\sigma\} = 0 \tag{6}$$

where, $[B]$ is the matrix with $(3m-n) \times (3m)$ order relating only to the material parameter and the local coordinates from form-finding analysis.

Combining Eqs. (4) and (6) we can obtain the transform relationship between the force densities of cable net and the stresses of membrane elements as follows

$$[G]\{\sigma\} = \{b\} \tag{7}$$

where, $[G] = [C\ B]^T$ is a matrix with $3m \times 3m$ order, and relates only to the material parameter, membrane thickness and the local coordinates from the form-finding analysis. $\{b\} = \{q\ 0\}^T$ is a vector with $3m$ order and relates only to the force-densities of cable nets.

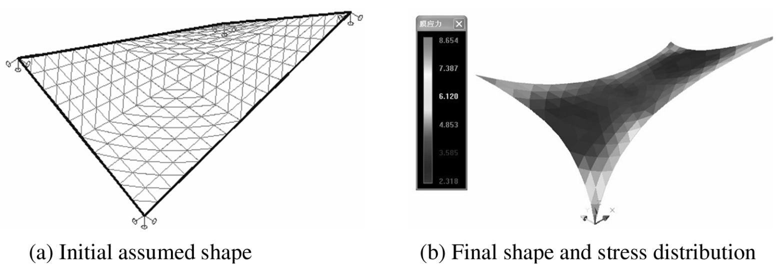
Eq. (7) is linear independant, therefore the matrix $[G]$ can be inversed. The stresses of membrane elements can be solved from force densities and geometry shapes of cable nets.

4. Numerical examples

4.1 A saddle membrane structure

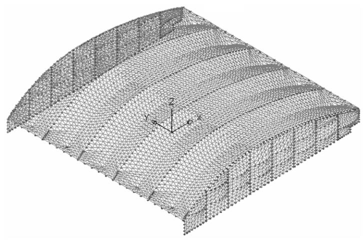
A saddle membrane structure, with the length 3000 mm, the width 2000 mm and the height 1000 mm, is considered here. Fig. 2(a) shows its initially assumed shape. The inital force density in the cable net is 1 and the initial force on the boundary cable is 10.

The meshes shown in Fig. 2 can be also thought of as a cable net. By the force-density method the form-finding analysis can be very easily and efficiently performed for such cable net. The obtained minimum surface is shown in Fig. 2(b) and the stress distribution on the membrane structure corresponding to the surface is also shown in Fig. 2(b) from the transform Eq. (7).

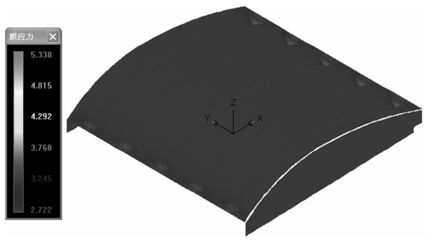


(a) Initial assumed shape (b) Final shape and stress distribution

Fig. 2 Saddle membrane structure



(a) Equilibrium shape



(b) Maximum main stress distribution

(c) Photo of Shanghai Zabei Tennis Ball Gymnasium

Fig. 3 Shanghai Zabei Tennis Ball Gymnasium

4.2 A practical project of membrane structure: Shanghai Zabei Tennis Ball Gymnasium

The form-finding of the membrane surface of Shanghai Zabei Tennis Ball Gymnasium is carried out by the equivalent method presented in this paper. Fig. 3(a) shows the equilibrium shape of the cable net obtained from the force-density method, which can be very efficiently solved. Fig. 3(b) shows the stress distribution of membrane surface transformed from the force densities of cable net. It would be very time-consuming due to many times of iterations to solve such stress distribution by using finite membrane element method. The finished structure is shown as in Fig. 3(c), which has been safely used for more than three years.

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