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The necessary number of profile lines for the analysis of concrete fracture surfaces

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Abstract. The article describes a technique for the measurement of the level of complexity of fracture surfaces by the method of vertical sections, and a performed statistical analysis of the effect of profile lines on the fractographic and fractal parameters of fractures, i.e. the profile line development factor, R_L, and the fracture surface development factor, R_S, (as defined by the cycloid method), as well as the fractal dimension, D_C, (as determined by the chord method), and the fractal dimension, D_{BC}, (as determined by the box method). The above-mentioned parameters were determined for fracture surfaces of basalt and gravel concretes, respectively, which had previously been subjected to fracture toughness tests. The concretes were made from mixtures of a water/cement ratio ranging from 0.41 to 0.61 and with a variable fraction of coarse aggregate to fine aggregate, $C_{agg.}/F_{agg.}$, in the range from 1.5 to 3.5. Basalt and gravel aggregate of a fraction to maximum 16 mm were used to the tests. Based on the performed analysis it has been established that the necessary number of concrete fracture profile lines, which assures the reliability of obtained testing results, should amount to 12.

Keywords: concrete; fractal dimension; pores structure; image analysis.

1. Introduction

In recent years, the interest of researchers has been focused on the use of fractal geometry (Mandelbrot 1977) in the testing of various constructional materials, and on the attempts to relate the fractal dimension, which defines the level of structure complexity, with the mechanical features of materials. In the 80s of the 20th century it was determined that the surfaces of cement pastes, mortars and concretes, formed as a result of failure (Winsolw 1985), are fractals. Subsequent years, up to the present day, have resulted in numerous publications in the scope of using fractal geometry in the testing of cement-based materials, e.g. (Saouma *et al.* 1990, Saouma and Barton 1994, Wu *et al.* 2000, Konkol and Prokopski 2004, 2005). The fractal analysis of concrete fracture surfaces is conducted by various methods. It can cover the surface (the 3D analysis, e.g. Zhou and Xie 2003) or, much more often used so far, the analysis of fracture profile lines (the 2D analysis). Until now, the fractal dimension of fracture profile lines has been more frequently analyzed, with the most often used methods being the chord method and the box method.

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The examination of profile lines aimed at the determination of the fractal dimension, or other characteristics of fracture surface morphology, necessitate the completion of a finite number of measurements, or the number of measurements sufficient for the unique description of the fracture surface, with the preset error of estimation of the average value. The number of measurements must also be appropriate from the point of view of economy, i.e. time and labour consumption. For concrete fractures, no studies have been carried out to date, which would aim at determining the necessary number of profile lines analyzed to obtain the reliable parameters of fracture surface morphology. Obviously, the number of profile lines subjected to examination should be selected so that both the requirements of mathematical statistics and economical considerations be satisfied. Furthermore, in view of the well-known high variability of the fractal dimension of concrete fracture profile lines over the fracture width, it is of purpose to establish an appropriate number of profile lines, so that too low their number do not lead to erroneous conclusions, and the analyses are not burdened with a large error.

So far, researchers have a priori set the number of profile lines examined. For example, Saouma *et al.* (1990), used 4, and in one instance 11 profile lines for determining the fractal dimension of concrete fracture profile lines by the box method. In doing this, they varied the direction of choosing profile lines (these were the lines parallel and perpendicular to the fracture direction), as well as the resolution, the measurement step length and the length of the lines examined.

Four profile lines, for determining the fractal dimension, were also used in work Saouma and Barton (1994). In addition to the average value of the fractal dimension, as determined by the box method, also the standard deviation of fractal dimension is given in that work.

Wu *et al*. (2005) have determined the fractal dimension of a surface as the sum of the fractal dimensions: D_1, obtained based on vertical sections of a direction coincident with that of the fracture, and D_2, determined from the vertical sections of a direction perpendicular to that of the fracture, with the values of D_1 and D_2 being estimated by them based on 6 measurement results.

In work Guinea *et al.* (2002) 10 profile lines are assumed as the sufficient number, and, in addition to the average value of the parameters: R_a (average roughness) and R_q (RMS roughness), a confidence interval of 68% is also given.

Compared to above-mentioned studies, a very large number of results were used in work Xie *et al.* (1999) for the determination of the average value of the fractal dimension. When examining fracture profiles in rocks, an analysis of change in the average value of the fractal dimension, as determined by the variogram method, was performed based on as many as 81 profile lines.

The examples of profile line number selection, quoted above, confirm the lack of a unified approach by the authors. The number of profile lines examined varied from 4 up to even 81 profile lines.

The purpose of investigation described in the present study was to determine the necessary number of profiles examined in order to obtain reliable average values of the concrete fracture surface morphology parameters (R_L, R_S, D_C and D_{BC}) being determined. The analyses were carried out with a preset error of estimation of the average value.

2. Description of tests and the analysis of obtained results

Concretes with a water/cement ratio, W/C, in the range from 0.41 to 0.61 and with a fraction of coarse aggregate to fine aggregate (sand), $C_{agg.}/F_{agg}$, ranging from 1.5 to 3.5 were subjected to

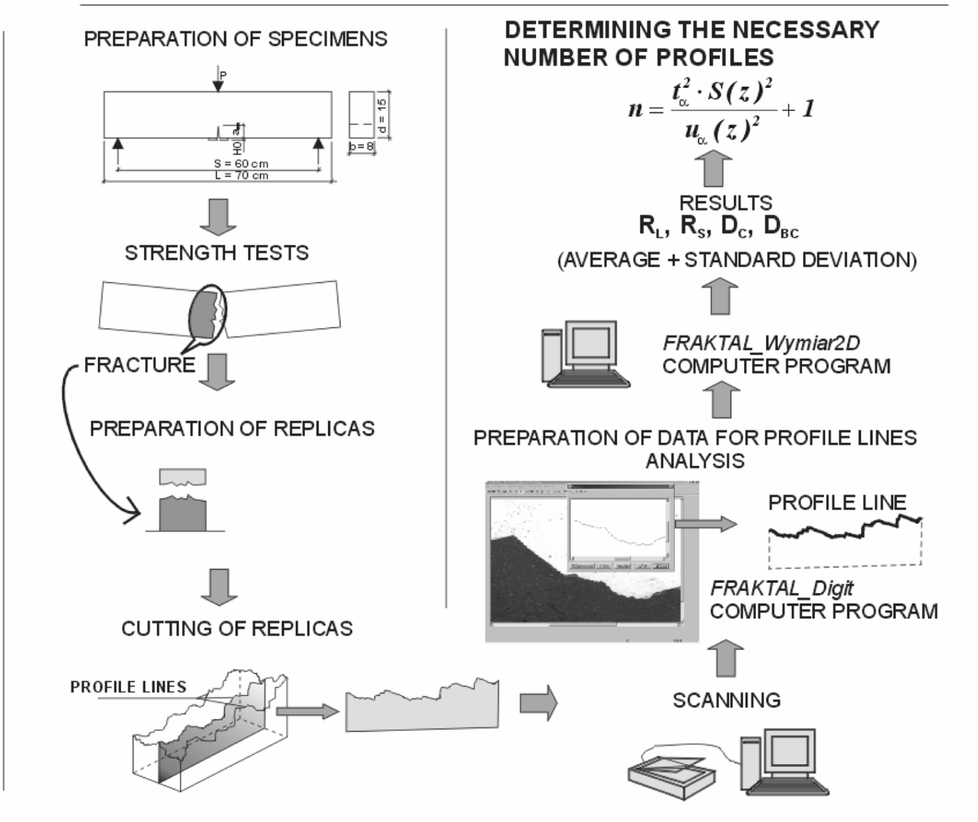

Fig. 1 Procedure

testing. Basalt and gravel aggregate of a fraction to maximum 16 mm were used to the tests. Overall, concretes obtained from 9 different concrete mixes were tested. The compositions of concretes and the values of the profile line development factor, R_L, and fracture surface development factor, R_S, obtained from the tests are published in work (Konkol and Prokopski 2004).

Fractographic and fractal tests were performed on specially prepared gypsum replicas of the fracture surfaces of concrete beams of dimensions of 8 × 15 × 70 cm, with one primary crack (Fig. 1), which had been previously used for fracture toughness tests according to mode I of fracture (tension at bending).

The preparation of specimens involved making of concrete fracture replicas of white gypsum, and then casting of stained gypsum onto them. The specimens were then cut along their longer side into 10 layers, each about 5 mm thick, thus obtaining 20 profile lines for each fracture (Fig. 1). The length of fracture surface is 80 mm. Result 10 × 5 mm = 50 mm it's the exclusively sum of thickness of layers got in result of cut of sample with diamond saw. Value 50 mm does not to take into account as a result of cut which made up lacking about 30 mm the decrease of material. The aim of adopting such a cutting direction was to obtain profile lines roughly consistent with the direction of crack propagation. The cut specimens were scanned at a resolution of 600 dpi., while separating an area of a size of 100 × 29 mm from each of them (Fig. 2). Computer images in the form of bitmaps served for obtaining information of the shape of profile lines. This data, after being entered to the *FRAKTAL_Wymiar2D*[1] software, enabled the computation of the parameters R_L, R_S, D_C and D_{BC}.

In the tests, the profile line development factor, R_L, and the fracture surface development factor, R_S, were determined (by the cycloid method, Wojnar 1990), whereas the fractal dimension, D, was determined, respectively, by the chord method (D_C) and the box method (D_{BC}).

The high variability of the parameters R_L, R_S, D_C and D_{BC} over the specimen width (Fig. 3) confirms the purposefulness of carrying out investigation aimed at determining the number of profiles necessary for obtaining an average value of $\overline{R}_L$, $\overline{R}_S$ and $\overline{D}$, which would be reliable from the statistical analysis point of view.

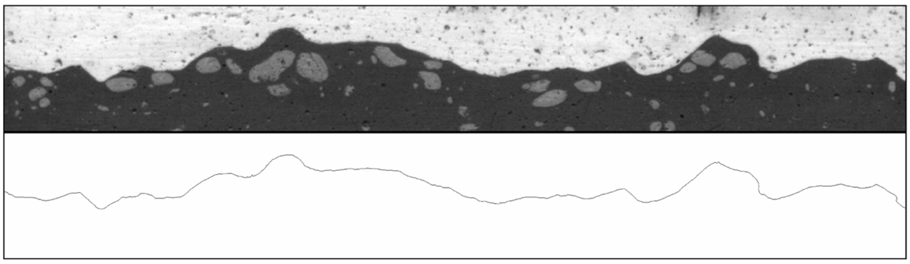

Fig. 2 Example of a fracture profile scanned at a resolution of 600 dpi (top) and the result of digitization (bottom) obtained using the *FRAKTAL_Digit*[2] software

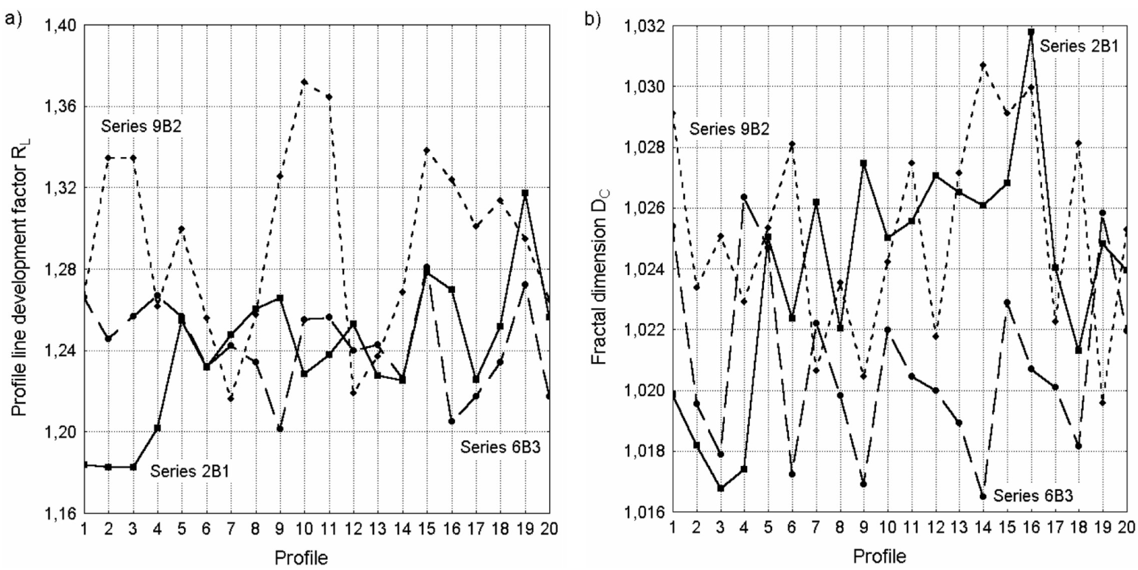

Fig. 3 Examples of diagrams of the effect of profile position on the specimen width on the values of: a) profile line development factor, R_L, b) fractal dimension, D_C

[1]Konkol, J.:*FRAKTAL_Wymiar2D*. A computer program.
[2]Konkol, J.: *FRAKTAL_Digit*. A computer program.

The diagrams shown (Fig. 3) were obtained from the examination of three profiles selected randomly from 45 replicas, for which 894 profile lines were obtained in total, with 472 profile lines from 24 replicas of basalt concrete fracture surfaces and 422 profile lines from 21 replicas of gravel concrete surfaces being subjected to analysis.

As follows from Fig. 3, the values of the fractal dimension on the specimen width are scattered in a random manner. A fitting of the fractal dimension value distribution on the specimen width to the normal distribution was made. Due to the unknown average value and unknown standard deviation, the Kolmogorov-Lilliefors and Shapiro-Wilk tests were employed. Examples of the obtained

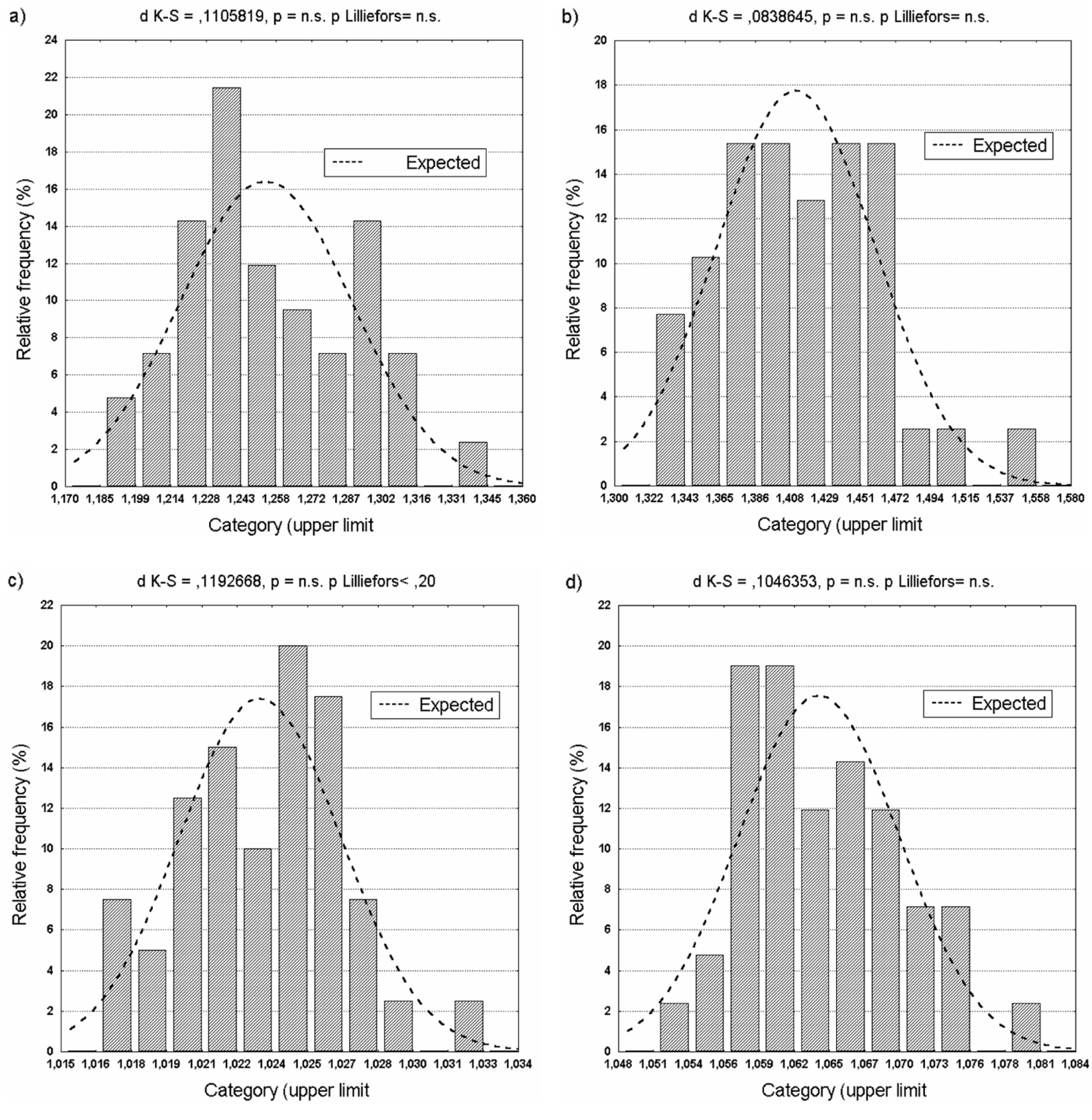

Fig. 4 Histograms of the distribution of the values of measured parameters on the specimen width, with the fitting of their distribution to the normal distribution (basalt concrete)
a) R_L (specimen 4B4), b) R_S (specimen 2B1), c) D_C (specimen 2B1), d) D_{BC} (specimen 4B4)

histograms are shown in Fig. 4. The number of classes k was determined from the formula below:

$$k = 1 + 3{,}322 \cdot \ln(n) \tag{1}$$

where
n: number of results.

In all cases, based on the performed fit goodness tests, a fitting of the distribution of measured parameters on the specimen width to the theoretical distribution assumed as the normal one was obtained. The fitting was made for results obtained from two fracture surfaces of the same specimen. The fitting of results obtained for a single fracture was also verified, where it was demonstrated that the distribution of the average value on the specimen width is consistent with the normal distribution.

As shown by the histograms (Fig. 4), the largest frequency of values of the parameters R_L, R_S, D_C and D_{BC} from the same class interval does not exceed 22%. These values gather at a frequency exceeding 70% (from 71 to 76%) in the 5 middle class bands (with the number of all class intervals being equal to 13). In the middle band (the values from the class interval are close to the average value), the frequency of the parameters R_L, R_S, D_C and D_{BC} is merely from about 10 to 20%, which is indicative of large scatters of results.

On the basis of the performed measurements, measurement inaccuracies were determined using the measures of the position and dispersion of

Table 1 Average values of statistics determined for all 894 testing results

Feature examined	Statistics*						
	$\bar{z}$	$R(z)$	$S(z)$	$S(\bar{z})$	$u_{0,05}(z)$	$u_{0,05\,\max}(z)$	υ
Basalt concrete							
R_L	1.251	0.130 (10.39%)	0.037 (2.96%)	0.008 (0.67%)	0.018 (1.42%)	0.031 (2.49%)	2.93%
R_S	1.443	0.217 (15.03%)	0.060 (4.16%)	0.014 (0.94%)	0.029 (2.01%)	0.055 (3.82%)	4.17%
D_C	1.0234	0.0134 (1.31%)	0.0036 (0.35%)	0.0008 (0.08%)	0.0018 (0.18%)	0.0027 (0.26%)	0.36%
D_{BC}	1.0644	0.0228 (2.14%)	0.0064 (0.60%)	0.0015 (0.14%)	0.0031 (0.29%)	0.0056 (0.53%)	0.61%
Gravel concrete							
R_L	1.257	0.143 (11.37%)	0.040 (3.18%)	0.009 (0.70%)	0.019 (1.51%)	0.028 (2.25%)	3.14%
R_S	1.450	0.232 (16.01%)	0.062 (4.28%)	0.014 (0.95%)	0.030 (2.04%)	0.045 (3.08%)	4.23%
D_C	1.0268	0.0174 (1.69%)	0.0048 (0.47%)	0.0011 (0.11%)	0.0023 (0.22%)	0.0034 (0.33%)	0.46%
D_{BC}	1.0646	0.0251 (2.36%)	0.0067 (0.63%)	0.0015 (0.14%)	0.0032 (0.30%)	0.0046 (0.43%)	0.63%

* - in brackets, the percentage value of a given statistic as related to the average value is given

✓ the average $\bar{z}$,
✓ the average range $R(z)$,
✓ the average standard deviation $S(z)$,
✓ the average standard error (standard deviation of the average) $S(\bar{z})$,
✓ the average confidence interval at a level of confidence $u_{0,05}(z)$ of 95%,
✓ the maximum confidence interval at a level of confidence $u_{0,05\,\max}(z)$ of 95%,
✓ the average coefficient of variation υ.

The obtained results of statistical analysis are summarized in Table 1.

Assuming the normal distribution of fractal dimension values on the specimen width of an unknown average and unknown standard deviation, and relating the standard deviation of measured values to the adopted confidence interval, the minimum number of profiles, n, was calculated from the following formula

$$n = \frac{t_\alpha^2 \cdot S(z)^2}{u_\alpha(z)^2} + 1 \tag{2}$$

where

t_α : the quantile of Student's distribution at $(n-1)$ degrees of freedom and the level of significance α,
$S(z)$: standard deviation estimated based on the tests carried out,
$u_\alpha(z)$: confidence interval, or the preset maximum error of estimation of the average value.

In order to determine the necessary number of profiles according to relationship (2), the results of standard deviation $S(z)$ values and confidence intervals $u_{0,05}(z)$ were used (Table 1).

Assuming the confidence interval to be greater by 1.4 times (it was assumed such, in order that the obtained value did not exceed the maximum value of the confidence interval $u_{0,05\,\max}(z)$), it was

Table 2 Determination of the necessary number of profile lines

Feature examined	Statistics*				
	Assumed	Determined			
	$u_{0,05}(z)$	t_α	n	$S(z)$	$S(\bar{z})$
			Basalt concrete		
R_L	0.025	2.201	12	0.038 (+1.49%)	0.011 (+29.05%)
R_S	0.041			0.059 (−1.22%)	0.017 (+28.07%)
D_C	0.0025	2.228	11	0.0036 (0.00%)	0.0011 (+30.00%)
D_{BC}	0.0043	2.201	12	0.0063 (−1.22%)	0.0019 (+26.63%)
			Gravel concrete		
R_L	0.027	2.201	12	0.040 (+0.21%)	0.012 (+31.49%)
R_S	0.041			0.061 (−1.22%)	0.018 (+28.56%)
D_C	0.0032			0.0047 (−1.22%)	0.014 (+26.82%)
D_{BC}	0.0045			0.0066 (−1.22%)	0.019 (+26.57%)

* - in brackets, percentage change in the value of a given statistic as related to the value obtained from the tests is given

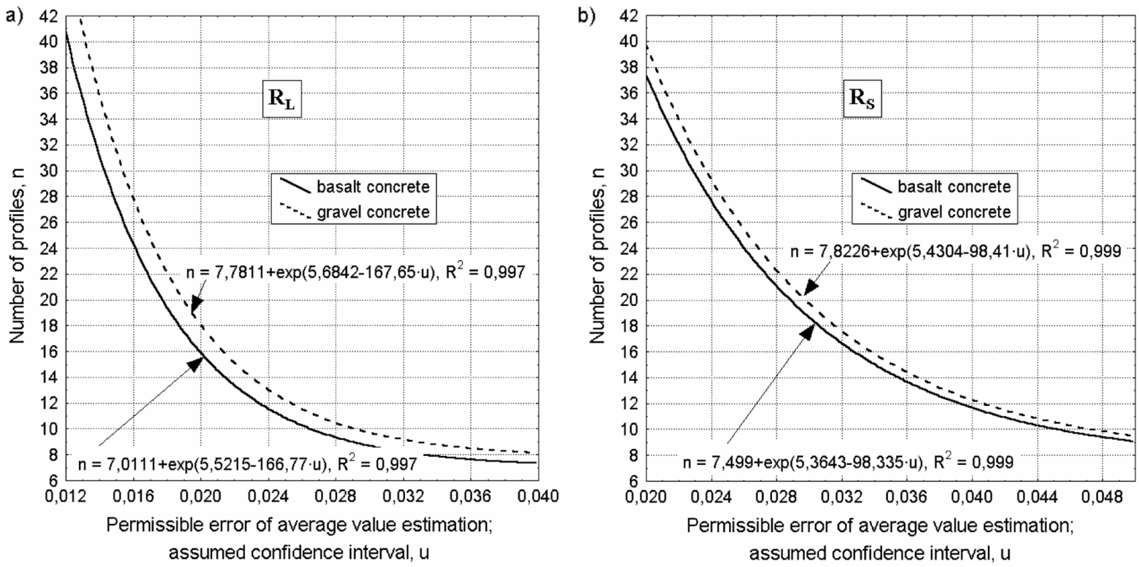

Fig. 5 Relationship of the number of profiles n versus the permissible error of estimation of the average value of: a) profile line development factor, R_L, b) fracture surface development factor, R_S

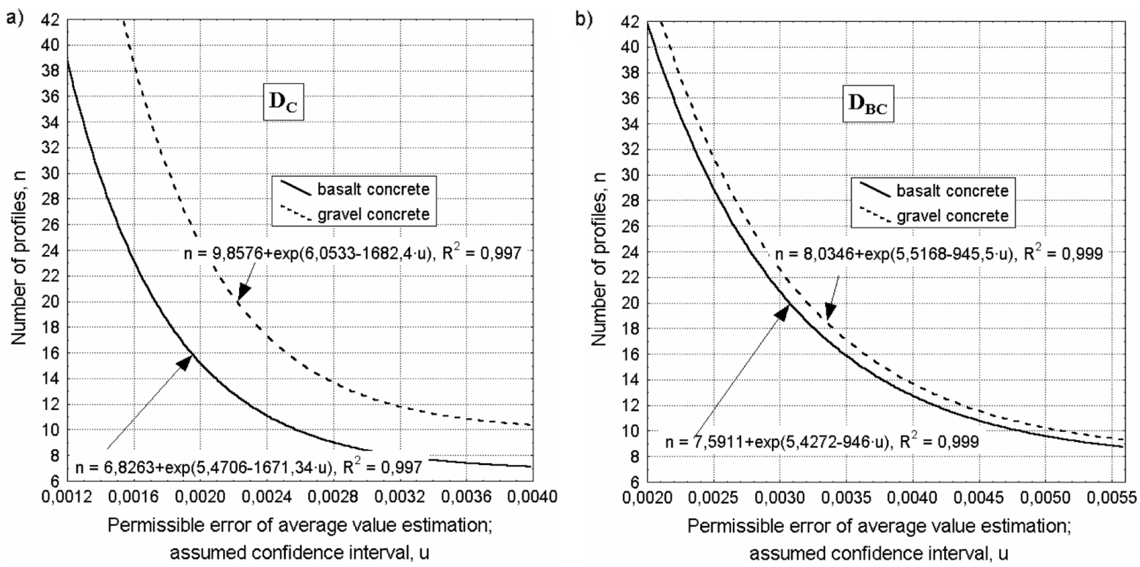

Fig. 6 Relationship of the number of profiles n versus the permissible error of estimation of the average value of the fractal dimension: a) D_C, determined by the chord method, b) D_{BC}, determined by the box method

found that the number of analyzed profiles should amount to at least 12 (Table 2). Proceeding similarly, i.e. taking the confidence interval value obtained from the tests, the necessary number of profiles was obtained to be equal to at least 20, whereas for the maximum confidence interval values this number ranged from 8 to 12 profiles.

Table 2 gives also the quantile of Student's t – distribution at $(n-1)$ degrees of freedom and a level of significance of $\alpha = 0.05$, and the values of standard deviation and standard error, as determined from the transformation of formula (2).

As follows from the comparison of the results shown in Tables 1 and 2, for both performed analyses, similar standard deviation values are obtained; however, the standard error value has changed by approx. 30%, which results in the possibility of a greater scatter of the average value.

Based on the standard deviations $S(z)$ (Table 1) and Eq. (1) obtained for both concretes, the necessary number of profiles n, as dependent on the permissible error of average value estimation (confidence interval u), was obtained using the method of successive approximations. The obtained functions are shown in Figs. 5 and 6.

The above relationships (Figs. 5 and 6) have been obtained by substituting the values of standard deviation determined from the tests to Eq. (2).

In the case of a larger and smaller scatter of results, or a different standard deviation value, than those assumed in the present study, it will be necessary to examine a larger number of profiles than resulting from Figs. 5 and 6. The number of profiles to be examined should then be calculated from the formula below

$$n = A_0 + \exp(A_1 + A_2 \cdot u + A_3 \cdot S(z) + A_4 \cdot u^2 + A_5 \cdot S(z)^2) \quad (3)$$

where

n : the necessary number of profiles,
u : permissible error of estimation of the average value,
$S(z)$: standard deviation value obtained from the tests,
A_i : coefficients for Eq. (3) (Table 3).

Note! In order to obtain results with the assumed scatter using formula (3), the values of n should be rounded off up.

The proposed relationship is valid within the following interval of variation of $S(z)$ and u

- ✓ for R_L, u in the range from 0.01 to 0.04, $S(z)$ in the range from 0.03 to 0.055,
- ✓ for R_S, u in the range from 0.02 to 0.05, $S(z)$ in the range from 0.05 to 0.08,
- ✓ for D_C, u in the range from 0.001 to 0.004, $S(z)$ in the range from 0.003 to 0.006,
- ✓ for D_{BC}, u in the range from 0.002 to 0.0055, $S(z)$ in the range from 0.0055 to 0.0085.

For the adopted model of regression (formula (2)), a very good fitting of the expected values to the observed values has been obtained; the coefficient of determination, R^2, is close to unity.

The relationships of $n = f(u, S(z))$, obtained for the parameters examined (R_L, R_S, D_C and D_{BC}),

Table 3 Summary of the coefficients A_i and the coefficients of determination R^2 for Eq. (3)

Feature examined	Coefficients of the approximating multinomial and the coefficients of determination						
	A_0	A_1	A_2	A_3	A_4	A_5	R^2
R_L	3.4640	3.0085	−218.3	98.40	2633	−585.6	0.999
R_S	3.1014	2.8622	−130.5	62.43	1022	−241.3	1.0
D_C	3.4552	3.1831	−2185.6	898.6	265490	−48666	0.999
D_{BC}	3.2354	2.9305	−1254	581.2	92844	−20858	1.0

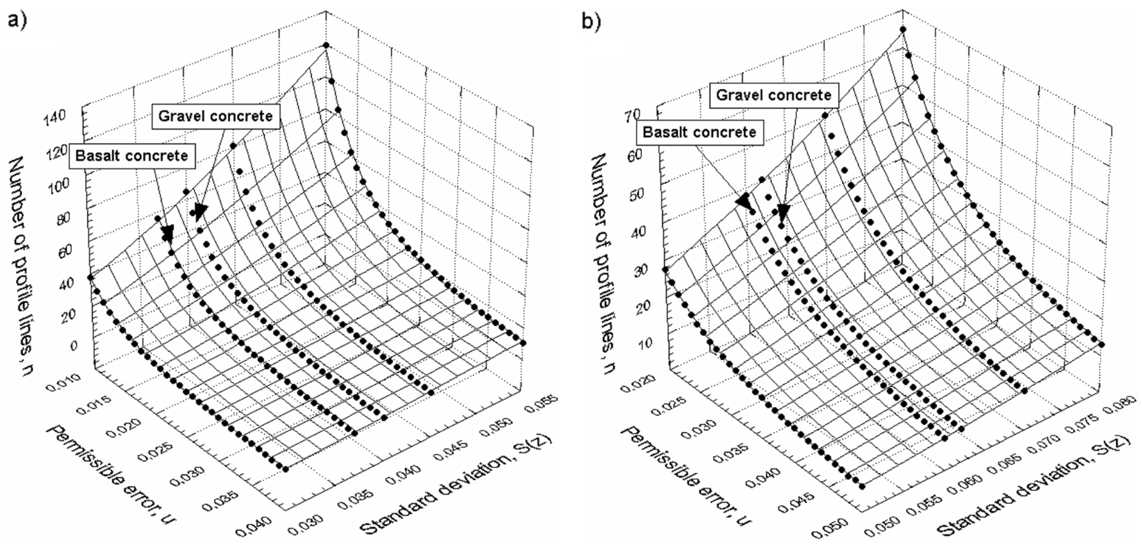

Fig. 7 Diagram of the function $n = f(u, S(z))$ obtained for: a) profile line development factor, R_L, b) fracture surface development factor, R_S

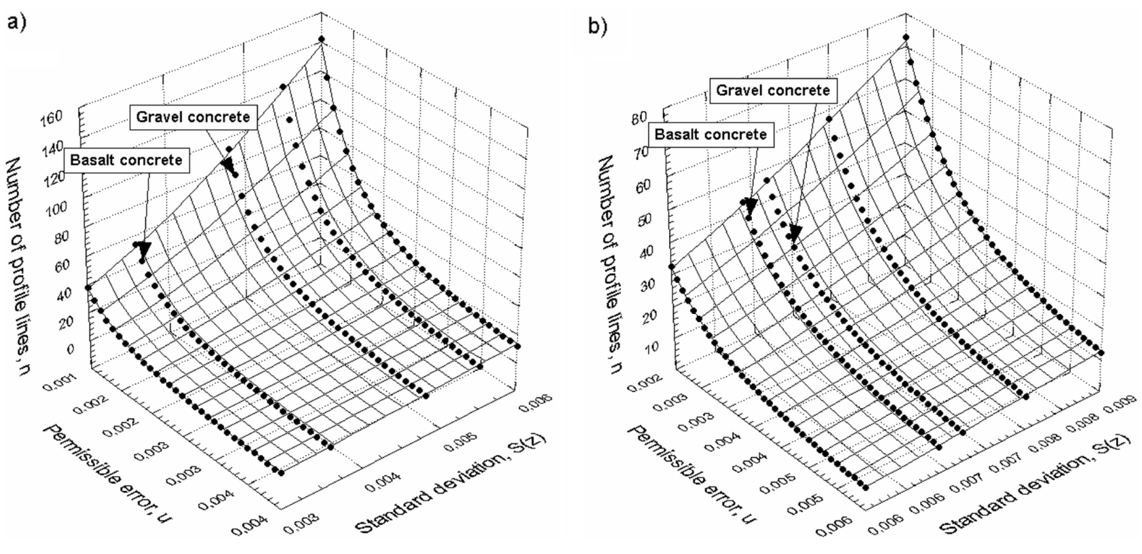

Fig. 8 Diagram of the function $n = f(u, S(z))$ obtained for the fractal dimensions, respectively: a) D_C, b) D_{BC}

are shown in Figs. 7 and 8. These graphs are obtained by substituting five different standard deviation values to Eq. (3), whereas two of them are the values obtained from the actual examination of the fracture surface profile lines of gravel and basalt concretes, respectively, one is the intermediate value ($R_L = 0.045$, $R_S = 0.07$, $D_C = 0.0055$ and $D_{BC} = 0.0075$), while the remaining two values constitute, respectively, the lower and upper limits of the standard deviation and are given above as the permissible interval of variability, $S(z)$.

3. Conclusions

1. Because of the high laboriousness of examination associated with the determination of the fracture surface morphology parameters of concrete specimens, it is purposeful to determine the necessary number of profiles enabling the reliable estimator of the average value of these parameters to be obtained.
2. By carrying out a large number of measurements (in total, 894 fracture profiles of basalt and gravel concretes were analyzed), it was possible to determine the average value of standard deviation and the average value of the confidence interval of the average value. The necessary number of profiles was determined by assuming that the examined features has the normal distribution of an unknown average value and an unknown standard deviation of the population, by using the obtained results of $S(z)$ and $u_{0,05}(z)$, and also assuming a priori the permissible error of estimation of the average value. For a permissible average value estimation error greater by 1.4 times than the one obtained from the examination of the confidence interval $u_{0,05}(z)$, the necessary number of profile lines was found to be at least 12.
3. Where no information of the value of standard deviation is available, the number of profiles n can be calculated by using the functions given in Figs. 5 and 6, depending on the assumed value of the permissible average value estimation error.
4. Where the standard deviation values have been estimated based on examination other than that presented in this study, the determination of the number of profiles n can be done by utilizing function (3), using the coefficients given in Table 3.
   It is recommended, however, to exercise care when using the above relationships beyond the indicated interval of variability of $S(z)$ and u.
5. On the basis of the performed analyses, the authors believe that the optimal number of concrete fracture profiles, being sufficient for analysis, is 12, which is a number of lines possible to be examined in a reasonable time and using the measurement technique described in the present study.

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