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An efficient three-dimensional fluid hyper-element for dynamic analysis of concrete arch dams

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Abstract. The accurate dynamic analysis of concrete arch dams relies heavily on employing a three-dimensional semi-infinite fluid element. The usual method for calculating the impedance matrix of this fluid hyper-element is dependent on the solution of a complex eigen-value problem for each frequency. In the present study, an efficient procedure is proposed which simplifies this procedure amazingly, and results in great computational time saving. Moreover, the accuracy of this technique is examined thoroughly and it is concluded that efficient procedure is incredibly accurate under all practical conditions.

Keywords: efficient fluid hyper-element; concrete arch dams; dynamic analysis.

1. Introduction

The dynamic analysis of concrete dams can be approached using several different techniques (Camara 2000, Dominguez and Maeso 1993, Maeso *et al.* 2002) However, the rigorous analysis of concrete arch dam-reservoir system is based on the FE-(FE-HE) method. This means, the dam is discretized by solid finite elements, while, the reservoir is divided into two parts, a near field region (usually an irregular shape) in the vicinity of the dam and a far field part (assuming uniform channel), which extends to infinity. The former region is discretized by fluid finite elements and the latter part is modeled by a three-dimensional fluid hyper-element. The analysis is carried out in frequency domain either by direct approach (Lotfi 2004), or sub-structuring techniques (Hall and Chopra 1983, Fok and Chopra 1986, Tan and Chopra 1995a, b). Regardless of the option selected among these rigorous techniques, a major portion of the numerical calculation time spent is due to the solution of a complex eigen-value problem related to fluid hyper-element, which must be solved at each frequency.

In this paper, an efficient procedure is proposed for the required impedance matrix calculation of the fluid hyper-element, which greatly reduces the computational time. In the following sections, the analysis technique (i.e., FE-(FE-HE) method) is reviewed briefly, while the fluid hyper-element impedance matrix theoretical background is described in detail. Subsequently, the efficient fluid hyper-element is introduced and its formulation is presented. Thereafter, a special purpose program (Lotfi 2001) is modified based on the proposed theory, and the response of Morrow Point arch dam is obtained for various conditions. This is utilized to investigate the accuracy of the efficient procedure thoroughly.

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2. Method of analysis

The analysis technique utilized in this study is based on the FE-(FE-HE) method, which is applicable for a general concrete arch dam-reservoir system.

The formulation can be explained much easier, if one concentrates initially on a dam with finite reservoir system (basically the same as a model of dam and reservoir near field), and subsequently add the effects of reservoir far field region for the general case. For this purpose, let us begin with this simpler formulation and then complete the formulation for the more general case on that basis.

2.1 Dam with finite reservoir system

This is the problem, which can be totally modeled by finite element method. It can be easily shown that in this case, the coupled equations of the system may be written as (Lotfi 2002):

$$\begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{B} & \mathbf{G} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{r}} \\ \ddot{\mathbf{p}} \end{bmatrix} + \begin{bmatrix} \mathbf{C} & \mathbf{0} \\ \mathbf{0} & \mathbf{L} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{r}} \\ \dot{\mathbf{p}} \end{bmatrix} + \begin{bmatrix} \mathbf{K} & -\mathbf{B}^T \\ \mathbf{0} & \mathbf{H} \end{bmatrix} \begin{bmatrix} \mathbf{r} \\ \mathbf{p} \end{bmatrix} = \begin{bmatrix} -\mathbf{MJa}_g \\ -\mathbf{BJa}_g \end{bmatrix} \tag{1}$$

M, **C**, **K** in this relation represent the mass, damping and stiffness matrices of the dam body, while **G**, **L**, **H** are assembled matrices of fluid domain. The unknown vector is composed of **r**, which is the vector of nodal relative displacements and the vector **p** that includes nodal pressures. Moreover, **J** is a matrix with each three rows equal to a 3×3 identity matrix (its columns correspond to unit rigid body motion in cross-canyon, stream, and vertical directions) and $\mathbf{a}_g$ denotes the vector of ground accelerations. Furthermore, **B** in the above relation is often referred to as interaction matrix.

For harmonic ground excitations $\mathbf{a}_g(t) = \mathbf{a}_g(\omega)e^{i\omega t}$ with frequency ω, displacements and pressures will all behave harmonic, and Eq. (1) can be expressed as:

$$\begin{bmatrix} -\omega^2\mathbf{M} + \mathbf{K}(1 + 2\beta_d i) & -\mathbf{B}^T \\ -\mathbf{B} & \omega^{-2}(-\omega^2\mathbf{G} + i\omega\mathbf{L} + \mathbf{H}) \end{bmatrix} \begin{bmatrix} \mathbf{r} \\ \mathbf{p} \end{bmatrix} = \begin{bmatrix} -\mathbf{MJa}_g \\ -\omega^{-2}\mathbf{BJa}_g \end{bmatrix} \tag{2}$$

In this relation, it is assumed that the damping matrix of the dam is of hysteretic type. This means:

$$\mathbf{C} = \frac{2\beta_d}{\omega}\mathbf{K} \tag{3}$$

where β_d is the constant hysteretic factor of the dam body. Relation (2) is the coupled equations of a dam with finite reservoir system in frequency domain. It should be also noted that the system of equation is made symmetric by multiplying the lower partition matrices by a factor of ω^{-2}.

2.2 Reservoir near field boundary conditions

Apart from water surface boundary condition (which is easily applied), there are three possible boundary condition types for the reservoir near field region. The condition of type I, is considered for the contact of fluid with flexible solid, such as the dam-reservoir interface. The second type of condition is the so-called approximate boundary condition. This can be imposed at the reservoir bottom and sidewalls. The last type of condition (III) is referred to as Sommerfeld boundary

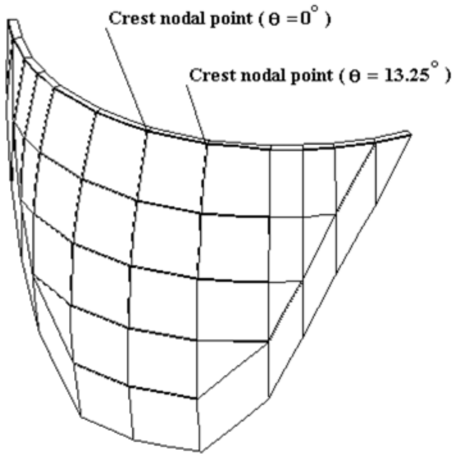

Fig. 1(a) Finite element mesh of the dam body

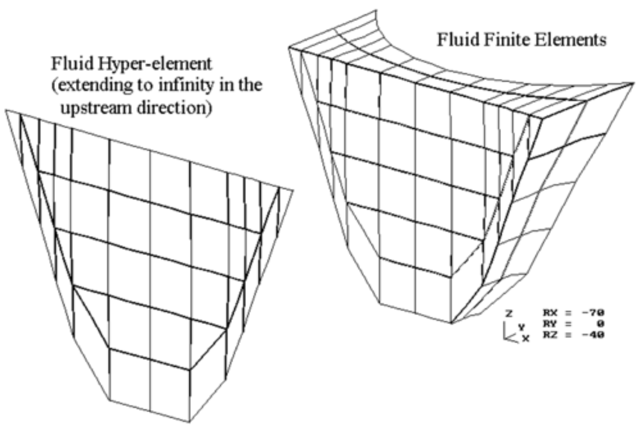

Fig. 1(b) Discretization of water domain (fluid finite elements ($L/H = 0.2$), and the fluid hyper-element)

condition. This is usually applied at the reservoir near field upstream boundary (in cases which far field region is not modeled), as a substitute for a precise transmitting boundary. However, when a fluid hyper-element is utilized, this condition is not required and waves are transmitted exactly through that semi-infinite element. The exact relations and detailed explanation about these conditions can be found in other studies (e.g., Lotfi 2004).

2.3 Fluid hyper-element

As mentioned, the three-dimensional fluid hyper-element is utilized to model the reservoir far-field region for the more general case. This part of the water domain, is assumed to be a uniform channel with an arbitrary geometric shape in the vertical plane which includes x, z-axes (see Fig. 1(b) for a typical discretization), and extends to infinity in the upstream direction (negative y-direction). Although, this is a three-dimensional semi-infinite fluid element, its discretization is performed in the vertical plane perpendicular to channel axis, which is referred to as the reference plane ($y = 0$).

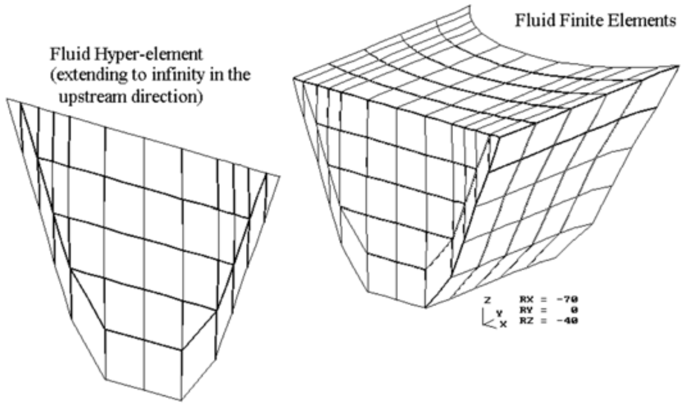

Fig. 1(c) Discretization of water domain (fluid finite elements ($L/H = 1.0$), and the fluid hyper-element)

Therefore, the element consists of several sub-channels, which extend to infinity and all the nodes of the hyper-element are located on that reference plane. The formulation of this element is presented as follows:

Assuming water to be linearly compressible and neglecting its viscosity, the small amplitude, irrotational motion of water due to harmonic excitation is governed by Helmholtz equation;

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} + \frac{\partial^2 P}{\partial z^2} + \frac{\omega^2}{C^2} P = 0 \tag{4}$$

where P is the amplitude of the hydrodynamic pressure (in excess of hydrostatic pressure) and C is the velocity of pressure waves in water.

By seeking solutions of the form e^{ky} in the stream direction, Eq. (4) becomes,

$$\lambda^2 P + \frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial z^2} = 0 \tag{5}$$

with the following definition of λ.

$$\lambda^2 = k^2 + \frac{\omega^2}{C^2} \tag{6}$$

By applying the variational method on Eq. (5), the following matrix relation is obtained at each sub-element level:

$$[-\lambda^2 \mathbf{A}^e + \mathbf{C}^e]\mathbf{P}^e = \mathbf{R}^e \tag{7}$$

where $\mathbf{P}^e$ is the vector of nodal pressure amplitudes for each sub-element with nodes located on the hyper-element reference plane (i.e., $y = 0$). Furthermore, matrices $\mathbf{A}^e$, $\mathbf{C}^e$ and vector $\mathbf{R}^e$ are defined below:

$$\mathbf{A}^e = \frac{1}{\rho} \int_A \mathbf{N}\mathbf{N}^T dA \tag{8a}$$

$$\mathbf{C}^e = \frac{1}{\rho}\int_A (\mathbf{N}_x\mathbf{N}_x^T + \mathbf{N}_z\mathbf{N}_z^T)dA \tag{8b}$$

$$\mathbf{R}^e = \frac{1}{\rho}\oint_s \mathbf{N}\frac{\partial P}{\partial n}ds \tag{8c}$$

In the above relations, $\mathbf{N}$ is the vector of shape functions, and $\mathbf{N}_x$, $\mathbf{N}_z$ denote derivatives of this vector with respect to x, z coordinates, respectively.

As for boundary conditions: neglecting gravity waves, one can write the condition

$$p = 0 \tag{9}$$

for the water surface. The condition at reservoir-foundation contact boundaries can be expressed by the approximate relation,

$$\frac{\partial P}{\partial n} = -\rho a_g^n(\omega) - i\omega q P \tag{10}$$

which allows for refraction of hydrodynamic pressure waves into the reservoir bottom materials or foundation rock medium. The admittance or damping coefficient q in this relation is related to a more meaningful wave reflection coefficient α,

$$\alpha = \frac{1 - qC}{1 + qC}$$

which is defined as the ratio of the amplitude of the reflected hydrodynamic pressure wave to the amplitude of a propagating pressure wave incident on the reservoir boundary, in the perpendicular direction (Fenves and Chopra 1984).

Imposing condition (10) on relation (8c) for sub-elements adjacent to the foundation contact surface, yields:

$$\mathbf{R}^e = -(\mathbf{D}^{ex}a_g^x + \mathbf{D}^{ez}a_g^z + i\omega q\mathbf{L}_h^e\mathbf{P}^e) \tag{11}$$

with the following definitions:

$$\mathbf{D}^{ex} = \int_s \mathbf{N}n_x ds \tag{12a}$$

$$\mathbf{D}^{ez} = \int_s \mathbf{N}n_z ds \tag{12b}$$

$$\mathbf{L}_h^e = \frac{1}{\rho}\int_s \mathbf{N}\mathbf{N}^T ds \tag{12c}$$

n_x, n_z are the components of a unit outward normal vector for the fluid sub-element boundary. Taking into account relations (9), and (11), the corresponding relation (7) for the hyper-element, is obtained by assembling contributions from different sub-elements:

$$[-\lambda^2\mathbf{A} + i\omega q\mathbf{L}_h + \mathbf{C}]\mathbf{P} = -(\mathbf{D}^x a_g^x + \mathbf{D}^z a_g^z) \tag{13}$$

P in this relation, is the vector of nodal pressure amplitudes. It includes all nodes of the fluid hyper-element below the water surface, which are located on the reference plane (i.e., $y = 0$).

Considering homogeneous boundary conditions in (13) corresponding to zero ground acceleration, leads to the following eigenvalue problem:

$$[-\lambda_j^2 \mathbf{A} + i\omega q \mathbf{L}_h + \mathbf{C}]\mathbf{X}_j = \mathbf{0} \tag{14}$$

$\lambda_j^2, \mathbf{X}_j$ are the jth eigenvalue and eigenvector of the fluid hyperelement.

There also exists particular solutions for relation (13) which corresponds to uniform unit acceleration of reservoir boundary in the ℓ-direction (x, or z-direction). In these case, the solution is independent of y-direction ($k = 0$), and considering relation (6), it yields:

$$\left[-\frac{\omega^2}{C^2}\mathbf{A} + i\omega q \mathbf{L}_h + \mathbf{C}\right]\mathbf{P}_p^\ell = -\mathbf{D}^\ell \tag{15}$$

The general solution for the amplitude of hydrodynamic pressures vector at an arbitrary y-coordinate is obtained by combinations of the eigenvectors and the particular solutions calculated from relations (14), (15). Considering the exponential form of the individual solutions in y-direction, one has:

$$\mathbf{P} = \sum_{j=1}^{n} \gamma_j \mathbf{X}_j e^{k_j y} + \mathbf{P}_p^x a_g^x(\omega) + \mathbf{P}_p^z a_g^z(\omega) \tag{16}$$

In this relation, γ_j is the participation factor for the jth mode, and $a_g^x(\omega), a_g^z(\omega)$ are included because unit vertical accelerations were assumed initially for calculation of particular solutions.

For the hyper-element reference plane (i.e., $y = 0$) which is denoted by h, the vector of pressure amplitudes (16) becomes:

$$\mathbf{P}_h = \sum_{j=1}^{n} \gamma_j \mathbf{X}_j + \mathbf{P}_p^x a_g^x(\omega) + \mathbf{P}_p^z a_g^z(\omega) \tag{17}$$

It can also be written in a more convenient matrix form,

$$\mathbf{P}_h = \mathbf{X}_h \Gamma + \mathbf{P}_p \mathbf{a}_g(\omega) \tag{18}$$

with the help of following definitions for reservoir hyper-element modal matrix, vector of participation factors and a matrix which includes particular solution vectors for different directions.

$$\mathbf{X}_h = [\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_n]$$

$$\Gamma = [\gamma_1, \gamma_2, \ldots, \gamma_n]^T$$

$$\mathbf{P}_p = [\mathbf{P}_p^x \;\; \mathbf{0} \;\; \mathbf{P}_p^z]$$

Moreover, the fluid y-direction (stream component) accelerations vector for an arbitrary vertical plane parallel to reference plane (constant y-plane), is obtained through differentiation of relation (16). It should be noted that acceleration in any direction is proportional to partial derivative of pressure in that direction (i.e., $\partial p/\partial n = -\rho\ddot{u}_n$). Thus:

$$\ddot{\mathbf{U}} = -\frac{1}{\rho}\sum_{j=1}^{n}\gamma_j k_j \mathbf{X}_j e^{k_j y} \tag{19}$$

For the reference plane, this vector becomes;

$$\ddot{\mathbf{U}}_h = -\frac{1}{\rho}\mathbf{X}_h\mathbf{K}_h\Gamma \tag{20}$$

where $\mathbf{K}_h$ is a diagonal matrix with the jth diagonal element being equal to k_j.

Solving for the participation vector from (18) by employing orthogonality condition of modal matrix and substituting in (20) yields:

$$\ddot{\mathbf{U}}_h = -\frac{1}{\rho}\mathbf{X}_h\mathbf{K}_h\mathbf{X}_h^T\mathbf{A}(\mathbf{P}_h - \mathbf{P}_p\mathbf{a}_g) \tag{21}$$

Multiplying both sides of this relation by $-\mathbf{A}$, one obtains:

$$\mathbf{R}_h = \mathbf{H}_h\mathbf{P}_h - \mathbf{R}_p\mathbf{a}_g(\omega) \tag{22}$$

by employing the following definitions:

$$\mathbf{R}_h = -\mathbf{A}\ddot{\mathbf{U}}_h \tag{23a}$$

$$\mathbf{H}_h = \frac{1}{\rho}\mathbf{A}\mathbf{X}_h\mathbf{K}_h\mathbf{X}_h^T\mathbf{A} \tag{23b}$$

$$\mathbf{R}_p = \mathbf{H}_h\mathbf{P}_p \tag{23c}$$

In relation (22), $\mathbf{R}_h$ represents a consistent vector equivalent to integration of inward horizontal acceleration (negative of stream component) for the hyper-element, and this vector contains essentially similar quantities as the components of the right hand side vectors of usual fluid finite elements (Lotfi 2004).

2.4 Dam-reservoir system

The formulation for a dam with finite reservoir was already presented. For the case where the reservoir extends to infinity, a hyper-element must be used along with the fluid finite elements utilized for reservoir near-field. Moreover, the governing relation for hyper-element was derived in previous section (relation (22)). Therefore, if the matrices of the hyper-element are assembled with the fluid finite elements, Eq. (2) becomes:

$$\begin{bmatrix} -\omega^2\mathbf{M} + \mathbf{K}(1+2\beta_d i) & -\mathbf{B}^T \\ -\mathbf{B} & \omega^{-2}((-\omega^2\mathbf{G} + i\omega\mathbf{L} + \mathbf{H}) + \overline{\mathbf{H}}_h(\omega)) \end{bmatrix}\begin{bmatrix}\mathbf{r}\\ \mathbf{p}\end{bmatrix} = \begin{bmatrix} -\mathbf{MJa}_g \\ \omega^{-2}(-\mathbf{BJa}_g + \overline{\mathbf{R}}_p(\omega)\mathbf{a}_g) \end{bmatrix} \tag{24}$$

Where $\overline{\mathbf{H}}_h(\omega)$ and $\overline{\mathbf{R}}_p(\omega)$ are the expanded form of $\mathbf{H}_h(\omega)$ and $\mathbf{R}_p(\omega)$ matrices which cover the entire fluid domain pressure degrees of freedom. Assuming that the pressure degrees of freedom related to hyper-element are ordered first in the unknown pressure vector, then these matrices have the following form:

$$\overline{\mathbf{H}}_h(\omega) = \begin{bmatrix} \mathbf{H}_h(\omega) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \tag{25a}$$

$$\overline{\mathbf{R}}_p(\omega) = \begin{bmatrix} \mathbf{R}_p(\omega) \\ \mathbf{0} \end{bmatrix} \tag{25b}$$

The relation (24) is the equation to be used instead of (2), when the reservoir is extended to infinity and one is considering the direct approach in frequency domain.

2.5 Fluid hyper-element (efficient procedure)

This element is basically formulated in the same manner as the usual fluid hyper-element. However, the main required matrix (i.e., $\mathbf{H}_h(\omega)$) is calculated based on an efficient method. The fundamental concepts of this technique is presented below:

Let us consider the formula, which is utilized to define matrix $\mathbf{H}_h(\omega)$:

$$\mathbf{H}_h(\omega) = \frac{1}{\rho}\mathbf{A}\mathbf{X}_h(\omega)\mathbf{K}_h(\omega)\mathbf{X}_h^T(\omega)\mathbf{A} \qquad \text{repeated} \tag{23b}$$

The calculation of the above matrix is very time consuming and cumbersome due to its ingredients $\mathbf{X}_h(\omega)$, $\mathbf{K}_h(\omega)$ which are both frequency dependent. These matrices depend on the solution of the above-mentioned eigenvalue problem (14), which may be rearranged as:

$$[\mathbf{C} + i\omega q\mathbf{L}_h]\mathbf{X}_j = \lambda_j^2\mathbf{A}\mathbf{X}_j \tag{26}$$

In general, the solution of this problem requires complex number arithmetic, except for the case in which damping coefficient $q = 0$ (i.e., $\alpha = 1$). Assuming the more general case (i.e., $q \neq 0$), the problem is simplified again only for the unique case of $\omega = 0$, which can be written as:

$$\mathbf{C}\overline{\mathbf{X}}_j = \overline{\lambda}_j^2\mathbf{A}\overline{\mathbf{X}}_j \tag{27}$$

Based on the solution of this standard eigen-problem, one can form the following matrices:

$$\overline{\mathbf{X}}_h = [\overline{\mathbf{X}}_1, \overline{\mathbf{X}}_2, \ldots, \overline{\mathbf{X}}_n] \tag{28a}$$

$$\overline{\Lambda} = \begin{bmatrix} \ddots & & \\ & \overline{\lambda}_j^2 & \\ & & \ddots \end{bmatrix} \tag{28b}$$

It is worth to mention that the bar superscript is denoting that these matrices are for the special case of $\omega = 0$. Furthermore, the orthogonality relations in this case are:

$$\overline{\mathbf{X}}_h^T \mathbf{A} \overline{\mathbf{X}}_h = \mathbf{I} \tag{29a}$$

$$\overline{\mathbf{X}}_h^T \mathbf{C} \overline{\mathbf{X}}_h = \overline{\Lambda} \tag{29b}$$

Similarly, in the general case of (26), the orthogonality relations are written as:

$$\mathbf{X}_h^T \mathbf{A} \mathbf{X}_h = \mathbf{I} \tag{30a}$$

$$\mathbf{X}_h^T (\mathbf{C} + i\omega q \mathbf{L}_h) \mathbf{X}_h = \Lambda \tag{30b}$$

with the following matrix definitions:

$$\mathbf{X}_h = [\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_n] \tag{31a}$$

$$\Lambda = \begin{bmatrix} \ddots & & \\ & \lambda_j^2 & \\ & & \ddots \end{bmatrix} \tag{31b}$$

By comparing relations (29) and (30), it is easy to see that $\mathbf{X}_h(\omega)$ is frequency independent and equal to $\overline{\mathbf{X}}_h$, if the following operation results in a diagonal matrix:

$$\overline{\mathbf{X}}_h^T (\mathbf{C} + i\omega q \mathbf{L}_h) \overline{\mathbf{X}}_h = [\ddots] \tag{32}$$

Let us investigate the conditions under which this is possible. By employing (29b), relation (32) can be written as:

$$\overline{\mathbf{X}}_h^T (\mathbf{C} + i\omega q \mathbf{L}_h) \overline{\mathbf{X}}_h = \overline{\Lambda} + i\omega q \mathbf{L}^* \tag{33}$$

$\overline{\Lambda}$ is a diagonal matrix. However, this is not the case for $\mathbf{L}^*$ (i.e., $\mathbf{L}^* = \overline{\mathbf{X}}_h^T \mathbf{L}_h \overline{\mathbf{X}}_h$) in general. Thus, let us define a diagonal matrix $\mathbf{L}_D^*$ as below:

$$\mathbf{L}_D^* = \text{Diagonal}[\mathbf{L}^*] \tag{34}$$

It is apparent that Eq. (32) is satisfied now, if $\mathbf{L}_h$ is replaced by $\hat{\mathbf{L}}_h$ in that equation, such that it yields:

$$\overline{\mathbf{X}}_h^T (\mathbf{C} + i\omega q \hat{\mathbf{L}}_h) \overline{\mathbf{X}}_h = \overline{\Lambda} + i\omega q \mathbf{L}_D^* \tag{35}$$

Of course, this requires the following equality:

$$\overline{\mathbf{X}}_h^T \hat{\mathbf{L}}_h \overline{\mathbf{X}}_h = \mathbf{L}_D^* \tag{36}$$

Although, the matrix $\hat{\mathbf{L}}_h$ is not required to be defined explicitly, this can be achieved through relation (36) by employing (29a):

$$\hat{\mathbf{L}}_h = \mathbf{A}\overline{\mathbf{X}}_h\mathbf{L}_D^*\overline{\mathbf{X}}_h^T\mathbf{A} \tag{37}$$

Therefore, it can be claimed that $\overline{\mathbf{X}}_j$ is also the j-th eigen-vector of the following eigen-problem:

$$[\mathbf{C} + i\omega q\hat{\mathbf{L}}_h]\overline{\mathbf{X}}_j = \lambda_j^2\mathbf{A}\overline{\mathbf{X}}_j \tag{38}$$

which is very similar to the original relation (26) except that $\mathbf{L}_h$ is replaced by an approximate form, $\hat{\mathbf{L}}_h$. The orthogonality relations corresponding to (38) may be written as:

$$\overline{\mathbf{X}}_h^T\mathbf{A}\overline{\mathbf{X}}_h = \mathbf{I} \tag{39a}$$

$$\overline{\mathbf{X}}_h^T(\mathbf{C} + i\omega q\hat{\mathbf{L}}_h)\overline{\mathbf{X}}_h = \Lambda \tag{39b}$$

Comparing relations (35), and (39b), it yields:

$$\Lambda = \overline{\Lambda} + i\omega q\mathbf{L}_D^* \tag{40}$$

The equality (40) may be presented in a different form taking into account the definitions (28b), and (31b):

$$\lambda_j^2 = \overline{\lambda}_j^2 + i\omega qL_{Dj}^* \tag{41}$$

Moreover, the wave number k_j, can be found from (6) by employing (41):

$$k_j = \sqrt{(\overline{\lambda}_j^2 - \omega^2/C^2 + i\omega qL_{Dj}^*)} \tag{42}$$

Based on the above discussion and following the same procedure, which leads to (23b), one can obtain the following equation:

$$\mathbf{H}_h(\omega) = \frac{1}{\rho}\mathbf{A}\overline{\mathbf{X}}_h\mathbf{K}_h(\omega)\overline{\mathbf{X}}_h^T\mathbf{A} \tag{43}$$

It must be emphasized that $\overline{\mathbf{X}}_h$ in this relation is frequency-independent, and corresponds to the modal matrix for the unique case of $\omega = 0$. The only frequency dependent matrix is $\mathbf{K}_h$, which is a diagonal matrix and its elements are calculated easily by Eq. (42) for any frequency ω. This incredibly simplifies the calculation of main contributing matrix of fluid hyper-element (i.e., $\mathbf{H}_h(\omega)$). In this manner, it is not required any more to solve a complex eigen-value problem for each frequency. In other words, the procedure depends on the solution of the initial eigen-problem (i.e., the one corresponding to $\omega = 0$). Note-worthy, this can be solved by standard eigen-solution routine, and it doesn't involve any complex number arithmetic.

3. Modeling and basic parameters

A computer program (Lotfi 2001) was enhanced based on the theory presented on the previous section. The program is based on the FE-(FE-HE) concept. This means, the dam is treated by solid finite elements, while the reservoir is divided into two parts, the near-field region in the vicinity of the dam, which is discretized by fluid finite elements, and the far-field part is modeled by a three-dimensional hyper-element. The program was initially relying on the exact method of fluid hyper-element impedance matrix calculation. However, a second alternative is also introduced to the program in the present study, which calculates this impedance matrix by the above-mentioned efficient procedure.

3.1 Models

An idealized symmetric model of Morrow Point arch dam is considered. The geometry of the dam may be found in (Hall and Chopra 1983).

The dam is discretized by 40 isoparametric 20-node solid finite elements (Fig. 1(a)). The water domain is divided into two regions (Fig. 1(b), or 1(c)). The near-field part is considered as a region, which extends to a specified length L, which is measured in upstream direction at dam mid-crest point. The far-field region starts from that point and extends to infinity in the upstream direction. Both these regions combined are assumed to form a uniform reservoir shape as in a previous study (Lotfi 2004). Two alternatives are considered as it can be observed in Figs. 1(b) and 1(c). These cases correspond to the specified length $L = 0.2\ H$ and H, respectively. H being the dam height or maximum water depth in the reservoir. The near-field region of these cases, is discretized by 80 or 200 (for $L/H = 0.2$ or 1.0) isoparametric 20-node fluid finite elements, while the far-field region in both cases is modeled by a fluid hyper-element which itself is constructed from 40 isoparametric 8-node sub-elements.

3.2 Basic parameters

The dam concrete is assumed to be homogeneous with isotropic linearly viscoelastic behavior and the following main characteristics:

Elastic modulus (E_d)	= 27.5 GPa
Poisson's ratio	= 0.2
Unit weight	= 24.8 kN/m^3
Hysteretic damping factor (β_d)	= 0.05

The impounded water is taken as inviscid, and compressible fluid with unit weight equal to 9.81 kN/m^3, and pressure wave velocity $C = 1440$ m/sec.

4. Results

The responses of dam crest are obtained due to upstream, vertical and cross-stream excitations for several values of wave reflection coefficient α, and L/H ratios. Initially, L/H ratio is kept constant and is taken equal to 0.2 (Fig. 1(b)). At this stage, α is varied as 0.75, 0.5, and 0.0. Later on, a final case is also considered, which corresponds to $\alpha = 0.0$, and $L/H = 1.0$.

It should be mentioned that the response quantities plotted are the amplitudes of the complex valued radial accelerations for two points located at dam crest (Fig. 1(a)). This is either the mid-crest point ($\theta = 0^o$) selected for upstream or vertical excitations or a point located at ($\theta = 13.25^o$) which is used for the case of cross-stream excitation. This is due to the fact that radial acceleration is zero at mid-crest for the cross-stream type of ground motion.

In each case, the amplitude of radial acceleration is plotted versus the dimensionless frequency for a significant range. The dimensionless frequency for upstream and vertical excitation is defined as ω/ω_1^S where ω is the excitation frequency and ω_1^S is the fundamental frequency of the dam on rigid foundation with empty reservoir for a symmetric mode. For the cross-stream excitation cases, the dimensionless frequency is defined as ω/ω_1^a, where ω_1^a is the fundamental resonant frequency of the dam on rigid foundation with empty reservoir for an anti-symmetric mode.

As mentioned above, several values of α, and L/H ratios are considered. The results for all these cases are obtained for different types of excitations and they are presented in Figs. 2-5. In each of these cases, the response based on the efficient procedure is compared against the exact method.

The first three cases is a very challenging test for the efficient procedure which correspond to a very small value of L/H = 0.2. The α value is varied as 0.75, 0.5, 0.0 for these cases, respectively (Figs. 2-4). In the first case (α = 0.75, L/H = 0.2), it is observed that there is practically no difference in response between the efficient procedure and the exact method of analysis for all three types of excitations (Fig. 2). For the second case (α = 0.50, L/H = 0.2), one notices small differences in response especially in the vicinity of the first major peak of the response for upstream ground motion (Fig. 3). In the third case (α = 0.0, L/H = 0.2), the difference in response becomes more

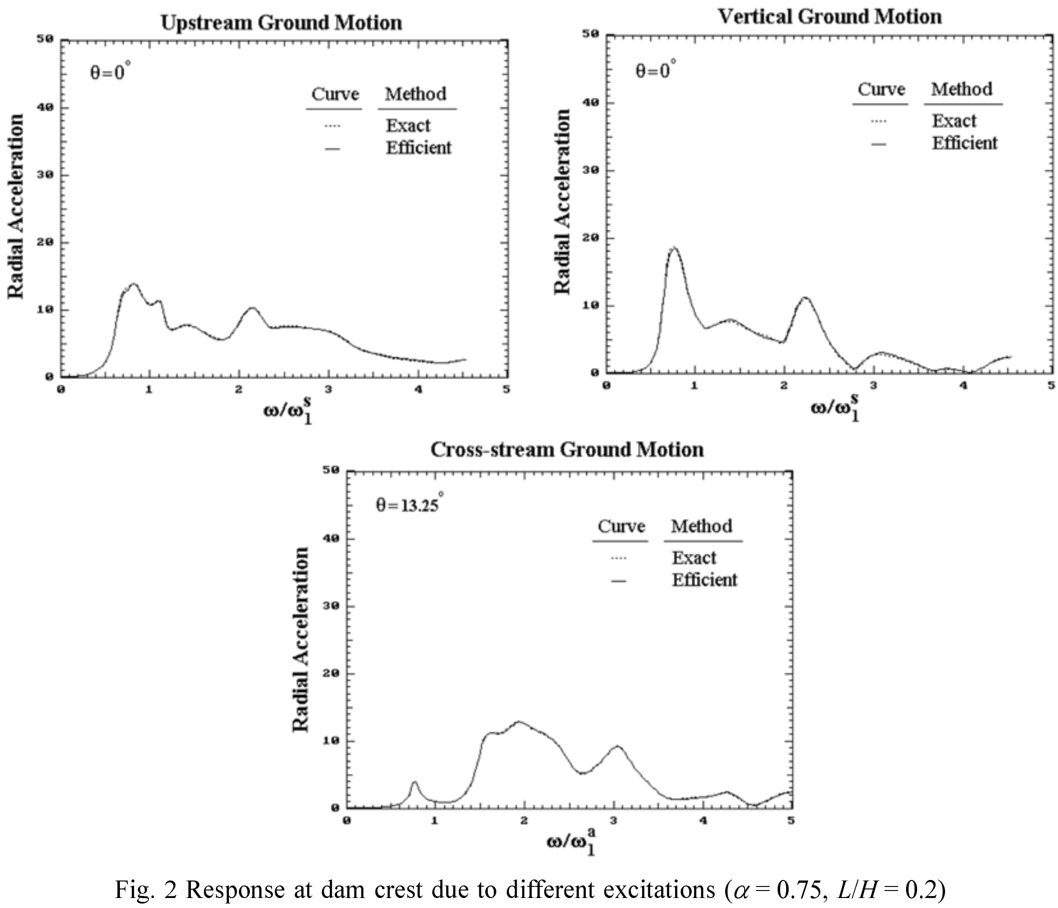

Fig. 2 Response at dam crest due to different excitations (α = 0.75, L/H = 0.2)

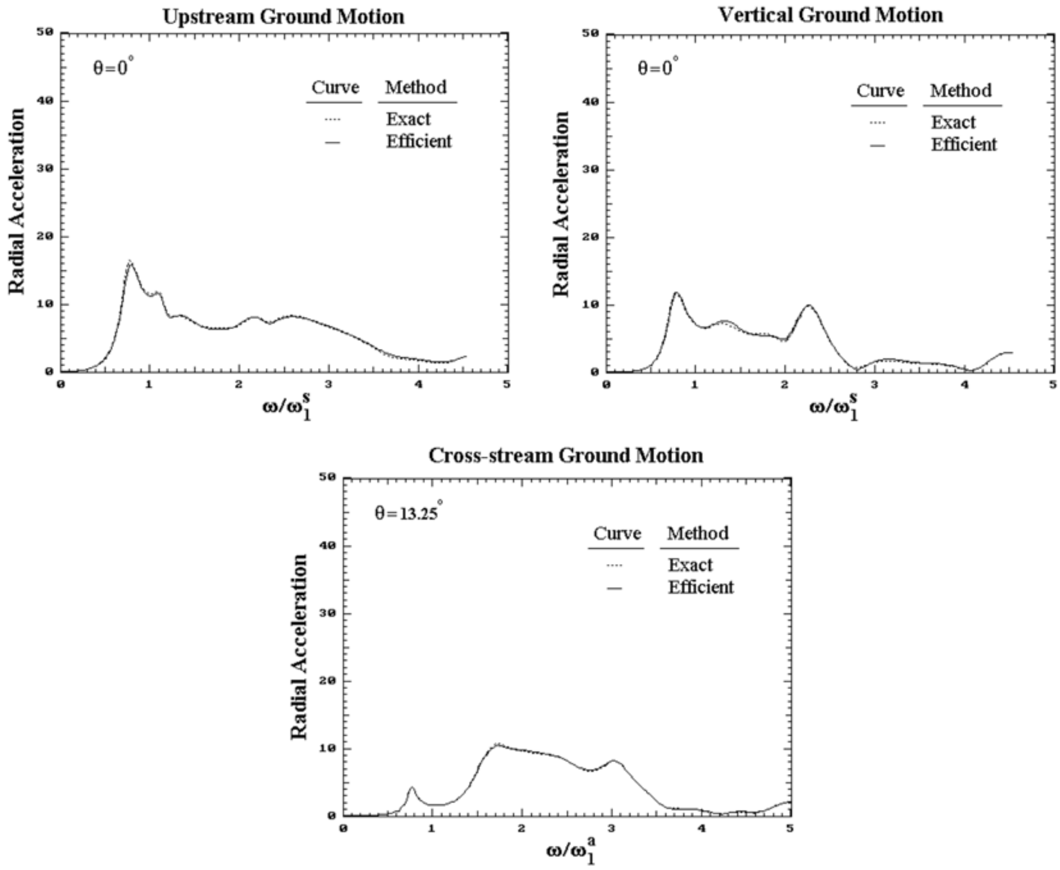

Fig. 3 Response at dam crest due to different excitations ($\alpha = 0.50$, $L/H = 0.2$)

significant, except for cross-stream excitation where the difference in response is still negligible (Fig. 4).

To make a more quantitative comparison, the percentage error is calculated at the frequency corresponding to the first major peak of the response in each case, and they are presented in Table 1. It is noticed that although a very small value of $L/H = 0.2$ is considered, the efficient procedure gives excellent results for all practical values of wave reflection coefficient (i.e., $\alpha \geq 0.5$), and the maximum error in response for all these cases is below 5%. The error becomes significant only for the very impractical case of $\alpha = 0.0$, which is about 10%, and this is merely for the upstream excitation. Furthermore, it should be also emphasized that there are no difference in response between the efficient procedure and the exact method when $\alpha = 1.0$ (for any L/H ratio), which is also included in Table 1.

Finally, a last case is considered which corresponds to ($\alpha = 0.0$, $L/H = 1.0$). This case is introduced to examine the effect of L/H ratio on the response. Although, $\alpha = 0.0$ is a very impractical value of wave reflection coefficient, this is selected because it was noticed that it gave the maximum error in response in combination with a L/H ratio of 0.2. Therefore, this would be a more challenging test to examine the reduction in the error in response for such a case by increasing the L/H ratio. The results for this case are depicted in Fig. 5, and the percentage error at the first

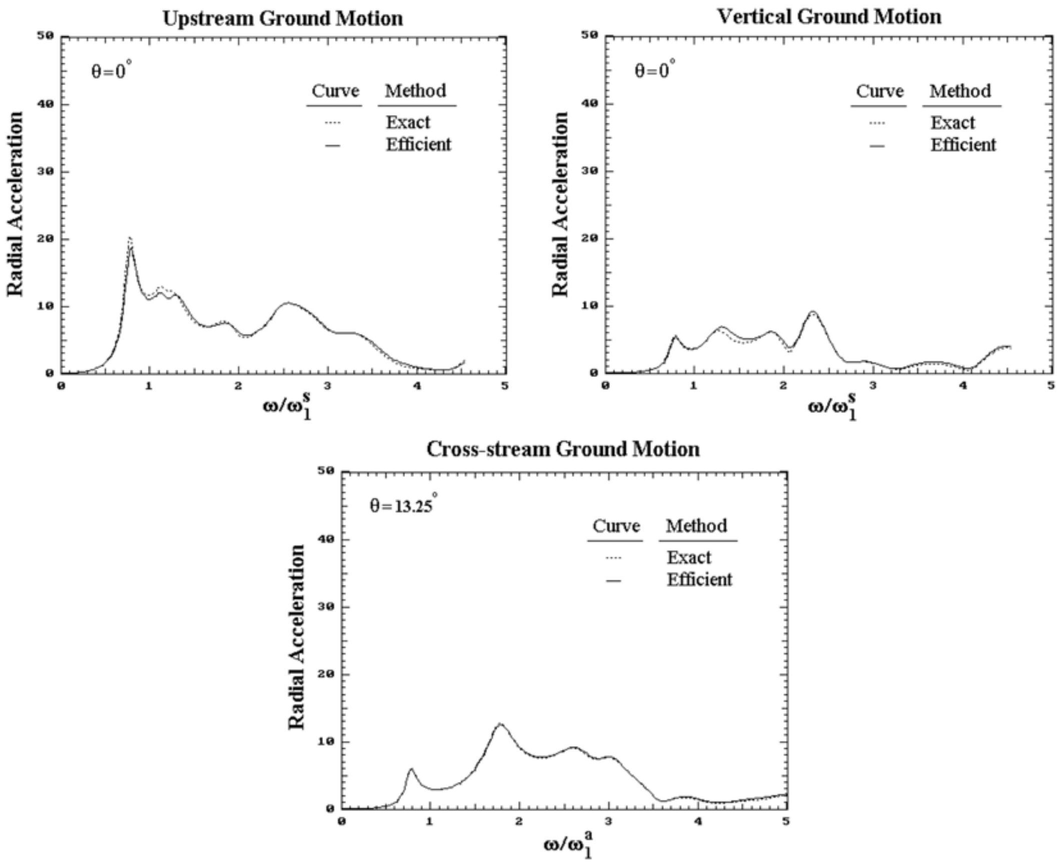

Fig. 4 Response at dam crest due to different excitations ($\alpha = 0.0$, $L/H = 0.2$)

Table 1 Percentage error at the first major peak of response

α	L/H	Upstream	Vertical	Cross-stream
1.00	Any	0.00	0.00	0.00
0.75	0.2	0.65	1.13	0.30
0.50	0.2	4.31	1.43	0.66
0.0	0.2	9.79	5.03	1.65
0.0	1.0	0.25	0.23	0.001

major peak of the response is given in Table 1. It is noticed that difference in response becomes negligible again, and the maximum percentage error in response is less than 1% for all three types of excitation considered for this case.

This means that even for the very low value of $\alpha = 0.0$ which is an impractical value, the efficient method produces excellent results for moderate values of L/H ratio (e.g., $L/H = 1.0$). It should be mentioned that in many cases, one has to consider a near-field region, which is much larger than this moderate value considered. Furthermore, $\alpha < 0.5$ are relatively impractical values of wave reflection coefficient for real cases occurring in the field.

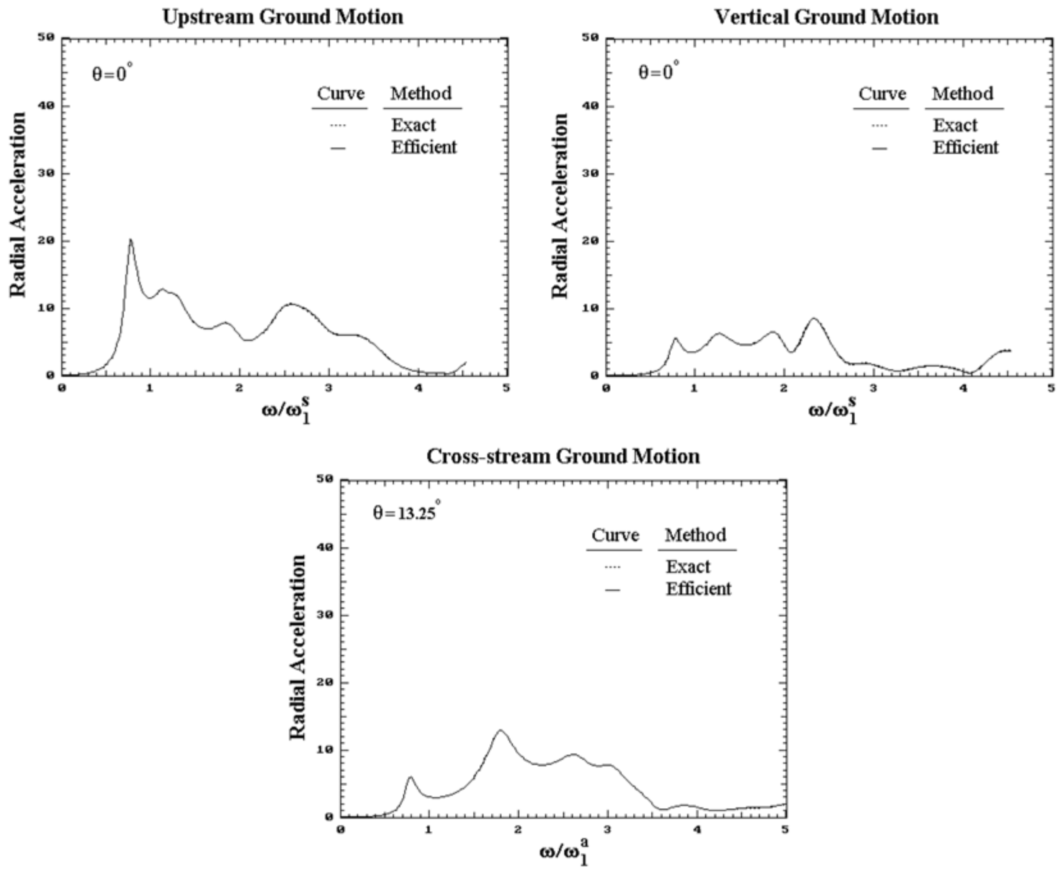

Fig. 5 Response at dam crest due to different excitations ($\alpha = 0.0$, $L/H = 1.0$)

5. Conclusions

The formulation based on FE-(FE-HE) procedure for dynamic analysis of concrete arch dam-reservoir systems, was explained briefly. An efficient procedure was proposed for the calculation of the impedance matrix of the three dimensional fluid hyper-element. A computer program was enhanced based on this methodology and the response of Morrow Point arch dam was studied for various combinations of wave reflection coefficient α, and L/H ratio for different excitations. In each case, the accuracy of the efficient procedure is examined against the exact method.

Overall, the main conclusions obtained by the present study can be listed as follows:

- Initially, the L/H ratio was selected as a small value of 0.2 to make the test for the efficient procedure very challenging. It was noticed that under this condition, the efficient procedure gave excellent results for all practical values of wave reflection coefficient considered (i.e., $\alpha \geq 0.5$). The maximum error in response for all these cases is below 5%. The error becomes significant only for the very impractical case of $\alpha = 0.0$, which is about 10%, and this is merely for the upstream excitation.

- A case was considered with $\alpha = 0.0$, $L/H = 1.0$. It is noticed that difference in response becomes negligible again, and the maximum percentage error in response is less than 1% for all three types of excitation considered for this case. This means that even for the very low value of $\alpha = 0.0$ which is an impractical value, the efficient method produces excellent results for moderate values of L/H ratio (e.g., $L/H = 1.0$). It should be mentioned that in many cases, one has to consider a near-field region, which is much larger than this moderate value considered. Furthermore, $\alpha < 0.5$ are relatively impractical values of wave reflection coefficient for real cases occurring in the field.
- The efficient procedure is proven to be a very accurate method under all practical conditions.

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